D-meson decay in the $1/N$ expansion

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Exclusive decays of the D^0 meson into $K^-\pi^+$, $K^-\rho^+$, $K^{*-}\pi^+$, $K^{*-}\rho^+$, and $K^-A_1^+$ are reanalyzed using the $1/N$ expansion, N being the number of colors, and recent data on semileptonic D decays. Except for the $D^0\to K^-A_1^+$ decay rate, which is smaller than the measured value by a factor of 4, the other decay rates are found to be more or less in agreement with experiment.

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Nonleptonic two-body D-meson decays, because of their dominant contribution to the total D-meson decay rate, could help us to understand the large D^+-D^0 lifetime difference, which has not been completely understood from an inclusive approach to D decay (the spectator model) [1]. The study of these exclusive D decay modes involves, however, knowledge of both the hadronic matrix elements of the effective nonleptonic decay Lagrangian and the strong final-state interaction, which lie outside the framework of perturbative QCD; therefore, it can only be done at the phenomenological level and must be based on the vacuum insertion approximation or factorization hypothesis, as was the attempt to explain the $\Delta I=\frac{1}{2}$ rule for nonleptonic K decays.

Though a large deviation from the vacuum insertion approximation is expected in nonleptonic K decays, for D-meson nonleptonic decays, such a deviation might not be important enough to produce a large discrepancy with data since a recent analysis of exclusive nonleptonic B decay [2] indicates that factorization should work better for a heavy-quark system. Thus factorization when combined with final-state interactions should provide a good description of nonleptonic D decays. Since final-state interactions cannot be taken into account easily, it would be simpler to neglect these effects altogether under certain approximations. Following the initial success of the factorization hypothesis for D decays obtained by Bauer *et al.* [3], we are led to the $1/N$ expansion approach advocated by Buras *et al.* [4]. Since the work of Ref. [3], new data have been accumulated for semileptonic and hadronic decays, and, with the recent measurements of the form factors in $D^+\to K^{*0}e\nu$ decays [5,6], it is now possible to carry out an analysis of many two-body D decays in the $1/N$ expansion without using the quark model for the form factors involved and to test the predictions against these new data.

The purpose of this paper is to present an analysis of the main hadronic decay modes of D^0 using information from both hadronic τ decays and the recent data on semileptonic D decays. We find that the $D^0\to K^-\pi^+$ decay rate is larger than the measured value by 66%; the $D^0\to K^-A_1^+$ decay rate is smaller than the measured value by a factor of 4. The $D^0\to K^{*-}\pi^+$ decay rate is also smaller than experiment by a factor of 2 if the new E653 data on the $D\to K^*$ form factors are used instead of the E691 data. For the decays $D^0\to K^-\rho^+$ and $D^0\to K^{*-}\rho^+$, the computed decay rates are in reasonably good agreement with experiment.

Our starting point is the following standard effective Lagrangian for the Cabibbo-favored charm decay [3]:

$$\mathcal{L}_{\mathrm{eff}}=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}[c_1(\bar{u}d)(\bar{s}c)+c_2(\bar{u}c)(\bar{s}d)] \ . \tag{1}$$

The coefficients c_1, c_2 represent the well-known QCD short-distance effects [7] defined in terms of c_+ and c_- by

$$c_1=\frac{c_++c_-}{2} \ , \quad c_2=\frac{c_+-c_-}{2} \ . \tag{2}$$

c_+ and c_1 are functions of Λ_{QCD} and the scale parameter μ with $c_+^2c_-=1$.

