Angular distribution and helicity dependence in $B \to K^{(*)} l^+ l^-$

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We calculate the decay rates for $B \to K l^+ l^-$ and $B \to K^* l^+ l^-$ as functions of the invariant mass of the $l^+ l^-$ pair and the center-of-mass angle in the $l^+ l^-$ center-of-mass frame. We obtain the helicity dependence of $B \to K^* l^+ l^-$ and calculate the contributions from the *CP*-odd and *CP*-even channels. In the calculations we use the heavy-quark and factorization approximations.

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I. INTRODUCTION

Decay rate measurements for $B \to K l^+ l^-$ and $B \to K^* l^+ l^-$, in addition to other rare B-meson decays, are expected to provide tests of the standard model. In addition, there is an interest in exploring *CP* violations in B decays. Therefore, it is important to utilize all the avenues of information to explore decay channels of B mesons. Experimentally it is important to know the angular distribution for a decaying particle to determine where detectors should be placed in order to get the maximum efficiency. This information can also be used to recognize the decay and to eliminate some of the background effects. Therefore, in this paper, our purpose is to obtain the angular dependence of the decays $B \to K l^+ l^-$ and $B \to K^* l^+ l^-$ (these decay processes are important because of their sensitivity to the top-quark mass). We do this first by looking at the decays in the $l^+ l^-$ center-of-mass frame, for which the configuration is shown in Fig. 1. Then we calculate the decay rates as functions of the center-of-mass angle $\vartheta_{\text{c.m.}}$ and the invariant mass s of the $l^+ l^-$ pair. Then we obtain the ratio of the $\cos^2\vartheta_{\text{c.m.}}$ term to the flat term as a function of s, which we call $\alpha(s)$. Then we proceed to obtain the ratio of the contribution from zero-helicity states and the ratio of the contribution from the *CP*-odd and *CP*-even states.

In doing the calculations mentioned above, we use the heavy-quark and factorization approximations [1–3]. Our aim is to provide a reference mark for experiments and, as a result, to see how well these approximations work for these decays.

II. KINEMATICS

The three-body differential decay rate for an unstable particle is given by

$$d\Gamma = \frac{1}{2M_B} \prod_{i=1}^{3} \left[\frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i} \right] \times (2\pi)^4 \delta^{(4)} \left[P - \sum_{i=1}^{3} p_i \right] |\mathcal{M}|^2 . \tag{1}$$

Here P is the four-momentum of the decaying particle and p_i are the four-momenta of the final-state particles; we choose p_1 and p_2 to be the four-momenta of l^+ and l^-, and $p_3 \equiv P_X$ to be the four-momentum of the remaining particle. $\mathcal{M}$ is the matrix element for the decay. By introducing the invariant mass $s = (p_1 + p_2)^2$ of the $l^+ l^-$ pair via

$$\begin{aligned} 1 &= \int ds\, \delta(s - p^2) d^4p\, \delta^{(4)}(p - p_1 - p_2) \\ &= \int ds \frac{d^3\mathbf{p}}{2E_\mathrm{p}} \delta^{(4)}(p - p_1 - p_2) , \end{aligned} \tag{2}$$

we obtain

$$\begin{aligned} \Gamma &= \int \frac{1}{2M_B} \frac{ds}{2\pi} \left[\frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^{(4)}(p - p_1 - p_2) \right] \left[\frac{d^3\mathbf{p}}{(2\pi)^3 2E_\mathrm{p}} \frac{d^3\mathbf{P}_X}{(2\pi)^3 2E_3} (2\pi)^4 \delta^{(4)}(P - P_X - p) \right] |\mathcal{M}|^2 \\ &= \int \frac{ds\, d\cos\vartheta}{2^9 \pi^3 M_B} \frac{\sqrt{\Lambda(s, m_1^2, m_2^2)}}{s} \frac{\sqrt{\Lambda(M_B^2, M_X^2, s)}}{M_B^2} |\mathcal{M}|^2 , \end{aligned} \tag{3}$$

or

$$\frac{d\Gamma}{ds\, d\cos\vartheta} = \frac{1}{2^9 \pi^3 s M_B^3} \sqrt{\Lambda(s, m_1^2, m_2^2)} \sqrt{\Lambda(M_B^2, M_X^2, s)} |\mathcal{M}|^2 , \tag{4}$$

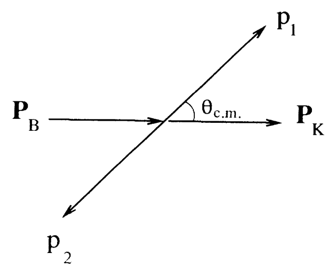

FIG. 1. Momentum configuration for the B-meson decay in the l^+l^- center-of-mass frame.

where

$$\Lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc \ . \qquad (5)$$

In the l^+l^- center-of-mass frame $\mathbf{p}_1=-\mathbf{p}_2, \mathbf{P}_B=\mathbf{P}_X$ (the configuration is as shown in Fig. 1),

$$\begin{aligned} s&=(p_1+p_2)^2=(P-P_X)^2\\ &=(E_1+E_2)^2=(E_B-E_X)^2 \ ,\\ s_{\min}&=(m_1+m_2)^2 \ ,\\ s_{\max}&=(M_B-M_X)^2 \ , \end{aligned} \qquad (6)$$

and

$$\begin{aligned} E_B&=\frac{M_B^2-M_X^2+s}{2\sqrt{s}} \ ,\\ E_X&=\frac{M_B^2-M_X^2-s}{2\sqrt{s}} \ ,\\ \mathbf{P}_B^2&=\mathbf{P}_X^2=\frac{1}{4s}\Lambda(M_B^2,M_X^2,s) \ . \end{aligned} \qquad (7)$$

In the calculations of the following section, we model the decay processes on the b-quark decay, and we consider the diagrams shown in Fig. 2. In the B-meson decay diagrams, we have introduced the factors $\mathcal{F}$ at each vertex. In general, the $\mathcal{F}$'s are functions of the masses and the decay rates of the mesons as well as the invariant mass of the l^+l^- pair. However, since our purpose is to obtain the angular distribution up to an overall factor, we leave them as parts of an overall factor.

