Smearing effects of ρ-meson width on $D \to PV$ decays

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We study the effects of mass averaging over the width of the ρ meson on branching ratios of $D \to PV$ decays. This smearing effect ensures lowering of these ratios in the required direction by about 25%. We calculate the smeared branchings for the Cabibbo-enhanced, -suppressed, and -doubly-suppressed $D/D_s \to P\rho$ decays in the presence of final-state interactions.

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I. INTRODUCTION

The weak nonleptonic decays provide useful information on the structure of hadrons and their interactions. However, purely hadronic decays are difficult to understand since nonperturbative quantum-chromodynamical (QCD) effects seem to play a major role in them. In the case of heavy-quark decays, it can be hoped that these nonperturbative effects are not so significant because of the large mass of the mesons. Unfortunately, to complicate the problem, the D-meson masses lie in the resonance region and nontrivial hadronization processes can modify the naive expectations substantially.

In the recent past, considerable effort has been made in understanding the decay mechanism of charmed mesons. Many phenomenological models [1–6] have been proposed, and initial work on the spectator model has yielded quite encouraging results. Refinements and improvements were applied to this approach, as the partial decay widths are found to be very sensitive to the values of the QCD coefficients which manifest variedly in the presence of weak annihilations, soft gluon radiations, final-state interactions (FSI's), etc. A simple and economical model, the Bauer-Stech-Wirbel (BSW) [4] model, which uses the factorization ansatz, has been reasonably successful in explaining most of the exclusive two-body decays. However, this approach alone is insufficient satisfactorily to fit data for all the charmed-meson decays, and it is essential to include FSI effects. Nevertheless, decays such as $D \to \bar{K} a_1$ are unexplained in spite of including FSI's. Recently, it has been suggested that a_1 being a broad resonance, smearing effects [7] due to its large width should be taken into consideration to bridge the gap between theory and experiment. In other words, it may be necessary to average its mass over the entire width rather than taking a fixed mass at its peak value. A similar treatment may also be required in the case of other decays where such wide resonances are produced.

In this paper we study the effects of the large ρ-meson width on the $D \to PV$ decays using the BSW framework [4]. In the BSW model, though most of the $D \to PV$ branchings fit within experimental limits, decays involving the ρ meson, in general, remain on the higher side. We calculate the smearing ratios for all $D \to P\rho$ decays in the Cabibbo-enhanced, -suppressed, and -doubly-suppressed modes. Though the smearing effect tends to lead the branching ratios in the right direction, inclusion of FSI's is imperative to explain the data.

II. SMEARING EFFECT

In general, the decay rate for $D \to PV$ is given

$$\Gamma(D \to PV) = \frac{k^3}{8\pi m_V^2} |A(D \to PV)|^2 , \tag{1}$$

where k is the three-momentum of final state particles in the rest frame of the D meson.

The ρ meson has a width of about 150 MeV, and this rather wide resonance results in an increased effective final-state phase space than the expected nominal value. This would imply that we consider a running mass m for calculating the decay rates. Using a measure, say, $\rho(m^2)$, with the normalization constraint

$$\int \rho(m^2) dm^2 = 1 , \tag{2}$$

we get the mass-averaged rate as

$$\bar{\Gamma}(D \to P\rho) = \int \rho(m^2) \Gamma(D \to P\rho(m^2)) dm^2 . \tag{3}$$

The form of the measure $\rho(m^2)$ may be calculated using Feynman rules to analyze the $\Gamma(D \to P2\pi)$ decays at a ρ pole. This suggests a choice of the following Breit-Wigner measure as derived in the Appendix:

$$\rho(m^2) = \frac{N}{\pi} \frac{m\Gamma_{\rm tot}(m^2)}{(m^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_{\rm tot}^2(m^2)} , \tag{4}$$

which seems most appropriate considering that the probability of a resonance being a physical particle is dictated by a shape that decreases on either side of the central value. N is the normalization factor introduced to ensure (2).

The total width of ρ is parametrized with respect to its main decay mode $\rho \to 2\pi$ to get

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$$\Gamma_{tot}(m) = \rho_{2\pi}(m)\Gamma_{2\pi}\Theta(m-2m_\pi) , \quad (5)$$

where the kinematical factor is

$$\rho_{2\pi}(m) = \frac{p_\pi^3(m)}{p_\pi^3(m_\rho)} , \quad (6)$$

and p_π is the center-of-mass momentum of π in the rest frame of ρ. $\Gamma_{2\pi}=0.150$ GeV, $m_\rho=0.77$ GeV is the mass at the peak value, and Θ is the step function which ensures the opening of the channel at the appropriate mass. The smearing ratio due to the running mass over the resonance width is defined as

$$\frac{\overline{\Gamma}(D\to P\rho)}{\Gamma(D\to P\rho)} . \quad (7)$$

We illustrate the calculations for one decay, i.e., the Cabibbo-enhanced $D^0 \to K^-\rho^+$ mode. The relevant weak Hamiltonian needed to describe this decay is

$$H_W = \frac{G_F\cos^2\theta_C}{\sqrt{2}}[a_1(\bar{u}d)_H(\bar{s}c)_H + a_2(\bar{u}c)_H(\bar{s}d)_H] . \quad (8)$$