By performing a Fierz reordering $\mathcal{L}_{\mathrm{eff}}$ can be written as a product of two color-singlet V-A currents [4,3,7]:

$$\mathcal{L}_H=\frac{G_f}{\sqrt{2}}V_{cs}^*V_{ud}[a_1(\bar{u}d)_H(\bar{s}c)H+a_2(\bar{u}c)_H(\bar{s}d)_H] \ , \tag{3}$$

where the index H indicates the change to hadron field operators and

$$a_1=c_1+\frac{1}{N}c_2 \ , \quad a_2=c_2+\frac{1}{N}c_1 \ . \tag{4}$$

In the factorization hypothesis (also called the vacuum insertion approximation) the nonleptonic decay matrix elements of $\mathcal{L}_H$ can be written as a product of two hadronic currents with the matrix elements taken in all possible ways among hadrons in the initial and final state. The nonleptonic decay amplitudes thus contain a charged-current term with a strength a_1 and a neutral-current term induced by the short-distance QCD effects with a strength a_2. Three classes of decays can then be classified as done by Bauer *et al.* [3]: those induced by the charged current (class I), those induced by the neutral current (class II), and those in which both charged and neutral currents participate (class III). Examples of decays are $D^0\to h_1^+h_2^-$ for class I, $D^0\to h_1^0h_2^0$ for class II

(h_1 and h_2 are light hadrons), and the D^+ decays for class III. The D decay amplitudes can now be directly obtained from the corresponding semileptonic D decays and from hadronic τ decays since the τ lepton and the D meson are nearly degenerate in mass. Since SU(3) is a good symmetry of strong interactions, class II and III decays can also be obtained in principle from the charged-current matrix elements via SU(3) symmetry.

It was noticed a long time ago [7] that if we use the value of a_2 given in Eq. (4) with $N=3$, class II decays are strongly suppressed relative to class I by a large factor due to the cancellation between the c_1 and c_2 terms in a_2. This is in disagreement with data which show that the neutral two-body decay modes of D^0 are not suppressed [8], and clearly indicates that the naive factorization of the nonleptonic weak interactions for charm decay into a product of hadronic currents at the tree level as given in Eq. (3) is not valid in general. As pointed out by Buras *et al.* [4], the factorization hypothesis is valid only in the limit $N\to\infty$. The $O(1/N)$ terms due to soft-gluon effects and final-state interactions of the hadrons in the decay product could become important for $N=3$, and therefore must be included in any calculation of exclusive D decays. This is not possible at present. A more pragmatic approach to D decay would be to ignore all terms of order $O(1/N)$, which, for $N=3$, should not be large enough to affect drastically the qualitative feature of D decays, especially for class I decays with a large a_1 strength. Then the D decay amplitudes can be obtained by the factorization contributions, while neglecting $O(1/N)$ terms in the coefficients a_1, a_2 [see Eq. (4)], as well as other $O(1/N)$ terms due to a soft gluon in the matrix elements of $\mathcal{L}_H$ and final-state interactions. This is the approach of Buras *et al.* [4]. They have analyzed a large number of two-body D decay modes in this way and obtained agreement with experiment to within a factor of 2. In the following we follow this approach using recent information from semileptonic D decays. Since hadronic D decays, by the factorization hypothesis, depend on the form factors of the vector and axial-vector currents obtained from semileptonic D decays, for convenience we give here expressions for the semileptonic $D\to Ke\nu$ and $D\to K^*e\nu$ decay rates. For the $D\to K^*e\nu$ decay, we use the expression for the decay rate in terms of the helicity amplitudes given in previous papers [9,10]:

$$\frac{d\Gamma}{dq^2}(D\to Ke\nu)=\frac{G_F^2|V_{cs}|^2\lambda^{3/2}(M^2,m^2,q^2)}{M^3}|f_+(q^2)|^2\ ; \tag{5}$$

for the $D\to K^*e\nu$ decay we have

$$\frac{d\Gamma}{dq^2}(D\to K^*e\nu)=\frac{d\Gamma_0}{dq^2}+\frac{d\Gamma_+}{dq^2}+\frac{d\Gamma_-}{dq^2}\ , \tag{6}$$

where $\lambda(x,y,z)$ is the usual phase-space factor defined as

$$\lambda(x,y,z,)=(x-y-z)^2-4yz\ , \tag{7}$$

and the differential partial helicity decay rates are

$$\frac{d\Gamma_{0,\pm}}{dq^2}=\frac{G_F^2|V_{cs}|^2}{192\pi^3}\frac{\lambda^{1/2}(M^2,m_1^2,q^2)}{M^3}Y_{0,\pm}(q^2)\ , \tag{8}$$