III. $B \to K l^+ l^-$

We can write the matrix element for the decay in Fig. 2 in the form

$$\mathcal{M}_{B\to Kl^+l^-}=\mathcal{M}_{\square}+\mathcal{M}^Z_{\triangle 1}+\mathcal{M}^Z_{\triangle 2}+\mathcal{M}^\gamma_{\triangle 1}+\mathcal{M}^\gamma_{\triangle 2} \ , \qquad (8)$$

where, for $i=u,c,t$,

$$\begin{aligned} \mathcal{M}_{\square}&=\left[\frac{g}{2\sqrt{2}}\right]^4 V_{si}^\dagger V_{ib}\frac{\mathcal{F}_B\mathcal{F}_K}{4M_BM_K}\int\frac{d^4k}{(2\pi)^4}\frac{T_{\square}^{\mu\nu}t_{\square\mu\nu}}{(k^2-m_i^2)(k_1^2-m^2)(k_2^2-M_W^2)(k_3^2-M_W^2)} \ ,\\ T_{\square}^{\mu\nu}&\equiv\mathrm{Tr}[\gamma_5(\not{P}_K+M_K)\gamma^\mu(1-\gamma_5)(\not{k}+m_i)\gamma^\nu(1-\gamma_5)(\not{P}_B+M_B)\gamma_5]\\ &=2(M_BP_K^\alpha+M_KP_B^\alpha)k^\beta\mathrm{Tr}[\gamma_\alpha\gamma^\mu\gamma_\beta\gamma^\nu(1-\gamma_5)] \ ,\\ t_{\square}^{\mu\nu}&\equiv\bar{q}_2\gamma^\nu(1-\gamma_5)(\not{k}_1+m)\gamma^\mu(1-\gamma_5)q_1=2\bar{q}_2\gamma^\nu\not{k}\gamma^\mu(1-\gamma_5)q_1 \ , \end{aligned} \qquad (9)$$

$$\begin{aligned} \mathcal{M}_{\triangle 1}&=\left[\frac{g}{2\sqrt{2}}\right]^2\left[\frac{-g^2}{4}\right]V_{si}^\dagger V_{ib}\frac{\mathcal{F}_B\mathcal{F}_K}{4M_BM_K}\int\frac{d^4k}{(2\pi)^4}\frac{[(k_3-k_2)_\lambda q_{\mu\nu}+(k_2-k_1)_\mu g_{\lambda\nu}+(k_1-k_3)_\nu g_{\lambda\mu}]T_{\triangle 1}^{\nu\lambda}t_{\triangle}^{\mu}}{(k^2-m_i^2)(k_1^2-M_W^2)(k_2^2-M_W^2)(s-M_Z^2)} \ ,\\ T_{\triangle 1}^{\mu\nu}&\equiv\mathrm{Tr}[\gamma_5(\not{P}_K+M_K)\gamma^\mu(1-\gamma_5)(\not{k}+m_i)\gamma^\nu(1-\gamma_5)(\not{P}_B+M_B)\gamma_5]\\ &=2(M_BP_K^\alpha+M_KP_B^\alpha)k^\beta\mathrm{Tr}[\gamma_\alpha\gamma^\mu\gamma_\beta\gamma^\nu(1-\gamma_5)] \ ,\\ t_{\triangle}^{\mu}&\equiv\bar{q}_2\gamma^\mu(a+b\gamma_5)q_1 \ , \end{aligned} \qquad (10)$$

$$\begin{aligned} \mathcal{M}_{\triangle 2}&=\left[\frac{g}{2\sqrt{2}}\right]^2\left[\frac{g}{4\cos\theta_W}\right]^2 V_{si}^\dagger V_{ib}\frac{\mathcal{F}_B\mathcal{F}_K}{4M_BM_k}\int\frac{d^4k}{(2\pi)^4}\frac{T_{\triangle 2}^{\mu}t_{\triangle\mu}}{(k^2-M_W^2)(k_1^2-m_i^2)(k_2^2-m_i^2)(s-M_Z^2)} \ ,\\ T_{\triangle 2}^{\mu}&\equiv\mathrm{Tr}[\gamma_5(\not{P}_K+M_K)\gamma^\nu(1-\gamma_5)(\not{k}_2+m_i)\gamma^\mu(c+d\gamma_5)(\not{k}_1+m_i)\gamma_\nu(1-\gamma_5)(\not{P}_B+M_B)\gamma_5] \ . \end{aligned} \qquad (11)$$

Here m_i is the mass of the internal quark and m is the mass of the internal lepton. To obtain the Z contribution, we substitute

$$\begin{aligned} a&=-1+4\sin^2\theta_W, \quad b=1 \ ,\\ c&=1-\tfrac{8}{3}\sin^2\theta_W, \quad d=1 \ ,\\ d-c&=\tfrac{8}{3}\sin^2\theta_W \ , \end{aligned} \qquad (12)$$

and to obtain the γ contribution, we use

$$\begin{aligned} &a=c=1, \quad b=d=0 \ ,\\ &M_Z\to 0, \quad \cos\theta_W\to 1 \ , \end{aligned} \qquad (13)$$

together with

$$\tfrac{1}{4}g^2\to e^2 \qquad (14)$$

in $\mathcal{M}_{\triangle 1}$ and

$$\left[\frac{g}{4\cos\theta_W}\right]^2 \to e^2 Q_i \tag{15}$$

in $\mathcal{M}_{\triangle 2}$, where Q_i is the charge of the internal quark line. The dominant term is the γ contribution since $s_{\max}=(M_B-M_K)^2 \ll M_W^2, M_Z^2$.

There is also a contribution when the internal quark lines form a resonance. The calculations for this case have already been done elsewhere [4–6].