The notation $(\bar{q}q)$ is an abbreviation for a color-singlet combination $\bar{q}\gamma_\mu(1-\gamma_5)q$, and the subscript H indicates them to be effective hadronic fields. The QCD coefficients a_1 and a_2 are taken as

$$a_1 = 1.2, \quad a_2 = -0.5 .$$

In the factorization approximation, the effects of long-range QCD are contained in the hadron masses, meson decay constants f_π, $f_\rho, \ldots$, and the form factors appearing in the matrix elements (ME's) of the weak currents. In the BSW model [5], using Lorentz invariance, the decompositions of hadronic ME's of currents are defined as

$$\langle P(k)|A_\mu(0)|0\rangle = -if_P k_\mu , \quad (9a)$$

$$\langle V(k)|V_\mu(0)|0\rangle = \varepsilon_\mu^* m_V f_V , \quad (9b)$$

$$\langle P(k)|J_\mu(0)|D(p)\rangle = \left[(k_D+k_P)_\mu - \frac{m_D^2-m_P^2}{q^2}q_\mu\right]F_1(q^2) + \frac{m_D^2-m_P^2}{q^2}q_\mu F_0(q^2) , \quad (9c)$$

$$\langle V(k)|J_\mu(0)|D(p)\rangle = \varepsilon_{\mu\nu\alpha\beta}P_D^\nu K_V^\beta \varepsilon^{*\alpha}\frac{2}{m_D+m_V}V(q^2) + i\left[\frac{\varepsilon^*\cdot q}{q^2}q_\mu 2m_V A_0(q^2) + \varepsilon_\mu^*(m_D+m_V)A_1(q^2) - \frac{\varepsilon^*\cdot q}{m_D+m_V}(k_D+k_V)_\mu A_2(q^2) - \frac{\varepsilon^*\cdot q}{q^2}q_\mu 2m_V A_3(q^2)\right] , \quad (9d)$$

where $q_\mu=(p-k)_\mu$ and ε_μ^* denotes the polarization of the vector meson. The form factors satisfy

$$2m_V A_3(q^2) = (m_D+m_V)A_1(q^2) - (m_D-m_V)A_2(q^2) ,$$
$$A_0(0) = A_3(0), \quad F_0(0) = F_1(0) , \quad (10)$$
$$F_1(q^2) = \frac{F_1(0)}{1-q^2/m_{1^-}^2}, \quad A_0(q^2) = \frac{A_0(0)}{1-q^2/m_{0^-}^2} .$$

In our calculation we have taken the values of the form factors at $q^2=0$ as given in Ref. [4].

The $D^0 \to K^-\rho^+$ decay amplitude (annihilation term neglected) is then given by

$$A(D^0\to K^-\rho^+) = \frac{G_F\cos^2\theta_C}{\sqrt{2}}[a_1\langle\rho^+|\bar{u}d|0\rangle\langle K^-|\bar{s}c|D^0\rangle] ,$$

which simplifies to

$$A(D^0\to K^-\rho^+) = \frac{G_F\cos^2\theta_C}{\sqrt{2}}(2m)a_1 f_\rho(m)F_1(m^2) . \quad (11)$$

Using formula (3) for mass averaging, the smearing ratio is calculated as

$$\frac{\overline{\Gamma}(D^0\to K^-\rho^+)}{\Gamma(D^0\to K^-\rho^+)} = \int \rho(m^2)\frac{k^3(m)}{k^3(m_\rho)}\frac{f_\rho(m)F_1^2(m^2)}{f_\rho(m_\rho)F_1^2(m_\rho^2)}dm^2 = 0.77 . \quad (12)$$

The mass dependence of different parts of the integrand in (12) is shown in Fig. 1(a). Similarly, we have calculated the smearing ratios for all the $D\to P\rho$ decays as given in column (ii) of Table I. In evaluating $\overline{\Gamma}$, similar to trends seen in $D\to\overline{K}a_1$, it was observed here too that the smearing ratio is quite sensitive to the range of integration. We used the range $2m_\pi \le m \le (m_D - m_P)$. We have scaled the decay constant

$$f_\rho(m) = \left[\frac{m_\rho}{m}\right]^{1/2} f_\rho(m_\rho) \ .$$

We note that the effect of smearing due to the broad width of the ρ meson is to decrease the branchings by about 20–25 %. Though we have performed our calculations in the BSW model framework, this decrease is expected in other models too [8]. This may be attributed to the shape of the Breit-Wigner measure [Fig. 1(b)]. This measure is increasing in the low-mass region $m < m_\rho$ and, though here $k(m)$ is larger than $k(m_\rho)$, the threshold factor dampens out the form factor and momentum contribution and the steep gradient nullifies its effect. In the region $m > m_\rho$, the measure is decreasing slowly, and the form factor appears to enhance the branching; however, $k(m) \to 0$ as $m \to (m_D - m_P)$, which sets kinematic constraints and suppresses further Breit-Wigner or any other contribution.