with

$$\begin{aligned} Y_0(q^2)&=\left[\frac{M+m_1}{2m_1}\right]^2\left[(M^2-m_1^2-q^2)A_1(q^2)\right.\\ &\qquad\left.-\frac{\lambda(M^2,m_1^2,q^2)}{(M+m_1)^2}A_2(q^2)\right]^2\ ,\\ Y_\pm(q^2)&=q^2(M+m_1)^2\\ &\quad\times\left[A_1(q^2)\mp\frac{\lambda^{1/2}(M^2,m_1^2,q^2)}{(M+m_1)^2}V(q^2)\right]^2\ . \end{aligned} \tag{9}$$

M, m, and m_1 are, respectively, the D, K, and K^* meson masses, and q^2 is the square of the momentum transfer, which is also the invariant mass of the lepton pair squared. In the expressions for Y, the quantities V, A_1, and A_2 are the three form factors for the vector- and axial-vector-current matrix elements between D and K^* [11]. These form factors have been measured by the E691 group [5] and more recently by the E653 group [6]. The q^2 dependences of the form factors f_+, V, A_1, A_2 are also measured [12]; they are consistent with a single-pole behavior with the pole mass $m_V\simeq 2.1$ GeV for f_+, V, and $m_A\simeq 25$ GeV for A_1, A_2. Normalizing the form factors at their $q^2=0$ values $f_+(0)$, $V(0)$, $A_1(0)$, $A_2(0)$, we have, for example,

$$f_+(q^2)=\frac{f_+(0)}{1-q^2/m_V^2}\ , \tag{10}$$

and similar expressions for $V(q^2)$, $A_1(q^2)$, and $A_2(q^2)$ with m_V and m_A as the pole mass for the V and A form factors, respectively. We then obtain the semileptonic decay rates

$$\Gamma(D\to Ke\nu)=1.527|V_{cs}|^2f_+^2(0)\times 10^{14}\ \mathrm{sec}^{-1}\ . \tag{11}$$

For the decay $D\to K^*e\nu$ we have

$$\Gamma(D\to K^*e\nu)=1.863|V_{cs}|^2F_{K^*e\nu}\times 10^{11}\ \mathrm{sec}^{-1}\ , \tag{12}$$

where

$$\begin{aligned} F_{K^*e\nu}&=A_1^2(0)+0.016V^2(0)+0.044A_2^2(0)\\ &\quad-0.332A_1(0)A_2(0)\ . \end{aligned} \tag{13}$$

In obtaining the above results, we have assumed a pole dominance for the form factors. Actually, because of phase-space suppression of the high lepton-pair momentum region, a large part of the semileptonic decay rate comes from the small q^2 region, and an expansion in powers of q^2 for the form factors can be used to compute the total semileptonic decay rates, which are thus not very sensitive to the detailed q^2 dependence of the form factors. We have used a linear expansion in q^2 for the form factors and found that the results obtained differ from the pole-dominance form by a few percent. The measured semileptonic decay rates [5,6,12] then give the following values for the form factors at $q^2=0$,

$$f_+(0)=0.71\pm0.06 \ , \tag{14}$$

and for the $D \to K^*$ form factors, the E691 measurements give

$$\begin{aligned} V(0)&=0.9\pm0.3\pm0.1 \ , \\ A_1(0)&=0.46\pm0.05\pm0.05 \ , \\ A_2(0)&=0.0\pm0.2\pm0.1 \ . \end{aligned} \tag{15}$$

The E653 group [6] more recently reported new measurements on the form factors in $D^+ \to K^{*0}\mu^+\nu$ decays and gave a large value for $A_2(q^2)$ relative to $A_1(q^2)$ for q^2 up to $0.6q^2_{\max}$. From previous measurements of the $D^+ \to K^{*0}e\nu$ branching ratio [13] and from the relative values of $V(q^2)$ and $A_2(q^2)$, we find

$$\begin{aligned} V(0)&=1.06\pm0.3\pm0.20 \ , \\ A_1(0)&=0.53\pm0.08\pm0.05 \ , \\ A_2(0)&=0.43\pm0.15\pm0.11 \ . \end{aligned} \tag{16}$$

We see that while the measured values for $A_1(0)$ and $V(0)$ in both measurements are comparable, there is a big difference in the value of $A_2(0)$, which is small in the E691 data.

