Photon polarization tensor and gauge dependence in three-dimensional quantum electrodynamics

Conrad J. Burden and Justin Praschifka
Department of Theoretical Physics, Research School of Physical Sciences and Engineering, Australian National University, Canberra ACT 2601, Australia

Craig D. Roberts
Physics Division, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439-4843
(Received 20 January 1992)

We calculate the three-dimensional QED (QED_3) photon polarization tensor using dressed fermion propagators and a fermion-photon vertex that satisfies the Ward-Takahashi identity. Irrespective of the structural details of the transverse part of the fermion-photon vertex the photon remains massless; i.e., there is no photon mass generation in the manner of the Schwinger mechanism. Our calculation suggests that QED_3 is confining when the polarization tensor is calculated using a dressed fermion propagator and fermion-photon vertex. The gauge parameter dependence of the fermion propagator, fermion-photon vertex, and photon polarization tensor is discussed in connection with the Landau-Khalatnikov gauge transformation laws.

PACS number(s): 12.20.Ds, 11.15.Tk, 11.30.Rd

I. INTRODUCTION

Quantum electrodynamics in two spatial and one temporal dimension (QED_3) is an ideal field-theoretical model for testing ideas that may be useful or relevant in quantum chromodynamics (QCD) and/or technicolor models [1]. Lattice simulations of QED_3 [2] indicate that there is a critical number of fermion flavors, $N=N_c$, above which there is no dynamical chiral-symmetry breaking (DCSB). Continuum studies using the Schwinger-Dyson equation (SDE) for the fermion self-energy [3–5] support this result. The import of this is that for $N>N_c$ a dynamically generated mass scale that could resolve the hierarchy problem is absent. However, recent studies [6,7] argue that this conclusion is an artifact of the $1/N$ expansion and that, in fact, chiral symmetry is dynamically broken for arbitrary N.

In continuum analyses the SDE for the fermion self-energy has been the main tool. However, in studying the N dependence of DCSB it is necessary to include fermion loop contributions in $\Pi_{\mu\nu}(k)$. In addition it has been realized that realistic studies of the fermion SDE should go beyond the bare vertex (or *rainbow*) approximation and include a fermion-photon vertex that satisfies the Ward-Takahashi identity and also includes a transverse piece. This latter piece is important because of its connection with multiplicative renormalizability [8–10] and gauge parameter independence of physical observables [11].

As an alternative to the $1/N$ expansion in the analysis of DCSB one can study the simultaneous solution of the coupled SDE's for the fermion self-energy and the photon polarization tensor which have been decoupled from the remaining tower of SDE's by using an ansatz for the fermion-photon vertex. As a first step toward the simultaneous solution of these equations we report and discuss herein the results of a calculation of the photon polarization tensor using a dressed fermion propagator and fermion-photon vertex. An important element of our discussion is a consideration of the gauge parameter dependence of the SDE approach which is almost universally ignored. In the context of a determination of N_c, for example, the SDE approach can only provide a reliable estimate if, in the truncation, an ansatz for the vertex which ensures the preservation of gauge covariance is employed.

A problem with this approach is that it is not obvious how to systematically improve the vertex ansatz which, at present, remains *ad hoc*. It does, however, have the virtue that it is a strictly nonperturbative approach. Progress relies on an improved understanding of the vertex and addressing this issue is an important part of this article. The predictions obtained from the simultaneous solution of the SDE for the polarization tensor and fermion self-energy will only be meaningful when the approximations and/or truncations associated with the vertex ansatz are understood.

Our paper is organized as follows. In Sec. II we discuss the regularization of the equation for the polarization tensor when using dressed Green's functions. In Sec. III it is shown that as long as the fermion-photon vertex satisfies the Ward identity there is no photon Schwinger-mass, i.e., no $1/k^2$ pole in the polarization scalar, irrespective of the details of the transverse part of the vertex. This analysis also makes it clear that including a dressed fermion-photon vertex at this level restores confinement to QED_3 in the sense that a logarithmic term reappears in the potential at large r. The gauge parameter dependence of the polarization tensor is also discussed. In Sec. IV we discuss the general transformation properties of Green's functions under a change in the gauge parameter, the Landau-Khalatnikov (LK) transformations [12], in the context of our equations. Our discussion makes it apparent that it is possible to ensure that the fermion propagator is gauge covariant and the polarization scalar gauge parameter independent simply by constraining the gauge transformation properties of the transverse part of the fermion-photon vertex. Equally, however, it reveals that general considerations regarding the essential features of this part of the vertex do not provide very tight constraints on its form. In Sec. V we summarize our results and conclusions.

