PHYSICAL REVIEW D VOLUME 46, NUMBER 6 15 SEPTEMBER 1992

On-shell expansion of effective action and quark-line diagrams in quantum chromodynamics

M. Komachiya and R. Fukuda
Department of Physics, Faculty of Science and Technology, Keio University, Yokohama 223, Japan
(Received 17 December 1991; revised manuscript received 27 March 1992)

The formalism of the on-shell expansion of the effective action allows us to obtain an exact expression for the scattering amplitude among the bound states which are excited above the nonperturbatively condensed vacuum. The on-shell expansion scheme is first generalized to the Grassmannian variables and then applied to get the two-body hadronic scattering amplitude within the framework of quantum chromodynamics. It automatically separates the diagrams which contribute to the condensation, wave function, and scattering parts. An exact expression for the quark-line diagrams are thus obtained.

PACS number(s): 11.10.Ef, 11.10.St, 12.38.Aw

I. INTRODUCTION

The effective action in field theory is one of the powerful tools for the investigation of symmetry-breaking problems [1]. The important feature is that the stationary condition of the effective action presents the nonperturbative vacuum solutions if they exist at all. Another important aspect of the effective action is its close relationship with particle scattering [2]. Namely, the connected S-matrix elements can be derived directly from the effective action once the corresponding particle modes, or the wave functions, are determined. The combination of these on-shell characters of the effective action may present a general formalism to study particle scattering in the interesting case where the ground state realizes nonperturbative condensations. However, so far less attention seems to have been paid to this possibility.

Recently, a novel way of studying the on-shell properties of the effective action has been developed in the form of the on-shell expansion [3–6]. It can be a basis for the investigation of the subjects listed above. Indeed, by using this method, the information on the following physical content of the theory is obtained step by step.

(1) The ground state (or the vacuum), whether or not it is a nonperturbatively condensed one.

(2) The particle modes above the vacuum determined in (1). If the vacuum realizes a condensation, the particle spectrum thus determined is the mode excited above the condensed vacuum.

(3) The scattering among the modes obtained in (2). The scattering diagrams are derived in the form of the tree-type graphs where the propagator and the vertex are written in terms of the second- and the higher-order derivatives of the effective action, respectively.

These exhaust all the physical information of the theory and the fact really shows that the effective action in field theory is a generating functional of the observable quantities.

The purpose of this paper is to apply this method to the problem of particle scattering above the condensed vacuum. In particular, we consider the application to quantum chromodynamics (QCD), which is one of the most interesting theories realizing the condensation phenomena: a quark-antiquark pair condenses leading to chiral-symmetry breaking and the gluon also condenses in a color-singlet combination. By starting from the formerly obtained expression of the effective action for QCD [7], we exemplify the derivation of the S-matrix elements of the two-body hadronic scattering within the level of the quark-line diagram. Based on the functional method including the effective action approach, there are several attempts to examine the observable information of QCD [8–10]. The study of this paper is concentrated on the formal application of an *exact* framework to the problem of hadronic scatterings. However, since the approach is *systematic,* it can be a useful starting point of the actual evaluation of the S-matrix element especially in the high-energy region. Although QCD is not solved at present, our expression may also serve as a firm basis of the phenomenological analysis.

The important role of the effective action lies in the fact that, when we define the effective action, the nontrivial change of the variable from the artificially introduced external source J to the expectation value of the field operator is made through the Legendre transformation. Graphically, the variable of the effective action includes an infinite series of the perturbative diagrams and this fact enables us to study the nonperturbative aspects of a given model in general. If we study the scattering diagrams by using only the perturbative Green's functions in the presence of J (i.e., without the Legendre transformation), the summations of the infinitely many subdiagrams are required in order to include the nonperturbative phenomena before setting $J=0$. Indeed, when we examine the general N-body channels, such resummations lead to the concept of the N-particle irreducibility of the diagram [11–13], which are nothing but the graphical meaning of the Legendre transformation or the definition of the effective action.

The essence of our approach is now summarized in the following two steps: (i) change the variables from the source to the expectation value to examine the nonperturbative phenomena; (ii) extract the physical quantities in the framework of the on-shell expansion. In sharp contrast to the conventional off-shell expansion, our formalism includes the wave function of the bound state, or the

Bethe-Salpeter (BS) amplitude $\phi_B(x,y)$ for a composite field, as the lowest-order on-shell variation. This wave function naturally arises as the deviation $\Delta\phi^{(1)}(x,y)$ from the selected stationary solution $\phi^{(0)}(x,y)$ of the effective action under the condition of the vanishing external source: $J=0$. The equation of motion for the lowest-order variation $\Delta\phi^{(1)}(x,y)$ takes the form of the eigenvalue equation and it determines the particle modes including the effects of the vacuum condensations. The obtained wave function automatically appears in the external legs of the scattering diagrams, which are the coefficients of the higher orders of the on-shell expansion in terms of $\Delta\phi^{(1)}(x,y)$. Since the particle modes are determined before examining their scatterings, the interactions for making up the condensed vacuum, the bound states, and also the interactions responsible for the scattering are clearly separated from each other in this formalism. Note that we are always on the physical trajectory $J=0$. So, in the case of gauge theories, the information thus obtained from the effective action is *gauge invariant*, since the formalism generates only the observable quantities. Even when the starting effective action is approximately truncated, the difference from the true value, including the gauge-dependent part, is small if the original form of the truncated series is well convergent. This situation is expected to be realized when we consider the high-energy scattering diagrams of QCD.

For a review of on-shell expansion, the reader is referred to the papers listed in [3–5] but our formalism itself can be understood by reading through Sec. II of the present paper.

The content is organized as follows. In Sec. II, the on-shell expansion is exemplified for a simple case of the Grassmann number variables. The formulation for the anticommutative variables is convenient when we study the scattering diagrams of the baryonic modes. Of course, the formulation thus obtained is also applicable to the ordinary number variables—the meson sector. Based on this formalism, Sec. III is devoted to the presentation of a formal derivation of the two-body hadronic scattering diagrams taking QCD as a basic theory. The relation to the lowest-order constituent rearrangement diagrams (CRD's) of quark-line physics [14] is discussed there and final comments are included in Sec. IV.

II. FORMALISM: ON-SHELL EXPANSION FOR GRASSMANN NUMBER CHANNEL

Here we present our basic formalism especially in the case of a fermionic field model. We have already discussed the derivation of the mode-determining equations for the Grassmann number channel by using the lowest-order condition of the on-shell expansion [7,15]. Including this case, below we generalize the previous arguments to the higher-order terms of the on-shell expansion—the scattering part among the excited modes.

Let us consider the generating functional $W[J]$ defined with the action $I[\Psi]$ for the Dirac field $\Psi=[\Psi,\overline{\Psi}]$ for simplicity:

$$\exp(iW[J])\equiv\int[d\Psi]\exp(i\{I[\Psi]+J_a\Psi_a\})\ , \qquad (2.1)$$

where Ψ_a and J_a are the Grassmann numbers. The subscript a indicates the species of the component field as well as the other degrees of freedom including space-time coordinates. (Summations and integrations over repeated indices are implied.) Apart from the irrelevant constant, Eq. (2.1) can also be written as

$$\exp(iW[J])=\langle 0|T\exp(iJ_a\hat{\Psi}_a)|0\rangle\ , \qquad (2.2)$$

by using the corresponding Heisenberg field operator $\hat{\Psi}$. The effective action $\Gamma[\psi]$ is then defined as [7]

$$\Gamma[\psi]\equiv W[J]-J_a\psi_a\ , \qquad (2.3)$$

$$\psi_a\equiv\frac{\overleftarrow{\delta}W[J]}{\delta J_a}\ , \qquad (2.4)$$

where the functional left (and right) derivative is given by the definition [16]

$$F[J+\delta J]-F[J]\equiv\delta J\frac{\overrightarrow{\delta}}{\delta J}F[J]\left[\equiv F[J]\frac{\overleftarrow{\delta}}{\delta J}\delta J\right]\ . \qquad (2.5)$$

The use of $\overleftarrow{\delta}/\delta J$ is convenient in the actual calculations because of our choice of the source term $J_a\Psi_a$ in (2.1). If we take the source term as Ψ_aJ_a, the right derivative $\overleftarrow{\delta}/\delta J$ will be used instead. For ψ derivatives, we employ $\overleftarrow{\delta}/\delta\psi$. In this case, the stationary condition of $\Gamma[\psi]$ becomes

$$\Gamma[\psi]\frac{\overleftarrow{\delta}}{\delta\psi_a}=-J_a=0\ , \qquad (2.6)$$

and the ground state or the vacuum is determined by the solution $\psi_a^{(0)}$ of (2.6). For the fermion-number-conserving solution, we get $\psi_a^{(0)}=0$, which is assumed throughout the paper although we keep $\psi_a^{(0)}$ in the following equations for the notational convenience.

