PHYSICAL REVIEW D VOLUME 46, NUMBER 1 1 JULY 1992

How good is the factorization approach in charm decays?

Xuan-Yem Pham
Laboratoire de Physique Théorique et Hautes Energies, Université Pierre et Marie Curie, Tour 16, 1er étage, 4 Place Jussieu, 75252 Paris Cedex 05, France

Xuan-Chi Vu
Institut Supérieur des Sciences et Techniques de l'Université de Picardie, 48 rue Raspail, 02109 Saint-Quentin CEDEX, France

(Received 3 February 1992)

An analysis of the factorization approach in charm hadronic decays is carried out by using the spectral-functions method and by exploiting the updated information on the semileptonic processes of both the D meson and the τ lepton. Not only the two-body but also the multiparticle exclusive decays can be computed for the first time. While for some channels agreement with experiment is somewhat acceptable, within a factor of 2, much larger variance is found for other modes: $D^0 \to K^- a_1^+(1260)$, $D^0 \to K^{-*}\pi^+$, $D^0 \to K^- \pi\pi\pi\pi$.

PACS number(s): 13.25.+m, 12.15.Ji, 13.20.Fc, 14.40.Jz

I. INTRODUCTION

By analogy with the semileptonic decay which is factorizable, i.e., its matrix element is a product of one hadronic and one leptonic current, the factorization approach [1–3] in nonleptonic processes is a natural extension asserting that the amplitudes are also the product of two hadronic currents, each one entered into an appropriate semileptonic mode. Whether or not this assumption might be justified in the $1/N_c$ expansion [4] is a different aspect of the problem not discussed here (N_c designates the number of colors and is three in QCD). We simply take it as a working hypothesis and ask ourselves how good this approximation is in charm decays by comparing theoretical predictions with data. Table I summarizes our results, and readers are invited to judge for themselves.

We begin with the standard effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{G}{\sqrt{2}} V_{cs}^* V_{ud}[c_1(\bar{u}d)(\bar{s}c) + c_2(\bar{u}c)(\bar{s}d)] , \qquad (1)$$

in which for simplicity we consider only the Cabibbo-favored modes. The coefficients $c_1 = (c_+ + c_-)/2$, $c_2 = (c_+ - c_-)/2$ (with $c_+^2 c_- = 1$) represent the well-known [5] hard-gluonic corrections and are functions of Λ_{QCD} and the scale μ at which the processes operate. Here $(\bar{u}d)$ is a brief notation for the colorless current $\bar{u}\gamma_\mu(1-\gamma_5)d$, and so on for the others. The usual procedure for computing the exclusive hadronic modes is to assume factorization and to sandwich $\mathcal{L}_{\text{eff}}$ between initial and final states in all possible ways, and to perform Fierz rearrangement (including colors) such that $\mathcal{L}_{\text{eff}}$ can be written as

$$\mathcal{L}_{\text{eff}}^{\text{had}} = \frac{G}{\sqrt{2}} V_{cs}^* V_{ud}[a_1(\bar{u}d)_H(\bar{s}c)_H + a_2(\bar{u}c)_H(\bar{s}d)_H] . \qquad (2)$$

Here the subscript H indicates the change to hadronic field operators [3], and $a_1 = c_1 + (1/N_c)c_2$, $a_2 = c_2 + (1/N_c)c_1$. The case $N_c = \infty$ (hence $a_1 = c_1$, $a_2 = c_2$) was suggested a long time ago [2] and subsequently put forward by other authors [3,4]. The decay amplitude is then a product of two hadronic currents in a way very similar to that of the semileptonic ones:

$$\mathcal{L}^{\text{lep}} = \frac{G}{\sqrt{2}} V_{ud}(\bar{l}\nu)(\bar{u}d)_H, \quad \frac{G}{\sqrt{2}} V_{cs}^*(\bar{s}c)_H(\bar{\nu}l) . \qquad (3)$$

From Eqs. (2) and (3) we immediately recognize that the hadronic decay modes best suited for factorization are those involving the charged currents proportional to a_1 (class I according to Ref. [3]). We can indeed take full advantage of information from both decays of the heavy lepton τ induced by the current $(\bar{u}d)_H$ on the one hand, and the semileptonic decays of the D meson into K and K^* supplied by the current $(\bar{s}c)_H$ on the other hand. We remark further that the D and the τ masses are nearly equal so that τ data can be fully exploitable in D decay. Clearly in the factorization approach, the knowledge of form factors contained in each hadronic current is the most important issue since the amplitudes are completely fixed by them. Therefore Sec. II will be devoted to the extraction of the form factors in semileptonic decays, which in turn determine the hadronic rates. These hadronic decays are treated in Sec. III (two-body modes) and Sec. IV (multibody channels).

II. THE FOUR FORM FACTORS IN $D \to K$ AND $D \to K^*$ SEMILEPTONIC TRANSITIONS

Since the $(\bar{s}c)_H$ current has zero isospin, the semileptonic decay rates of D^+ and D^0 are equal; it is not necessary to specify them.

While the decay $D \to Ke\nu$ is similar to the old K_{l3},

 ©1992 The American Physical Society

$$\frac{d\Gamma}{dq^2}(D \to Ke\nu) = \frac{G^2|V_{cs}|^2}{192\pi^3} \frac{\lambda^3(M^2,m^2,q^2)}{M^3} |f_+^2(q^2)| \,, \tag{4}$$

and depends on only one vector form factor $f_+(q^2)$, the mode $D \to K^* e\nu$ is more complicated. Many years ago [6], a general and convenient formalism for analyzing this latter mode in terms of three helicity amplitudes $(0,\pm)$ was suggested with an emphasis on looking for angular correlations. The method was subsequently derived by other authors [7] and especially used by the E691 experimental group [8] in their recent investigation. Let us write

$$\frac{d\Gamma}{dq^2}(D \to K^* e\nu) = \sum_{i=0,\pm} \frac{d\Gamma_i}{dq^2} = \sum_{i=0,\pm} \frac{G^2|V_{cs}|^2}{192\pi^3} \frac{\lambda^3(M^2,m_1^2,q^2)}{M^3} Y_i(q^2) \,, \tag{5}$$

with

$$Y_0(q^2) = \left[\frac{M+m_1}{2m_1}\right]^2 \left[\frac{M^2-m_1^2-q^2}{\lambda(M^2,m_1^2,q^2)} A_1(q^2) - \frac{\lambda(M^2,m_1^2,q^2)}{(M+m_1)^2} A_2(q^2)\right]^2 , \tag{6}$$

$$Y_\pm(q^2) = \frac{q^2(M+m_1)^2}{\lambda^2(M^2,m_1^2,q^2)} \times \left[A_1(q^2) \mp \frac{\lambda(M^2,m_1^2,q^2)}{(M+m_1)^2} V(q^2)\right]^2 . \tag{7}$$