II. FERMION-PHOTON VERTEX AND PHOTON POLARIZATION TENSOR

In our study we employ Euclidean metric and, so that we may properly describe chiral symmetry [13], we use 4×4 Euclidean Dirac matrices chosen such that they satisfy the algebra $\{\gamma_\mu,\gamma_\nu\}=2\delta_{\mu\nu}$. With these conventions the inverse of the photon propagator is

$$D_{\mu\nu}^{-1}(k)=\delta_{\mu\nu}k^2-k_\mu k_\nu(1-1/\xi)+\Pi_{\mu\nu}(k) \tag{1}$$

with ξ a gauge-fixing parameter and $\Pi_{\mu\nu}(k)$ the photon polarization tensor:

$$\Pi_{\mu\nu}(k)=-e^2\int\frac{d^3q}{(2\pi)^3}\mathrm{tr}[\gamma_\mu S(q+\tfrac{1}{2}k)\Gamma_\nu(q+\tfrac{1}{2}k,q-\tfrac{1}{2}k)S(q-\tfrac{1}{2}k)] \ . \tag{2}$$

In Eq. (2), $S(k)$ is the dressed fermion propagator $[S^{-1}(k)=i\gamma\cdot kA(k)+B(k)]$ which satisfies [11]

$$S^{-1}(p)=i\gamma\cdot p+e^2\int\frac{d^3q}{(2\pi)^3}D_{\mu\nu}(p-q)\Gamma_\mu(p,q)\times S(q)\gamma_\nu \ . \tag{3}$$

Equations (2) and (3) are, respectively, Schwinger-Dyson equations for the photon polarization tensor and fermion self-energy.

In Eqs. (2) and (3), $\Gamma_\mu(p,q)$ is the dressed fermion-photon vertex. The minimal constraint on this function is that it satisfy the Ward-Takahashi identity:

$$i(p-q)_\mu\Gamma_\mu(p,q)=S^{-1}(p)-S^{-1}(q) \ . \tag{4}$$

This coupled with the requirement that kinematic singularities be absent [14] leads us to the ansatz

$$\Gamma_\mu(p,q)=\tfrac{1}{2}[A(p)+A(q)]\gamma_\mu+\frac{(p+q)_\mu}{p^2-q^2}\{[A(p)-A(q)]\tfrac{1}{2}(\gamma\cdot p+\gamma\cdot q)-i[B(p)-B(q)]\} \ . \tag{5}$$

This vertex, which ensures [11] that, for $\xi\in[0,1]$,

$$\frac{d}{d\xi}\langle\bar\psi\psi\rangle\approx0 \tag{6}$$

is unique up to the addition of a transverse piece which cannot affect Eq. (4) and may be constrained by requiring multiplicative renormalizability of Eq. (3) [8]. A similar form has also been used in phenomenological studies of QCD [15,16]. We emphasize, however, that satisfying Eq. (4) does not guarantee the gauge covariance of particle propagators [17].

Gauge invariance requires that $k_\nu\Pi_{\mu\nu}(k)=0$. This is ensured as long as $\Gamma_\mu(p,q)$ satisfies the Ward-Takahashi identity and Eq. (2) is regularized properly in which case it follows that $\Pi_{\mu\nu}(k)=(k^2\delta_{\mu\nu}-k_\mu k_\nu)\Pi(k)$. Hereafter we shall refer to $\Pi(k)$ as the polarization scalar. The dressed photon propagator can now be written as

$$D_{\mu\nu}(k)=\left[\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right]\frac{1}{k^2[1+\Pi(k)]}+\xi\frac{k_\mu k_\nu}{k^4} \ . \tag{7}$$

In Ref. [3] it was argued that to leading order in a $1/N$ expansion (with N the number of fermion flavors) $A(k)\equiv1$ in Eq. (3). The flavor dependence of DCSB was then studied by evaluating Eq. (2) using bare fermion propagators and $\Gamma_\nu=\gamma_\nu$. (This neglects the influence of the fermion self energy on $\Pi_{\mu\nu}$.) In this case one finds easily [18] that, for N massless fermions,

$$\Pi(k)=N\frac{e^2}{8k} \ . \tag{8}$$

Combining these approximations with Eq. (3) one determines that, at leading order in a $1/N$ expansion, chiral symmetry is not dynamically broken for $N>N_c=32/\pi^2$. At the next order in the $1/N$ expansion a critical value persists but it is found to be larger: $N_c=\frac{4}{3}(32/\pi^2)$ [4].

In contrast with this it has been reported [6] that QED_3 exhibits DCSB for arbitrary N. Therein it is argued that such a result is not inconsistent with lattice gauge theory analyses and that it is suggestive that the $1/N$ expansion is not an appropriate tool in the study of a nonperturbative phenomenon such as DCSB. Herein we will not address this disagreement but remark only that it is perhaps premature to draw strong conclusions from the coupled SDE approach given the uncertainties associated with the vertex ansatz.

We are interested in evaluating Eq. (2) using the vertex of Eq. (5) and the propagator obtained as a solution of Eq. (3) with the same vertex and with $\Pi(k)=0$. [This is the first step in the iterative solution of the coupled pair of Eqs. (2) and (3).] It is then important to also consider the gauge parameter dependence of the polarization scalar (it should, in fact, be gauge parameter independent). To address this we use the solution of Eq. (3) described in detail in Ref. [11]. We report below only those features of this solution that are important herein:

$$A(k\to\infty)=1+\frac{\xi e^2}{16k}\,, \tag{9a}$$

$$B(k\to\infty)=\frac{\lambda}{k^2} \tag{9b}$$

with ξ the gauge-fixing parameter and λ a gauge-parameter-dependent constant. These results are obtained using an analytic asymptotic analysis of the Dirac vector and scalar parts of Eq. (3). In Landau gauge ($\xi=0$) the lowest-order term in Eq. (9a) is of $O(1/k^2)$. Both $A(k)$ and $B(k)$ are nonzero at $k^2=0$.

