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Dynamical symmetry breaking and phase transitions in a three-dimensional Gross-Neveu model in a strong magnetic field

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The three-dimensional Gross-Neveu model in a strong magnetic field and at finite density is considered. It is shown that the phase structure of the model depends crucially on the filling of the threshold levels. In particular, with "asymmetric filling" of the threshold levels, a magnetic field induces a transition to a phase with broken chiral symmetry.

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I. INTRODUCTION

Four-fermion models in (2+1)-dimensional space-time have been a subject of considerable interest for both quantum field theory and its solid-state applications. First and foremost, four-fermion couplings have been known to induce spontaneous chiral-symmetry breaking [1] and hence could provide realistic fermion masses in QCD (see, e.g., [2]) and, perhaps, electroweak theory [3]. Another relevant fact is that given solid-state models (for example, the $S=\frac{1}{2}$ quantum antiferromagnet Heisenberg model) in the continuum limit reduce to four-fermion theories [4].

In this paper, we study the influence of a background magnetic field and finite density on dynamical symmetry breaking in a very popular four-fermion theory, namely, the Gross-Neveu model [5]. We show that the phase structure of the model is intimately related to its chiral properties.

To avoid possible confusion, we remind the reader that the use of the generally accepted 2×2 Dirac matrices in this model (which are in fact the Pauli matrices) leaves no room for a chiral symmetry, because there is no other Hermitian 2×2 matrix that anticommutes with all the γ's. Therefore, to obtain chiral symmetry, one has to consider 4×4 Dirac matrices [6]. We will study both versions referring to them as "minimal" and "chiral," respectively.

The very possibility of undergoing a transition from a massive phase to a massless one is determined by whether or not the model is chiral. (Chiral symmetry, if any, is obviously broken by the presence of a magnetic field.) However, the finite density of the fermions changes the situation drastically. The key point here is whether or not threshold levels [7] in a fermionic energy spectrum exist and are occupied.

II. VACUUM OF THE GROSS-NEVEU MODEL IN A STRONG MAGNETIC FIELD

We start with a brief recapitulation of the most important features of the Gross-Neveu model.

(i) It describes the dynamics of N fermion species Ψ_j with a $\frac{1}{2}g^2(\bar{\Psi}\Psi)^2$ self-interaction. Introducing an auxiliary field $\lambda=g\bar{\Psi}\Psi$ makes the Lagrangian quadratic in Ψ:

$$\mathcal{L}=\sum_{j=1}^{N}\bar{\Psi}_j(i\gamma^\mu\partial_\mu-g\lambda)\Psi_j+\frac{\lambda^2}{2}\ . \tag{1}$$

(ii) The model is usually treated by employing a $1/N$ expansion. The renormalizability is proven in [8].

(iii) For a sufficiently strong coupling g the field λ acquires a nonzero vacuum expectation value and a dynamical generation of a fermionic mass $m=g\langle\lambda\rangle$ occurs [5,8].

(iv) Since m should be found at the stationary point of the respective effective potential V_{eff}, m depends on a variety of external parameters (e.g., finite temperature [9] and the Aharonov-Bohm background field [10]). The aim of this investigation is to examine how an external magnetic field affects the dynamically generated mass.

The fermionic spectrum in a uniform magnetic field H is known exactly (from this point on, we assume $eH>0$):

$$\varepsilon_{n,\sigma}^2=eH(2n+1+\sigma)+m^2\ , \tag{2}$$

where $\sigma=\pm1$ and $n=0,1,2,\ldots$. The effective potential can easily be evaluated by summing $\varepsilon_{n,\sigma}$ over the Dirac sea. However, there is one subtlety for the minimal model [7]: the value of σ is fully determined by the sign of energy. One should require $\sigma=-1$ for $\varepsilon>0$ and $\sigma=1$ for $\varepsilon<0$. The computational details are given in [11]. The renormalization procedure is discussed at length in [8,10]. The resultant expression is

$$N^{-1}V_{\rm eff}^{(\rm min)}=-\frac{m^2m_0}{4\pi}-\frac{(eH)^{3/2}}{\pi\sqrt{2}}\zeta\left[-\frac{1}{2};\frac{m^2}{2eH}+1\right], \tag{3}$$

where $\zeta(s,v)$ is the generalized Riemann function [12].

In the chiral version, both values of spin σ are allowed for all energies. Consequently, we obtain

$$N^{-1}V_{\rm eff}^{(\rm ch)}=2N^{-1}V_{\rm eff}^{(\rm min)}-\frac{eH}{2\pi}m\ . \tag{4}$$

The last term in Eq. (4) is nothing but the famous threshold level energy, $\varepsilon_{0,-}=-m$, times its degeneracy $eH/2\pi$ per unit area. This term plays a crucial role in whatever happens at large H. Since the mass m depends upon H, via the equilibrium condition $\partial V_{\rm eff}/\partial m=0$, it is energetically favorable for m to increase at large H.