We write the three dimensionless helicity amplitudes $Y_i(q^2)$ in terms of three form factors $V(q^2)$, $A_2(q^2)$, $A_2(q^2)$ defined in Ref. [3] and first measured by the E691 group via the double angular (θ_v, θ_e) distributions [8]. As previously noted [6], the relative $\mp$ sign in the right-hand side of Eq. (7) reflects the $V-A$ character of the charmed current $(\bar{s}c)$. Here M, m, m_1 are, respectively, the D, K, K^* masses; $\sqrt{q^2}$ is the momentum transfer, which is also the lepton-pair invariant mass, and $\lambda(a,b,c) = (a^2+b^2+c^2-2ab-2ac-2bc)^{1/2}$ is the totally symmetric Källen function. As remarked before, the knowledge of the form factors—not only their normalizations at one point (say at $q^2=0$) but also their q^2 dependence—is of great importance for accurate calculations of the hadronic rates. The q^2 dependence, as measured by many groups [9] are all consistent with a simple pole behavior with mass $\Lambda_v \simeq 2.1$ GeV for $f_+(q^2)$ and $V(q^2)$ and $\Lambda_a \simeq 2.5$ GeV for $A_{1,2}(q^2)$.

We now turn to the form-factor normalizations at $q^2=0$ that essentially fix the rates. For this purpose, integrations over q^2 of Eqs. (4) and (5) must be done. In the linear approximation of the form factors,

$$F^2(q^2) = F^2(0)\left[1-\frac{q^2}{\Lambda^2}\right]^{-2} \simeq F^2(0)\left[1+2\frac{q^2}{\Lambda^2}\right] ,$$

the integrations can be analytically performed. This linear approximation is largely adequate, not only because $q^2/\Lambda^2 \ll 1$ in the whole range of integration [from 0 to $(M-m_j)^2$] but mainly because of the kinematic $\lambda(M^2,m_j^2,q^2)$ term, which practically ensures that only the very small q^2 region contributes to the integrals; the most unfavorable case—when q^2 reaches its maximum value $(M-m_j)^2$—is completely suppressed by the kinematic $\lambda(M^2,m_j^2,q^2)$ term.

Then we get, in units of $\hat{\Gamma} = (G^2M^5/192\pi^3)|V_{cs}|^2 = 7.42\times 10^{11}\ \text{s}^{-1}$,

$$\Gamma(D \to Ke\nu) = \frac{1}{4} f_+^2(0)\left[g_1\left[\frac{m}{M}\right] + \left[\frac{M^2}{\Lambda_v^2}\right]\bar{g}_1\left[\frac{m}{M}\right]\right]\hat{\Gamma}, \quad \Gamma(D \to K^* e\nu) = \Gamma_{\text{trans}} + \Gamma_{\text{long}} \,, \tag{8}$$

$$\Gamma_{\text{trans}} = \Gamma_+ + \Gamma_- = \left\{V^2(0)\left[g_2\left[\frac{m_1}{M}\right] + \left[\frac{M^2}{\Lambda_v^2}\right]\bar{g}_2\left[\frac{m_1}{M}\right]\right] + A_1^2(0)\left[g_3\left[\frac{m_1}{M}\right] + \left[\frac{M^2}{\Lambda_a^2}\right]\bar{g}_3\left[\frac{m_1}{M}\right]\right]\right\}\hat{\Gamma} \,, \tag{9}$$

$$\Gamma_{\text{long}} = \Gamma_0 = \left\{A_1^2(0)\left[g_4\left[\frac{m_1}{M}\right] + \left[\frac{M^2}{\Lambda_a^2}\right]\bar{g}_4\left[\frac{m_1}{M}\right]\right] + A_2^2(0)\left[g_5\left[\frac{m_1}{M}\right] + \left[\frac{M^2}{\Lambda_a^2}\right]\bar{g}_5\left[\frac{m_1}{M}\right]\right] - A_1(0)A_2(0)\left[g_6\left[\frac{m_1}{M}\right] + \left[\frac{M^2}{\Lambda_a^2}\right]\bar{g}_6\left[\frac{m_1}{M}\right]\right]\right\}\hat{\Gamma} \,, \tag{10}$$

with

$$g_1(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x \tag{11}$$

$$\bar{g}_1(x) = \tfrac{2}{5} - 6x^2 - 32x^4 + 32x^6 + 6x^8 - \tfrac{2}{5}x^{10} - 48x^4(1+x^2)\ln x \,, \tag{12}$$

$$g_2(x)=\frac{1}{4(1+x)^2}\bar{g}_1(x)\ , \tag{13}$$

$$\bar{g}_2(x)=\frac{1}{(1+x)^2}[\tfrac{1}{15}-\tfrac{8}{5}x^2-25x^4+25x^8+\tfrac{8}{5}x^{10}-\tfrac{1}{15}x^{12}-8x^4(3+8x^2+3x^4)\ln x]\ , \tag{14}$$

$$g_3(x)=(1+x)^2[\tfrac{1}{3}+3x^2-3x^4-\tfrac{1}{3}x^6+4x^2(1+x^2)\ln x]\ , \tag{15}$$

$$\bar{g}_3(x)=(1+x)^2[\tfrac{1}{3}+\tfrac{28}{3}x^2-\tfrac{28}{3}x^6-\tfrac{1}{3}x^8+8x^2(1+3x^2+x^4)\ln x]\ , \tag{16}$$

$$g_4(x)=\left[\frac{1+x}{x}\right]^2[\tfrac{1}{16}-\tfrac{1}{3}x^2+\tfrac{3}{2}x^4-x^6-\tfrac{11}{48}x^8+\tfrac{1}{2}x^4(1+4x^2)\ln x]\ , \tag{17}$$

$$\bar{g}_4(x)=\left[\frac{1+x}{x}\right]^2[\tfrac{1}{40}-\tfrac{5}{24}x^2+\tfrac{8}{3}x^4+2x^6-\tfrac{103}{24}x^8-\tfrac{23}{120}x^{10}+x^4(1+9x^2+4x^4)\ln x]\ , \tag{18}$$

$$g_5(x)=\frac{1}{24x^2(1+x)^2}(1-9x^2+45x^4-45x^8+9x^{10}-x^{12}+120x^6\ln x)\ , \tag{19}$$

$$\bar{g}_5(x)=\frac{1}{84x^2(1+x)^2}[1-14x^2+126x^4+525x^6-525x^8-126x^{10}+14x^{12}-x^{14}+840x^6(1+x^2)\ln x]\ , \tag{20}$$

$$g_6(x)=\frac{1}{16x^2}[2(1-x^2)g_1(x)-\bar{g}_1(x)]\ , \tag{21}$$

$$\bar{g}_6(x)=\left[\frac{1+x}{2x}\right]^2[2(1-x^2)g_2(x)-\bar{g}_2(x)]\ . \tag{22}$$