III. BRANCHING RATIOS WITH FINAL-STATE INTERACTIONS (FSI's)

Since charmed-meson masses lie in the resonance region, rescattering effects of outgoing mesons become particularly important. Strong interactions may include FSI effects which can modify naive expectations for exclusive two-body charm decays substantially. For on-mass-shell FSI's, the bare amplitude A^0 should be corrected [4] as

$$A = S^{1/2} A^0 \ ,$$

where $S^{1/2}$ denotes the square root of the S matrix for hadron-hadron scattering, which induces phase factors and mixing of the decay channels having the same quantum numbers. Since little is known about the many open channels, $S^{1/2}$ cannot be easily estimated. Nevertheless, an isospin-level analysis provides us sufficient insight into

TABLE I. Smearing ratios and corresponding smeared branchings.

Decay constants	Form factors	Phase angles
$f_\rho = 0.221$ GeV	$A_0^{D\to\rho}(0) = 0.669$	$\delta^{K\rho} \approx \delta^{\bar{K}\rho} = 20°$
$f_\pi = 0.133$ GeV	$F_1^{D\to\pi}(0) = 0.692$	$\delta^{\pi\rho} \approx \delta'^{\pi\rho} = 45°$
$f_K = 0.162$ GeV	$F_1^{D\to K}(0) = 0.762$	
$f_{\eta_d} = 0.068$ GeV	$F_1^{D\to\eta}(0) = 0.681$	
$f_{\eta_s} = 0.092$ GeV	$F_1^{D\to\eta'}(0) = 0.655$	$\eta-\eta'$ mixing angle $= -19°$ $(-11°)$
$f_{\eta'_d} = 0.065$ GeV	$F_1^{D_s\to\eta}(0) = 0.723$	
$f_{\eta'_s} = 0.096$ GeV	$F_1^{D_s\to\eta'}(0) = 0.704$	

Decay	Smearing ratio	Smeared branching with FSI's (%)	Experimental Results (%) [9]
$\Delta C = \Delta S = -1$			
$D^0 \to K^- \rho^+$	0.77	7.98	7.8 ± 1.1
$D^0 \to \bar{K}^0 \rho^0$	0.76	0.43[a]	$0.43^{+0.31}_{-0.19}$
$D^+ \to \bar{K}^0 \rho^+$	0.78	11.75	6.6 ± 1.7
$D_s^+ \to \pi^0 \rho^+$	0.85	0.00	
$D_s^+ \to \pi^+ \rho^0$	0.85	0.00	< 0.21
$D_s^+ \to \eta \rho^+$	0.77	3.27 (4.58)	5.29 ± 2.7 [11]
$D_s^+ \to \eta' \rho^+$	0.77	1.62 (1.28)	1.51 ± 0.75 [11]
$\Delta C = -1$, $\Delta S = 0$			
$D^0 \to \pi^- \rho^+$	0.79	0.50	
$D^0 \to \pi^+ \rho^-$	0.77	0.12	
$D^0 \to \pi^0 \rho^0$	0.78	0.11	
$D^+ \to \pi^+ \rho^0$	0.79	0.07[a]	0.07 ± 0.05 [10]
$D^+ \to \pi^0 \rho^+$	0.80	0.38	
$D^0 \to \eta \rho^0$	0.81	0.001 (0.0003)	
$D^0 \to \eta' \rho^0$	0.83	0.002 (0.002)	
$D^+ \to \eta \rho^+$	0.77	0.14 (0.10)	
$D^+ \to \eta' \rho^+$	0.83	0.036 (0.043)	
$D_s^+ \to K^0 \rho^+$	0.78	0.63	
$D_s^+ \to K^+ \rho^0$	0.78	0.07	
$\Delta C = -\Delta S = -1$			
$D^0 \to K^+ \rho^-$	0.75	$2.82 \times \tan^4\theta_C$	
$D^0 \to K^0 \rho^0$	0.75	$0.30 \times \tan^4\theta_C$	
$D^+ \to K^+ \rho^0$	0.75	$3.58 \times \tan^4\theta_C$	
$D^+ \to K^0 \rho^+$	0.75	$1.37 \times \tan^4\theta_C$	

[a]Input.