To regularize Eq. (2) one might first attempt simply to add and subtract the perturbative result (which has exactly the same degree of divergence). This, however, is insufficient in all but the Landau gauge since for $\xi\neq0$ this procedure does not ensure that the Ward-Takahashi identity for the photon polarization tensor is satisfied. In fact,

$$k_\nu\Pi_{\mu\nu}(k)=-e^2\int\frac{d^3q}{(2\pi)^3}\mathrm{tr}\{\gamma_\mu[S(q-\tfrac{1}{2}k)-S_0(q-\tfrac{1}{2}k)]-\gamma_\mu[S(q+\tfrac{1}{2}k)-S_0(q+\tfrac{1}{2}k)]\} \tag{10}$$

with $S_0(k)$ the massless bare fermion propagator but, because of the asymptotic behavior of $A(k)$ described in Eq. (9a), each of the integrals in the difference is logarithmically divergent for $\xi\neq0$. Consequently one may not perform a shift in integration variables and, in fact, for $\xi\neq0$, $k_\nu\Pi_{\mu\nu}(k)\neq0$. Proceeding with this prescription leads to a spurious photon mass [19]. (In the Landau gauge, the integrals are finite and the Ward-Takahashi identity is satisfied so this subtraction is sufficient to regularize the integral properly. In this case the photon remains massless.)

In order to avoid this difficulty and to have a procedure that is valid independent of ξ, the problem of regularization is best handled by contracting the polarization tensor with

$$\mathcal{P}_{\mu\nu}=\delta_{\mu\nu}-3\frac{k_\mu k_\nu}{k^2} \tag{11}$$

which, in three dimensions, is orthogonal to $\delta_{\mu\nu}$. The divergent part of this integral is proportional to $\delta_{\mu\nu}$ and hence this contraction projects out only the finite part of the integrand. (This is the approach used in Ref. [20].)

Following this procedure we find that

$$\Pi(k)=-\frac{e^2}{2k^2}\int\frac{d^3q}{(2\pi)^3}\mathrm{tr}\left[\gamma_\mu S(q+\tfrac{1}{2}k)\Gamma_\mu(q+\tfrac{1}{2}k,q-\tfrac{1}{2}k)S(q-\tfrac{1}{2}k)-3\frac{\gamma\cdot k}{k^2}S(q+\tfrac{1}{2}k)k_\nu\Gamma_\nu(q+\tfrac{1}{2}k,q-\tfrac{1}{2}k)S(q-\tfrac{1}{2}k)\right]. \tag{12}$$

Using Eqs. (5), (9a), and (9b) it is clear that this integral is finite independent of the gauge and it also reproduces Eq. (8) when bare, massless fermion propagators are used.

III. MASSLESS PHOTON AND GAUGE DEPENDENCE OF Π

It is interesting now to address the question of photon mass generation since there are a number of lattice studies of the gauge-boson propagator in QED_4 [21] and QCD [22] which find a nonzero gauge-boson mass after fixing a lattice Landau gauge. To study this one must consider $\Pi(0)$ since a $1/k^2$ pole in the polarization scalar signals mass generation in the manner of the Schwinger mechanism [23] which is the supposed nature of mass generation in the lattice simulations. From Eq. (8) it is clear that there is no such mass generation in perturbation theory.

To proceed we write

$$k^2\Pi(k)=-N\frac{e^2}{\pi^2}f(k) \tag{13}$$

and using the differential form of the Ward identity $[\Gamma_\mu(q,q)=-i\partial^q_\mu S^{-1}(q)]$ one finds

$$f(0)=\lim_{k^2\to0}\frac{i}{16\pi}\int\frac{d^3q}{(2\pi)^3}\times\mathrm{tr}\left[\left[\gamma_\mu-3\frac{\gamma\cdot kk_\mu}{k^2}\right]\partial^q_\mu S(q)\right], \tag{14}$$

where the final limit must be taken with care after the Dirac trace is evaluated. Using Green's theorem

$$f(0)=\lim_{k^2\to0}\frac{1}{4\pi}\int_{S_2}d^2S\left[1-3\frac{(k\cdot q)^2}{k^2q^2}\right]\times\frac{A(q)}{q^2A^2(q)+B^2(q)} \tag{15}$$

and since $\int_0^\pi d\theta\sin\theta[1-3\cos^2\theta]=0$ one has

$$f(0)=0\,. \tag{16}$$

A similar proof of this result was communicated to us in

Ref. [20].