In Eqs. (11)–(22) the $g_i(x)$ represent contributions for constant form factors, while $\bar{g}_i(x)$ are those due to the q^2 dependence. It turns out that the q^2 dependence enhances the width by 24% in $D\to Ke\nu$ and 10% in $D\to Ke^*\nu$. We get numerically

$$\Gamma(D\to Ke\nu)=0.195f_+^2(0)\hat{\Gamma}=1.45f_+^2(0)10^{11}\ \mathrm{s}^{-1}\ , \tag{23}$$

$$\begin{aligned}\Gamma(D\to K^*e\nu)&=[0.238A_1^2(0)+0.0105A_2^2(0)-0.078A_1(0)A_2(0)+0.0039V^2(0)]\hat{\Gamma}\\&=[1.769A_1^2(0)+0.078A_2^2(0)-0.578A_1(0)A_2(0)+0.029V^2(0)]\times10^{11}\ \mathrm{s}^{-1}\ ,\end{aligned} \tag{24}$$

$$R=\frac{\Gamma_{\mathrm{long}}}{\Gamma_{\mathrm{trans}}}=\frac{2.10A_1^2(0)+0.137A_2^2(0)-1.013A_1(0)A_2(0)}{A_1^2(0)+0.051V^2(0)}\ . \tag{25}$$

The average rate $\Gamma(D\to Ke\nu)$ is known [9] to be $(7.1\pm0.6)\times10^{10}\ \mathrm{s}^{-1}$, from which [9] $f_+(0)=0.71\pm0.06$, in striking agreement with previous predictions [6].

For $D\to K^*e\nu$, the large difference in magnitude of the coefficients accompanying the form factors $A_1(0)$, $A_2(0)$, and $V(0)$ in Eqs. (24) and (25) indicates that the contributions of the vector $V(q^2)$ are negligible in both the absolute rate Γ and the ratio R. Only the axial vector $A_1(q^2)$ and its interference with $A_2(q^2)$ are significant. This observation suggests that the form factors $A_1(0)$ and $A_2(0)$ can be safely determined by using Γ and R as the contour constraints represented, respectively, by an ellipse and a straight line in the (A_1, A_2) plane illustrated in Fig. 1. The plot could, to some extent, bypass the double angular measurements (θ_v,θ_e) utilized to extract the form factors [8]. Only the single-angle θ_v distribution is needed to measure R. Once this quantity is known, combining it with the absolute rate Γ is then sufficient for fixing $A_1(0)$ and $A_2(0)$.

While data on Γ are in good agreement among six experiments, with the average [9] $\Gamma(D\to K^*e\nu)=(4.3\pm0.6)\times10^{10}\ \mathrm{s}^{-1}$, the experimental situation [9] is still unclear for the ratio R ranging from $1.8^{+0.6}_{-0.4}\pm0.3$ (E691) [8] to $0.47^{+0.95}_{-0.12}\,{}^{+0.90}_{-0.15}$ (Mark III) [10].

In spite of the uncertainty in R, it is remarkable that the $A_1(0)$ form factor which we find to be around 0.55 ± 0.10 is quite stable from all experiments[1] as indicated by the ellipse of Fig. 1. This is due to the fact that the rate Γ depends almost on $A_1(0)$. The value $A_1(0)=0.55\pm0.10$ which we obtain is only about half of what was predicted by many theoretical models

[1]Among the six experimental groups ARGUS, CLEO, E653, E691, Mark III, and WA82, the Mark III collaboration has a higher Γ rate than the five others. Also their ratio $R=\Gamma_{\mathrm{long}}/\Gamma_{\mathrm{trans}}$ is smaller than that of E691 and E653 but similar to WA82. Combining Γ and R of the Mark III data, we get $A_1(0)=0.85\pm0.10$ and $A_2(0)=1.4-1.8$ using the $V(0)=0.9$ of E691, while we get $A_1(0)=0.55\pm0.10$ from the average data of the five other groups.

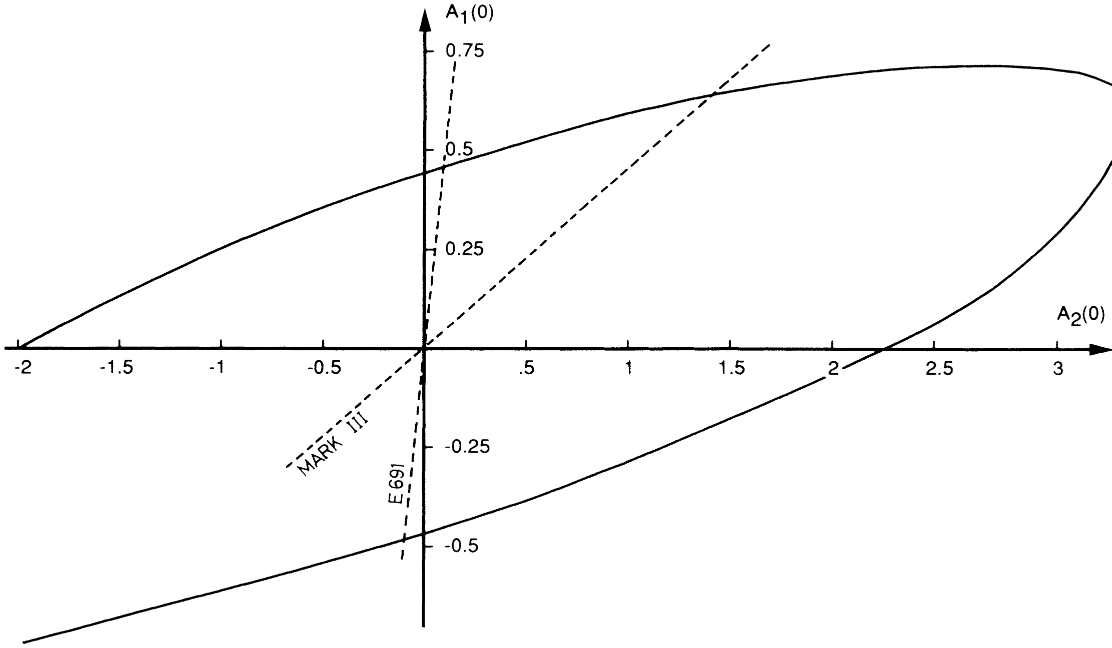

FIG. 1. Contour constraints in the (A_1, A_2) plane. The ellipse, from Eq. (24), is for the rate $\Gamma(D\to K^*e\nu)$. The dashed straight lines, from Eq. (25), are for the ratio $R=\Gamma_{\rm long}/\Gamma_{\rm trans}$. The E691 and the Mark III central values of R are indicated along the straight lines.

[3,6,7,11].

On the other hand, the form factor $A_2(0)$ depends essentially on R as shown by the straight lines of Fig. 1. From the central value of the Mark III data on R, we obtain $A_2(0)=1.45$,[1] very different from the E691 result $A_2(0)=0\pm0.1\pm0.2$. The uncertainties in R are reflected in the uncertainties in $A_2(0)$, in agreement with Ref. [12].