If the Dirac sea does not contain the $\varepsilon_{0,-}$ level, then by analogy with the theory of superconductivity [13] (which can be considered as a nonrelativistic example of dynamical symmetry breaking), one is led to the conclusion that there exists a critical value H_c such that $m(H_c)=0$. This is exactly what occurs in the minimal model. The related formulas follow.

The equilibrium equation is

$$m_0+\left[\frac{eH}{2}\right]^{1/2}\zeta\left[\frac{1}{2};\frac{m^2}{2eH}+1\right]=0\ . \tag{5}$$

This implicitly defines the function $m(H)$ with $m(0)=m_0$ (plotted in Fig. 1). For weak fields $eH\ll m_0^2$

$$m\approx m_0\left[1-\frac{1}{2}\frac{eH}{m_0^2}\right]. \tag{6}$$

The critical field is

$$eH_e=\frac{m_0^2}{\zeta^2(1/2)}\approx 0.92m_0^2\ . \tag{7}$$

Here, $\zeta(s)$ is the Riemann zeta function [12]. Recall that the phase transition is, in a sense, just a formality since the massless theory has no more continuous symmetry than the massive one. For the chiral model, a threshold mode exists [see the last term in Eq. (4)]:

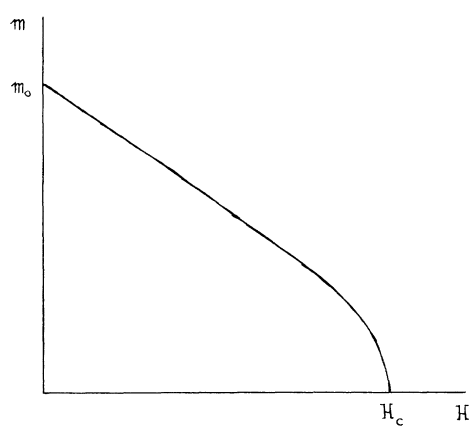

FIG. 1. The dynamical fermion mass m versus the magnetic field in the minimal model; $eH_c\simeq 0.92m_0^2$.

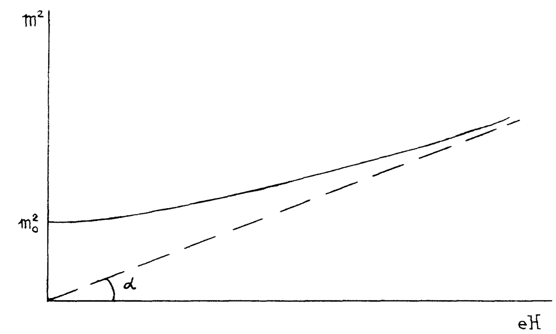

FIG. 2. The dynamical fermion mass m as a function of the magnetic field H in the chiral model; $\tan\alpha\simeq 0.2$.

$$m_0+\left[\frac{eH}{2}\right]^{1/2}\zeta\left[\frac{1}{2};\frac{m^2}{2eH}+1\right]+\frac{eH}{2m}=0\ . \tag{8}$$

Consequently, for the weak field ($eH\ll m_0^2$),

$$m\approx m_0\left[1+\frac{1}{12}\left[\frac{eH}{m_0^2}\right]^2\right] \tag{9}$$

and, for the strong-field limit,

$$m^2\approx keH\ , \tag{10}$$

where the factor $k\simeq 0.2$ is the root of the following equation

$$\sqrt{2k}\,\zeta\left[\frac{1}{2};\frac{k}{2}\right]=1\ . \tag{11}$$

The corresponding m vs H behavior, shown in Fig. 2, clearly demonstrates that the chiral symmetry will not be restored. Even if we set $m_0=0$, any nonzero H induces the fermion mass, $m^2=keH$ [see Eq. (11)] and hence breaks the symmetry.

III. HIGH-DENSITY PHASE TRANSITION IN A STRONG MAGNETIC FIELD

Thus far we have assumed that the particle density is zero. Let us examine the way in which finite density ρ modifies the theory. In this case, several positive-energy levels are added to the Dirac sea, including the threshold energy level $\varepsilon_{0,+}=m$ (unlike the $\varepsilon_{0,-}$ one, it does exist for both versions). As a result, the minimal model acquires a threshold level. In contrast, the two levels $\varepsilon_{0,\pm}$ "cancel out" of the chiral model. So, we expect the properties of the models to be interchanged.