Hence, as long as the Ward identity is satisfied the photon remains massless independent of the gauge parameter and details of the transverse part of the vertex. This is a general result in Abelian theories, independent of the details of the interaction and, suitably modified, it also holds in QED_4. We expect, therefore, that in the continuum limit lattice simulations should also yield this result.

We have calculated $\Pi(k)$ for $N=1$ and have plotted it for a range of values of ξ in Fig. 1. Our result can be summarized by the fit

$$\Pi(k)=\frac{e^2}{8(k^2+e^4a^2)^{1/2}}+be^{-ck^2/e^4} \tag{17}$$

with ξ dependent fitting parameters a, b, and c specified below (ξ,a,b,c):

$$(0.0,0.2044,8.760\times10^{-2},7.767)\ , \tag{18a}$$

$$(0.5,0.2541,5.055\times10^{-2},11.70)\ , \tag{18b}$$

$$(1.0,0.2778,-9.349\times10^{-3},0.5285)\ . \tag{18c}$$

[Of course, in this calculation, a trivial multiplicative factor of N appears on the right-hand side of Eq. (17) if we use N fermions. A complicated dependence on N can only arise when one carries out the simultaneous solution of Eqs. (2) and (3).]

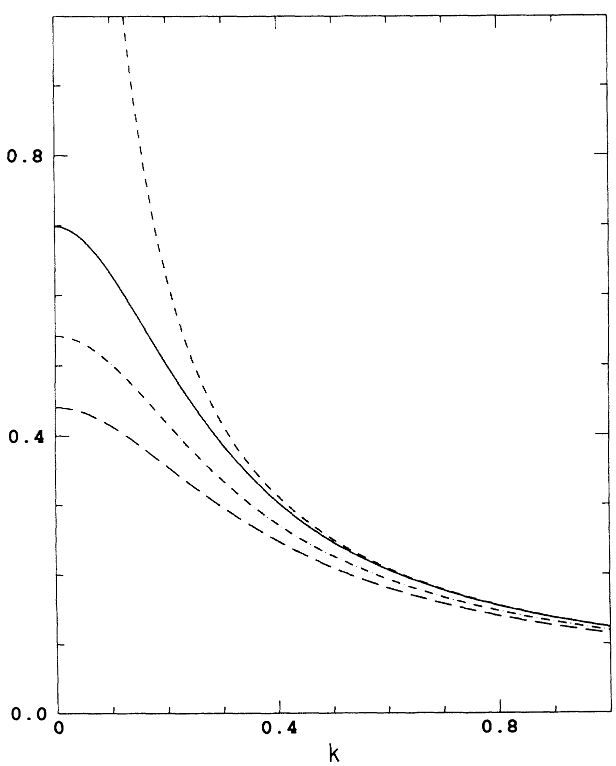

FIG. 1. $\Pi(k)$ is plotted for $\xi=0$: solid line; $\xi=0.5$: dash-dot line; $\xi=1.0$: dashed line. (We fix the mass scale by setting $e^2=1$.) The plot illustrates the deviation of our result from the perturbative one at small k. On the scale of this plot the fitting function of Eq. (17) lies exactly on top of the numerical results. The perturbative result of Eq. (8) is the short-dash line.

One observes from this fit that, at large k^2, our result returns to the perturbative one of Eq. (8). For small k^2, $\Pi(k)$ differs from this perturbative result which was used in the analyses of Refs. [3–5] and primarily in Ref. [6]. The difference is simply a quantitative softening of the infrared behavior of $\Pi(k)$ and it has been reported [6] that this has little effect on DCSB when used simply as input to Eq. (3). This difference is not unimportant, however. Simply assuming that $\Pi(k)$ is bounded (in absolute value) and continuously differentiable on $(0,\infty)$ and that $\Pi(k)\sim 1/k$ for $k\to\infty$ is enough to ensure that, for large r, the potential associated with the dressed photon propagator [11] is (see the Appendix)

$$V(r)=\frac{e^2}{[1+\Pi(0)]2\pi}\ln e^2 r+\text{const}+h(r)\ , \tag{19}$$

where $h(r)$ falls off at least as fast as $1/r$. Hence, the large r behavior of the potential is dominated by a confining logarithmic term. [A numerical calculation of the potential using our fit of Eq. (17) is consistent with this.] It will be recalled that this was true of the bare photon propagator but not of the perturbatively dressed photon propagator [11]. The inclusion of a dressed fermion-photon vertex and dressed fermion propagator (obtained with this vertex) at this level has thus restored the confining nature of QED_3.

Our calculated result (in the Landau gauge), evaluated with the complete vertex of Eq. (5), differs from that in Eq. (15) of Ref. [6] [which appears to be missing an additive factor of 1 in the last line: $\mathcal{P}(k)=1+\Pi(k)$]. The difference is the large-k behavior: we find $\Pi(k)\sim 1/k$ whereas Ref. [6] reports that $\Pi(k)=0$ for $k>e^2/8$ $(N=1)$. The small-k behavior is broadly in agreement with ours although we would not choose to define an electron mass as the solution of $\mathcal{M}(m)=m$. These calculations are performed in Euclidean space and an electron mass pole should appear on the negative real k^2 axis. We would advocate a definition that corresponds to $\mathcal{M}(im)=m$ [24]. Not surprisingly, if we choose to fit the last line of Eq. (15) in Ref. [6] to our form of $\Pi(k)$ we obtain a value of $m=0.076$ which can be compared with $B(0)/A(0)=0.094$, this latter expression being an estimate by linear extrapolation of the location of the pole in our electron propagator on the negative k^2 axis.