We then look for another solution of (2.6) in the form $\psi_a=\psi_a^{(0)}+\Delta\psi_a$ and write $\Delta\psi_a$ as

$$\Delta\psi_a\equiv\Delta\psi_a^{(1)}+\Delta\psi_a^{(2)}+\Delta\psi_a^{(3)}+\cdots\ , \qquad (2.7)$$

assuming that $\Delta\psi_a^{(n)}$ is the order of $(\Delta\psi_a^{(1)})^n$. The variation $\Delta\psi_a$ is determined to satisfy the condition

$$\Gamma[\psi]\frac{\overleftarrow{\delta}}{\delta\psi_a}\bigg|_{\psi^{(0)}+\Delta\psi}=\left[\Gamma[\psi]\frac{\overleftarrow{\delta}}{\delta\psi_a}\right]\frac{\overleftarrow{\delta}}{\delta\psi_b}\Delta\psi_b\bigg|_{\psi^{(0)}}+\frac{1}{2!}\left[\Gamma[\psi]\frac{\overleftarrow{\delta}}{\delta\psi_a}\right]\frac{\overleftarrow{\delta}}{\delta\psi_b}\Delta\psi_b\frac{\overleftarrow{\delta}}{\delta\psi_c}\Delta\psi_c\bigg|_{\psi^{(0)}}+\cdots\ . \qquad (2.8)$$

Since the combination $(\overleftarrow{\delta}/\delta\psi)\Delta\psi$ commutes with each other, we find that each $\Delta\psi_a^{(n)}$ is successively given by the following set of equations:

$$\left[\Gamma[\psi]\frac{\overleftarrow{\delta}}{\delta\psi_a}\right]\frac{\overleftarrow{\delta}}{\delta\psi_b}\Delta\psi_b^{(1)}\Bigg|_0=0\ , \tag{2.9a}$$

$$\left[\Gamma[\psi]\frac{\overleftarrow{\delta}}{\delta\psi_a}\right]\frac{\overleftarrow{\delta}}{\delta\psi_b}\Delta\psi_b^{(2)}\Bigg|_0+\frac{1}{2!}\left[\Gamma[\psi]\frac{\overleftarrow{\delta}}{\delta\psi_a}\right]\frac{\overleftarrow{\delta}}{\delta\psi_b}\Delta\psi_b^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_c}\Delta\psi_c^{(1)}\Bigg|_0=0\ , \tag{2.9b}$$

$$\left[\Gamma[\psi]\frac{\overleftarrow{\delta}}{\delta\psi_a}\right]\frac{\overleftarrow{\delta}}{\delta\psi_b}\Delta\psi_b^{(3)}\Bigg|_0+2\times\frac{1}{2!}\left[\Gamma[\psi]\frac{\overleftarrow{\delta}}{\delta\psi_a}\right]\frac{\overleftarrow{\delta}}{\delta\psi_b}\Delta\psi_b^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_c}\Delta\psi_c^{(2)}\Bigg|_0 +\frac{1}{3!}\left[\Gamma[\psi]\frac{\overleftarrow{\delta}}{\delta\psi_a}\right]\frac{\overleftarrow{\delta}}{\delta\psi_b}\Delta\psi_b^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_c}\Delta\psi_c^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_d}\Delta\psi_d^{(1)}\Bigg|_0=0\ , \tag{2.9c}$$

etc. The notation $|_0$ in (2.9) means the substitution of the stationary solution $\psi_a=\psi_a^{(0)}$.

In order to study the physical modes and their scattering, it is convenient to use the identities of the Legendre transformation [4], which can be derived by operating $(\delta J_a\cdot\overrightarrow{\delta}/\delta J_a)^n$ on both sides of the first equality of (2.6). Under the variation $J_a\to J_a+\delta J_a$, the following relation holds for an arbitrary functional $F[\psi]$:

$$F\left[\psi_a(J)+\delta J_b\frac{\overrightarrow{\delta}\psi_a(J)}{\delta J_b}\right]-F[\psi_a(J)]=F[\psi]\frac{\overleftarrow{\delta}}{\delta\psi_a}\delta J_b\frac{\overrightarrow{\delta}\psi_a(J)}{\delta J_b}$$
$$=F[\psi]\frac{\overleftarrow{\delta}}{\delta\psi_a}\delta J_b\frac{\overrightarrow{\delta}}{\delta J_b}\left[\frac{\overrightarrow{\delta}W}{\delta J_a}\right]\ . \tag{2.10}$$

Therefore, the operator $\overleftarrow{\delta}/\delta\psi_a\cdot\delta J_b(\overrightarrow{\delta}/\delta J_b)\cdot(\overrightarrow{\delta}W/\delta J_a)$ will be used for the functional of ψ, while we use $\delta J_a\cdot\overrightarrow{\delta}/\delta J_a$ for the functional of J. We notice that both are ordinary number operators. The identities are then obtained as follows:

$$\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_a}\left[\frac{\overleftarrow{\delta}}{\delta\psi_{b'}}\delta J_b\frac{\overrightarrow{\delta}}{\delta J_b}\left[\frac{\overrightarrow{\delta}W}{\delta J_{b'}}\right]\right]=-\delta J_b\delta_{ab}\ , \tag{2.11a}$$

$$\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_a}\left[\frac{\overleftarrow{\delta}}{\delta\psi_{b'}}\delta J_b\frac{\overrightarrow{\delta}}{\delta J_b}\left[\frac{\overrightarrow{\delta}W}{\delta J_{b'}}\right]\right]\left[\frac{\overleftarrow{\delta}}{\delta\psi_{c'}}\delta J_c\frac{\overrightarrow{\delta}}{\delta J_c}\left[\frac{\overrightarrow{\delta}w}{\delta J_{c'}}\right]\right]+\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_a}\left[\frac{\overleftarrow{\delta}}{\delta\psi_{b'}}\delta J_b\frac{\overrightarrow{\delta}}{\delta J_b}\delta J_c\frac{\overrightarrow{\delta}}{\delta J_c}\left[\frac{\overrightarrow{\delta}W}{\delta J_{b'}}\right]\right]=0\ , \tag{2.11b}$$

$$\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_a}\left[\frac{\overleftarrow{\delta}}{\delta\psi_{b'}}\delta J_b\frac{\overrightarrow{\delta}}{\delta J_b}\left[\frac{\overrightarrow{\delta}W}{\delta J_{b'}}\right]\right]\left[\frac{\overleftarrow{\delta}}{\delta\psi_{c'}}\delta J_c\frac{\overrightarrow{\delta}}{\delta J_c}\left[\frac{\overrightarrow{\delta}W}{\delta J_{c'}}\right]\right]\left[\frac{\overleftarrow{\delta}}{\delta\psi_{d'}}\delta J_d\frac{\overrightarrow{\delta}}{\delta J_d}\left[\frac{\overrightarrow{\delta}W}{\delta J_{d'}}\right]\right]$$
$$+3\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_a}\left[\frac{\overleftarrow{\delta}}{\delta\psi_{b'}}\delta J_b\frac{\overrightarrow{\delta}}{\delta J_b}\delta J_c\frac{\overrightarrow{\delta}}{\delta J_c}\left[\frac{\overrightarrow{\delta}W}{\delta J_{b'}}\right]\right]\left[\frac{\overleftarrow{\delta}}{\delta\psi_{d'}}\delta J_d\frac{\overrightarrow{\delta}}{\delta J_d}\left[\frac{\overrightarrow{\delta}W}{\delta J_{d'}}\right]\right]$$
$$+\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_a}\left[\frac{\overleftarrow{\delta}}{\delta\psi_{b'}}\delta J_b\frac{\overrightarrow{\delta}}{\delta J_b}\delta J_c\frac{\overrightarrow{\delta}}{\delta J_c}\delta J_d\frac{\overrightarrow{\delta}}{\delta J_d}\left[\frac{\overrightarrow{\delta}W}{\delta J_{b'}}\right]\right]=0\ , \tag{2.11c}$$

etc. By convention, $[\overleftarrow{\delta}/\delta\psi\cdots]$'s in (2.11) are assumed to operate only on the effective action $\Gamma[\psi]$ so that they commute with each other. From (2.11a), we get

$$\left[\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_a}\frac{\overleftarrow{\delta}}{\delta\psi_c}\right]\left[\frac{\overrightarrow{\delta}}{\delta J_b}\frac{\overrightarrow{\delta}}{\delta J_c}W\right]=-\varepsilon_{a\leftrightarrow b}\varepsilon_{c\leftrightarrow b}\delta_{ab}\ , \tag{2.12}$$