With these measured values for the form factors, we now compute the class-I hadronic decay modes of D^0 in the $1/N$ approximation. For a single-particle state X, the two-body D decay rates are given by a generalized expression [10] similar to the expression for hadronic τ decays. For the decay $D^0 \to K^-X$, we have

$$\Gamma(D^0 \to K^-X)=K\int_0^{(M-m)^2} dq^2\lambda^{1/2}(M^2,m^2,q^2)f_+^2(q^2)\{\lambda(M^2,m^2,q^2)[v_1(q^2)+a_1(q^2)]+(M^2-m^2)^2a_0(q^2)\} \ , \tag{17}$$

where

$$K=\frac{c_1^2G_F^2|V_{cs}^*V_{ud}|^2}{64\pi^2M^3} \ . \tag{18}$$

The quantities v_J, a_J ($J=0.1$) are, respectively, the vector and the axial-vector spectral functions associated with the hadronic state X, having total angular momentum J defined as [14]

$$\sum_X \langle 0|J_\mu(0)|X\rangle\langle X|J_\nu(0)|0\rangle(2\pi)^4\delta^4(q-p_X)=(-q^2g_{\mu\nu}+q_\mu q_\nu)[v_1(q^2)+a_1(q^2)]+q_\mu q_\nu a_0(q^2) \ , \tag{19}$$

and can be extracted from the measured Cabibbo-favored hadronic τ decays for which the rate is given by

$$\Gamma(\tau^- \to \nu_\tau X)=\frac{G_F^2|V_{ud}|^2}{32\pi^2m_\tau^3}\int_0^{m_\tau^2} dq^2(m_\tau^2-q^2)^2\{(m_\tau^2+2q^2)[v_1(q^2)+a_1(q^2)]+m_\tau^2a_0(q^2)\} \ . \tag{20}$$

Similarly, the $D^0 \to K^{*-}X$ decay rates are given by

$$\Gamma(D^0 \to K^{*-}X)=K\int_0^{(M-m_1)^2} dq^2\lambda^{1/2}(M^2,m_1^2,q^2)\{Y(q^2)[v_1(q^2)+a_1(q^2)]+H_0^2\} \ , \tag{21}$$

with

$$\begin{aligned} Y(q^2)&=Y_+(q^2)+Y_-(q^2)+Y_0(q^2) \ , \\ H_0^2&=\lambda(M^2,m_1^2,q^2)\left[\frac{M+m_1}{2m_1}\right]\left[A_1(q^2)-\frac{M-m_1}{M+m_1}A_2(q^2)\right]^2 a_0(q^2) \ . \end{aligned} \tag{22}$$

We note in passing that if X is a multibody hadronic state, Eqs. (17) and (21) are not the only contributions to the multibody $D^0 \to K^-X$ and $D^0 \to K^{*-}X$ decay rates. There are additional terms in which the $\bar{c}s$ currents lead to emission of K^- and other nonstrange light hadrons. These terms may be obtained from the inclusive semileptonic decays $D \to KXe\nu$ and $D \to K^*Xe\nu$ and from hadronic τ decays. Only in the case where these decays are much less important than the decays $D \to Ke\nu$ and $D \to K^*e\nu$ can the multibody D decay rate be obtained to a good approximation from Eqs. (17) and (21). If this is the case, then many features of τ decays (e.g., hadron multiplicity) should also be qualitatively reproduced in class-I D^0 decays. In particular, the dominance of the one-prong over the three-prong events in τ decays is translated into the dominance of the two-prong events with one K^- in D^0 decays. Actually, the analysis is more complicated since events with a K^- can be contaminated by the decays $D^0 \to K^{*-}X$ (class I) and $D^0 \to K^{*0}X$ (class II) followed by the $K^* \to K\pi$ strong decays.