We summarize the present situation of the form factors: $f_+(0)=0.71\pm0.06$ and $A_1(0)=0.55\pm0.10$ can be considered as settled.[1] $V(0)$ is insignificant for our purposes in both the leptonic and hadronic modes, as will be discussed later. Only $A_2(0)$ is still not fixed; it ranges from $0\pm0.1\pm0.2$ to 1.45 ± 0.4, depending on our choice of the E691 or the Mark III data.[1]

III. NONLEPTONIC DECAYS

By factorization, the rate for D^0 decays into a K^- and a nonstrange hadronic system X (X can be one-particle or multiparticle) can be written as

$$\frac{d\Gamma}{dq^2}(D^0\to K^-X)=a_1^2\frac{G^2|V_{ud}V_{cs}^*|^2}{32\pi M^3}f_+^2(q^2)\lambda(M^2,m^2,q^2)\times\{(M^2-m^2)^2a_0(q^2)+\lambda^2(M^2,m^2,q^2)[v_1(q^2)+a_1(q^2)]\}\ , \tag{26}$$

which is completely similar to the hadronic decay of the heavy lepton τ, as given by [13]

$$\frac{d\Gamma}{dq^2}(\tau^+\to\bar\nu_\tau X)=\frac{G^2|V_{ud}|^2}{16\pi M_\tau^3}(M_\tau^2-q^2)^2\{M_\tau^2a_0(q^2)+(M_\tau^2+2q^2)[v_1(q^2)+a_1(q^2)]\}\ . \tag{27}$$

Here $v_J(q^2)$, $a_J(q^2)$ $(J=0,1)$ are, respectively, the dimensionless vector and axial-vector spectral functions [13] associated with the hadronic state X having a total angular momentum J. They are defined as

$$\sum_X\langle 0|\bar u\gamma_\mu(1-\gamma_5)d|X\rangle\langle X|\bar d\gamma_\nu(1-\gamma_5)u|0\rangle(2\pi)^3\delta^4(q-p_x)=(-q^2g_{\mu\nu}+q_\mu q_\nu)[v_1(q^2)+a_1(q^2)]+q_\mu q_\nu a_0(q^2)\ . \tag{28}$$

The advantage of these general formulas is that not only the two-body modes but also the continuum and multibody channels can be treated on the same footing. The two-body modes are only a particular case in which the spectral functions $v_J(q^2)$ and $a_J(q^2)$ are replaced, in the narrow-width approximation, by the δ function $\delta(q^2-m_X^2)$ times the corresponding decay constant squared. Some examples are given below:

Particle X	Dimensionless spectral functions
π	$a_0(q^2)=f_\pi^2\delta(q^2-m_\pi^2)$
$\rho(770)$	$v_1(q^2)=f_\rho^2\delta(q^2-m_\rho^2)\to\frac{f_\rho^2\gamma_\rho m_\rho}{\pi}\frac{1}{(q^2-m_\rho^2)^2+\gamma_\rho^2 m_\rho^2}$
$a_1(1260)$	$a_1(q^2)=f_A^2\delta(q^2-m_A^2)\to\frac{f_A^2\gamma_A m_A}{\pi}\frac{1}{(q^2-m_A^2)^2+\gamma_A^2 m_A^2}$.

(29)

For the $\rho(770)$ and especially for the broad $a_1(1260)$ mesons, the inadequate narrow-width approximation must be improved by replacing the δ functions in Eq. (29) with the Breit-Wigner form in which the full widths γ are taken into account. The substitution is made according to the prescription [13]

$$\delta(q^2-m^2)\to\frac{\gamma m}{\pi}\frac{1}{(q^2-m^2)^2+\gamma^2m^2}\ , \tag{30}$$

which represents, to a certain degree, the final-state interactions.

In Eq. (29), f_π is the usual pion decay constant ($\simeq 132$ MeV), f_ρ is defined by $\langle 0|V_\mu|\rho^\pm\rangle=\epsilon_\mu f_\rho m_\rho$, and f_A is defined similarly. The constant f_ρ is $\sqrt{2}$ times the ρ^0 decay coupling measured by the rate $\rho^0\to e^+e^-$, which gives $f_\rho\simeq 215$ MeV. Unlike f_π and f_ρ, the decay constant f_A is not so accurately known. Direct information comes from the decay $\tau\to\nu a_1(1260)$. In the narrow-width approximation of $\rho(770)$ and $a_1(1260)$, we have

$$\frac{B(\tau\to\nu a_1)}{B(\tau\to\nu\rho)}=\left[\frac{f_A}{f_\rho}\right]^2\frac{(M_\tau^2-m_A^2)^2(M_\tau^2+2m_A^2)}{(M_\tau^2-m_\rho^2)^2(M_\tau^2+2m_\rho^2)}$$
$$=0.55\left[\frac{f_A}{f_\rho}\right]^2 ,$$

from which we get $(f_A/f_\rho)^2=0.86$, where data on τ are given in a recent review [14]. Taking into account the ρ and $a_1(1260)$ widths by putting the Breit-Wigner forms, Eq. (29), into Eq. (27), and after integration over q^2, we get numerically (for $\gamma_A=400$ MeV)

$$\Gamma(\tau\to\nu a_1)=0.393\frac{G^2|V_{ud}|^2}{16\pi}M_\tau^3 f_A^2\ ,$$
$$\Gamma(\tau\to\nu\rho)=0.804\frac{G^2|V_{ud}|^2}{16\pi}M_\tau^3 f_\rho^2\ . \tag{31}$$

The Breit-Wigner–corrected coefficients 0.393 and 0.804 in Eq. (31) replace, respectively, the coefficients $(1-m_A^2/M_\tau^2)^2(1+2M_A^2/M_\tau^2)=0.5$ and $(1-m_\rho^2/M_\tau^2)^2(1+2M_\rho^2/M_\tau^2)=0.91$ of the narrow-width approximation. From Eq. (31) we then get $(f_A/f_\rho)^2=0.97$. Both values 0.86 and 0.97 for $(f_A/f_\rho)^2$ are consistent with the first Weinberg sum rule [15], written in our notation as $f_\rho^2-f_A^2=f_\pi^2$ and giving $(f_A/f_\rho)^2=0.62$. In the following we will take $(f_A/f_\rho)^2=0.97$ corresponding to $f_A=212$ MeV.

Furthermore, from the conserved vector current, the continuum spectral function $v_1(q^2)$ can be related to the cross section of e^+e^- annihilation into the isospin-1 hadrons [13]

$$v_1(q^2)=\frac{q^2\sigma_{I=1}(e^++e^-\to\text{hadrons})}{8\pi^3\alpha^2}\ . \tag{32}$$

Similarly, the rate $\Gamma(D^{-0}\to K^*X)$ can also be written as

$$\frac{d\Gamma}{dq^2}(D^0\to K^{-*}X)=a_1^2\frac{G^2|V_{cs}^*V_{ud}|^2}{32\pi M^3}\lambda^3(M^2,m_1^2,q^2)\times\left[Y(q^2)[v_1(q^2)+a_1(q^2)]+\left[\frac{M+m_1}{2m_1}\right]^2\left[A_1(q^2)-\frac{M-m_1}{M+m_1}A_2(q^2)\right]^2a_0(q^2)\right] , \tag{33}$$

where $Y(q^2)=Y_0(q^2)+Y_+(q^2)+Y_-(q^2)$.