where $\varepsilon_{a\leftrightarrow b}$ denotes the sign factor caused by the interchange of the components a and b. For the Grassmann variables, $\varepsilon_{a\leftrightarrow b}=-1$. Here we notice that Eq. (2.12) can be used when the ordinary number components are further included. In such a case, by setting $\psi=\psi^{(0)}$ or $J=0$, the components of $\Gamma\overleftarrow{\delta}/\delta\psi_a\cdot\overleftarrow{\delta}/\delta\psi_c$ and $\overrightarrow{\delta}/\delta J_b\cdot\overrightarrow{\delta}/\delta J_c W$, which are of Grassmannian character, vanish under our choice of the vacuum $\psi^{(0)}=0$. Equation (2.12) then simply becomes

$$\left[\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_a}\frac{\overleftarrow{\delta}}{\delta\psi_c}\right]_0\left[\frac{\overrightarrow{\delta}}{\delta J_b}\frac{\overrightarrow{\delta}}{\delta J_c}W\right]_0=-\delta_{ab}\ , \tag{2.13}$$

or, by using the notation $\Gamma\overleftarrow{\delta}/\delta\psi_a\cdot\overleftarrow{\delta}/\delta\psi_c \equiv\overrightarrow{\delta}/\delta\psi_c(\overrightarrow{\delta}/\delta\psi_a\cdot\Gamma)$,

$$\frac{\overrightarrow{\delta}}{\delta J_b}\left[\frac{\overrightarrow{\delta}W}{\delta J_c}\right]\Bigg|_0\frac{\overrightarrow{\delta}}{\delta\psi_c}\left[\frac{\overrightarrow{\delta}\Gamma}{\delta\psi_a}\right]\Bigg|_0 =\frac{\overrightarrow{\delta}}{\delta\psi_a}\left[\frac{\overrightarrow{\delta}\Gamma}{\delta\psi_c}\right]\Bigg|_0\frac{\overrightarrow{\delta}}{\delta J_c}\left[\frac{\overrightarrow{\delta}W}{\delta J_b}\right]\Bigg|_0=-\delta_{ab}\ . \tag{2.14}$$

To change the order of the operation in (2.14), we have used the fact that the components a, b, and c should be chosen so that all of them are the Grassmann or the ordinary number variables. Other combinations vanish for

$J=0$. When the variable ψ in (2.14) is replaced by the composite one satisfying, for example, $\psi_{ij}=\psi_{ji}$ or $-\psi_{ji}$, the δ function in the right-hand side will be changed into the symmetric part

$$\delta^{(s)}_{ij,i'j'}\equiv(1/2!)(\delta_{ii'}\delta_{jj'}+\delta_{ij'}\delta_{ji'})$$

or the antisymmetric part

$$\delta^{as}_{ij,i'j'}\equiv(1/2!)(\delta_{ii'}\delta_{jj'}-\delta_{ij'}\delta_{ji'})$$

of the unit tensor, respectively. In more general cases, a properly mixed symmetry will be taken into account in the identity (2.14).

As a special case of (2.10), the variation $\delta\psi_a$ and δJ_a are related in their lowest order by

$$\delta\psi_a=\delta J_b\frac{\vec{\delta}}{\delta J_b}\left[\frac{\vec{\delta}W}{\delta J_a}\right] . \tag{2.15}$$

On the other hand, what we want to consider here is the variation $\Delta\psi_a$ defined under the condition of the vanishing external sources:

$$\Delta\psi_a^{(1)}\equiv\Delta J_b\frac{\vec{\delta}}{\delta J_b}\left[\frac{\vec{\delta}W}{\delta J_a}\right]\Bigg|_0 . \tag{2.16}$$

By using (2.14) and (2.16), we find

$$\Delta J_a=-\Delta\psi_b^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_b}\left[\frac{\overleftarrow{\delta}\Gamma}{\delta\psi_a}\right]\Bigg|_0\equiv K_a , \tag{2.17}$$

which means that, for the existence of the nonzero variation $\Delta\psi_a^{(1)}\neq 0$, the artificial external source K_a should be introduced in order to change the original boundary states [5] as will be shown below.

With these preliminaries, first we consider the lowest-order equation (2.9a) which determines the particle modes. Writing the component of ψ as $[\langle\psi\rangle,\langle\bar{\psi}\rangle]$, Eq. (2.9a) takes the matrix form

$$\Delta\psi_b^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_b}\left[\frac{\overleftarrow{\delta}\Gamma}{\delta\psi_a}\right]\Bigg|_0=[\Delta\langle\psi_j(y)\rangle^{(1)},\Delta\langle\bar{\psi}_j(y)\rangle^{(1)}]\begin{bmatrix}0 & -Z_2^{-1}(i\partial\!\!\!/_x-m)_{ij}\delta^4(x-y)\\ Z_2^{-1}(i\partial\!\!\!/_y-m)_{ji}\delta^4(y-x) & 0\end{bmatrix}$$
$$=0 . \tag{2.18}$$

Here Z_2 is the renormalization factor of the ψ field and m is the observed mass. We want to remark here that Eq. (2.18) with the proper boundary conditions is formally equivalent to (2.17) where the information of the boundary states is considered to be given by the source $-\Delta J_a\equiv -K_a$. As the solutions of (2.18), the variations $\Delta\langle\psi\rangle^{(1)}$ and $\Delta\langle\bar{\psi}\rangle^{(1)}$ are then represented with the arbitrary Grassmann variables $C^{\pm}$ as follows:

$$\begin{aligned}\Delta\langle\psi(x)\rangle^{(1)}&=\int\frac{d^3p}{(2\pi)^3}\frac{m}{p^0}[C^+(\mathbf{p},\sigma)u(p,\sigma)e^{-ip\cdot x}+C^-(\mathbf{p},\sigma)v(p,\sigma)e^{ip\cdot x}]\\ &\equiv\int d^3p[\tilde{C}^+(\mathbf{p},\sigma)u(p,\sigma)e^{-ip\cdot x}+\tilde{C}^-(\mathbf{p},\sigma)v(p,\sigma)e^{ip\cdot x}] ,\end{aligned} \tag{2.19}$$

$$\begin{aligned}\Delta\langle\bar{\psi}(x)\rangle^{(1)}&=\int\frac{d^3p}{(2\pi)^3}\frac{m}{p^0}[C^{+*}(\mathbf{p},\sigma)\bar{u}(p,\sigma)e^{ip\cdot x}+C^{-*}(\mathbf{p},\sigma)\bar{v}(p,\sigma)e^{-ip\cdot x}]\\ &\equiv\int d^3p[\tilde{C}^{+*}(\mathbf{p},\sigma)\bar{u}(p,\sigma)e^{ip\cdot x}+\tilde{C}^{-*}(\mathbf{p},\sigma)\bar{v}(p,\sigma)e^{-ip\cdot x}] ,\end{aligned} \tag{2.20}$$

where $p^0=(\mathbf{p}^2+m^2)^{1/2}$, $\tilde{C}^{\pm}(\mathbf{p},\sigma)\equiv[m/(2\pi)^3p^0]C^{\pm}(\mathbf{p},\sigma)$ and $u(p,\sigma)$, $v(p,\sigma)$ are the Dirac spinors with fixed helicity σ satisfying $(p\!\!\!/-m)u(p,\sigma)=0$ and $(p\!\!\!/+m)v(p,\sigma)=0$, respectively. The summation over σ has been implied in (2.19) and (2.20).

In order to write the higher-order variation $\Delta\psi_a^{(n)}$ by using the derivatives of $W[J]$, the identities (2.11b), (2.11c), etc., are utilized under the condition of $J_a=0$ (or $\psi_a=\psi_a^{(0)}=0$), where δJ_a is replaced by ΔJ_a given in (2.17). In this case,

$$\left[\frac{\overleftarrow{\delta}}{\delta\psi_a}\Delta J_b\frac{\vec{\delta}}{\delta J_b}\left[\frac{\vec{\delta}W}{\delta J_a}\right]\Bigg|_0\right]=\frac{\overleftarrow{\delta}}{\delta\psi_a}\Delta\psi_a^{(1)} . \tag{2.21}$$

From (2.9b), (2.9c), etc., we then obtain

$$\Delta\psi_a^{(n)}=\frac{1}{n!}\left\{\left[-\Delta\psi_{a_1}^{(1)}\left[\frac{\overleftarrow{\delta}}{\delta\psi_{a_1}}\frac{\overleftarrow{\delta}\Gamma}{\delta\psi_{a_1'}}\right]\frac{\vec{\delta}}{\delta J_{a_1'}}\right]\cdots\left[-\Delta\psi_{a_n}^{(1)}\left[\frac{\overleftarrow{\delta}}{\delta\psi_{a_n}}\frac{\overleftarrow{\delta}\Gamma}{\delta\psi_{a_n'}}\right]\frac{\vec{\delta}}{\delta J_{a_n'}}\right]\frac{\vec{\delta}W}{\delta J_a}\right\}_0 . \tag{2.22}$$

In deriving (2.22), we have used the fact that, except for the fixed order of the variables and operators, all the coefficients in (2.9) and (2.11) are the same as in the boson field case. The result is, of course, also available when the ordinary number variables are included.