It will be observed from Fig. 1 and our parametrization that even though the Ward-Takahashi identities are satisfied the polarization scalar is gauge parameter dependent. In QED_3 the polarization scalar should be independent of ξ as manifest in the perturbative result [18]. The ξ dependence of our result is a direct consequence of the implicit ξ dependence of $A(k)$ and $B(k)$ since there is no explicit ξ dependence in Eq. (2). In Ref. [11] it was shown that the implicit ξ dependence of these functions, obtained as solutions of Eq. (3) with Eq. (5), almost canceled in the calculation of the fermion condensate ensuring Eq. (6). Unfortunately, a similar result is not observed in $\Pi(k)$. As we discuss in the following section, this difficulty is the result of an inadequate ansatz for the fermion-photon vertex and its gauge parameter dependence. [It will be observed from Fig. 1 that, at large k, $\Pi(k)$ is independent of ξ, as one would expect given Eqs.

(9a) and (9b): i.e., nonperturbative effects are not important at large k.]

IV. PROPAGATOR AND VERTEX GAUGE TRANSFORMATION PROPERTIES

The transformation laws describing the response of the fermion and photon propagators and fermion-photon vertex to a change in the gauge parameter were first discussed by Landau and Khalatnikov [12], and subsequently obtained from a refined calculation in Ref. [25]. These laws are most simply specified in terms of the Minkowski coordinate space functions $S_F(x), D_{\mu\nu}(x)$, and $\Gamma_\mu(x,y,z)$, related to their momentum space counterparts by

$$S_F(p)=\int d^3x\, e^{ip\cdot x}S_F(x)\ , \tag{20}$$

$$D_{\mu\nu}(p)=\int d^3x\, e^{ip\cdot x}D_{\mu\nu}(x)\ , \tag{21}$$

$$(2\pi)^3\delta^3(p-q-r)\Gamma_\mu(p,q) =\int d^3x\, d^3y\, d^3z\, e^{i(p\cdot x-q\cdot y-r\cdot z)}\Gamma_\mu(x,y,z)\ . \tag{22}$$

The photon propagator in an arbitrary gauge is given in terms of the Landau gauge photon propagator $D_{\mu\nu}(x;0)$ by

$$D_{\mu\nu}(x;\Delta)=D_{\mu\nu}(x;0)+\partial_\mu\partial_\nu\Delta(x)\ , \tag{23}$$

where, in the covariant gauge-fixing procedure we have used herein,

$$\Delta(x)=-\xi e^2\int\frac{d^3k}{(2\pi)^3}\frac{e^{-ik\cdot x}}{k^4}\ . \tag{24}$$

The corresponding gauge transformation rule for the photon polarization scalar $\Pi(k)$ is

$$\Pi(x,\Delta)=\Pi(x,0)\ ; \tag{25}$$

i.e., it is gauge parameter independent as discussed above; for the fermion propagator,

$$S_F(x;\Delta)=S_F(x;0)e^{-i[\Delta(0)-\Delta(x)]}\ , \tag{26}$$

and, for the fermion-photon vertex,

$$B_\mu(x,y,z;\Delta)=B_\mu(x,y,z;0)e^{-i[\Delta(0)-\Delta(x-y)]} -iS_F(x-y;0)e^{-i[\Delta(0)-\Delta(x-y)]} \times\frac{1}{e^2}\frac{\partial}{\partial z_\mu}[\Delta(x-z)-\Delta(z-y)]\ , \tag{27}$$

where $B_\mu(x,y,z)$ is the nonamputated vertex which is related to the bare vertex Γ_μ by

$$B_\mu(x,y,z)=\int d^3x'\,d^3y'\,d^3z'\,S_F(x-x') \times\Gamma_\nu(x',y',z')S_F(y-y')D_{\mu\nu}(z-z')\ . \tag{28}$$

One can show that the transformations of Eqs. (25)–(27) leave the Minkowski space Ward-Takahashi identity

$$(p-q)_\mu\Gamma^\mu(p,q)=S_F^{-1}(p)-S_F^{-1}(q)\ , \tag{29}$$

and SD equations

$$S_F^{-1}(p)=\not p+ie^2\int\frac{d^3q}{(2\pi)^3}\Gamma_\mu(p,q)D^{\mu\nu}(p-q)S_F(q)\gamma_\nu\ , \tag{30}$$

$$\Pi_{\mu\nu}(p)=ie^2\int\frac{d^3q}{(2\pi)^3}\mathrm{tr}[\gamma_\mu S_F(q+\tfrac{1}{2}k)\Gamma_\nu(q+\tfrac{1}{2}k,q-\tfrac{1}{2}k)S_F(q-\tfrac{1}{2}k)] \tag{31}$$

invariant. To verify this for the Ward-Takahashi identity, it is easier to use the equivalent form

$$(p-q)^\mu B_\mu(p,q;\Delta)=[S_F(q;\Delta)-S_F(p;\Delta)](p-q)^2\Delta(p-q)\ . \tag{32}$$