Within these matrix elements, we have momentum integrals of the form

$$\mathcal{I}=\int \frac{d^4k}{(2\pi)^4}\frac{1}{\prod_i (k_i^2-a_i^2)} ,$$
$$\mathcal{I}^\mu=\int \frac{d^4k}{(2\pi)^4}\frac{k_i^\mu}{\prod_i (k_i^2-a_i^2)} , \tag{16}$$
$$\mathcal{I}^{\mu\nu}=\int \frac{d^4k}{(2\pi)^4}\frac{k_i^\mu k_j^\nu}{\prod_i (k_i^2-a_i^2)} .$$

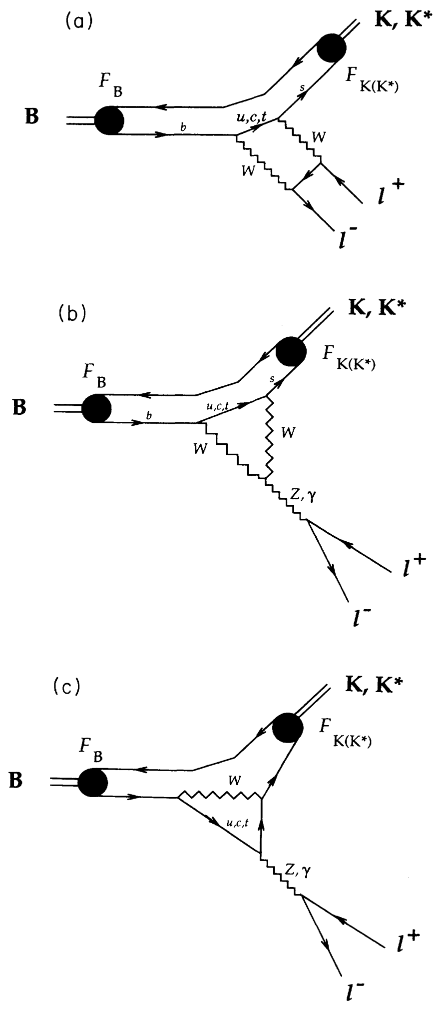

FIG. 2. Decay diagrams for $B\to K^* l^+ l^-$ and $B\to K l^+ l^-$.

In the limit where the external momenta are negligible with respect to the internal momenta, $k_i \to k$ up to an overall sign, and these integrals, up to an overall sign, reduce to

$$\mathcal{I}=\int \frac{d^4k}{(2\pi)^4}\frac{1}{\prod_i (k^2-a_i^2)} ,$$
$$\mathcal{I}^\mu=\int \frac{d^4k}{(2\pi)^4}\frac{k^\mu}{\prod_i (k^2-a_i^2)}=0 , \tag{17}$$
$$\mathcal{I}^{\mu\nu}=\int \frac{d^4k}{(2\pi)^4}\frac{k^\mu k^\nu}{\prod_i (k^2-a_i^2)} = \frac{1}{4}g^{\mu\nu}\int \frac{d^4k}{(2\pi)^4}\frac{k^2}{\prod_i (k^2-a_i^2)} .$$

After some algebra, we obtain the following expressions for the matrix elements:

$$\mathcal{M}_\square=\left[\frac{g}{2\sqrt{2}}\right]^4 V_{si}^\dagger V_{ib}\frac{\mathcal{F}_B\mathcal{F}_K}{4M_BM_K}\frac{8\mathcal{I}_\square}{s-M_Z^2}\bar{q}_2\gamma^\mu(1-\gamma_5)q_1\wp_\mu ,$$
$$\mathcal{M}_{\triangle 1}=\left[\frac{g}{2\sqrt{2}}\right]^2\left[\frac{-g^2}{4}\right]V_{si}^\dagger V_{ib}\frac{\mathcal{F}_B\mathcal{F}_K}{4M_BM_K}\frac{-24\mathcal{I}_{\triangle 1}}{s-M_Z^2}t^\mu\wp_\mu , \tag{18}$$
$$\mathcal{M}_{\triangle 2}=\left[\frac{g}{2\sqrt{2}}\right]^2\left[\frac{g}{4\cos\theta_W}\right]^2 \times V_{si}^\dagger V_{ib}\frac{\mathcal{F}_B\mathcal{F}_K}{4M_BM_K}\frac{8\mathcal{I}_{\triangle 2}}{s-M_Z^2}t^\mu\wp_\mu ,$$
$$\wp^\mu=(M_BP_K+M_KP_b)^\mu .$$

In getting these expressions, we have used

$$[\gamma_\nu\gamma_\beta\gamma_\mu(1-\gamma_5)][\gamma^\mu\gamma^\beta\gamma^\nu(v-a\gamma_5)] =[\gamma_\mu(1-\gamma_5)]\{10[\gamma^\mu(v-a\gamma_5)] -6[\gamma^\mu(a-v\gamma_5)]\} ,$$
$$\gamma_\nu\gamma_\beta\gamma_\mu=g_{\nu\beta}\gamma_\mu+g_{\mu\beta}\gamma_\nu-g_{\nu\mu}\gamma_\beta-i\epsilon_{\nu\beta\mu\alpha}\gamma_5\gamma^\alpha , \tag{19}$$
$$-6i\gamma_5\gamma^\alpha=\epsilon^{\nu\beta\mu\alpha}\gamma_\nu\gamma_\beta\gamma_\mu .$$

We have also defined

$$\mathcal{I}_\square\equiv\int \frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^2-M_W^2)(k^2-m_I^2)(k^2-m^2)} ,$$
$$\mathcal{I}_{\triangle 1}\equiv\int \frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^2-M_W^2)^2(k^2-m_I^2)} , \tag{20}$$
$$\mathcal{I}_{\triangle 2}\equiv\int \frac{d^4k}{(2\pi)^4}\frac{(c-d)k^2-2(c+d)m_I^2}{(k^2-M_W^2)(k^2-m_I^2)^2} .$$

The matrix element for $B\to Kl^+l^-$ can be written as

$$\mathcal{M}=[\mathcal{V}(\bar{q}_2\gamma^\mu q_1)+\mathcal{A}(\bar{q}_2\gamma^\mu\gamma_5 q_1)]\wp_\mu . \tag{21}$$