Let us examine the physical meaning of the variation $\Delta\psi_a$. For this purpose, the asymptotic fields [17] are introduced with $p_0=(\mathbf{p}^2+m^2)^{1/2}$:

$$\hat{\Psi}_{\text{in(out)}}(x)\equiv\int\frac{d^3p}{(2\pi)^3}\frac{m}{p^0}[\hat{b}_{\text{in(out)}}(\mathbf{p},\sigma)u(p,\sigma)e^{-ip\cdot x}+\hat{d}^{\dagger}_{\text{in(out)}}(\mathbf{p},\sigma)v(p,\sigma)e^{ip\cdot x}]\ , \tag{2.23}$$

$$\hat{\overline{\Psi}}_{\text{in(out)}}(x)\equiv\int\frac{d^3p}{(2\pi)^3}\frac{m}{p^0}[\hat{b}^{\dagger}_{\text{in(out)}}(\mathbf{p},\sigma)\overline{u}(p,\sigma)e^{ip\cdot x}+\hat{d}_{\text{in(out)}}(\mathbf{p},\sigma)\overline{v}(p,\sigma)e^{-ip\cdot x}]\ , \tag{2.24}$$

under the limit

$$\hat{\Psi}(x)\to Z_2^{1/2}\hat{\Psi}_{\text{in(out)}}(x)\quad [x^0\to-\infty(+\infty)]\ . \tag{2.25}$$

Consider the lowest-order variation $\Delta\psi_a^{(1)}$, for instance. Equation (2.22) for the trivial case of $n=1$ can be transformed as

$$\begin{aligned}\Delta\psi_a^{(1)}&=-\Delta\psi_b^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_b}\left[\frac{\overrightarrow{\delta}\Gamma}{\delta\psi_c}\right]\Bigg|_0\frac{\overrightarrow{\delta}}{\delta J_c}\left[\frac{\overrightarrow{\delta}W}{\delta J_a}\right]\Bigg|_0\\&=i\left\langle 0\left|T\left[-\Delta\psi_b^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_b}\left[\frac{\overrightarrow{\delta}\Gamma}{\delta\psi_c}\right]\Bigg|_0\hat{\overline{\Psi}}_c\hat{\Psi}_a\right]\right|0\right\rangle\\&=\left\langle 0\left|T\left[-iZ_2^{-1}\int d^4x[\Delta\langle\overline{\psi}_i(x)\rangle^{(1)}(i\overrightarrow{\partial\!\!\!/}_x-m)_{ij}\hat{\Psi}_j(x)+\hat{\overline{\Psi}}_i(x)(-i\overleftarrow{\partial\!\!\!/}_x-m)_{ij}\Delta\langle\psi_j(x)\rangle^{(1)}]\hat{\Psi}_a\right]\right|0\right\rangle\ ,\end{aligned} \tag{2.26}$$

where we have used

$$\frac{\overrightarrow{\delta}}{\delta J_c}\left[\frac{\overrightarrow{\delta}W}{\delta J_a}\right]\Bigg|_0=i\langle 0|T\hat{\overline{\Psi}}_c\hat{\Psi}_a|0\rangle_{J=0}\ , \tag{2.27}$$

and (2.18). The integrations over x in (2.26) are carried out, respectively:

$$\begin{aligned}-iZ_2^{-1}\int d^4x\,\Delta\langle\overline{\psi}(x)\rangle^{(1)}(i\overrightarrow{\partial\!\!\!/}-m)\hat{\Psi}(x)&=-iZ_2^{-1}\int d^4x\,i\partial_{x^0}[\Delta\langle\overline{\psi}(x)\rangle^{(1)}\gamma_0\hat{\Psi}(x)]\\&=Z_2^{-1}\int d^3x(\lim_{x^0\to\infty}-\lim_{x^0\to-\infty})\\&\quad\times\left[\int d^3p[\tilde{C}^{+*}(\mathbf{p},\sigma)up\ \ ,\sigma)e^{ip\cdot x}+\tilde{C}^{-*}(\mathbf{p},\sigma)\overline{v}(p,\sigma)e^{-ip\cdot x}]\gamma_0\hat{\Psi}(x)\right]\\&=Z_2^{-1/2}\int d^3p\{\tilde{C}^{+*}(\mathbf{p},\sigma)[\hat{b}_{\text{out}}(p,\sigma)-\hat{b}_{\text{in}}(p,\sigma)]\\&\quad+\tilde{C}^{-*}(\mathbf{p},\sigma)[\hat{d}^{\dagger}_{\text{out}}(p,\sigma)-\hat{d}^{\dagger}_{\text{in}}(p,\sigma)]\}\end{aligned} \tag{2.28}$$

$$\begin{aligned}-iZ_2^{-1}\int d^4x\hat{\overline{\Psi}}(x)(-i\overleftarrow{\partial\!\!\!/}_x-m)\Delta\langle\psi(x)\rangle^{(1)}&=-iZ_2^{-1}\int d^4x(-i\partial_{x^0})[\hat{\overline{\Psi}}(x)\gamma_0\Delta\langle\psi(x)\rangle^{(1)}]\\&=-Z_2^{-1}\int d^3x(\lim_{x^0\to\infty}-\lim_{x^0\to-\infty})\\&\quad\times\left[\hat{\overline{\Psi}}(x)\gamma_0\int d^3p[\tilde{C}^{+}(\mathbf{p},\sigma)u(p,\sigma)e^{-ip\cdot x}+\tilde{C}^{-}(\mathbf{p},\sigma)v(p,\sigma)e^{ip\cdot x}]\right]\\&=Z_2^{-1/2}\int d^3p\{\tilde{C}^{+}(\mathbf{p},\sigma)[\hat{b}^{\dagger}_{\text{out}}(p,\sigma)-\hat{b}^{\dagger}_{\text{in}}(p,\sigma)]\\&\quad+\tilde{C}^{-}(\mathbf{p},\sigma)[\hat{d}_{\text{out}}(p,\sigma)-\hat{d}_{\text{in}}(p,\sigma)]\}\ .\end{aligned} \tag{2.29}$$

These just correspond to the inverse process of the original proof of the Lehmann-Symanzik-Zimmermann well-known reduction formula [17]. Then we find, after taking the proper time ordering,

$$\begin{aligned}\Delta\psi_a^{(1)}(x)&=Z_2^{-1}\int d^3p\langle 0|[\tilde{C}^{+*}(\mathbf{p},\sigma)\hat{b}_{\text{out}}(p,\sigma)+\tilde{C}^{-}(\mathbf{p},\sigma)\hat{d}_{\text{out}}(p,\sigma)]\hat{\Psi}_a|0\rangle\\&\quad+Z_2^{-1}\int d^3p\langle 0|\hat{\Psi}_a[\hat{b}^{\dagger}_{\text{in}}(p,\sigma)\tilde{C}^{+}(\mathbf{p},\sigma)+\hat{d}^{\dagger}_{\text{in}}(p,\sigma)\tilde{C}^{-*}(\mathbf{p},\sigma)]|0\rangle\\&\equiv\langle 1_f^-|\hat{\Psi}_a|0\rangle+\langle 0|\hat{\Psi}_a|1_f^+\rangle\ .\end{aligned} \tag{2.30}$$

Equation (2.30) is the general representation of the wave function of the Dirac field $\hat{\Psi}$.