[The Fourier transforms of B and Δ are defined in a similar way to Eqs. (20) and (22).] To verify invariance of the SD equations it is convenient to use the coordinate space forms [26]

$$\delta^3(x-y)=(i\not\partial-m)S_F(x-y)-ie^2\gamma_\mu B^\mu(x,y,x)\ , \tag{33}$$

$$(g_{\mu\nu}\partial^2-\partial_\mu\partial_\nu)\Pi(x-y)=ie^2\int d^3z_1d^3z_2\mathrm{tr}[\gamma_\mu S_F(x-z_1)\Gamma_\nu(z_1,z_2,y)S_F(z_2-x)]\ . \tag{34}$$

It follows that the required transformation rules, Eqs. (25) and (26), can be guaranteed to hold for solutions of the SD equations, Eqs. (30) and (31), provided an ansatz is chosen for the vertex Γ_μ which gauge transforms according to Eq. (27) and which satisfies the Ward-Takahashi identity. In practice, however, explicitly specifying such a vertex (for an arbitrary value of the gauge parameter ξ, say) is hampered by the complexity of this transformation rule.

The vertex ansatz we have used Eq. (5) was chosen by the criteria that it should respect the Ward-Takahashi identity and be free of kinematic light-cone singularities.

While a direct comparison of the implicit ξ dependence (via the functions A and B) of this vertex with the rule (27) is not a simple matter, it is not difficult to check *a posteriori* whether quenched approximation solutions obtained using this vertex [11] satisfy the more straightforward rules in Eqs. (25) and (26).

For the polarization scalar this is simple: from Eq. (25) it follows that the Euclidean momentum space function $\Pi(k)$ found earlier should be independent of ξ. As noted earlier, however, the solutions obtained have a strong dependence on the gauge parameter.

To examine the fermion propagator we first transform the Euclidean momentum space form $[S^{-1}(p)=i\gamma\cdot pA(p)+B(p)]$ to its Euclidean coordinate space counterpart

$$S(x)=\gamma\cdot xX(x^2)+Y(x^2)\ , \tag{35}$$

where

$$X(x^2)=\frac{1}{2\pi x^2}\delta(x) - \frac{1}{2\pi^2}\int_0^\infty dp\frac{pV(p)\sin px-[p^2V(p)-1]x\cos px}{x^3}\ , \tag{36}$$

with $V(p)=A(p)/[p^2A^2(p)+B^2(p)]$ and

$$Y(x^2)=\frac{1}{2\pi^2}\int_0^\infty p\,dp\frac{\sin px}{x}\frac{B(p)}{p^2A^2(p)+B^2(p)}\ . \tag{37}$$

The Euclidean phase occurring in Eq. (26) is, from Eq. (24) [27],

$$\Delta(0)-\Delta(x)=-i\xi e^2\int\frac{d^3k}{(2\pi)^3}\frac{1-e^{-ik\cdot x}}{k^4}=-\frac{i\xi}{8\pi}e^2x \tag{38}$$

and, hence, under a gauge transformation,

$$X(x^2;\xi)=X(x^2;0)e^{(\xi/8\pi)e^2x}\ ,$$
$$Y(x^2;\xi)=Y(x^2;0)e^{(\xi/8\pi)e^2x}\ . \tag{39}$$

In Fig. 2 we plot the scalar part $Y(x^2)$ of the propagator obtained by numerically integrating Eq. (37) using solutions A and B obtained in Ref. [11] at $\xi=0.0$, 0.5, and 1.0 and also its LK transform to Landau ($\xi=0$) gauge. If the plot in the main body of the figure had each curve lying one upon the other then $Y(x^2)$ would be seen to transform correctly under the LK transformations; i.e., it would be gauge covariant. In Fig. 3 we present a similar plot for the vector part of the propagator, $X(x^2)$. The figures complement the observation of Eq. (6) and illustrate that the implicit ξ dependence of $A(k)$ and $B(k)$ provides a large part of the necessary vertex gauge dependence but not all of it. The missing gauge dependence in the vertex is responsible for the gauge parameter dependence of the polarization scalar and the residual noncovariance of the fermion propagator evident in Figs. 2 and 3.

We point out also that, given the Landau and Khalatnikov transformation laws described above we can, at least in a formal sense, write down the most general ansatz for the vertex satisfying the requirements that (1) the vertex and corresponding propagators obtained by solving the fermion and photon SDE's respect gauge covariance in the sense of transforming according to Eqs. (23), (26), and (27), (2) the Ward-Takahashi identity is satisfied, and (3) the vertex has the same transformation properties under space parity, time reversal and charge conjugation as the bare vertex γ_μ.