Consequently,

$$\begin{aligned}|\mathcal{M}|^2=&\{|\mathcal{V}|^2\mathrm{Tr}[(\not p_1-m)\gamma^\mu(\not p_2+m)\gamma^{\mu'}]+|\mathcal{A}|^2\mathrm{Tr}[(\not p_1-m)\gamma^\mu\gamma_5(\not p_2+m)\gamma^{\mu'}\gamma_5]\}\wp_\mu\wp_{\mu'}\\&+\{\mathcal{V}^\dagger\mathcal{A}\,\mathrm{Tr}[(\not p_1-m)\gamma^\mu(\not p_2+m)\gamma^{\mu'}\gamma_5]+\mathcal{A}^\dagger\mathcal{V}\,\mathrm{Tr}[(\not p_1-m)\gamma^\mu\gamma_5(\not p_2+m)\gamma^{\mu'}]\}\wp_\mu\wp_{\mu'}\\&=4\{(|\mathcal{V}|^2+|\mathcal{A}|^2)[2(p_1\cdot\wp)(p_2\cdot\wp)-(p_1\cdot p_2)(\wp\cdot\wp)]+m^2(|\mathcal{V}|^2-|\mathcal{A}|^2)(\wp\cdot\wp)\}\ . \qquad (22)\end{aligned}$$

In the l^+l^- center-of-mass frame, we have

$$\begin{aligned}&P_B=(E_B,0,0,P_K),\quad P_K=(E_K,0,0,P_K)\ ,\\&p_1=(E,\mathbf{p}),\quad p_2=(E,-\mathbf{p})\ ,\\&\mathbf{P}_B\cdot\mathbf{p}=P_Kp\cos\vartheta_{\mathrm{c.m.}}\ . \qquad (23)\end{aligned}$$

As a result

$$\begin{aligned}&p_1\cdot\wp=M_K(E_BE-P_Kp\cos\vartheta_{\mathrm{c.m.}})+M_B(E_KE-P_Kp\cos\vartheta_{\mathrm{c.m.}})\ ,\\&p_2\cdot\wp=M_K(E_BE+P_Kp\cos\vartheta_{\mathrm{c.m.}})+M_B(E_KE+P_Kp\cos\vartheta_{\mathrm{c.m.}})\ ,\\&(p_1\cdot\wp)(p_2\cdot\wp)=E^2(M_BE_K+M_KE_B)^2-(M_B+M_K)^2P_K^2p^2\cos^2\vartheta_{\mathrm{c.m.}}\ ,\\&(\wp\cdot\wp)=(M_BE_K+M_KE_B)^2-(M_B+M_K)^2P_K^2\ ,\\&(p_1\cdot p_2)=\tfrac{1}{2}s-m^2\ ,\\&p^2=\tfrac{1}{4}s-m^2\ ,\\&E^2=\tfrac{1}{4}s\ . \qquad (24)\end{aligned}$$

Substituting these equations in (22) gives

$$\begin{aligned}|\mathcal{M}|^2=4&\left[(|\mathcal{V}|^2+|\mathcal{A}|^2)(M_B+M_K)^2P_K^2\left[\frac{s}{2}-m^2\right]\right]\left[1-\left[\frac{s-4m^2}{s-2m^2}\right]\cos^2\vartheta_{\mathrm{c.m.}}\right]\\&+4m^2\{|\mathcal{A}|^2[2(M_BE_K+M_KE_B)^2-(M_B+M_K)^2P_K^2]+|\mathcal{V}|^2(M_B+M_K)^2P_K^2\}\ . \qquad (25)\end{aligned}$$

For $4m^2\le s\le(M_B-M_K)^2$, $|\mathcal{V}|^2\gg|\mathcal{A}|^2$; thus,

$$\begin{aligned}&|\mathcal{M}|^2\simeq2|\mathcal{V}|^2[(M_B+M_K)^2P_K^2s][1+\alpha(s)\cos^2\vartheta_{\mathrm{c.m.}}]\ ,\\&\alpha(s)\equiv-\left[1-\frac{4m^2}{s}\right]\ , \qquad (26)\end{aligned}$$

and

$$\begin{aligned}\frac{d\Gamma}{ds\,d\cos\vartheta}=&\frac{1}{2^9\pi^3M_B^3}(2\sqrt{s}P_K)^3\frac{1}{2}|\mathcal{V}|^2(M_B+M_K)^2\\&\times[1+\alpha(s)\cos^2\vartheta_{\mathrm{c.m.}}]\ . \qquad (27)\end{aligned}$$

In the limit $m\to0$, the expression reduces to

$$\frac{d\Gamma}{ds}=\frac{1}{2^9\pi^3M_B^3}[\Lambda(M_B^2,M_K^2,s)]^{3/2}\frac{1}{3}|\mathcal{V}|^2(M_B+M_K)^2\ . \qquad (28)$$

At this point, we use the calculations of Ref. [7] to obtain the expressions for $\mathcal{F}$'s. Hence, our final expression is given by

$$\begin{aligned}\frac{d\Gamma}{ds\,d\cos\vartheta}=&\frac{\mathcal{G}(s)}{2^9\pi^3M_B^3}[\Lambda(M_B^2,M_K^2,s)]^{3/2}(M_B+M_K)^2\\&\times[1+\alpha(s)\cos^2\vartheta_{\mathrm{c.m.}}]\ , \qquad (29)\end{aligned}$$

where

$$\begin{aligned}\mathcal{G}(s)=&\frac{G_F^2\alpha^2|s_3+s_2e^{i\delta}|^2}{2\pi^2(M_B+M_K)^2}\\&\times[|2m_bc_7(m_b)h(s)+c_8(m_b)f_+(s)|^2\\&+|c_9(m_b)f_+(s)|^2] \qquad (30)\end{aligned}$$

and c_7, c_8, c_9, f_+, and h are as defined in Ref. [7]. We see that as $m\to0$, $d\Gamma/ds\,d\cos\vartheta\propto1+\alpha\cos^2\vartheta_{\mathrm{c.m.}}$ with $\alpha=-1$. But as $s\to4m^2$, $\alpha(s)\to0$. This is an important conclusion. For example, at the Collider Detector at Fermilab (CDF), the efficiency for $\alpha=+1$ is about 75% of the efficiency for $\alpha=-1$.[1]

IV. $B\to K^*l^+l^-$

In a similar manner, we can calculate the differential partial decay rate for $B\to K^*l^+l^-$. In this case we need to make the substitutions

$$\begin{aligned}&M_K\to M_{K^*}\ ,\\&P_K\to P_{K^*}=\left[\frac{1}{4s}\Lambda(M_B^2,M_{K^*}^2,s)\right]^{1/2}\ ,\\&\mathcal{F}_K\to\mathcal{F}_{K^*}\ ,\\&\gamma_5(\not P_K+M_K)\to\not\epsilon_{K^*}(\not P_{K^*}+M_{K^*})\ . \qquad (31)\end{aligned}$$

[1] J. Mueller (private communication).