Repeatedly using the process discussed above, the total form of $\Delta\psi_a$ is obtained as

$$\Delta\psi_a=\frac{\langle\theta_f^-|\hat{\Psi}_a|\theta_f^+\rangle}{\langle\theta_f^-|\theta_f^+\rangle}\ , \tag{2.31}$$

$$|\theta_f^+\rangle\equiv\exp\left[Z_2^{-1/2}\int d^3p[\hat{b}^{\dagger}_{\text{in}}(p,\sigma)\tilde{C}^{+}(\mathbf{p},\sigma)+\hat{d}^{\dagger}_{\text{in}}(p,\sigma)\tilde{C}^{-*}(\mathbf{p},\sigma)]\right]|0\rangle\ , \tag{2.32}$$

$$\langle\theta_f^-|\equiv\langle 0|\exp\left[Z^{-1/2}\int d^3p[\tilde{C}^{+*}(\mathbf{p},\sigma)\hat{b}_{\text{out}}(p,\sigma)+\tilde{C}^{-}(\mathbf{p},\sigma)\hat{d}_{\text{out}}(p,\sigma)]\right]\ . \tag{2.33}$$

Let us introduce the new generating functional $W_{\theta_f^-\theta_f^+}[J]$:

$$\exp(iW_{\theta_f^-\theta_f^+}[J]) \equiv \left\langle \theta_f^- \left| T\exp\left[i\int_{x^0=T_i}^{x^0=T_f} d^4x\, J(x)\cdot\hat{\Psi}(x) \right] \right| \theta_f^+ \right\rangle , \tag{2.34}$$

where the limits $T_i \to -\infty$ and $T_f \to \infty$ are assumed to be considered. The effective action $\Gamma_{\theta_f^-\theta_f^+}[\psi]$ is further defined from $W_{\theta_f^-\theta_f^+}[J]$:

$$\Gamma_{\theta_f^-\theta_f^+}[\psi^*] \equiv W_{\theta_f^-\theta_f^+}[J] - J_a\psi_a^* , \tag{2.35}$$

$$\psi_a^* \equiv \frac{\vec{\delta} W_{\theta_f^-\theta_f^+}[J]}{\delta J_a}, \quad [(\psi_a^*)_{J=0} = \Delta\psi_a] . \tag{2.36}$$

We then utilize the relation

$$\left\langle \theta_f^- \left| T\exp(i\textstyle\int J_a\hat{\Psi}_a) \right| \theta_f^+ \right\rangle = e^{\langle 1_f^- | 1_f^+ \rangle} \left\langle 0 \left| T\exp\left[i\textstyle\int (J_a+K_a)\hat{\Psi}_a \right] \right| 0 \right\rangle , \tag{2.37}$$

$$\langle 1_f^- | 1_f^+ \rangle = Z_2^{-1} \int d^3p \frac{p^0}{m}(2\pi)^3 [\tilde{C}^{+*}(\mathbf{p},\sigma)\tilde{C}^{+}(\mathbf{p},\sigma) + \tilde{C}^{-}(\mathbf{p},\sigma)\tilde{C}^{-*}(\mathbf{p},\sigma)] , \tag{2.38}$$

and find, from (2.35), that $i\Gamma_{\theta_f^-\theta_f^+}[\psi^*]$ itself becomes the generating functional $S^{(c)}$ of the connected S-matrix element under the condition of $J=0$:

$$i\Gamma_{\theta_f^-\theta_f^+}[\Delta\psi] = \langle 1_f^- | 1_f^+ \rangle + iW[J_a \equiv K_a] \equiv S^{(c)}(\tilde{C}^{+*},\tilde{C}^{-};\tilde{C}^{+},\tilde{C}^{-*}) . \tag{2.39}$$

Again, by using (2.37) and from (2.39), the effective action $\Gamma[\psi]$ in (2.3) now can be related to the generating functional of the connected T-matrix element $T^{(c)}$, or equivalently, $iW[J_a = K_a]$ in (2.39), in the following way:

$$i\Gamma[\Delta\psi] + iK_a\Delta\psi_a = T^{(c)}(\tilde{C}^{+*},\tilde{C}^{-};\tilde{C}^{+},\tilde{C}^{-*}) . \tag{2.40}$$

The second term on the left-hand side of (2.40) cancels the same contribution contained in $i\Gamma[\Delta\psi]$. So, if we set the corresponding term equal to zero when we expand $\Gamma[\Delta\psi]$ around the stationary solution $\psi^{(0)}=0$, we can also use the compact formula $i\Gamma[\Delta\psi] = T^{(c)}$ in the actual calculations. The inclusion of composite variables is now straightforward as in the case of the boson field model discussed in Ref. [5].

III. QCD

Based on the formalism discussed in the previous section, here we examine the construction of the connected S-matrix element of the two-body hadronic scattering in QCD theory. Exactly solving the nonperturbative quantities of QCD is, of course, a difficult problem but it is not necessary to see how the physical variables introduced by the Legendre transformation appear in the obtained scattering diagrams. The vacuum solutions may be taken as given quantities to be determined by experiments. The effective form of the lowest-order on-shell variation or the wave function of hadrons can also be assumed in the actual calculations including the mixing parameters. Apart from these nonperturbative quantities, the scattering matrix element among hadrons is diagrammatically expanded in a systematic way by the higher-order terms of the on-shell expansion. In this respect, the present formalism will be a useful basis for the approximated numerical evaluation of the high-energy hadronic scatterings. The story is summarized in the following three steps.

(i) We start from the effective action $\Gamma[\psi]$ defined as in (2.3) and (2.4):

$$\Gamma[\psi] \equiv \Gamma[\langle q_iq_jq_k\rangle, \langle \bar{q}_i\bar{q}_j\bar{q}_k\rangle, \langle A_\mu A_\nu A_\rho\rangle, \langle \bar{q}_i A_\mu q_j\rangle, \langle \bar{\eta}_i A_\mu \eta_j\rangle, \langle A_\mu A_\nu\rangle, \langle q_i\bar{q}_j\rangle, \langle \eta_i\bar{\eta}_j\rangle] , \tag{3.1}$$

where q, η, and A are the quark field, the ghost field, and the gluon field, respectively. The subscripts represent all the attributes of the field including the color and the flavor indices. The notation $\langle O\rangle$ implies $\langle 0|T\hat{O}|0\rangle_{J\neq 0}$ and the external sources J_a of $W[J]$ are assumed to have been introduced for two- and three-body channels of baryon, meson-glueball sectors. The vacuum expectation values are first determined by the stationary requirement of the effective action:

$$\frac{\overleftarrow{\delta}\Gamma}{\delta\langle q_iq_jq_k\rangle} = \frac{\overleftarrow{\delta}\Gamma}{\delta\langle \bar{q}_i\bar{q}_j\bar{q}_k\rangle} = \frac{\delta\Gamma}{\delta\langle A_\mu A_\nu A_\rho\rangle} = \cdots = \frac{\delta\Gamma}{\delta\langle \eta_i\bar{\eta}_j\rangle} = 0 . \tag{3.2}$$

The solutions for meson and glueball channels $\langle A_\mu A_\nu\rangle^{(0)}$, $\langle q_i\bar{q}_j\rangle^{(0)}$, etc., are expected to include the effects of chiral and gluon condensations. For baryon and antibaryon channels we get

$$\langle q_iq_jq_k\rangle^{(0)} = \langle \bar{q}_i\bar{q}_j\bar{q}_k\rangle^{(0)} = 0 .$$

(ii) The physical modes of the baryon, meson, and glueball are next examined by the lowest-order condition (2.9a) which is given in the matrix form

$$\begin{bmatrix} \text{baryon part} & 0 \\ 0 & \begin{matrix}\text{meson and}\\ \text{glueball part}\end{matrix} \end{bmatrix}_0 \begin{bmatrix} \Delta\langle qqq\rangle^{(1)} \\ \Delta\langle \bar{q}\,\bar{q}\,\bar{q}\rangle^{(1)} \\ \Delta\langle AAA\rangle^{(1)} \\ \Delta\langle \bar{q}Aq\rangle^{(1)} \\ \Delta\langle \bar{\eta}A\eta\rangle^{(1)} \\ \Delta\langle AA\rangle^{(1)} \\ \Delta\langle q\bar{q}\rangle^{(1)} \\ \Delta\langle \eta\bar{\eta}\rangle^{(1)} \end{bmatrix} = 0 . \qquad (3.3)$$

The subscript 0 in (3.3) means the substitution of the selected (condensed) vacuum solutions. Because of the assumed color confinement, the finite mass eigenvalue to (3.3) is expected only for color-singlet combinations $\Delta\langle q_A q_B q_C\rangle^{(1)} \propto \varepsilon_{ABC}$, $\Delta\langle q_A \bar{q}_B\rangle^{(1)} \propto \delta_{AB}$, $\Delta\langle A_\alpha A_\beta\rangle^{(1)} \propto \delta_{\alpha\beta}$, etc., with A, B, C and α, β denoting the color degrees of freedom. The mode-determining equation for baryon channel is given in the form of the Bethe-Salpeter-type wave equation for three quarks [7] where $\Delta\langle qqq\rangle^{(1)}$ plays the role of the BS amplitude. For meson and glueball channels Eq. (3.3) presents a set of coupled equations. For massless quarks, only the flavor-singlet channels of the meson and glueball will mix with each other. The mixing in the pseudoscalar channel is expected to be related to the U(1) problem [18].