Returning to the two-body D^0 decays in the $1/N$ expansion the decay rates now can be easily obtained from Eqs. (17) and (21) with the single-particle π,ρ, A_1 contributions to the spectral functions [14]:

$$a_0(q^2)=2\pi f_\pi^2\delta(q^2)\ ,$$
$$v_1(q^2)=2\pi f_\rho^2\frac{\delta(q^2-m_\rho^2)}{m_\rho^2}\ , \tag{23}$$
$$a_1(q^2)=2\pi f_A^2\frac{\delta(q^2-m_A^2)}{m_A^2}\ ,$$

where $f_\pi \simeq 132$ MeV is the usual pion decay constant and f_ρ is defined by

$$\langle 0|V_\mu(0)|\rho^\pm\rangle=f_\rho\epsilon_\mu\ , \tag{24}$$

and similarly for f_A. f_ρ is related to the ρ^0 decay constant extracted from the decay rate of $\rho^0\to e^+e^-$, which gives

$$f_\rho\simeq\sqrt{2}m_\rho f_\pi\times 1.137\ . \tag{25}$$

The spectral functions $v_1(q^2)$ and $a_1(q^2)$ given in Eq. (23) are only valid in the zero-width approximation. Because of the importance of the finite-width effects on the τ and D decay rates for a broad resonance such as the A_1 meson with a width of 400 MeV, instead of the δ-function approximation for the spectral function, we shall use the following Breit-Wigner forms of $v_1(q^2)$ and $a_1(q^2)$ for the ρ and A_1 meson:

$$v_1(q^2)=\frac{1}{\pi}\frac{f_\rho^2}{m_\rho^2}\left[\frac{m_\rho\Gamma_\rho}{(q^2-m_\rho^2)^2+m_\rho^2\Gamma_\rho^2}\right]\ ,$$
$$a_1(q^2)=\frac{1}{\pi}\frac{f_A^2}{m_A^2}\left[\frac{m_A\Gamma_A}{(q^2-m_A^2)^2+m_A^2\Gamma_A^2}\right]\ . \tag{26}$$

For the $D^0\to K^-\pi^+$ decay rate we then obtain

$$\Gamma(D^0\to K^-\pi^+)=\frac{c_1^2G_F^2}{32\pi}M^3\left[1-\frac{m^2}{M^2}\right]^3|V_{cs}^*V_{ud}|^2f_+^2(0)\ , \tag{27}$$

or, numerically,

$$\Gamma(D^0\to K^-\pi^+)=1.865c_1^2|V_{cs}^*V_{ud}|^2f_+^2(0)\times10^{11}\ \mathrm{sec}^{-1}\ . \tag{28}$$

If we assume a pole dominance for the $D\to K$ form factor f_+, we can express the $D^0\to K^-\pi^+$ decay rate in terms of the semileptonic $D\to Ke\nu$ decay rate. Using Eq. (10), we find

$$\Gamma(D^0\to K^-\pi^+)=1.221c_1^2|V_{ud}|^2\Gamma(D^0\to K^-e^+\nu)\ . \tag{29}$$

As mentioned earlier, this result does not strongly depend on the q^2 dependence of the form factor $f_+(q^2)$ and, thus, can be used to predict the $D^0\to K^-\pi^+$ decay rate in the $1/N$ expansion in terms of the semileptonic $D\to Ke\nu$ decay rate.

For the decay $D^0\to K^-\rho^+$, we have, similarly,

$$\Gamma(D^0\to K^-\rho^+)=\frac{c_1^2G_F^2}{32\pi M^3}|V_{cs}^*V_{ud}|^2\lambda^{3/2}(M^2,m^2,m_\rho^2)\times\frac{f_\rho^2}{m_\rho^2}f_+^2(m_\rho^2) \tag{30}$$

in the zero-width approximation for the ρ meson.