Putting Eq. (29) into Eqs. (26) and (33), the principal two-body decay rates are [in units of $(G^2|V_{cs}^*V_{ud}|^2/32\pi)M^5=6\pi^2|V_{ud}|^2\hat{\Gamma}$]:

$$\Gamma(D^0\to K^-\pi^+)=a_1^2\left[\frac{f_\pi}{M}\right]^2\lambda^3\left[1,\frac{m^2}{M^2},\frac{m_\pi^2}{M^2}\right]f_+^2(m_\pi^2)\ , \tag{34}$$

$$\Gamma(D^0\to K^-\rho^+)=a_1^2\left[\frac{f_\rho}{M}\right]^2\left[\frac{\gamma_\rho m_\rho}{\pi M^2}\int_{(2m_\pi/M)^2}^{[(M-m)/M]^2}dx\, f_+^2(M^2x)\frac{\lambda^3(1,m^2/M^2,x)}{(x-m_\rho^2/M^2)^2+\gamma_\rho^2m_\rho^2/M^4}\right] \tag{35}$$

$$\to a_1^2\left[\frac{f_\rho}{M}\right]^2\lambda^3\left[1,\frac{m^2}{M^2},\frac{m_\rho^2}{M^2}\right]f_+^2(m_\rho^2)\ , \tag{35'}$$

$$\Gamma(D^0\to K^-a^+)=a_1^2\left[\frac{f_A}{M}\right]^2\left[\frac{\gamma_A m_A}{\pi M^2}\int_{(3m_\pi/M)^2}^{[(M-m)/M]^2}dx\, f_+^2(M^2x)\frac{\lambda^3(1,m^2/M^2,x)}{(x-m_A^2/M^2)^2+\gamma_A^2m_A^2/M^4}\right] \tag{36}$$

$$\to a_1^2\left[\frac{f_A}{M}\right]^2\lambda^3\left[1,\frac{m^2}{M^2},\frac{m_A^2}{M^2}\right]f_+^2(m_A^2)\ , \tag{36'}$$

$$\Gamma(D^0\to K^{*-}\pi^+)=a_1^2\left[\frac{f_\pi}{M}\right]^2\lambda^3\left[1,\frac{m_1^2}{M^2},\frac{m_\pi^2}{M^2}\right]\left[\frac{M+m_1}{2m_1}\right]^2\left[A_1(m_\pi^2)-\frac{M-m_1}{M+m_1}A_2(m_\pi^2)\right]^2, \tag{37}$$

$$\Gamma(D^0\to K^{*-}\rho^+)=a_1^2\left[\frac{f_\rho}{M}\right]^2\left[\frac{\gamma_\rho m_\rho}{\pi M^2}\int_{(2m_\pi/M)^2}^{[(M-m_1)/M]^2}dx\frac{\lambda^3(1,m_1^2/M^2,x)}{(x-m_\rho^2/M^2)^2+\gamma_\rho^2 m_\rho^2/M^4}Y(M^2x)\right] \tag{38}$$

$$\to a_1^2\left[\frac{f_\rho}{M}\right]^2\lambda^3\left[1,\frac{m_1^2}{M^2},\frac{m_\rho^2}{M^2}\right]Y(m_\rho^2)\ . \tag{38'}$$

Equations (35′), (36′), and (38′) correspond to the zero-width approximation. Numerically Eqs. (38) and (38′) are, respectively [the term $a_1^2(f_\rho/M)^2$ is not included],

$$0.717A_1^2(0)+0.0145A_2^2(0)-0.1462A_1(0)A_2(0)+0.0146V^2(0)\ ,$$

$$0.88A_1^2(0)+0.0116A_2^2(0)-0.1478A_1(0)A_2(0)+0.0192V^2(0)\ .$$

As in the leptonic modes, the vector form factor $V(q^2)$ does not contribute to $D\to K^*\pi$. Its contribution is also insignificant in the $K^*\rho$ mode where, practically, only $A_1(q^2)$ and $A_2(q^2)$ count.

Since all the class-I decay rates depend on the (hard-gluon-corrections) coefficient a_1, which is sensitive to both Λ_{QCD} and the scale μ, it is more convenient to factor it out and to compare, in the third column of Table I, all the branching ratios with the $D^0\to K^-\pi^+$ one taken as a starting-point input. For the $K^-\rho^+$, $K^-a_1^+(1260)$, and

TABLE I. Comparison of experimental data (column 2) with theoretical calculations [column 3 for $a_1=1.02$, column 4 for $a_1=1.11$ ($N_c=3$), column 5 for $a_1=1.30$ ($N_c=\infty$)]. Theoretical predictions in parentheses correspond to the narrow-width approximation.

	Experiments[a]	Branching ratios (percent), theoretical predictions		
Modes	Branching ratios (percent)	$a_1=1.02$ (from fit to the $D^0\to K^-\pi^+$ data taken as input)	$a_1=c_1+\frac{1}{3}c_2$ = 1.11 ($N_c=3$)	$a_1=c_1=1.3$ ($N_c=\infty$)
$D^0\to K^-\pi^+$	3.71±0.25	3.71	4.39	6.02
$D^0\to K^-\rho^+$	7.8±1.1	5.53 (6.31)	6.54 (7.47)	8.98 (10.25)
$D^0\to K^-a_1^+(1260)$	9±0.9±1.7[b]	1.29[c] 1.28 (0.86) 1.25	1.52[c] 1.51 (1.02) 1.48	2.06[c] 2.05 (1.39) 2.01
$D^0\to K^{-*}\pi^+$	4.6±0.4	2.13[d] 0.38[e]	2.52[d] 0.45[e]	3.46[d] 0.61[e]
$D^0\to K^{-*}\rho^+$	6.2±2.3±2[b]	4.00[d] (4.92) 5.98[e] (7.83)	4.73[d] (5.82) 7.09[e] (9.27)	6.49[d] (7.99) 9.72[e] (12.71)
$D^0\to K^-\pi^+\pi^0$	9.1±1.2[f] 7.3±3.6±0.9[g]	5.56	6.58	9.03
$D^0\to K^-\pi^+\pi^-\pi^+\pi^0$	$3.8^{+2}_{-1.5}$[g] 5.2±0.7±0.6[h]	0.5	0.59	0.81
$D^0\to K^-\pi^+\pi^0\pi^0\pi^0$		0.07	0.08	0.10

[a] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990) (PDG 1990).
[b] The Mark III Collaboration [10].
[c] The three values in decreasing order are obtained from the γ_A width taken, respectively, to be 500, 400, and 300 MeV.
[d] Calculations using the E691 form factors [8] $A_1=0.46, A_2=0, V=0.9$.
[e] Calculations using the form factors $A_1=0.70, A_2=1.45, V=0.9$ extracted from the Mark III data [10]. Very similar results are obtained with another set of form factors compatible with Mark III data: $A_1=0.85, A_2=1.8, V=0.9$.
[f] This branching ratio is obtained (PDG 1990) when $D^0\to K^{-*}\pi^+$ and $D^0\to K^{0*}\pi^0$ followed by $K^{-*}\to K^-\pi^0$, and $K^{0*}\to K^-\pi^+$ are subtracted.
[g] ACCMOR Collaboration [22].
[h] E691 Collaboration [23].