(iii) The scattering among the physical modes in (ii) is then determined by the higher-order terms of the on-shell expansion. The hadron-hadron scattering amplitude is obtained by the terms proportional to $(\Delta\langle\psi_a\rangle^{(1)})^4$ which are given by expanding $\Gamma[\psi^{(0)}+\Delta\psi]$ around the chosen solution $\psi^{(0)}$. Let us examine a simplified case where the physical modes are determined by the following 2×2 on-shell condition:

$$\begin{bmatrix} \Gamma^{(2)}_{11} & \Gamma^{(2)}_{21} \\ \Gamma^{(2)}_{12} & \Gamma^{(2)}_{22} \end{bmatrix} \begin{bmatrix} \Delta\langle\psi_1\rangle^{(1)} \\ \Delta\langle\psi_2\rangle^{(1)} \end{bmatrix} = 0 . \qquad (3.4)$$

The subscripts 1 and 2 denote the species of the ordinary number field and the second derivative $\Gamma^{(2)}$ of Γ is assumed to have been evaluated at the chosen vacuum solutions. By eliminating $\Delta\langle\psi_1\rangle^{(1)}$ or $\Delta\langle\psi_2\rangle^{(1)}$ and by using the identity of the Legendre transformation,

$$\begin{bmatrix} \Gamma^{(2)}_{11} & \Gamma^{(2)}_{21} \\ \Gamma^{(2)}_{12} & \Gamma^{(2)}_{22} \end{bmatrix} \begin{bmatrix} W^{(2)}_{11} & W^{(2)}_{21} \\ W^{(2)}_{12} & W^{(2)}_{22} \end{bmatrix} = -\mathbb{1} , \qquad (3.5)$$

we find, from (3.4) that,

$$[(\Gamma^{(2)}_{12}\Gamma^{(2)\,-1}_{22}\Gamma^{(2)}_{21}) - \Gamma^{(2)}_{11}]\Delta\langle\psi_1\rangle^{(1)} = (W^{(2)}_{11})^{-1}\Delta\langle\psi_1\rangle^{(1)} = 0 , \qquad (3.6a)$$

$$[(\Gamma^{(2)}_{21}\Gamma^{(2)\,-1}_{11}\Gamma^{(2)}_{12}) - \Gamma^{(2)}_{22}]\Delta\langle\psi_2\rangle^{(1)} = (W^{(2)}_{22})^{-1}\Delta\langle\psi_2\rangle^{(1)} = 0 , \qquad (3.6b)$$

which really shows that $\Delta\langle\psi_1\rangle^{(1)}$ and $\Delta\langle\psi_2\rangle^{(1)}$ are determined by the pole structure of the corresponding Green's functions. The solutions may be written in Fourier space,

$$\Delta\langle\psi_1(p)\rangle^{(1)} = f_a(p)\delta(p^2-m_a^2) + f_b(p)\delta(p^2-m_b^2) + \cdots , \qquad (3.7a)$$

$$\Delta\langle\psi_2(p)\rangle^{(1)} = g_a(p)\delta(p^2-m_a^2) + g_b(p)\delta(p^2-m_b^2) + \cdots , \qquad (3.7b)$$

with arbitrary functions $f_{a,b}(p)$ and $g_{a,b}(p)$. Assuming that

$$|f_a| > |f_b| \quad \text{and} \quad |g_a| < |g_b| , \qquad (3.8)$$

the modes a and b in (3.7) are physically identified with the particle corresponding to the field 1 and 2, respectively. In this sense, we call $\Delta\langle\psi_{1(2)}(p)\rangle^{(1)}$ the proper wave function of the mode a (b). The S-matrix element among the modes a's is now given as follows. On the basis of the on-shell expansion for $\Delta\langle\psi_{1,2}\rangle^{(1)}$, first eliminate $\Delta\langle\psi_2\rangle^{(1)}$, for example, in order that the proper wave function graphically appears in each external leg. Similarly as in (2.19) or (2.20), $\tilde{C}_a^+$ and $\tilde{C}_a^-$ for the bosonic mode a are defined from $f_a(p)$ of (3.7a) in this case. The derivatives of (2.40) (with $\Gamma[\psi]$ of this model) are then taken in terms of $\tilde{C}_a^\pm$ to get the a-mode's scattering matrix element. $\Delta\langle\psi_1\rangle^{(1)}$ can be eliminated instead of $\Delta\langle\psi_2\rangle^{(1)}$ or both can be used without eliminating either of them. The correct S-matrix element appears in all cases, of course, if we take the on-shell projection corresponding to the a mode.

Our interest here lies in the final step (iii) of the above procedure.

A. Effective action

We first recapitulate the formerly derived expression of the effective action for QCD theory [7] which is necessary for the following argument. In order to examine the baryon and meson-glueball channels, we consider the Legendre transformation for two- and three-body composite channels. The effective action with a three-body composite variable is not so familiar in particle physics compared with the one [19] and two-body [20] cases; however, it is easily obtained by slightly modifying the Legendre transformation rule given by De Dominicis and Martin [21]. According to their approach, we write the bare QCD action including the artificial external sources J_ν ($\nu=1,2,3$) in the form

$$I[\Psi,J] \equiv J_1(a)\Psi(a) + \frac{1}{2!}[J_2(a,b)+v_2^0(a,b)]\Psi(a)\Psi(b) + \frac{1}{3!}[J_3(a,b,c)+v_3^0(a,b,c)]\Psi(a)\Psi(b)\Psi(c) + \frac{1}{4!}v_4^0(a,b,c,d)\Psi(a)\Psi(b)\Psi(c)\Psi(d) , \qquad (3.9)$$

where $\Psi=[\bar{q},q,\bar{\eta},\eta,A]$. The vertex v_ν^0 ($\nu=2,3,4$) comes from the original QCD action. For convenience, here the external sources are introduced for all the combinations of the field up to three-body channels. The sources are assumed to be antisymmetric for Grassmann components and symmetric for the others. ($J_\nu/\nu!$ corresponds to J_a in the previous section.) The generating functional $W[J]$ is defined as in (2.1) and the variables of Γ are then introduced as

$$G_\nu(a,b,\ldots)\equiv\nu!\frac{\vec{\delta}W[J]}{\delta J_\nu(a,b,\ldots)} \quad (\nu\equiv 1,2,3) . \tag{3.10}$$

The notation $\tilde{G}_\nu$ will be used for the connected part of G_ν:

$$\tilde{G}_2(a,b)\equiv G_2(a,b)-G_1(a)G_1(b) , \tag{3.11}$$

$$\tilde{G}_3(a,b,c)\equiv G_3(a,b,c)-\sum_{P(a,b,c)}\varepsilon^{P(a,b,c)}\left[\frac{1}{2!}G_1(a)\tilde{G}_2(b,c)+\frac{1}{3!}G_1(a)G_1(b)G_1(c)\right] , \tag{3.12}$$

where $P(a,b,c)$ denotes the permutation of a,b,c and $\varepsilon^{P(a,b,c)}$ is the corresponding sign factor appropriate for the Grassmann field. Also, we introduce C_3 and $\tilde{C}_3$:

$$\tilde{G}_3(a,b,c)\equiv\varepsilon^{P(a',b',c';a,b,c)}C_3(a',b',c')\tilde{G}_2(a',a)\tilde{G}_2(b',b)\tilde{G}_2(c',c) , \tag{3.13}$$

$$\Psi(a)\Psi(b)\Phi(c)\tilde{C}_3(a,b,c)\equiv C_3(a,b,c)\Psi(a)\Psi(b)\Psi(c) . \tag{3.14}$$

The sign factor $\varepsilon^{P(a',b',c';a,b,c)}$ in (3.13) is defined by

$$\Psi(a')\Psi(b')\Psi(c')\Psi(a)\Psi(b)\Psi(c)\equiv\varepsilon^{P(a',b',c';a,b,c)}\Psi(a')\Psi(a)\Psi(b')\Psi(b)\Psi(c')\Psi(c) . \tag{3.15}$$