Because of phase-space limitation and the large ρ-meson width, a more accurate calculation has to be performed with the Breit-Wigner form for the spectral function $v_1(q^2)$. Using the pole-dominance form factor and the Breit-Wigner form for $v_1(q^2)$, we obtain numerically, for $m_\rho=0.768$ GeV and $\Gamma_\rho=0.148$ GeV,

$$\Gamma(D^0\to K^-\rho^+)=2.694c_1^2|V_{cs}^*V_{ud}|^2f_+^2(0)\times10^{11}\ \mathrm{sec}^{-1}\ . \tag{31}$$

Instead of giving the decay rates in terms of the form factor $f_+(0)$, we can also express the rates in terms of the $D\to Ke\nu$ decay rate. We then find

$$\Gamma(D^0\to K^-\rho^+)=1.764c_1^2|V_{ud}|^2\Gamma(D^0\to Ke\nu)\ . \tag{32}$$

We remark that the δ-function approximation for the spectral function [given in Eq. (23)] gives a $D^0\to K^-\rho^+$ decay rate larger than the above value by 14%. This reduction in the $D^0\to K^-\rho^+$ decay rate relative to the δ-function approximation is similar to the reduction of the $\tau\to\nu\rho$ decay rate found previously [15].

To compute the $D^0\to K^-A_1^+$ decay rate we shall use the value of f_A extracted from the $\tau\to\nu A_1$ decay. For convenience we give here the $\tau\to\nu A_1$ decay rate in terms of the $\tau\to\nu\rho$ decay rate. Using a Breit-Wigner form for the spectral functions $v_1(q^2)$ and $a_1(q^2)$ [Eq. (26)], we find

$$\Gamma(\tau\to\nu A_1)=0.495\frac{f_A^2}{f_\rho^2}\frac{m_\rho^2}{m_A^2}\Gamma(\tau\to\nu\rho)\ . \tag{33}$$

From the branching ratios of $\tau\to\nu A_1$ and $\tau\to\nu\rho$ measured recently [8],

$$B(\tau\to\nu\rho)=(22.7\pm0.9)\%\ ,$$
$$B(\tau\to\nu A_1)=(10.8\pm3.4)\%\ , \tag{34}$$

we obtain

$$\frac{f_A^2}{m_A^2}=(0.96\pm0.33)\frac{f_\rho^2}{m_\rho^2}\ . \tag{35}$$

We note that this value for f_A, extracted from the τ decay, is consistent with the theoretical value given by the first Weinberg sum rule, which gives, using Eq. (25),

$$\frac{f_A^2}{m_A^2}=0.61\frac{f_\rho^2}{m_\rho^2}\ . \tag{36}$$

The $D^0\to K^-A_1^+$ decay rate can now be computed in term of f_A. The δ-function approximation for $a_1(q^2)$ gives

$$\Gamma(D^0 \to K^- A_1^+) = \frac{c_1^2 G_F^2}{32\pi M^3} |V_{cs}^* V_{ud}|^2 \times \frac{f_A^2}{m_A^2} \lambda^{3/2}(M^2, m^2, m_A^2) f_+^2(m_A^2) \ . \tag{37}$$

A more accurate result is obtained by using the Breit-Wigner form for the spectral function $a_1(q^2)$ given in Eq. (26) for the $D^0 \to K^- \rho^+$ decay rate. We obtain, for $m_A = 1.260$ GeV and $\Gamma_A = 0.400$ GeV and with the value of f_A obtained from $\tau \to \nu A_1$ as given by Eq. (35),

$$\Gamma(D^0 \to K^- A_1^+) = (0.62 \pm 0.21) c_1^2 |V_{cs}^* V_{ud}|^2 \times f_+^2(0) \times 10^{11} \ \mathrm{sec}^{-1} \ , \tag{38}$$

which gives, in terms of the $D \to K e \nu$ decay rate,

$$\Gamma(D^0 \to K^- A_1^+) = (0.40 \pm 0.14) c_1^2 |V_{ud}|^2 \Gamma(D^0 \to K e \nu) \ , \tag{39}$$

which is consistent with the value obtained with the sum-rule value for f_A. In fact, from Eq. (36), we get

$$\Gamma(D^0 \to K^- A_1^+) = 0.253 c_1^2 |V_{ud}|^2 \Gamma(D^0 \to K e \nu) \ . \tag{40}$$