These substitutions give

$$\begin{aligned} T^{\mu\nu}_{*\Box} &\equiv \mathrm{Tr}\not{\epsilon}_{K^*}(\not{P}_{K^*}+M_{K^*})\gamma^\mu(1-\gamma_5)(\not{k}+m_I) \\ &\quad\times\gamma^\nu(1-\gamma_5)(\not{P}_B+M_B)\gamma_5 , \\ T^{\mu\nu}_{*\triangle 1} &\equiv \mathrm{Tr}\not{\epsilon}_{K^*}(\not{P}_{K^*}+M_{K^*})\gamma^\mu(1-\gamma_5)(\not{k}+m_I) \\ &\quad\times\gamma^\nu(1-\gamma_5)(\not{P}_B+M_B)\gamma_5 , \\ T^{\mu}_{*\triangle 2} &\equiv \mathrm{Tr}\not{\epsilon}_{K^*}(\not{P}_{K^*}+M_{K^*}) \\ &\quad\times\gamma^\nu(1-\gamma_5)(\not{k}_2+m_I)\gamma^\mu(c+d\gamma_5) \\ &\quad\times(\not{k}_1+m_I)\gamma_\nu(1-\gamma_5)(\not{P}_B+M_B)\gamma_5 . \end{aligned} \tag{32}$$

After some algebra, we obtain

$$\begin{aligned} T^{\mu\nu}_{*\Box}t_{\Box\mu\nu} &= \frac{k^2}{2}[\bar{q}_2\gamma_\mu(1-\gamma_5)q_1] \\ &\quad\times 32[(\varepsilon_{K^*}\cdot P_B)P^\mu_{K^*}-(P_B\cdot P_{K^*}+M_BM_{K^*})\epsilon^\mu_{K^*} \\ &\quad\quad+i\epsilon^{\alpha\beta\mu\nu}\epsilon_{K^*\alpha}P_{K^*\beta}P_{B\nu}] , \\ T^{\mu\nu}_{*\triangle 1}&[(k_3-k_2)_\lambda g_{\mu\nu}+(k_2-k_1)_\mu g_{\lambda\nu}+(k_1-k_3)_\nu g_{\lambda\mu}] \\ &= -24\frac{k^2}{2}[(\varepsilon_{K^*}\cdot P_B)P^\mu_{K^*}-(P_B\cdot P_{K^*}+M_BM_{K^*})\varepsilon^\mu_{K^*} \\ &\quad\quad+i\epsilon^{\alpha\beta\mu\nu}\varepsilon_{K^*\alpha}P_{K^*\beta}P_{B\nu}] , \\ T^{\mu}_{*\triangle 2} &= 2[(c-d)k^2-2(c+d)m_I^2] \\ &\quad\times[(\varepsilon_{K^*}\cdot P_B)P^\mu_{K^*}-(P_B\cdot P_{K^*}+M_BM_{K^*})\varepsilon^\mu_{K^*} \\ &\quad\quad+i\epsilon^{\alpha\beta\mu\nu}\varepsilon_{K^*\alpha}P_{K^*\beta}P_{B\nu}] . \end{aligned} \tag{33}$$

Now we write the matrix element in the form

$$\mathcal{M}=(\mathcal{V}[\bar{q}_2\gamma_\mu q_1]+\mathcal{A}[\bar{q}_2\gamma_\mu\gamma_5 q_1])(v^\mu+a^\mu) , \tag{34}$$

where we have defined

$$\begin{aligned} a^\mu &\equiv i\zeta\epsilon^{\alpha\beta\mu\nu}\varepsilon_{K^*\alpha}P_{K^*\beta}P_{B\nu} , \\ v^\mu &\equiv [\xi(\varepsilon_{K^*}\cdot P_B)P^\mu_{K^*}-(P_B\cdot P_{K^*}+M_BM_{K^*})\varepsilon^\mu_{K^*}] , \end{aligned} \tag{35}$$

and introduced[2] ζ and ξ. In the l^+l^- frame, the states corresponding to helicity λ of K^* are given by

$$\begin{aligned} \varepsilon^{\lambda=0}_{K^*} &= \frac{1}{M_{K^*}}(P_{K^*},0,0,E_{K^*}) , \\ \varepsilon^{\lambda=\pm}_{K^*} &= \frac{1}{\sqrt{2}}(0,\mp 1,-i,0) . \end{aligned} \tag{36}$$

When we perform the calculations we obtain

$$\begin{aligned} |\mathcal{M}|^2 &= 4|\mathcal{V}|^2\{[2(p_1\cdot v)(p_2\cdot v)-(p_1\cdot p_2+m^2)(v\cdot v)]+[2(p_1\cdot a)(p_2\cdot a)-(p_1\cdot p_2+m^2)(a\cdot a)]\} \\ &\quad+4|\mathcal{A}|^2\{[2(p_1\cdot v)(p_2\cdot v)-(p_1\cdot p_2-m^2)(v\cdot v)]+[2(p_1\cdot a)(p_2\cdot a)-(p_1\cdot p_2-m^2)(a\cdot a)]\} , \end{aligned} \tag{37}$$

where

$$\begin{aligned} v\cdot v &= \{\xi^2sP^2_{K^*}-\tfrac{1}{4}[(M_B+M_{K^*})^2-s]^2\}\delta_{\lambda 0}-\tfrac{1}{4}[(M_B+M_{K^*})^2-s]^2\delta_{\lambda\pm}, \quad a\cdot a=-\zeta^2sP^2_{K^*}\delta_{\lambda\pm} , \\ (p_1\cdot a)(p_2\cdot a) &= -\zeta^2sP^2_{K^*}p^2\sin^2\vartheta\delta_{\lambda\pm}=-\zeta^2sP^2_{K^*}p^2(1-\cos^2\vartheta)\delta_{\lambda\pm} ; \\ (p_1\cdot v)(p_2\cdot v) &= \left[\xi E_{k^*}\sqrt{s}-\frac{1}{2}[(M_B+M_{K^*})^2-s]\right]^2\frac{P^2_{K^*}E^2}{M^2_{K^*}}\delta_{\lambda 0} \\ &\quad-\left[\xi\sqrt{s}P^2_{K^*}-\frac{1}{2}[(M_B+M_{K^*})^2-s]E_{K^*}\right]^2\frac{p^2\cos^2\vartheta}{M^2_{K^*}}\delta_{\lambda 0}-\frac{1}{8}[(M_B+M_{K^*})^2-s]^2p^2\sin^2\vartheta\delta_{\lambda\pm} . \end{aligned} \tag{38}$$