Now the effective action is obtained in the form [7,21]

$$\begin{aligned}\Gamma[G_1,\tilde{G}_2,C_3]&\equiv W[J]-\sum_{\nu=1}^{3}\frac{1}{\nu!}J_\nu(a,b,\ldots)G_\nu(a,b,\ldots)\\ &=\frac{1}{2!}v_2^0(a,b)G_2(a,b)+\frac{1}{3!}v_3^0(a,b,c)G_3(a,b,c)+\frac{i}{2}\mathrm{STr}\ln\tilde{G}_2\\ &\quad+\frac{1}{2\cdot 3!}\sum_{\{a,b,c,\,a',b',c'\}}\varepsilon^{P(a,b,c;a',b',c')}C_3(a,b,c)\tilde{G}_2(a,a')\tilde{G}_2(b,b')\tilde{G}_2(c,c')\tilde{C}_3(a',b',c')-i\kappa ,\end{aligned} \tag{3.16}$$

where STr denotes the supertrace [16] and κ is the sum of the possible one-, two-, and three-particle-irreducible (1,2,3PI) vacuum diagrams constructed out of G_1, $\tilde{G}_2$ (propagator), C_3 (three-point vertex), and the original gluon four-point vertex $iv_4^0(A,A,A,A)$. By convention, even when a graph is disconnected by cutting three internal lines, we call it three-particle irreducible provided one and only one of the disconnected parts is a C_3 vertex itself. Also, κ does not include the graphs built out of a single C_3 vertex [21]. Some of the diagrams of κ are shown in Fig. 1. Note that $\langle qqq\rangle$, $\langle\bar{q}\bar{q}\bar{q}\rangle$, $\langle AAA\rangle$, $\langle\bar{q}Aq\rangle$, $\langle\bar{\eta}A\eta\rangle$, $\langle AA\rangle$, $\langle q\bar{q}\rangle$, and $\langle\eta\bar{\eta}\rangle$ of (3.1) correspond to the De Dominicis–Martin variables $\tilde{G}_3(q,q,q,)$, $\tilde{G}_3(\bar{q},\bar{q},\bar{q})$, $\tilde{G}_3(A,A,A)$, $\tilde{G}_3(\bar{q},A,q)$, $\tilde{G}_3(\bar{\eta},A,\eta)$, $\tilde{G}_2(A,A)$, $\tilde{G}_2(q,\bar{q})$, and $\tilde{G}_2(\eta,\bar{\eta})$, respectively. In the following a C_3 vertex will be used instead of $\tilde{G}_3$ for convenience but this does not affect the obtained scattering amplitudes themselves. S-matrix elements are independent of the field variables which are chosen to describe them.

Since we are only interested in two- and three-body channels, here we neglect the variables introduced for the elementary field $\Psi(a)$ in (3.16). Notice that the vacuum solution $\langle\Psi(a)\rangle^{(0)}\equiv G_1^{(0)}(a)$ is zero and $\Delta G_1^{(1)}(a)$ does not exist as a physical mode because of the expected confinement of the color degrees of freedom. As we briefly mentioned in case (3) of the Introduction, scattering diagrams are obtained in tree graphs made up with proper vertices $\Gamma^{(n)}$ $(n>2)$ and the propagator $-(\Gamma^{(2)})^{-1}(=W^{(2)})$. Therefore, the higher-order terms including $G_1(A_\nu)$ derivatives of Γ, for example, apparently vanish when the color-singlet projections are

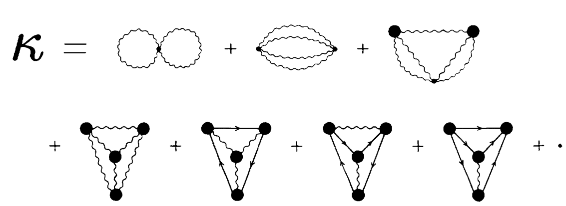

FIG. 1. Some of the diagrams included in the κ part of the QCD effective action (3.16). A solid line denotes $\tilde{S}$ and a wavy line $\tilde{D}$. They are quark and the gluon full propagator under the external sources, respectively. Each three-point vertex is given by an appropriate component of $C_3(a,b,c)$. A four-point vertex represents the original four-gluon vertex $iv_4^0(A,A,A,A)$.

taken for all external legs of the diagrams. The components of $\tilde{G}_2(a,b)$ and $C_3(a,b,c)$ which decouple from baryon and meson-glueball parts [$\tilde{G}_2(q,q)$, $\tilde{G}(\eta,\eta)$, $C_3(q,q,A)$, etc.] are also omitted. These assumptions correspond to the situation where the external sources are not introduced for the corresponding channels in (3.9). The remaining part of (3.16) is written explicitly as below in terms of the quark, gluon, and the ghost field:

$$\begin{aligned}\Gamma[G_1,\tilde{G}_2,C_3]=&\tfrac{1}{2}iD_{0\,\mu\nu}^{-1}\tilde{D}^{\mu\nu}-iS_{0\,ij}^{-1}\tilde{S}_{ji}-i\Delta_{0\,ij}^{-1}\tilde{\Delta}_{ji}\\&+\frac{1}{3!}\left[v_3^0(A_\mu,A_\nu,A_\rho)+\frac{i}{2}C_3(A_\mu,A_\nu,A_\rho)\right]\tilde{D}^{\mu\mu'}\tilde{D}^{\nu\nu'}\tilde{D}^{\rho\rho'}C_3(A_{\mu'},A_{\nu'},A_{\rho'})\\&-\left[v_3^0(\bar{q}_i,A_\mu,q_{i'})+\frac{i}{2}C_3(\bar{q}_i,A_\mu,q_{i'})\right]\tilde{S}_{ji}\tilde{D}^{\mu\nu}\tilde{S}_{i'j'}C_3(\bar{q}_{j'},A_\nu,q_j)\\&-\left[v_3^0(\bar{\eta}_i,A_\mu,\eta_{i'})+\frac{i}{2}C_3(\bar{\eta}_i,A_\mu,\eta_{i'})\right]\tilde{\Delta}_{ji}\tilde{D}^{\mu\nu}\tilde{\Delta}_{i'j'}C_3(\bar{\eta}_{j'},A_\nu,\eta_j)\\&+\frac{i}{3!}C_3(q_i,q_j,q_k)\tilde{S}_{ii'}\tilde{S}_{jj'}\tilde{S}_{kk'}C_3(\bar{q}_{i'},\bar{q}_{j'},\bar{q}_{k'})+i\,\mathrm{Tr}\ln\tilde{S}+i\,\mathrm{Tr}\ln\tilde{\Delta}\\&-\frac{i}{2}\mathrm{Tr}\ln\tilde{D}-i(\text{corresponding terms of }\kappa)\,,\end{aligned}\tag{3.17}$$

where we have employed the simplified notation $\tilde{D}\equiv\tilde{G}_2(A,A)$, $\tilde{S}\equiv\tilde{G}_2(q,\bar{q})$, and $\tilde{\Delta}\equiv\tilde{G}_2(\eta,\bar{\eta})$. The bare propagators for gluon, quark, and ghost field are denoted by D_0, S_0, and Δ_0, respectively.

B. Derivation of the S-matrix element

Based on the effective action (3.17), let us consider a formal derivation of the connected S-matrix element of the hadron-hadron scattering. The advantage of our formalism is that the nonperturbative effects are automatically taken into account in the expansion series.

1. Meson-meson scattering

Consider the meson-meson scattering case. Expanding the left-hand side of (2.40) with the effective action (3.17) around the vacuum solutions for each component, we find that the corresponding S-matrix elements appear in

$$\left[\frac{1}{4!}\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_Y}\Delta\psi_Y^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_Z}\Delta\psi_Z^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_W}\Delta\psi_W^{(1)}+\frac{3}{3!}\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_Y}\Delta\psi_Y^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_Z}\Delta\psi_Z^{(2)}+\frac{1}{2!}\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(2)}\frac{\overleftarrow{\delta}}{\delta\psi_Y}\Delta\psi_Y^{(2)}\right]_0,\tag{3.18}$$

where the subscripts $X,Y,\ldots$ imply the components which couple to the meson and glueball channels. (The left and right derivatives are the same in this case.) We then pick up the terms which are proportional to the fourth order of the proper wave functions of mesons $(\Delta\tilde{S}^{(1)})^4$ from (3.18). The color indices of the variation $\Delta\tilde{S}^{(1)}$ are supposed to be selected in their singlet channel. Other degrees of freedom, such as flavor combinations and total spin of $\Delta\tilde{S}^{(1)}$, are to be determined when we project out the proper mode of the observed mesons. The second-order variation $\Delta\psi_X^{(2)}$ of (3.18) is explicitly given by (2.9b) and (2.14):

$$\begin{aligned}\Delta\psi_X^{(2)}&=\frac{1}{2!}\left[\frac{\vec{\delta}}{\delta J_Y}\frac{\vec{\delta}W}{\delta J_X}\right]_0\left[\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_Y}\frac{\overleftarrow{\delta}}{\delta\psi_a}\Delta\psi_a^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_b}\Delta\psi_b^{(1)}\right]_0\\&=\frac{1}{2!}\left[\frac{\vec{\delta}}{\delta J_Y}\frac{\vec{\delta}W}{\delta J_X}\right]_0\left[\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_Y}\frac{\overleftarrow{\delta}}{\delta\psi_Z}\Delta\psi_Z^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_W}\Delta\psi_W^{(1)}\right.\\&\qquad\left.+2\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_Y}\frac{\overleftarrow{\delta}}{\delta C_3(q,q,q)}\Delta C_3(q,q,q)^{(1)}\frac{\overleftarrow{\delta}}{\delta C_3(\bar{q},\bar{q},\bar{q})}\Delta C_3(\bar{q},\bar{q},q)^{(1)}+\cdots\right]_0,\end{aligned}\tag{3.19}$$

where the notation $(\vec{\delta}/\delta J\cdot\vec{\delta}W/\delta J)_0$ is conveniently used instead of writing it as the inverse of $(-\Gamma\overleftarrow{\delta}/\delta\psi\cdot\overleftarrow{\delta}/\delta\psi)_0$. [Recall that the source J_a here symbolically represents J_1, $J_2/2!$, and $J_3/3!$ introduced in (3.9).]