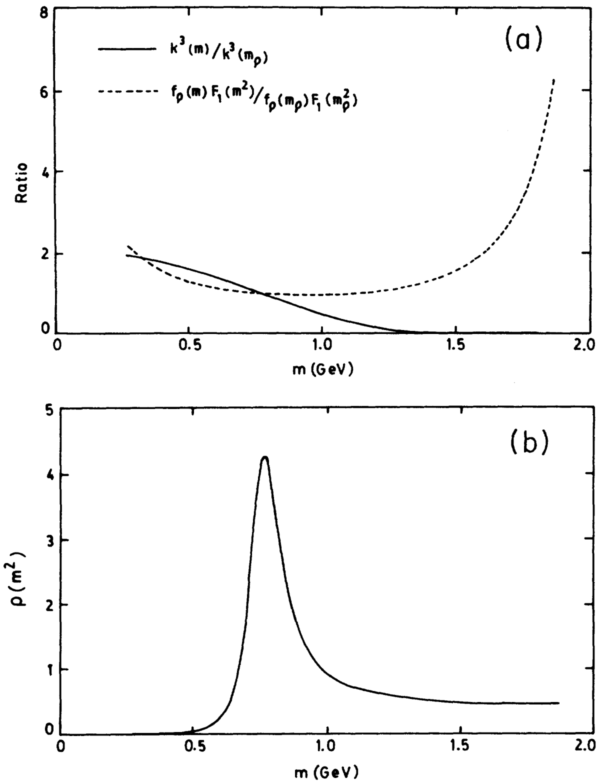

FIG. 1. (a) Mass dependence of center-of-mass momentum, decay constants, and form factors. (b) Breit-Wigner measure for the ρ meson.

the elastic FSI. In the weak amplitude, the parts corresponding to different isospin channels each pick up a phase appropriate to scattering in that particular isospin state. The net amplitude may be modified depending on the phases. We apply FSI's along with smearing effects to the study of $D \to \bar{K}\rho/\pi\rho/K\rho$ and $D_s \to K\rho$ modes. For $D/D_s \to \eta\rho/\eta'\rho$ decays, as there is only one isospin final state, elastic FSI effects are absent. We present their smeared branchings in column (iii) of Table I. The paucity of data in the Cabibbo-suppressed modes limits extractable information on the phase factors.

A. Cabibbo-enhanced mode

1. $D \to \bar{K}\rho$

For these decays the isospin analysis gives

$$\begin{aligned} A(D^0 \to K^-\rho^+) &= \frac{A_{3/2} e^{i\delta_{3/2}^{\bar{K}\rho}}}{\sqrt{3}} [1 + \sqrt{2} r e^{i\delta^{\bar{K}\rho}}] , \\ A(D^0 \to \bar{K}^0\rho^0) &= \frac{A_{3/2} e^{i\delta_{3/2}^{\bar{K}\rho}}}{\sqrt{3}} [\sqrt{2} - r e^{i\delta^{\bar{K}\rho}}] , \\ A(D^+ \to \bar{K}^0\rho^+) &= \frac{3 A_{3/2} e^{i\delta_{3/2}^{\bar{K}\rho}}}{\sqrt{3}} , \end{aligned} \tag{13}$$

where $r = A_{1/2}/A_{3/2}$ is the ratio of reduced amplitudes arising from isospin-$\frac{1}{2}$ and -$\frac{3}{2}$ channels. The phase difference is defined as

$$\delta^{\bar{K}\rho} = \delta_{1/2}^{\bar{K}\rho} - \delta_{3/2}^{\bar{K}\rho} . \tag{14}$$

In the BSW framework, the reduced amplitudes (up to a scale factor $2mG_F\cos^2\theta_C/\sqrt{2}$) are expressed as functions of running mass m:

$$\begin{aligned} A_{1/2}(m) &= \frac{1}{\sqrt{6}} [2a_1 f'_\rho - a_2 f'_k] , \\ A_{3/2}(m) &= \frac{1}{\sqrt{3}} [a_1 f'_\rho + a_2 f'_k] , \end{aligned} \tag{15}$$

where

$$f'_\rho = f_\rho(m) F_1^{D \to P}(m^2) \quad \text{and} \quad f'_P = f_P A_0^{D \to \rho}(m_P^2) .$$

At $m = m_\rho$, $A_{1/2} = 0.214$ GeV and $A_{3/2} = 0.101$ GeV.

The present data [9] on D^0 fixes $\delta^{\bar{K}\rho} = 20° \pm 25°$. For $\delta^{\bar{K}\rho} = 20°$ the smeared branchings for these decays are given in column (iii) of Table I. We see that though the D^0 decays satisfy the experimental observations with the inclusion of FSI's, $D^+ \to \bar{K}^0\rho^+$ is still on the higher side. We may remark here that the naive BSW model prediction for $B(D^+ \to \bar{K}^0\rho^+) = 15.09\%$ is reduced to 11.75% in the right direction because of the smearing effect. One may note that this decay is not affected by the elastic FSI's considered above. However, $\bar{K}\rho$ and $\bar{K}^*\pi$ channels can mix through the probable inelastic FSI's. Because of the inelastic FSI's, the decay amplitudes of different channels can couple and communicate with each other, leading to enhancement or depletion of these modes. Using the coupled-channel approach, Kamal, Sinha, and Sinha [6] have obtained the following branchings for the $D \to \bar{K}\rho$ and $\bar{K}^*\pi$ decays:

$$\begin{aligned} &B(D^0 \to K^-\rho^+) = 8.95\%, \quad \text{expt}(7.8 \pm 1.1)\% , \\ &B(D^0 \to \bar{K}^0\rho^0) = 1.10\%, \quad \text{expt}(0.43^{+0.31}_{-0.19})\% , \\ &B(D^+ \to \bar{K}^0\rho^+) = 10.04\%, \quad \text{expt}(6.6 \pm 1.7)\% , \\ &B(D^0 \to K^{*-}\pi^+) = 3.70\%, \quad \text{expt}(4.6 \pm 0.6)\% , \\ &B(D^0 \to \bar{K}^{*0}\pi^0) = 2.79\%, \quad \text{expt}(2.0 \pm 0.6)\% , \\ &B(D^+ \to \bar{K}^{*0}\pi^+) = 1.11\%, \quad \text{expt}(1.7 \pm 0.8)\% . \end{aligned} \tag{16}$$

Applying smearing factors to $D \to \bar{K}\rho$ further lowers the branchings to a nice agreement with experimental data:

$$\begin{aligned} &B(D^0 \to K^-\rho^+) = 6.89\% , \\ &B(D^0 \to \bar{K}^0\rho^0) = 0.84\% , \\ &B(D^+ \to \bar{K}^0\rho^+) = 7.83\% . \end{aligned} \tag{17}$$

B. Cabibbo-suppressed mode

1. $D \to \pi\rho$

Weak amplitudes at the isospin level are

$$A(D^0 \to \pi^-\rho^+) = \left[\frac{1}{2\sqrt{3}} B_2 e^{i\delta_2^{\pi\rho}} + \frac{1}{\sqrt{6}} A_0 e^{i\delta_0^{\pi\rho}} - \frac{1}{2}(A_1 + B_1) e^{i\delta_1^{\pi\rho}} \right] ,$$

$$A(D^0\to\pi^+\rho^-)=\left[\frac{1}{2\sqrt{3}}B_2e^{i\delta_2^{\pi\rho}}+\frac{1}{\sqrt{6}}A_0e^{i\delta_0^{\pi\rho}}+\frac{1}{2}(A_1+B_1)e^{i\delta_1^{\pi\rho}}\right],$$

$$A(D^0\to\pi^0\rho^0)=\left[-\frac{1}{\sqrt{3}}B_2e^{i\delta_2^{\pi\rho}}+\frac{1}{\sqrt{6}}A_0e^{i\delta_0^{\pi\rho}}\right],\tag{18}$$

$$A(D^+\to\pi^+\rho^0)=-\frac{1}{2\sqrt{2}}[\sqrt{3}B_2e^{i\delta_2^{\pi\rho}}+(2A_1-B_1)e^{i\delta_1^{\pi\rho}}],$$

$$A(D^+\to\pi^0\rho^+)=-\frac{1}{2\sqrt{2}}[\sqrt{3}B_2e^{i\delta_2^{\pi\rho}}-(2A_1-B_1)e^{i\delta_1^{\pi\rho}}],$$

where A_i and B_i are the weak reduced amplitudes corresponding to isospin channels i arising from $\Delta I=\frac{1}{2},\frac{3}{2}$ parts of the weak Hamiltonian, respectively. In terms of the BSW parameters, the amplitudes (up to a scale $2mG_F\cos\theta_C\sin\theta_C/\sqrt{2}$) are expressible as functions of the running mass m:

$$\begin{aligned}A_0(m)&=\frac{1}{\sqrt{6}}(a_2-2a_1)(f'_\rho+f'_\pi),\\A_1(m)&=\tfrac{1}{3}(2a_1-a_2)(f'_\rho-f'_\pi),\\B_1(m)&=\tfrac{1}{3}(a_1+a_2)(f'_\rho-f'_\pi),\\B_2(m)&=-\frac{1}{\sqrt{3}}(a_1+a_2)(f'_\rho+f'_\pi).\end{aligned}\tag{19}$$

The values of these reduced amplitudes at $m=m_\rho$ in GeV are $A_0=-0.318$, $A_1=0.087$, $B_1=0.021$, and $B_2=-0.109$.

The smeared branchings are calculated in terms of the phase difference

$$\delta^{\pi\rho}=\delta_1^{\pi\rho}-\delta_2^{\pi\rho}\quad\text{and}\quad\delta'^{\pi\rho}=\delta_0^{\pi\rho}-\delta_2^{\pi\rho}.$$

We obtain two phase-independent relations for the smeared branchings:

$$\begin{aligned}&B(D^0\to\pi^+\rho^-)+B(D^0\to\pi^-\rho^+)+B(D^0\to\pi^0\rho^0)=0.73\%,\\&B(D^+\to\pi^+\rho^0)+B(D^+\to\pi^0\rho^+)=0.45\%.\end{aligned}\tag{20}$$

For the $D\to\pi\rho$ sector, only one mode has been measured [10]:

$$B(D^+\to\pi^+\rho^0)=(0.07\pm0.05)\%.\tag{21}$$

In addition to that [9]

$$B(D^0\to\pi^+\pi^-\pi^0)=(1.2\pm0.4)\%\tag{22}$$

may be used to set an upper limit for $B(D^0\to\pi\rho)$.