$K^{-*}\rho^+$ modes we have computed rates taking into account the $a_1(1260)$ and $\rho(770)$ widths via the Breit-Wigner factor. Predictions from the zero-width approximation are also indicated in parentheses. For the $a_1^+(1260)$ meson, three values of γ_A (500, 400, and 300 MeV) are used in Eq. (36), and the corresponding rates are given in decreasing order. As can be seen in Table I and Eq. (31), for all decays $\tau\to\nu a_1$, $\tau\to\nu\rho$, $D\to K\rho$, and $D\to K^*\rho$ the rates calculated with the Breit-Wigner form are always smaller than the ones using the zero-width approximation, except for the $D\to Ka_1$ mode where the inverse is true. The reason may be the tiny phase space available in the latter case.

Also, for the $K^*\pi$ and $K^*\rho$ modes, two theoretical predictions are given using either E691 form factors or those extracted from the Mark III data (see Sec. II, footnote 1). We note that for the latter data, a too small $D\to K^*\pi$ rate is obtained. This small rate prediction is a very general feature of factorization when the longitudinal helicity amplitude $Y_0(q^2)$ is small compared to the transverse ones $Y_\pm(q^2)$. In the case of small $R=\Gamma_{\rm long}/\Gamma_{\rm trans}$, the form factor $A_2(q^2)$ is comparable to or larger than $A_1(q^2)$, as can be seen directly from Eq. (6). Moreover, we realize that if factorization holds, the ratio R is intimately related to the ratio $(D^0\to K^*\pi)/(D^0\to K^*\rho)$. This quantity is in fact proportional to $R/(1+R)$ averaged. Further precise measurements of $D\to K^*l\nu$ are needed to determine the R parameter and, consequently, to enable a more consistent test of factorization.

In the fourth and fifth columns of Table I, the coefficient a_1 is not fitted to the $D^0\to K^-\pi^+$ data but is taken to be 1.11 and 1.3, corresponding, respectively, to the number of colors ($N_c=3$ and $N_c=\infty$). The resulting branching ratios follow. Several processes have been previously considered [2–4] in the $N_c=\infty$ case, without information on the recently available form factors. We finally remark that variance between experiment and theory is more pronounced in the two modes $D^0\to K^-a_1^+(1260)$ and $D^0\to K^{-*}\pi^+$ than in the others.

IV. THE MULTIBODY DECAY MODES

Our approach does not only provide a tool for analyzing two-body decays. The multichannel decay $D^0\to K^-(2n\pi)^+$ can be treated on the same footing. We first discuss the $D^0\to K^-\pi^+\pi^0$ case. Within the factorization assumption, the absolute rate and the dipion invariant-mass distribution can be unambiguously predicted and checked experimentally.

From Eqs. (26) and (32) the dipion invariant-mass y distribution is given by ($y=\sqrt{q^2}$)

$$\frac{d\Gamma_2}{dy}=a_1^2\frac{G^2|V_{cs}^*V_{ud}|^2}{384\pi^3M^3}f_+^2(y)\lambda^3(M^2,m^2,y^2)\times y\left[1-\frac{4m_\pi^2}{y^2}\right]^{3/2}|F_\pi(y)|^2 =a_1^2|V_{cs}^*V_{ud}|^2f_+^2(0)\frac{d\tilde{\Gamma}_2}{dy}\,, \quad (39)$$

where $F_\pi(q^2)$ is the pion electromagnetic form factor measured from $e^++e^-\to\pi^++\pi^-$:

$$q^2\sigma(e^++e^-\to\pi^+\pi^-)=\frac{\pi\alpha^2}{3}\left[1-\frac{4m_\pi^2}{q^2}\right]^{3/2}|F_\pi(q^2)|^2\,. \quad (40)$$

Following the $\tau^+\to\bar{\nu}_\tau\pi^+\pi^0$ analyses [16], in the region $0.3<y<0.9$ GeV we use the pion form factor measured at Orsay [17], and for $0.9<y<1.34$ GeV the Novosibirsk data [18] are taken as input. In Fig. 2 we plot the normalized $d\tilde{\Gamma}_2/dy$. Measurement of the dipion mass distribution in $D^0\to K^-\pi^+\pi^0$ provides clearly an independent test of factorization. Of course, the contamination from $D^0\to K^*\pi$ followed by $K^*\to K\pi$ must be subtracted. Numerical integration over the whole range of y yields $\tilde{\Gamma}_2=27.5\times10^{10}$ s^{-1}, giving $\Gamma(D^0\to K^-\pi^+\pi^0)=12.47a_1^2\times10^{10}$ s^{-1}; hence $B(D^0\to K^-\pi^+\pi^0)/B(D^0\to K^-\pi^+)$ is predicted to be 1.50 if factorization holds (see Table I).

We next discuss $D^0\to K^-(4\pi)^+$. Following the treatment of τ decay into four pions [16], we get from the isospin decomposition [16]

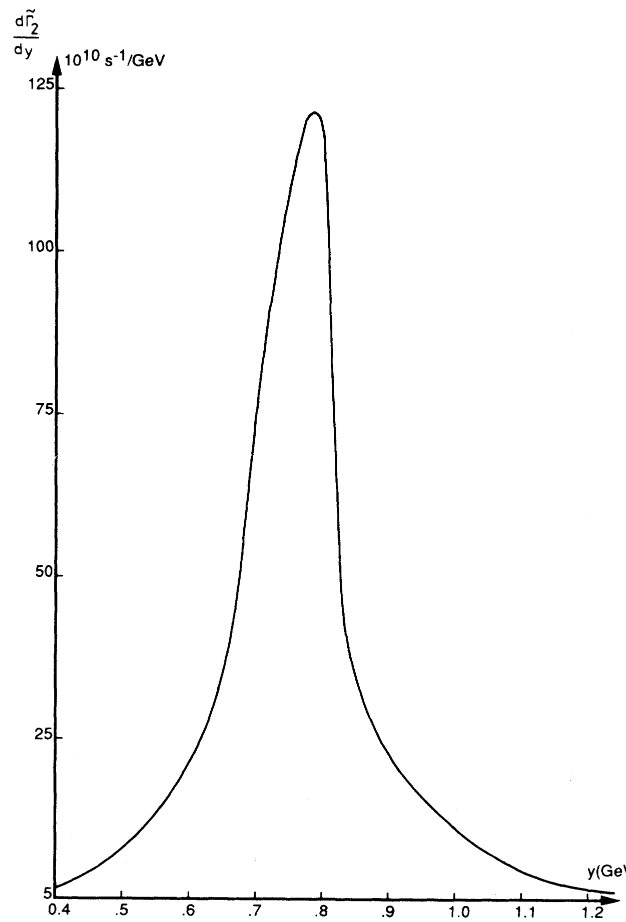

FIG. 2. The normalized $d\tilde{\Gamma}_2/dy$ two-pion invariant-mass y distribution [Eq. (39)].