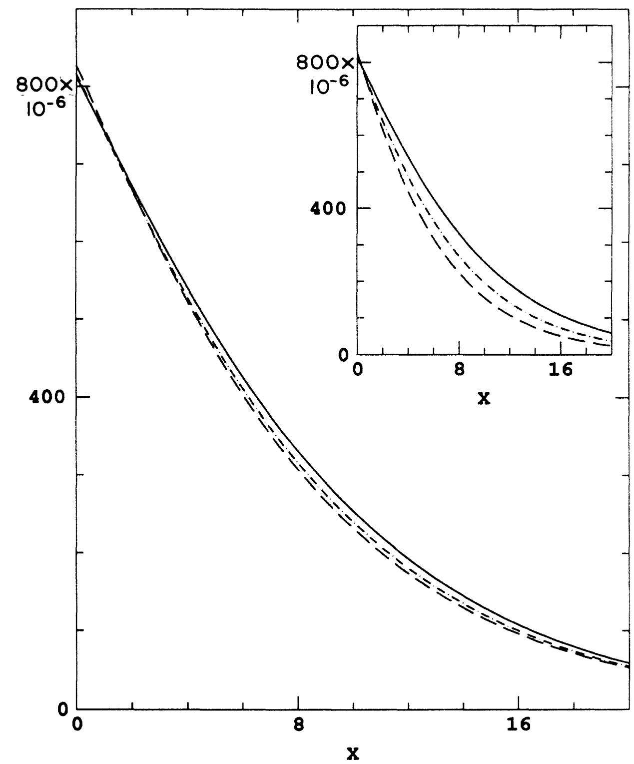

FIG. 2. In the inset we plot $Y(x^2)$ for $\xi=0$: solid line; $\xi=0.5$: dash-dot line; $\xi=1.0$: dashed line. In the main body of the figure we plot the Landau-Khalatnikov transforms of these functions back to Landau ($\xi=0$) gauge.

To do this, one first specifies the ansatz in a particular gauge, say the Landau gauge, as

$$\Gamma_\mu(p,q;0)=\Gamma^0_\mu(p,q)+\sum_{i=1}^{8}f^i(p^2,q^2,p\cdot q)T^i_\mu(p,q)\ , \tag{40}$$

where $\Gamma^0_\mu(p,q)$ is the vertex of Eq. (5) and T^i_μ are the eight transverse tensors of Eq. (3.4) in Ref. [14] of which those with $i=1,2,3$ are symmetric under $p\leftrightarrow q$ and the remainder are antisymmetric. The requirement that this vertex transform in the same way as the bare vertex under charge conjugation implies that all of the f^i are symmetric except for f^6 which is antisymmetric. One may then decide upon the ansatz that the gauge dependence of the vertex is defined by Eq. (27); i.e., the vertex at another value of the gauge parameter is given simply by applying this transformation law. This reduces the problem to finding the best ansatz in a single gauge.

It is worth remarking that determining the eight arbitrary functions f^i in the transverse part of Γ_μ amounts to

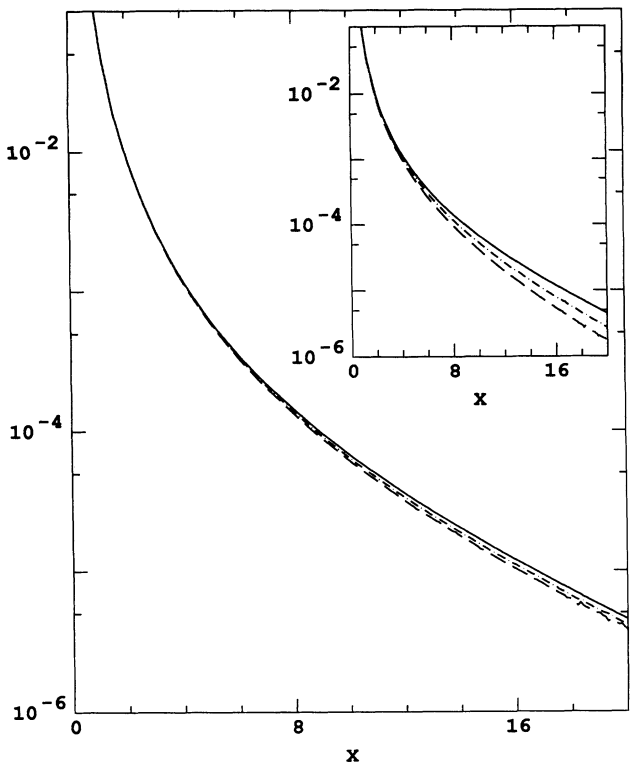

FIG. 3. In the inset we plot $X(x^2)$ [minus the $\delta(x)$ piece] for $\xi=0$: solid line; $\xi=0.5$: dash-dot line; $\xi=1.0$: dashed line. In the main body of the figure we plot the Landau-Khalatnikov transforms of these functions back to Landau ($\xi=0$) gauge.

solving the remaining vertex SDE. The important point to appreciate is that no further restrictions can be placed on the general form of the vertex by the application of only the above constraints in the fermion and photon SDE's. For this reason,the argument [8] that multiplicative renormalizability of Eq. (3) allows $f^i=0$ for $i\neq 6$ in Eq. (40) is welcome. Nevertheless, in the sense described above, the form of this function remains arbitrary.