As a result, with the definitions

$$\begin{aligned} \sum_{\lambda=0,\pm}\mathcal{Q}^\lambda_{v1} &= \mathcal{Q}_{v1}\equiv[2(p_1\cdot v)(p_2\cdot v)-(p_1\cdot p_2+m^2)(v\cdot v)] , \\ \sum_{\lambda=0,\pm}\mathcal{Q}^\lambda_{v2} &= \mathcal{Q}_{v2}\equiv[2(p_1\cdot v)(p_2\cdot v)-(p_1\cdot p_2-m^2)(v\cdot v)] , \\ \sum_{\lambda=0,\pm}\mathcal{Q}^\lambda_{a1} &= \mathcal{Q}_{a1}\equiv[2(p_1\cdot a)(p_2\cdot a)-(p_1\cdot p_2+m^2)(a\cdot a)] , \\ \sum_{\lambda=0,\pm}\mathcal{Q}^\lambda_{a2} &= \mathcal{Q}_{a2}\equiv[2(p_i\cdot a)(p_2\cdot a)-(p_1\cdot p_2-m^2)(a\cdot a)] , \end{aligned} \tag{39}$$

[2]Here we actually have

$$a^\mu=i\epsilon^{\alpha\beta\mu\nu}\varepsilon_{K^*\alpha}P_{K^*\beta}P_{B\nu} ,$$

$$v^\mu=[(\varepsilon_{K^*}\cdot P_B)P^\mu_{K^*}-(P_B\cdot P_{K^*}+M_BM_{K^*})\varepsilon^\mu_{K^*}] ,$$

but for the sake of generality, we keep ζ and ξ in the following calculations.

we obtain

$$\mathcal{Q}_{v1}^{\lambda=0}=\frac{s}{2}\left[\left[\xi E_{K^*}\sqrt{s}-\frac{1}{2}[(M_B+M_{K^*})^2-s]\right]^2\frac{P_{K^*}^2}{M_{K^*}^2}-\left[\xi^2 sP_{K^*}^2-\frac{1}{4}[(M_B+M_{K^*})^2-s]^2\right]\right]$$
$$-\frac{s}{2}\left[1-\frac{4m^2}{s}\right]\left[\xi\sqrt{s}P_{K^*}^2-\frac{1}{2}[(M_B+M_{K^*})^2-s]E_{K^*}\right]^2\frac{1}{M_{K^*}^2}\cos^2\vartheta\ ,$$

$$\mathcal{Q}_{v1}^{\lambda=\pm}=\frac{s}{16}\left[1+\frac{4m^2}{s}\right][(M_B+M_{K^*})^2-s]^2\left[1+\left[\frac{s-4m^2}{s+4m^2}\right]\cos^2\vartheta\right]\ ,$$

$$\mathcal{Q}_{v2}^{\lambda=0}=\frac{s}{2}\left[\xi E_{K^*}\sqrt{s}-\frac{1}{2}[(M_B+M_{K^*})^2-s]\right]^2\frac{P_{K^*}^2}{M_{K^*}^2}-\frac{s}{2}\left[1-\frac{4m^2}{s}\right]\left[\xi^2 sP_{K^*}^2-\frac{1}{4}[(M_B+M_{K^*})^2-s]^2\right]$$
$$-\frac{s}{2}\left[1-\frac{4m^2}{s}\right]\left[\xi\sqrt{s}P_{K^*}^2-\frac{1}{2}[(M_B+M_{K^*})^2-s]E_{K^*}\right]^2\frac{1}{M_{K^*}^2}\cos^2\vartheta\ , \tag{40}$$

$$\mathcal{Q}_{v2}^{\lambda=\pm}=\frac{s}{16}\left[1-\frac{4m^2}{s}\right][(M_B+M_{K^*})^2-s]^2(1+\cos^2\vartheta)\ ,\quad \mathcal{Q}_{a1}^{\lambda=0}=\mathcal{Q}_{a2}^{\lambda=0}=0\ ,$$

$$\mathcal{Q}_{a1}^{\lambda=\pm}=\xi^2\frac{s^2}{16}P_{K^*}^2\left[1-\left[\frac{4m^2}{s}\right]^2\right]\left[1+\left[\frac{s-4m^2}{s+4m^2}\right]\cos^2\vartheta\right]\ ,\quad \mathcal{Q}_{a2}^{\lambda=\pm}=\xi^2\frac{s^2}{16}P_{K^*}^2\left[1-\frac{4m^2}{s}\right]^2(1+\cos^2\vartheta)\ .$$

Thus, the differential decay rate for the helicity λ of K^* is given by

$$\left[\frac{d\Gamma}{ds\,d\cos\vartheta}\right]_\lambda=\frac{[\Lambda(M_B^2,M_{K^*}^2,s)]^{1/2}}{2^9\pi^3M_B^3}4|\mathcal{V}|^2[(\mathcal{Q}_{v1}^\lambda+\mathcal{Q}_{a1}^\lambda)+\eta^2(\mathcal{Q}_{a2}^\lambda+\mathcal{Q}_{v2}^\lambda)]$$
$$=\frac{\mathcal{G}_*(s)}{2^9\pi^3M_B^3}8[\Lambda(M_B^2,M_{K^*}^2,s)]^{1/2}[(\mathcal{Q}_{v1}^\lambda+\mathcal{Q}_{a1}^\lambda)+\eta^2(\mathcal{Q}_{a2}^\lambda+\mathcal{Q}_{v2}^\lambda)]\ , \tag{41}$$

where

$$\eta^2\equiv\frac{|\mathcal{A}|^2}{|\mathcal{V}|^2}\ . \tag{42}$$

Equivalently, after summing over the helicities,

$$\frac{d\Gamma}{ds\,d\cos\vartheta}=\left[\frac{d\Gamma}{ds\,d\cos\vartheta}\right]_0[1+\alpha_{K^*}(s)\cos^2\vartheta]\ , \tag{43}$$