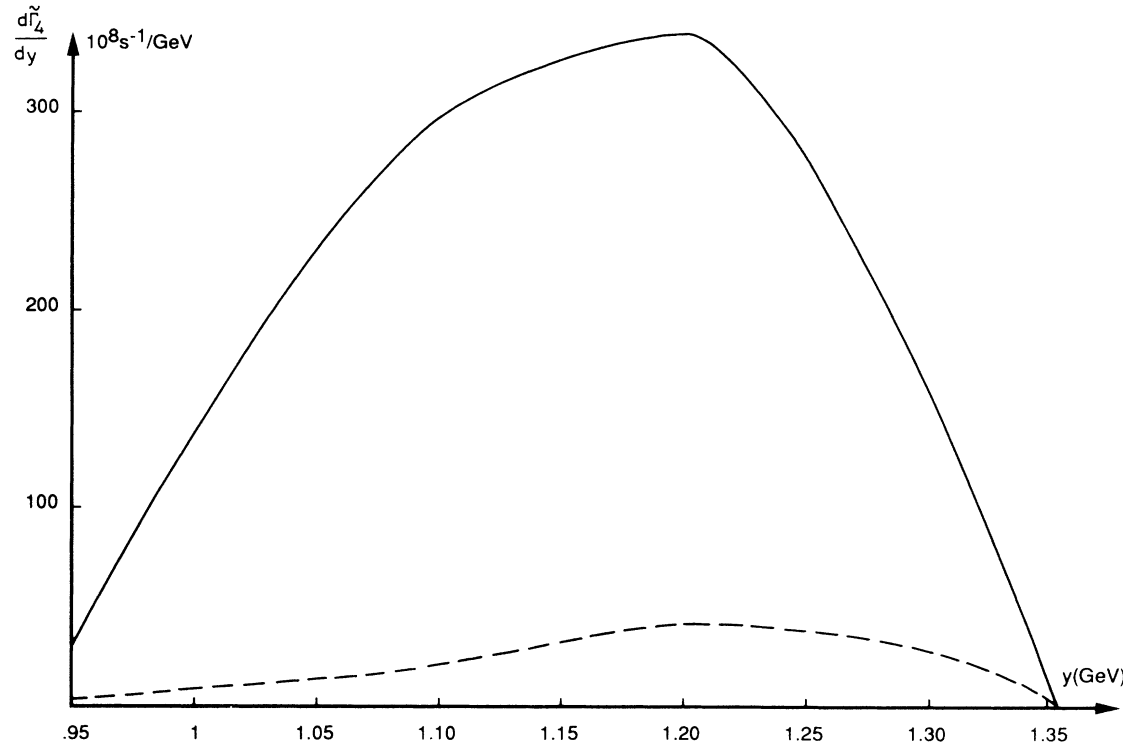

FIG. 3. The normalized $d\tilde{\Gamma}_4/dy$ four-pion invariant-mass y distribution [Eq. (44)]: solid line, $d\tilde{\Gamma}_4/dy(D^0\to K^-\pi^+\pi^-\pi^+\pi^0)$; dashed line, $d\tilde{\Gamma}_4/dy(D^0\to K^-\pi^+\pi^0\pi^0\pi^0)$.

$$\frac{d\Gamma_4}{dq^2}(D^0\to K^-4\pi)=a_1^2|V_{cs}^*V_{ud}|^2\frac{G^2}{256\alpha^2M^3\pi^4}\frac{dF}{dq^2}\ , \tag{41}$$

with

$$\frac{dF}{dq^2}(D^0\to K^-\pi^+\pi^0\pi^0\pi^0) =f_+^2(q^2)\lambda^3(M^2,m^2,q^2) \times[q^2\tfrac{1}{2}\sigma(e^+e^-\to\pi^+\pi^-\pi^+\pi^-)]\ , \tag{42}$$

$$\frac{dF}{dq^2}(D^0\to K^-\pi^+\pi^-\pi^+\pi^0) =f_+^2(q^2)\lambda^3(M^2,m^2,q^2)q^2 \times[\tfrac{1}{2}\sigma(e^+e^-\to\pi^+\pi^-\pi^+\pi^-) +\sigma(e^+e^-\to\pi^+\pi^-\pi^0\pi^0)]\ . \tag{43}$$

Data for $e^+e^-\to\pi^+\pi^-\pi^+\pi^-$ and $e^+e^-\to\pi^+\pi^-\pi^0\pi^0$ are taken from Ref. [16] in which the Frascati [19], Novosibirsk [18], and Orsay [17] results were used: both cross sections are very well measured from the threshold to the kinematic limit $M-m\simeq 1.4$ GeV in contradistinction with $\tau\to\nu(4\pi)$ case considered in Ref. [16] where higher-energy e^+e^- cross sections are needed (but measured with big errors). The normalized four-pion invariant-mass distributions $d\tilde{\Gamma}_4/dy$, defined as ($y=\sqrt{q^2}$)

$$\frac{d\Gamma_4}{dy}=2\sqrt{q^2}\frac{d\Gamma_4}{dq^2}=a_1^2|V_{cs}^*V_{ud}|^2f_+^2(0)\frac{d\tilde{\Gamma}_4}{dy}\ , \tag{44}$$

are plotted in Fig. 3, from which we obtain, after numerical integrations,

$$\Gamma(D^0\to K^-\pi^+\pi^0\pi^0\pi^0)=(0.12\times 10^{10}\ \mathrm{s}^{-1})a_1^2\ ,$$

$$\Gamma(D^0\to K^-\pi^+\pi^-\pi^+\pi^0)=(0.91\times 10^{10}\ \mathrm{s}^{-1})a_1^2\ ,$$

giving $B(D^0\to K^-4\pi)\simeq 0.5\%$.

Interpretation of $D^0\to K^-4\pi$ in terms of the class-I D^0 decay must be cautious because of possible contamination by two- and/or three-body subresonant modes coming from both charged and neutral currents (classes I and II of Ref. [3]). Contamination from class II, $D^0\to\bar{K}^{*0}\eta$, $\bar{K}^{*0}\omega$ followed by $\bar{K}^{*0}\to K^-\pi^+$ and $(\eta,\omega)\to\pi\pi\pi$, have been already subtracted in these experiments. In Table I, results for both $D^0\to K^-(2\pi)$ and $D^0\to K^-(4\pi)$ are given.