We proceed with the minimal version first. Assume that N positive-energy levels [see Eq. (2)] are completely filled and denote the fractional filling of the $n=N$ level by $\nu(0\le\nu<1)$ so that the density is

$$\rho=\frac{eH}{2\pi}(N+\nu)\ . \tag{12}$$

The total energy (which is $V_{\rm eff}$ plus the sum of $\varepsilon>0$ over the occupied levels) should be minimized as a function of m yielding

$$m_0+\left[\frac{eH}{2}\right]^{1/2}\zeta\left[\frac{1}{2};\frac{m^2}{2eH}+N\right]-\frac{eH}{m}-\frac{\nu eH}{\sqrt{m^2+2eHN}}=0\ . \tag{13}$$

To the lowest order in eH/m_0^2, we obtain

$$\frac{m^2}{m_0^2}\approx 1-\frac{4\pi\rho}{m_0^2}-\frac{eH}{m_0^2}\left[\frac{2m_0}{\sqrt{m_0^2-4\pi\rho}}-1\right]\ . \tag{14}$$

Now we take a closer look at the positive threshold level. For $\rho=0$, it is empty and there is a critical field given by Eq. (7). If we inject fermions, the $\varepsilon_{0,+}$ level contributes to the total energy, and a divergence would prevent us from setting $m=0$ in Eq. (13). Instead, all the particles in the strong field ($eH \gg m_0^2$ with ρ kept fixed) occupy the lowest level, $\varepsilon_{0,+}$, where $N=0$ and $\nu=2\pi\rho/eH \ll 1$. This implies $m^2 \propto eH$.

Obviously, there is no chiral-symmetry breaking in this case. Such a behavior of the dynamical mass should be understood as a manifestation of a well-known spectral asymmetry [7]. Note also that, in a zero magnetic field, when a "band" structure of the spectrum dissolves, one has $m=\sqrt{m_0^2-4\pi\rho}$ and there is a critical density $\rho_0=m_0^2/4\pi$ [9].

Finally, let us turn our attention to the chiral model. It is convenient to separate the contribution of the $\varepsilon_{0,+}$ level and denote its filling by ν_0 $(0\le\nu_0\le 1)$. The particle density is given by

$$\rho=\frac{eH}{2\pi}(\nu_0+2N+\nu)\ , \tag{15}$$

where ν is the fractional filling of the top level; $0\le\nu\le 2$, due to spin degeneracy.

To consider the spontaneous symmetry breaking, one should again evaluate the total energy and minimize with respect to m. The respective stationary equation,

$$2m_0+\sqrt{2eH}\,\zeta\left[\frac{1}{2},\frac{m^2}{2eH}+N+1\right]+\frac{(1-\nu_0)eH}{m}\delta_{N,0}-\frac{(1-\delta_{N,0})\nu eH}{\sqrt{m^2+2eH(N+1)}}=0\ , \tag{16}$$

defines the dependence of m on density ρ and the magnetic field H. (Here $\delta_{N,0}$ is the Kronecker delta.) The weak-field expansion ($eH \ll m_0^2$) is of the form

$$\frac{m^2}{m_0^2}=1-\frac{2\pi\rho}{m_0^2}+\frac{1}{6}\left[\frac{eH}{m_0^2}\right]^2+\begin{cases}\dfrac{(1-\nu_0)eH}{m_0^2}\left[\dfrac{m_0}{\sqrt{m_0^2-2\pi\rho}}-1\right] & \text{if } N=\nu=0\ ,\\[2ex] -\dfrac{1}{4}\left[\dfrac{eH}{m_0^2}\right]^2(\nu-1)^2 & \text{if } \nu_0=1\ .\end{cases} \tag{17}$$

Notice that the magnetic field breaks the chiral symmetry, whereas the finite density tends to restore it. (In zero field, the critical density is $\rho_c=m_0^2/2\pi$.) The total effect can readily be traced in Eqs. (16) and (17). Unless the positive threshold level is exactly filled, the dynamically generated mass m grows linearly with H, and setting $m=0$ in Eq. (16) makes the third term diverge. At larger densities ($\nu_0=1$) the contributions of $\varepsilon_{0,\pm}$ cancel so that the chiral symmetry can be restored. If, for example, N positive-energy levels are exactly filled, the critical values are

$$eH_e=\frac{2m_0^2}{\zeta^2(\frac{1}{2},N+1)}\ , \tag{18}$$

and

$$\rho_c=\frac{eH_e}{2\pi}(2N+1)\ . \tag{19}$$

IV. SUMMARY

We have studied the $(2+1)$-dimensional Gross-Neveu model in a uniform magnetic field. It was demonstrated that strong field behavior of dynamical fermion mass m crucially depends on the existence and the filling of the threshold energy levels $\varepsilon_{0,\pm}$. When only one of these two levels is filled (asymmetrical filling) the influence of magnetic field results in the increasing of m even if the bare mass $m_0=0$. Thus we have formulated particular conditions under which a magnetic field in $2+1$ dimensions can induce the broken-symmetry phase.

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