where

$$\left[\frac{d\Gamma}{ds\,d\cos\vartheta}\right]_0\equiv\left[\frac{d\Gamma}{ds\,\cos\vartheta}\right]_{\cos^2\vartheta=0} \tag{44}$$

and

$$\alpha_{K^*}=\frac{(1+\eta^2)(s-4m^2)}{2[(\mathcal{Q}_{v1}+\mathcal{Q}_{a1})+\eta^2(\mathcal{Q}_{a2}+\mathcal{Q}_{v2})]_{\cos^2\vartheta=0}}$$
$$\times\left[\frac{[(M_B+M_{K^*})^2-s]^2}{8}+\xi^2P_{K^*}^2\left[\frac{s-4m^2}{8}\right]\right.$$
$$\left.-\left[\xi\frac{\sqrt{s}P_{K^*}^2}{2M_{K^*}}-\frac{[(M_B+M_{K^*})^2-s]E_{K^*}}{4M_{K^*}}\right]^2\right]\ . \tag{45}$$

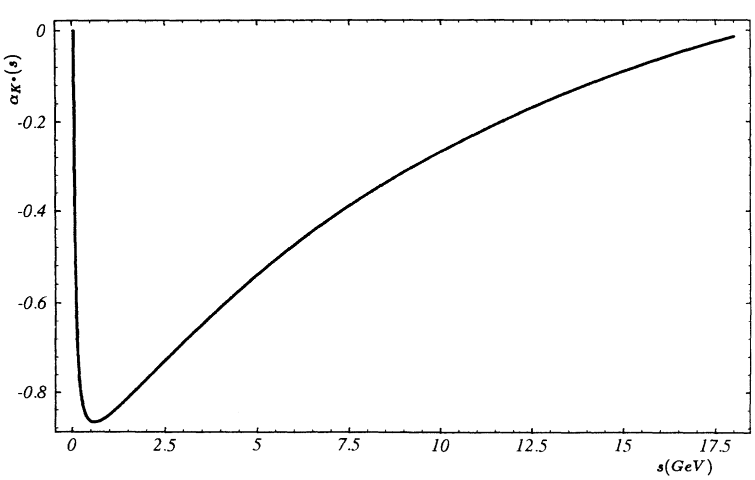

FIG. 3. Plot of $\alpha_{K^*}(s)$ as a function of s.

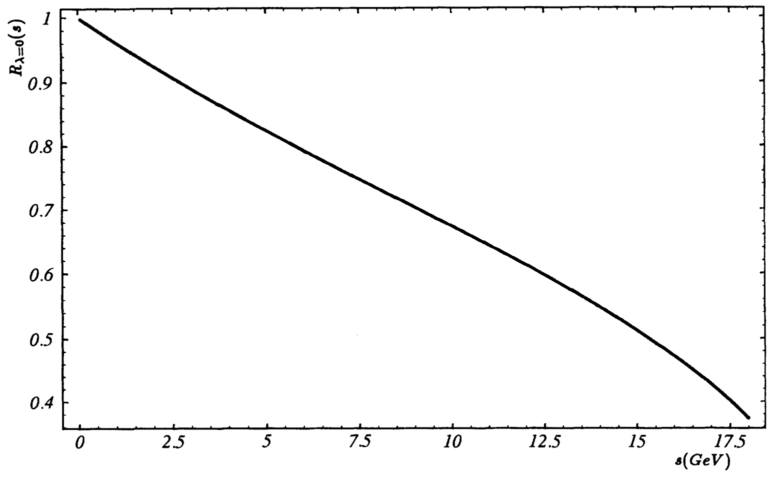

FIG. 4. Plot of $R_{\lambda=0}(s)$ as a function of s.

We plot $\alpha_{K^*}(s)$ in Fig. 3 for $\zeta^2=\xi^2=1$ and $\eta=0$ [since for $4m^2 \le s \le (M_B-M_K)^2, \eta^2 \ll 1$]. In Fig. 4, we plot the ratio of the contribution from the zero-helicity state:

$$R_{\lambda=0} \equiv \frac{(d\Gamma/ds)_{\lambda=0}}{(d\Gamma/ds)_{\lambda=0}+(d\Gamma/ds)_{\lambda=+}+(d\Gamma/ds)_{\lambda=-}} \quad (46)$$

As can be seen both in Figs. 3 and 4, the dominant contribution comes from the $\lambda=0$ state. It should be kept in mind that $R_{\lambda=0}$ is a helicity rate; therefore, it would be affected by the QCD corrections since only the total angular momentum would be conserved in general. Similarly, we can extract the contributions of CP-odd and CP-even states. We define the ratio of the contribution from the odd CP states to the overall contribution by

$$R_{\text{odd}}(s) \equiv \frac{(d\Gamma/ds)_{CP=\text{odd}}}{(d\Gamma/ds)_{CP=\text{odd}}+(d\Gamma/ds)_{CP=\text{even}}} \quad (47)$$

Then we obtain

$$R_{\text{odd}}(s) = \frac{\frac{1}{8}\zeta^2 s^2 P_{K^*}^2 + \frac{1}{16}[(M_B+M_{K^*})^2-s]^2[s+\frac{4}{3}(s-4m^2)]}{\int d\cos\vartheta[\mathcal{Q}_{v1}(s,\cos\vartheta)+\mathcal{Q}_{a1}(s,\cos\vartheta)]} \quad (48)$$

We plot this ratio in Fig. 5. From Fig. 5 we conclude that CP-odd states contribute less than 1.2% of the total differential decay rate for a given s. Thus the CP-odd contribution is considerably less than the CP-even contribution. Since QCD conserves CP, R_{odd} would not be affected by the QCD corrections.