V. SUMMARY AND CONCLUSION

In Sec. II, we made a detailed analysis of the $D\to K^*e\nu$ decay in order to extract information on the form factors. We found that $A_1(0)$ must be around 0.55 ± 0.10 from all experiments except the Mark III Collaboration from which we get a higher value $A_1(0)=0.85$. As for $A_2(0)$, the present situation is still unclear; it ranges from 0 to 1.45 due to the uncertainties of the ratio $\Gamma_{\rm long}/\Gamma_{\rm trans}$. The contributions of the vector form factor $V(q^2)$ are negligible, and $f_+(0)$ in $D \to Ke\nu$ is well determined to be 0.71 ± 0.06. Equipped with these form factors, the two-body hadronic decays were discussed in Sec. III with results summarized in Table I. While for some modes agreement between experiment and theory is fair (within a factor of 2), in the others such as $D^0 \to K^{-*}\pi^+$ and $D^0 \to K^- a_1^+(1260)$, much larger differences are found. In Sec. IV, the multichannel decays $D^0 \to K^-(2\pi)$ and $D^0 \to K^-(4\pi)$ were analyzed, providing a further test of factorization. Compared to experiment, the $D^0 \to K^-(2\pi)$ prediction is rather acceptable, while the $D^0 \to K^-(4\pi)$ one is found to be too small. (See Table I.) However, for the latter case, the comparison should be considered with caution since possible resonant subcomponent contamination coming from both charged and neutral currents must be subtracted. It should be noted, however, that in these experiments, the subtraction has already been done for the two principal sources of contamination, $\bar{K}^{*0}\eta$ and $\bar{K}^{*0}\omega$.

Before any firm conclusion can be drawn on the factorization approach, the problem of form factors must be unambiguously settled. Since the uncertainty involves the K^* via the $A_2(q^2)$ form factor, any statement on the $D^0 \to K^{-*}\pi^+$ mode seems premature. On the other hand, for the $D^0 \to K^- a_1^+(1260)$ and $D^0 \to K^-(4\pi)$ decays, the theoretical predictions are much smaller than experimental values. For the former case, one might think that such enhancement—as suggested by data—could come from the annihilation mechanism implemented with a resonance near the D mass according to the scenario $D^0 \to_{\text{weak}} V^* \to_{\text{strong}} K^- a_1(1260)$, where V^* denotes a $J^P = 1^-$ vector meson. A candidate for this V^* intermediate state is the $K^*(1680)$. However, such a scenario is untenable since the mode $K^-\pi^+$ must be enhanced by the same order (or an even larger order due to phase space). This has already been ruled out by experiments. One then turns to the eternal problem of final-state interactions (which is not the purpose of our paper), remembering, however, that the mode $D^0 \to K^0\bar{K}^0$ is the cleanest example in which final-state interactions are shown to be important [20] and experimentally confirmed [21]. It should be finally mentioned that the factorization approach may be only suitable for energetic two-body (or quasi-two-body) transitions; for $D \to Ka_1(1260)$ with very little energy release, the approximation is questioned.

ACKNOWLEDGMENTS

Laboratoire de Physique Théorique et Hautes Energies, Paris is Laboratoire Associe au C.N.R.S., U.A. 280.

[1] D. Fakirov and B. Stech, Nucl. Phys. **B133**, 315 (1978).
[2] X. Y. Pham, Acta Phys. Pol. B **11**, 735 (1980).
[3] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987); **29**, 637 (1985).
[4] A. J. Buras, J. M. Gérard, and R. Rückl, Nucl. Phys. **B268**, 16 (1986).
[5] M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33**, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. **52B**, 351 (1974).
[6] R. Nabavi, X. Y. Pham, and W. N. Cottingham, J. Phys. G **3**, 1485 (1977); X. Y. Pham and J.-M. Richard, Nucl. Phys. **B138**, 453 (1978).
[7] J. G. Körner and G. A. Schuler, Z. Phys. C **46**, 93 (1990); F. J. Gilman and R. L. Singleton, Phys. Rev. D **41**, 142 (1990).
[8] J. C. Anjos *et al.*, Phys. Rev. Lett. **65**, 2630 (1990).
[9] R. Kass, invited talk at the 4th International Symposium on Heavy Flavour Physics, Orsay, 1991 (unpublished); M. S. Witherell, *ibid.*; D. Potter, in *Proceedings of the Joint International Lepton-Photon Symposium and European Conference on High Energy Physics,* Geneva, Switzerland, 1991, edited by S. Hegarty, K. Potter, and E. Quercigh (World Scientific, Singapore, 1992).
[10] Z. Bai *et al.*, Phys. Rev. Lett. **66**, 1011 (1991); D. Coffman *et al.*, Phys. Rev. D **45**, 2196 (1992).
[11] B. Grinstein, N. Isgur, D. Scora, and M. B. Wise, Phys. Rev. D **39**, 799 (1989); J. G. Korner and G. A. Schuler, Z. Phys. C **38**, 511 (1988); **41**, 690(E) (1988).
[12] J. G. Körner, K. Schilcher, M. Wirbel, and Y. L. Wu, Z. Phys. C **48**, 663 (1990).
[13] Y. S. Tsai, Phys. Rev. D **4**, 2821 (1971).
[14] M. L. Perl, Rep. Prog. Phys. (to be published).
[15] S. Weinberg, Phys. Rev. Lett. **18**, 507 (1967).
[16] F. J. Gilman and S. H. Rhie, Phys. Rev. D **31**, 1066 (1985).
[17] D. Benaksas *et al.*, Phys. Lett. **40B**, 685 (1972).
[18] V. Sidorov, quoted in F. J. Gilman and S. H. Rhie, Phys. Rev. D **31**, 1066 (1985).
[19] C. B. Bacci *et al.*, Nucl. Phys. **B152**, 215 (1979).
[20] X. Y. Pham, Phys. Lett. B **193**, 331 (1987); the same result is also reached by L. L. Chau and H. Y. Cheng, UCD PHYS-Pub-91-26 (1991).
[21] The experimental situation is reviewed by B. Gittleman, in *Proceedings of the 25th International Conference on High Energy Physics,* Singapore, 1990, edited by K. K. Phua and Y. Yamaguchi (World Scientific, Singapore, 1991); P. E. Karchin, in *Proceedings of the XIVth International Symposium on Lepton and Photon Interactions,* Stanford, California, 1989, edited by M. Riordan (World Scientific, Singapore, 1990); see also R. Kass, invited talk at the 4th International Symposium on Heavy Flavour Physics, Orsay, 1991 (unpublished).
[22] S. Barlag *et al.*, Phys. Lett. B **232**, 561 (1989).
[23] J. C. Anjos *et al.*, Phys. Rev. D **42**, 2414 (1990).