V. SUMMARY AND CONCLUSIONS

Herein we have calculated the QED_3 photon polarization tensor using a dressed fermion propagator obtained as a solution of the Schwinger-Dyson equation and demonstrated that, as long as the Schwinger-Dyson equation for the polarization tensor is regularized such that the Ward identity $k_\nu \Pi_{\mu\nu}(k)=0$ is preserved then the photon remains massless irrespective of the details of the structure of the vertex; i.e., there is no photon mass generation in the manner of the Schwinger mechanism. Our calculation also demonstrates that including the dressed fermion-photon vertex and dressed fermion propagator (obtained with this vertex) in the calculation of the polarization tensor restores confinement to QED_3 in the sense that the photon propagator obtained with this again generates a potential with infinite ionization energy.

The Landau-Khalatnikov transformation laws, which specify the gauge parameter dependence of the propagators and vertex and ensure the covariance of the Schwinger-Dyson equations, were discussed in detail. We pointed out that the requirements of gauge covariance, preservation of the Ward-Takahashi identity and appropriate transformation properties under space parity, time reversal, and charge conjugation do not provide very tight constraints on the form of the transverse piece of the fermion-photon vertex. The Landau-Khalatnikov transformation laws do, however, provide a useful means of specifying the gauge parameter dependence of a given ansatz for the vertex. In this connection, as mentioned above, a simple ansatz, with no explicit gauge parameter dependence but with an implicit dependence through the functions $A(k)$ and $B(k)$ which specify the quark propagator, is inadequate.

ACKNOWLEDGMENTS

We thank M. R. Pennington for pointing out an error in the regularization procedure for the photon polarization tensor in our original manuscript and a simple manner in which to correct it. The work of C.D.R. was supported by the Department of Energy, Nuclear Physics Division under Contract No. W-31-109-ENG-38.

APPENDIX: POTENTIAL FROM NONPERTURBATIVE POLARIZATION SCALAR

Following convention, the potential in the present case is

$$V(\mathbf{r})=-e^2\int\frac{d^2k}{4\pi^2}e^{i\mathbf{k}\cdot\mathbf{r}}\frac{1}{k^2}\frac{1}{1+\Pi(k)}$$
$$=-\frac{e^2}{2\pi}\int_0^\infty dk\frac{1}{k}\frac{1}{1+\Pi(k)}J_0(kx)\,, \qquad \text{(A1)}$$

where J_0 is a Bessel function of order 0. From this it follows that

$$\frac{dV}{dr}=\frac{e^2}{2\pi}\int_0^\infty dk\frac{J_1(kr)}{1+\Pi(k)} \qquad \text{(A2)}$$

where J_1 is a Bessel function of order one.

An integration by parts gives

$$\int_0^\infty dk\frac{J_1(kr)}{1+\Pi(k)}=\frac{1}{r}\left[\frac{1}{1+\Pi(0)}+\epsilon(r)\right]\,, \qquad \text{(A3)}$$

where

$$\epsilon(r)=\int_0^\infty dk\,J_0(kr)f(k) \qquad \text{(A4)}$$

with

$$f(k)=\frac{d}{dk}\frac{1}{1+\Pi(k)}\,. \qquad \text{(A5)}$$

Now, if we assume that $\Pi(k)$ is bounded (in absolute value) and continuously differentiable on $(0,\infty)$ and that $\Pi(k)\sim 1/k$ for $k\to\infty$ (an intuitive assumption that is consistent with our numerical calculation), then it is always possible to choose constants a and b such that

$$f(k)<\frac{a}{b^2+k^2} \qquad \text{(A6)}$$

$\forall k \in (0,\infty)$. This being the case then

$$\epsilon(r) < \epsilon_m(r) = ar\int_0^\infty dz \frac{J_0(z)}{(rb)^2+z^2} . \qquad (A7)$$

The last integral can be evaluated (Ref. [28]: Equations 6.532.1, 8.339.2, and 8.583.1) and, at large r, we have

$$\epsilon_m(r) = \frac{a}{b^2 r}\left[1+\sum_{n=1}^{\infty}(-)^n \frac{1}{(br)^{2n}}[(2n-1)!!]^2\right] . \qquad (A8)$$

Now, $\epsilon(r) < \epsilon_m(r)\ \forall r$ and, hence, at large r,

$$\frac{dV}{dr} = \frac{e^2}{2\pi}\left[\frac{1}{r}\frac{1}{1+\Pi(0)} + \text{correction}\right] , \qquad (A9)$$

where the correction term is bounded above by

$$\frac{a}{(br)^2} + O\left[\frac{1}{r^3}\right] . \qquad (A10)$$

This entails Eq. (19).

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