Using (21), we find that a nonzero phase $\delta^{\pi\rho}=45°\pm15°$ is required, which in turn predicts

$$B(D^+\to\pi^0\rho^+)=(0.38\pm0.05)\%.\tag{23}$$

In the absence of experimental numbers on $D^0\to\pi\rho$ states, $\delta'^{\pi\rho}$ cannot be fixed. However, a choice for this may be made from the observation that both $I=0$ and 1 $\pi\rho$ final states, having nonexotic quantum numbers, may resonate. Correspondingly, we do not expect $\delta_0^{\pi\rho}$ to be very different from $\delta_1^{\pi\rho}$. Taking $\delta^{\pi\rho}=\delta'^{\pi\rho}=45°$, we calculate $D^0\to\pi\rho$ branchings, as given in column (iii) of Table I.

2. $D_s^+\to K\rho$

The amplitudes at the isospin level are

$$\begin{aligned}A(D_s^+\to K^0\rho^+)&=\frac{1}{\sqrt{3}}[-\sqrt{2}A_{1/2}e^{i\delta_{1/2}^{K\rho}}+B_{3/2}e^{i\delta_{3/2}^{K\rho}}],\\A(D_s^+\to K^+\rho^0)&=-\frac{1}{\sqrt{3}}[A_{1/2}e^{i\delta_{1/2}^{K\rho}}+\sqrt{2}B_{3/2}e^{i\delta_{3/2}^{K\rho}}],\end{aligned}\tag{24}$$

where $A_{1/2}$ and $B_{3/2}$ denote the reduced amplitudes appearing for $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ parts of the weak Hamiltonian. These are given by

$$\begin{aligned}A_{1/2}(m)&=\frac{1}{\sqrt{6}}[2a_1-a_2]f'_\rho,\\B_{3/2}(m)&=-\frac{1}{\sqrt{3}}[a_1+a_2]f'_\rho.\end{aligned}\tag{25}$$

At $m=m_\rho$, $A_{1/2}=0.234$ GeV and $B_{3/2}=-0.080$ GeV.

The phase-independent relation for the branching ratios is

$$B(D_s^+\to K^+\rho^0)+B(D_s^+\to K^0\rho^+)=0.70\%.\tag{26}$$

The smeared branchings calculated in terms of the phase difference

$$\delta^{K\rho}=\delta_{1/2}^{K\rho}-\delta_{3/2}^{K\rho}$$

are given in column (iii) of Table I.

C. Cabibbo-doubly-suppressed mode

1. $D\to K\rho$

The amplitudes at the isospin level are

$$\begin{aligned}A(D^0\to K^+\rho^-)&=\frac{\sqrt{2}}{3}[A_{3/2}e^{i\delta_{3/2}^{K\rho}}+(A_{1/2}+\sqrt{3}B_{1/2})e^{i\delta_{1/2}^{K\rho}}],\\A(D^0\to K^0\rho^0)&=\tfrac{2}{3}[A_{3/2}e^{i\delta_{3/2}^{K\rho}}-\tfrac{1}{2}(A_{1/2}+\sqrt{3}B_{1/2})e^{i\delta_{1/2}^{K\rho}}],\\A(D^+\to K^+\rho^0)&=-\tfrac{2}{3}[A_{3/2}e^{i\delta_{3/2}^{K\rho}}-\tfrac{1}{2}(A_{1/2}-\sqrt{3}B_{1/2})e^{i\delta_{1/2}^{K\rho}}],\\A(D^+\to K^0\rho^+)&=\frac{\sqrt{2}}{3}[A_{3/2}e^{i\delta_{3/2}^{K\rho}}+(A_{1/2}-\sqrt{3}B_{1/2})e^{i\delta_{1/2}^{K\rho}}],\end{aligned}\tag{27}$$

where $A_{1/2}$ and $A_{3/2}$ denote the reduced amplitudes appearing for $\Delta I=1$ part of the weak Hamiltonian and $B_{1/2}$ denotes reduced amplitude for $\Delta I=0$ piece. In the BSW framework, these are given (up to a scale factor $-2mG_F\sin^2\theta_C/\sqrt{2}$) and can be seen to be independent of the ρ mass:

$$A_{1/2}=\frac{1}{2\sqrt{2}}[a_1+a_2]f'_K=0.029\ \mathrm{GeV}\ ,$$
$$B_{1/2}=\frac{\sqrt{3}}{2\sqrt{2}}[a_1-a_2]f'_K=0.121\ \mathrm{GeV}\ , \qquad (28)$$
$$A_{3/2}=\frac{1}{\sqrt{2}}[a_1+a_2]f'_K=0.058\ \mathrm{GeV}\ .$$