In the same manner, we can express the $D^0 \to K^{*-} \pi^+$ and $D^0 \to K^{*-} \rho^+$ decay rates in terms of the $D^0 \to K^{*-} e^+ \nu$ decay rate. From Eqs. (21) and (22), and using the spectral function $a_0(q^2)$ in Eq. (23) and $v_1(q^2)$ in Eq. (26), we get

$$\Gamma(D^0 \to K^{*-} \pi^+) = 2.537 c_1^2 |V_{cs}^* V_{ud}|^2 F_{K^*\pi} \times 10^{11} \ \mathrm{sec}^{-1} \ , \tag{41}$$

where

$$F_{K^*\pi} = [A_1(0) = 0.35 A_2(0)]^2 \ . \tag{42}$$

In terms of the $D^0 \to K^{*-} e^+ \nu$ decay rate, we find, from Eqs. (12) and (41),

$$\Gamma(D^0 \to K^{*-} \pi^+) = 1.382 c_1^2 |V_{ud}|^2 \frac{F_{K^*\pi}}{F_{K^* e\nu}} \Gamma(D^0 \to K^{*-} e^+ \nu) \ . \tag{43}$$

For the $D^0 \to K^{*-} \rho^+$ decay rate we obtain

$$\Gamma(D^0 \to K^{*-} \rho^+) = 4.550 c_1^2 |V_{cs}^* V_{ud}|^2 F_{K^*\rho} \times 10^{11} \ \mathrm{sec}^{-1} \ , \tag{44}$$

with

$$F_{K^*\rho} = A_1^2(0) + 0.019 V^2(0) + 0.019 A_2^2(0) - 0.193 A_1(0) A_2(0) \ . \tag{45}$$

In terms of the $D^0 \to K^{*-} e^+ \nu$ decay, we have

$$\Gamma(D^0 \to K^{*-} \rho^+) = 2.442 c_1^2 |V_{ud}|^2 \frac{F_{K^*\rho}}{F_{K^* e\nu}} \Gamma(D^0 \to K^{*-} e^+ \nu) \ . \tag{46}$$

As mentioned above, the finite-width effects reduce the $D^0 \to K^- \rho^+$ and $D^0 \to K^{*-} \rho^+$ decay rates relative to the zero-width approximation by 14% and 15%, respectively. The situation is different for the decay $D^0 \to K^- A_1^+$, where the large width of the A_1 meson and the small phase space increase the decay rate by 50% (for $\Gamma_A = 0.400$ GeV).

To obtain the numerical values for the above decay rates, we shall use the value of c_1 given by perturbative QCD. For a value of the renormalization scale parameter $\mu = 1.4$ GeV, previous calculations of Buras *et al.* [4] give $c_1 = 1.18$, 1.24, and 1.30 for $\Lambda_{\mathrm{QCD}} = 100$, 200, and 300 MeV, respectively. Thus, different values for Λ_{QCD} within experimental errors only change the computed values for the decay rates by 10–20%, which is smaller than the uncertainties in the $1/N$ approximation. In the following we shall take $c_1 = 1.25$ in our computed decay rates. With $|V_{ud}| = 0.9744$ as given by the Particle Data Group [8]; the recently measured branching ratio for the semileptonic decay $D^0 \to K^{*-} e^+ \nu$ [12],

$$B(D^0 \to K^{*-} e^+ \nu) = (0.46 \pm 0.13) B(D^0 \to K^- e^+ \nu) \ ; \tag{47}$$

and the measured value of $(3.4 \pm 0.4)\%$ for $B(D^0 \to K^- e^+ \nu)$ [8]; we find

$$\begin{aligned} B(D^0 \to K^- \pi^+) &= (6.16 \pm 0.72)\% \ , \\ B(D^0 \to K^0 \rho^+) &= (8.89 \pm 1.04)\% \ , \\ B(D^0 \to K^- A_1^+) &= (1.98 \pm 0.23)\% \ , \end{aligned} \tag{48}$$

where the value for the $D^0 \to K^- A_1^+$ decay rate is obtained with f_A given by the Weinberg sum rule.