For completeness, we quote our results for $B \to K^{(*)}J/\psi, J/\psi \to l^+l^-$. (The detailed calculations for this case are done in Refs. [4–6], including QCD corrections.) This process takes place via the decay $b \to c\bar{c}q$. The decay rate for $B \to KJ/\psi$ is given by

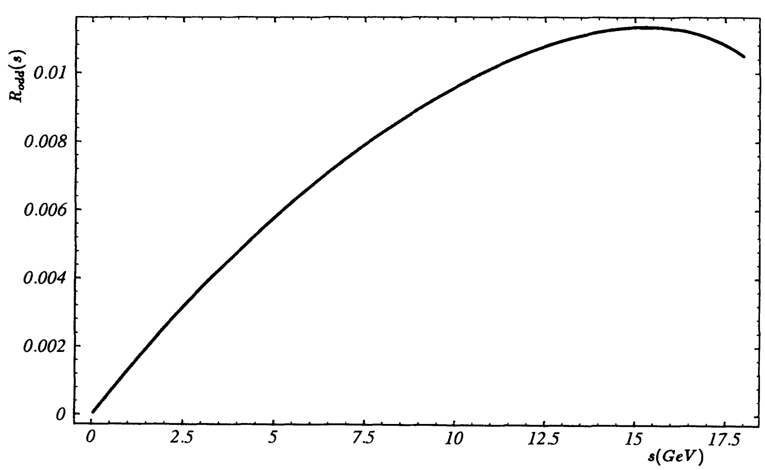

FIG. 5. Plot of $R_{\text{odd}}(s)$ as a function of s.

$$\Gamma^{\lambda_\psi}_{B\to J/\psi K}=\frac{P^3}{8\pi}\left|\frac{16}{3}\left[\frac{ig}{2\sqrt{2}}\right]^2\frac{V^\dagger_{qc}V_{cb}}{K^2-M_W^2}\frac{\mathcal{F}_B\mathcal{F}_K\mathcal{F}_{J/\psi}}{8M_BM_\psi M_K}\right|^2[(M_B+M_K)^2]\delta_{\lambda_\psi,0}\ . \tag{49}$$

The requirement that $\lambda_\psi=0$ follows from the conservation of angular momentum. The B meson, as well as the K meson, is a spin-0 particle; therefore, after the decay the orbital angular momentum of the K-J/ψ system would be perpendicular to the decay plane, which would require J/ψ to have its spin perpendicular to its direction of motion, i.e., $\lambda_\psi=0$. The ratios of the contributions from the different helicity states of K^* for $B\to J/\psi K^*$ is given by

$$\frac{\Gamma^{[0,0]}}{\Gamma^{[\pm,\pm]}}\simeq\frac{[(E_\psi E_{K^*}+P^2_{K^*})(E_{K^*}+M_{K^*})-M_BP^2_{K^*}]^2}{M^2_{K^*}M^2_\psi(E_{K^*}+M_{K^*}\pm\rho^{-1}P_{K^*})^2}\simeq\frac{1}{\frac{1}{2}(1\pm0.55\rho^{-1})^2}\ , \tag{50}$$

where ρ measures the relative strength of the vector and the axial-vector contributions. Similarly, for the ratio of the CP-odd to CP-all contributions, we have

$$R_{\text{odd}}\simeq\frac{M^2_{K^*}M^2_\psi\rho^{-2}P^2_{K^*}}{[(E_\psi E_{K^*}+P^2_{K^*})(E_{K^*}+M_{K^*})-M_BP^2_{K^*}]^2+M^2_{K^*}M^2_\psi[\rho^{-2}P^2_{K^*}+(E^2_{K^*}+M^2_{K^*})]}\simeq\frac{1}{2+13\rho^2}\ . \tag{51}$$

And the total decay rate is given by

$$\Gamma_{B\to K^*J/\psi}=\frac{P_{K^*}}{8\pi M^2_{K^*}}\left[\frac{16}{2}\left[\frac{ig}{2\sqrt{2}}\right]^2\frac{V^\dagger_{qc}V_{cb}}{k^2-M_W^2}\frac{\mathcal{F}_B\mathcal{F}_\psi\mathcal{F}_{K^*}}{8M_BM_\psi M_{K^*}}\right]^2$$
$$\times\{[(E_\psi E_{K^*}+P^2_{K^*})(E_{K^*}+M_{K^*})-M_BP^2_{K^*}]^2+M^2_{K^*}M^2_\psi[\rho^{-2}P^2_{K^*}+(E^2_{K^*}+M^2_{K^*})]\}\ . \tag{52}$$

According to these results, the $\lambda_{K^*}=0$ contribution would account for at most 80% of the decay rate, which corresponds to $\rho\gtrsim 10$, and the CP-odd channel would account for at most 50% of the decay rate, which corresponds to $\rho=0$. The preliminary results from the ARGUS Collaboration support the dominance of the $\lambda_{K^*}=0$ channel while the CLEO results do not [5,6].

V. CONCLUSION

In this paper, we have calculated the decay rates for $B\to Kl^+l^-$ and $B\to K^*l^+l^-$, modeling these processes by the quark decay $b\to sl^+l^-$ in the l^+l^- center-of-mass frame as functions of the center-of-mass angle $\vartheta_{\text{c.m.}}$ and the invariant mass s of the l^+l^- pair. Then we obtained the ratio of the $\cos^2\vartheta_{\text{c.m.}}$ term to the flat term as a function of s, which we call $\alpha(s)$ for K and $\alpha_*(s)$ for K^*. We have concluded that both $\alpha(s)$ and $\alpha_*(s)$ are negative in the allowed region of s, which implies that the contribution from the zero-helicity states dominate these decay channels. It is important to have explicit expressions for $\alpha(s)$ and $\alpha_*(s)$ since they help in identifying the decay channels as well as in getting the best efficiency (as mentioned in the text, in the case of the CDF the efficiency for $\alpha=+1$ is about 75% of the efficiency for $\alpha=-1$). We found that the weight factor $\alpha_{K^*}(s)$ of the $\cos^2\vartheta$ term takes its largest values for small s values. We have also calculated the contributions from the different helicity states of K^* and observed that the $\lambda_{K^*}=0$ channel dominates the decay. Our calculations have also shown that CP-odd part of the decay accounts for only a very small part of the decay rate ($\leq 1.2\%$). We do not expect the QCD corrections to effect R_{odd} since QCD conserves CP, but we do expect them to effect $R_{\lambda=0}$ since only the total angular momentum is conserved in QCD in general.

In our calculations above, we used the heavy-quark and factorization approximations. The results are independent of the model used for form factors. These results should serve as a reference mark for experiments.

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