The smeared branchings are to be calculated in terms of the phase difference

$$\delta^{K\rho}=\delta^{K\rho}_{1/2}-\delta^{K\rho}_{3/2}\ .$$

However, as no experimental information is available on $D\to K\rho$, the phase factor can not be determined. If one assumes particle-antiparticle symmetry, the phase factor $\delta^{K\rho}$ may be taken to be the same as that for the $D\to\bar{K}\rho$ decays. However, two phase-independent relations for the D^0 and D^+ decays can be obtained as

$$B(D^0\to K^0\rho^0)+B(D^0\to K^+\rho^-)=3.12\times\tan^4\theta_C\%\ ,$$
$$B(D^+\to K^0\rho^+)+B(D^+\to K^+\rho^0)=4.95\times\tan^4\theta_C\%\ .$$

Predictions for smeared branchings for $D/D_s\to K\rho$ decays for $\delta^{K\rho}=20°$ are given in column (iii) of Table I.

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APPENDIX: DERIVATION OF THE SMEARING FORMULA USING THE $D\to P2\pi$ ANALYSIS AT A ρ POLE

Let us first consider the decay process $D\to K\rho\to K\pi\pi$ as two separate processes: the weak decay $D\to K\rho$ part and the strong decay $\rho\to\pi\pi$ part.

1. Weak decay $D^0\to K^-\rho^+$

Assuming the weak-decay amplitude to be A_W and all particles to be on shell, the decay rate in terms of Lorentz-invariant phase space (LIPS) is given by

$$\Gamma(D^0\to K^-\rho^+)=\int\frac{1}{2m_D}\sum_{\mathrm{pol}}|A_W|^2X_{\mathrm{LIPS}}(P_D^2;q,k)\ ,$$

where

$$X_{\mathrm{LIPS}}(P_D^2;q,k)=(2\pi)^4\delta^4(P_D-q-k)\frac{d^3q}{(2\pi)^3 2q^0}\frac{d^3k}{(2\pi)^3 2k^0}\ ,$$

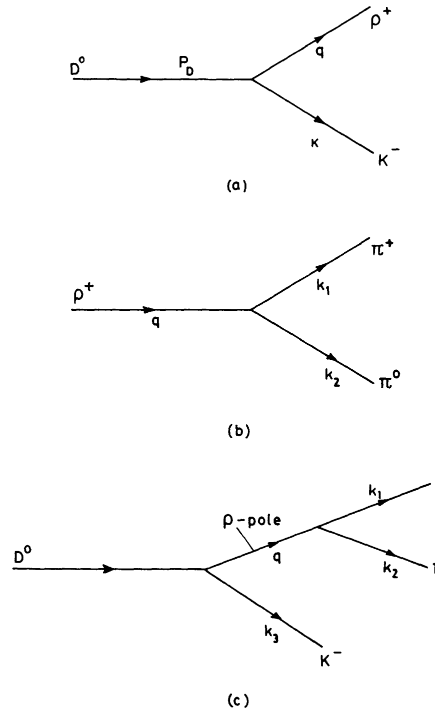

FIG. 2. (a) Weak decay $D\to K\rho$, (b) strong decay $\rho\to\pi\pi$, and (c) weak decay $D\to K\pi\pi$.

with $P_D^2=m_D^2$, $q^2=m_\rho^2$, and $k^2=m_K^2$ [Fig. 2(a)].

2. Strong decay $\rho^+\to\pi^+\pi^0$

If we represent the strong-decay amplitude by A_S, then

$$A_S=g_{\rho\pi\pi}\varepsilon\cdot(k_1-k_2)$$

and the width is

$$\Gamma(\rho^+\to\pi^+\pi^0)=\frac{g^2_{\rho\pi\pi}}{2m_\rho}\int\sum_{\mathrm{pol}}|\varepsilon\cdot(k_1-k_2)|^2X_{\mathrm{LIPS}}(q^2;k_1,k_2)\ ,$$

where $g_{\rho\pi\pi}$ is the strong-coupling constant and $q^2=m_\rho^2$, $k_1^2=k_2^2=m_\pi^2$, and $q=k_1+k_2$ [Fig. 2(b)].