Comparing with the measured values [8,16]

$$\begin{aligned} B(D^0 \to K^- \pi^+)_{\mathrm{expt}} &= (3.71 \pm 0.25)\% \ , \\ B(D^0 \to K^- \rho^+)_{\mathrm{expt}} &= (7.8 \pm 1.1)\% \ , \\ B(D^0 \to K^- A_1^+)_{\mathrm{expt}} &= (7.8 \pm 1.5)\% \ , \end{aligned} \tag{49}$$

we see that the $1/N$ expansion reproduces reasonably well the $D^0 \to K^- \rho^+$ decay rate. The prediction for $D^0 \to K^- \pi^+$ is somewhat larger than experiment (by 66%), which is not unexpected since final-state interactions are important as indicated by the fact that the decay amplitudes possess a phase [3]. For the decay $D^0 \to K^- A_1^+$, the disagreement with experiment is much more serious since the $1/N$ expansion gives a decay rate too low by a factor of 4 compared with experiment. Contrary to the previous result by Stech and co-workers as quoted by Coffman *et al.* [16], we find that the finite-width corrections do not increase the decay rate by a large amount. This indicates that the $1/N$ expansion is insufficient to account for the large measured $D^0 \to K^- A_1^+$ decay rate. For the other decays, if we use the form factors V, A measured by the E691 group as given in Eq. (15), from Eqs. (41) and (44),

$$B(D^0 \to K^{*-}\pi^+) = (3.01 \pm 1.18)\% ,$$
$$B(D^0 \to K^{*-}\rho^+) = (5.67 \pm 2.26)\% . \tag{50}$$

With the new E653 data given in Eq. (16), we get

$$B(D^0 \to K^{*-}\pi^+) = (1.98 \pm 0.77)\% ,$$
$$B(D^0 \to K^{*-}\rho^+) = (5.74 \pm 2.29)\% . \tag{51}$$

We remark that the computed decay rates are sensitive to the $A_1(0)A_2(0)$ interference term, which would strongly suppress the $K^{*-}\pi^+$ mode relative to $K^{*-}\rho^+$ if the form factor A_2 is comparable to A_1, as predicted by the quark model [3] and found in the E653 measurements [6]. Further measurements on the semileptonic $D \to K^* e\nu$ could resolve the inconsistency between the E691 and the E653 data on the form factor $A_2(q^2)$ and provide us with an unambiguous prediction for the $D^0 \to K^{*-}\pi^+$ decay rate in the $1/N$ expansion. We note also that the branching ratio we found for the decay $D^0 \to K^{*-}\rho^+$ is smaller than the value of Bauer *et al.* [3] by a factor of 4 due to the small values of $A_1(q^2)$, $A_2(q^2)$, and $V(q^2)$ we took from the E691 data. Our computed value for the E691 data more or less agrees with the Bauer-Stech-Wirbel set 1 (BSW1) as quoted by Coffman *et al.* [16], who give $(6.1 \pm 2.3)\%$ for the $D^0 \to K^{*-}\rho^+$ branching ratio, which is somewhat bigger than our value obtained with finite-width correction for the ρ meson. Comparing with the recently measured decay rates [8,16]

$$B(D^0 \to K^{*-}\pi^+)_{\text{expt}} = (4.6 \pm 0.6)\% ,$$
$$B(D^0 \to K^{*-}\rho^+)_{\text{expt}} = (6.2 \pm 2.3 \pm 2.0)\% , \tag{52}$$

we find that the $1/N$ expansion also reproduces quite well the $K^{*-}\rho^+$ decay modes, within experimental errors. The prediction for the $K^{*-}\pi^+$ mode is, however, smaller than measurement by a factor of 2 if the new E653 data is used. If the form factor $A_2(q^2)$ turns out to be large and comparable to $A_1(q^2)$, as found in the E653 data, then the $1/N$ expansion somewhat underestimates the $D^0 \to K^{*-}\pi^+$ decay. This again indicates that, as shown by Bauer *et al.* [3] with the $D^0 \to K^-\pi^+$ decay, important final-state interactions must be present to account for the large measured decay rates for these two decay modes.

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