3. Weak decay $D^0\to K^-\pi^+\pi^0$

Now consider the decay process $D^0\to K^-\pi^+\pi^0$ through a ρ pole, which, having a large width, is taken with the Breit-Wigner correction in the propagator. Thus the amplitude for this process is given as [Fig. 2(c)]

$$A=g_{\rho\pi\pi}\varepsilon\cdot(k_1-k_2)\frac{1}{(q^2-m_\rho^2)+i\Gamma(q^2)m_\rho}A_W$$

and the resulting decay width is

$$\Gamma(D^0\to K^-\pi^+\pi^0)=\frac{1}{2m_D}\int|A|^2X_{\mathrm{LIPS}}(P_D^2;k_1,k_2,k_3)\ .$$

Now $d^4q\,\delta^4(q-k_1-k_2)=1$ by definition [12]. Therefore we can write

$$X_{\rm LIPS}(P_D^2;k_1,k_2,k_3)=(2\pi)^4\delta^4(P_D-k_1-k_2-k_3)d^4q\,\delta^4(q-k_1-k_2)\frac{d^3k_1}{(2\pi)^3 2k_1^0}\frac{d^3k_2}{(2\pi)^3 2k_2^0}\frac{d^3k_3}{(2\pi)^3 2k_3^0}\ .$$

Let $q^2=s=m_\rho^2$,

$$dX_{\rm LIPS}(q)=\frac{d^4q}{(2\pi)^3}\delta(q^2-s)\ .$$

By definition

$$dX_{\rm LIPS}(P)=\frac{d^3P}{(2\pi)^3 2E}=\frac{d^4P}{(2\pi)^3}\delta(P^2-m^2)\ .$$

We make q^2 or s an explicit integration variable by writing

$$d^4q=d^4q\,\delta(q^2-s)ds=(2\pi)^3 dX_{\rm LIPS}(q)ds\ ,$$

i.e.,

$$X_{\rm LIPS}(P_D^2;k_1,k_2,k_3)=(2\pi)^4\delta^4(P_D-q-k_3)\frac{d^3q}{(2\pi)^3 2q^0}\frac{d^3k_3}{(2\pi)^3 2k_3^0}dq^2$$
$$\times\frac{1}{2\pi}(2\pi)^4\delta^4(q-k_1-k_2)\frac{d^3k_1}{(2\pi)^3 2k_1^0}\frac{d^3k_2}{(2\pi)^3 2k_2^0}\ ,$$

$$X_{\rm LIPS}(m_D^2;k_1,k_2,k_3)=\frac{1}{2\pi}\int X_{\rm LIPS}(m_D^2;q,k_3)dq^2 X_{\rm LIPS}(q^2;k_1,k_2)\ .$$

Substituting in $\Gamma(D^0\to K^-\pi^+\pi^0)$,

$$\Gamma(D^0\to K^-\pi^+\pi^0)=\frac{1}{2\pi}\int g_{\rho\pi\pi}^2\sum_{\rm pol}|\varepsilon\cdot(k_1-k_2)|^2 X_{\rm LIPS}(q^2;k_1,k_2)$$
$$\times\frac{dq^2}{(q^2-m_\rho^2)^2+\Gamma^2(q^2)m_\rho^2}\frac{1}{2m_D}|A_W|^2 X_{\rm LIPS}(m_D^2;q,k_3)\ .$$

From Secs. 1 and 2, we get

$$\Gamma(\rho(q^2)\to\pi\pi)=\frac{g_{\rho\pi\pi}^2}{2(q^2)^{1/2}}\int\sum_{\rm pol}|\varepsilon\cdot(k_1-k_2)|^2 X_{\rm LIPS}(q^2;k_1,k_2)\ ;$$

here $(q^2)^{1/2}$ is the ρ mass and

$$\Gamma(D\to K\rho(q^2))=\int\frac{1}{2m_D}\sum_{\rm pol}|A_W(D\to K\rho(q^2))|^2 X_{\rm LIPS}(m_D^2;q,k_3)\ .$$

Here the ρ meson is off shell and hence the q^2 dependence is carried in the form factors. This leads to

$$\Gamma(D^0\to K^-\pi^+\pi^0)=\frac{1}{\pi}\int\Gamma(\rho(q^2)\to\pi\pi)\frac{(q^2)^{1/2}dq^2}{(q^2-m_\rho^2)^2+\Gamma^2(q^2)m_\rho^2}\Gamma(D\to K\rho(q^2))\ ,$$

$$\Gamma(D^0\to K^-\pi^+\pi^0)=\frac{1}{\pi}\int\Gamma(D\to K\rho(q^2))\frac{(q^2)^{1/2}\Gamma(q^2)dq^2}{(q^2-m_\rho^2)^2+\Gamma^2(q^2)m_\rho^2}\ .$$

As the $B(\rho\to 2\pi)$ is $\approx 100\%$, therefore, $\Gamma(q^2)$ in the numerator, which is the off-shell ρ width, is taken to be the same as in the denominator. Actually, the numerator represents the partial width with respect to the mode. This suggests the form of the Breit-Wigner measure,

$$\frac{1}{\pi}\frac{m\Gamma(m^2)}{(m^2-m_\rho^2)^2+\Gamma^2(m^2)m_\rho^2}\ ,$$

which, in a narrow resonance approximation, gives

$$\Gamma(D^0\to K^-\pi^+\pi^0)=\Gamma(D\to K\rho(m_\rho^2))\ .$$

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