The variations $\Delta\psi_a^{(1)}$ for meson and glueball channels mix with each other in their mode-determining equation. In order to see the relation between the proper wave function $\Delta\tilde{S}^{(1)}$ and the other variation $\Delta\psi_O^{(1)}$ with an arbitrary fixed variable $O(\neq\tilde{S})$ of the meson-glueball part, we write the on-shell condition (3.3) in the form of (3.4) after eliminating the variables except for $\Delta\tilde{S}^{(1)}$ and $\Delta\psi_O^{(1)}$:

$$\begin{bmatrix} (\gamma_O)_{\tilde{S},\tilde{S}} & (\gamma_O)_{O,\tilde{S}} \\ (\gamma_O)_{\tilde{S},O} & (\gamma_O)_{O,O} \end{bmatrix} \begin{bmatrix} \Delta\tilde{S}^{(1)} \\ \Delta\psi_O^{(1)} \end{bmatrix} = 0 \ . \tag{3.20}$$

The derivative $\overleftarrow{\delta}/\delta\psi_X\cdot\Delta\psi_X^{(1)}$ is then rewritten in terms of $\Delta\tilde{S}^{(1)}$:

$$\begin{aligned} \frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(1)} &= \frac{\overleftarrow{\delta}}{\delta\tilde{S}}\Delta\tilde{S}^{(1)} - \frac{\overleftarrow{\delta}}{\delta\psi_{\bar{D}}}(\gamma_{\bar{D}})^{-1}_{\bar{D},\bar{D}}(\gamma_{\bar{D}})_{\bar{D},\tilde{S}}\Delta\tilde{S}^{(1)} - \cdots \\ &\equiv \frac{\overleftarrow{\delta}}{\delta\psi_X}\hat{\gamma}_{X,\tilde{S}}\Delta\tilde{S}^{(1)} \ . \end{aligned} \tag{3.21}$$

It is convenient to suppose that the mixing matrix $\hat{\gamma}_{X,\tilde{S}}$ itself is also a given quantity. Combinging (3.18) with (3.19) and (3.21), the general full order form of the S-matrix element of the meson-meson scattering is obtained as follows:

$$\begin{aligned} &\Bigg[\frac{1}{4!}\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\hat{\gamma}_{X,\tilde{S}}\Delta\tilde{S}^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_Y}\hat{\gamma}_{Y,\tilde{S}}\Delta\tilde{S}^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_Z}\hat{\gamma}_{Z,\tilde{S}}\Delta\tilde{S}^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_W}\hat{\gamma}_{W,\tilde{S}}\Delta\tilde{S}^{(1)} \\ &\quad + \frac{3}{4!}\left\{\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\frac{\overleftarrow{\delta}}{\delta\psi_{X'}}\hat{\gamma}_{X',\tilde{S}}\Delta\tilde{S}^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_{X''}}\hat{\gamma}_{X'',\tilde{S}}\Delta\tilde{S}^{(1)}\right\}\left\{\frac{\overrightarrow{\delta}}{\delta J_Y}\frac{\overrightarrow{\delta}W}{\delta J_X}\right\}\left\{\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_Y}\frac{\overleftarrow{\delta}}{\delta\psi_{Y'}}\hat{\gamma}_{Y',\tilde{S}}\Delta\tilde{S}^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_{Y''}}\hat{\gamma}_{Y'',\tilde{S}}\Delta\tilde{S}^{(1)}\right\}\Bigg]_0 \ . \end{aligned} \tag{3.22}$$

Here $\overleftarrow{\delta}/\delta\psi_X$, etc., operate only on the effective action Γ.

Let us calculate the lowest-order term of (3.22), which produces the conventional quark-line diagrams. It is given by keeping only $i\,\mathrm{Tr}\ln\tilde{S}$ in (3.17). Then (3.22) becomes nonzero only for $X=Y=\cdots=Y''=\tilde{S}$. Since

$$(i\,\mathrm{Tr}\ln\tilde{S})\frac{\overleftarrow{\delta}}{\delta\tilde{S}_{ij}}\frac{\overleftarrow{\delta}}{\delta\tilde{S}_{kl}} = -i\tilde{S}_{jk}^{-1}\tilde{S}_{li}^{-1} \ ,$$

the symbolical notation $\overrightarrow{\delta}/\delta J_{\tilde{S}_{ij}}(\overrightarrow{\delta}W/\delta J_{\tilde{S}_{kl}})$ can be replaced by $-i\tilde{S}_{il}\tilde{S}_{kj}$ owing to the identity (2.14) which reads, in this case,

$$\begin{aligned} \left\{\Gamma\frac{\overleftarrow{\delta}}{\delta\tilde{S}_{ij}}\frac{\overleftarrow{\delta}}{\delta\tilde{S}_{mn}}\right\}_0\left\{\frac{\overrightarrow{\delta}}{J_{\tilde{S}_{kl}}}\frac{\overrightarrow{\delta}}{J_{\tilde{S}_{mn}}}W\right\}_0 &= \left\{\frac{\overrightarrow{\delta}}{J_{\tilde{S}_{ij}}}\frac{\overrightarrow{\delta}}{J_{\tilde{S}_{mn}}}W\right\}_0\left\{\Gamma\frac{\overleftarrow{\delta}}{\delta\tilde{S}_{kl}}\frac{\overleftarrow{\delta}}{\delta\tilde{S}_{mn}}\right\}_0 \\ &= -\delta_{\tilde{S}_{ij}\tilde{S}_{kl}} = -\delta_{ik}\delta_{jl} \ . \end{aligned} \tag{3.23}$$

The general expression (3.22) now simply becomes

$$i\frac{6}{4!}\tilde{S}_{jk}^{(0)\,-1}\tilde{S}_{lm}^{(0)\,-1}\tilde{S}_{no}^{(0)\,-1}\tilde{S}_{pi}^{(0)\,-1}\Delta\langle q_k\bar{q}_l\rangle^{(1)}\Delta\langle q_m\bar{q}_n\rangle^{(1)}\Delta\langle q_o\bar{q}_p\rangle^{(1)}\Delta\langle q_i\bar{q}_j\rangle^{(1)} \ , \tag{3.24}$$

where $\Delta\langle q_i\bar{q}_j\rangle^{(1)} = \Delta\tilde{S}_{ij}^{(1)}$ and $\tilde{S}^{(0)}$ is the stationary solution of $\tilde{S}$. Graphically, the result is given by the quark-line diagram shown in Fig. 2(a). Each line naturally represents the full propagator $\tilde{S}^{(0)}$ in our case. When we try to calculate $\tilde{S}^{(0)}$ directly from the effective action after some approximations, not only $i\,\mathrm{Tr}\ln\tilde{S}$ but also the remaining terms of (3.17) should properly be taken into account, of course, for the correct estimation of the nonperturbative ground state or the low-energy behavior of QCD.

The scattering matrix element is extracted as follows. Consider, for example, the mesons A,B and C,D in the initial and the final states, respectively. As in the case of the bosonic model briefly discussed at the beginning of this section, the S-matrix element is obtained by operating $\delta/\delta\tilde{C}_A^+\cdot\delta/\delta\tilde{C}_B^+\cdot\delta/\delta\tilde{C}_C^-\cdot\delta/\delta\tilde{C}_D^-$ to (3.24) with $\tilde{C}_i^{\pm}$ defined for the mesonic bound-state mode i. The result becomes

$$\begin{aligned} i\tilde{S}_{jk}^{(0)\,-1}\tilde{S}_{lm}^{(0)\,-1}\tilde{S}_{no}^{(0)\,-1}\tilde{S}_{pi}^{(0)\,-1}(&A_{kl}C_{mn}D_{op}B_{ij} + A_{kl}C_{mn}B_{op}D_{ij} + A_{kl}B_{mn}C_{op}D_{ij} \\ &+ A_{kl}B_{mn}D_{op}C_{ij} + A_{kl}D_{mn}B_{op}C_{ij} + A_{kl}D_{mn}C_{op}B_{ij}) \ , \end{aligned} \tag{3.25}$$

where A, B, C, or D symbolically represents the wave function of each mesonic mode projected out from the whole index space of $\Delta\langle q_i\bar{q}_j\rangle^{(1)}$. The first three terms in the parentheses of (3.25) are graphically shown in Figs. 2(b)–2(d). The rest are given by the same types of diagram but with the propagator in the opposite direction. They appear in equal weight with the factor 1. Each diagram of Figs. 2(b)–2(d) corresponds to X, H, and Z types

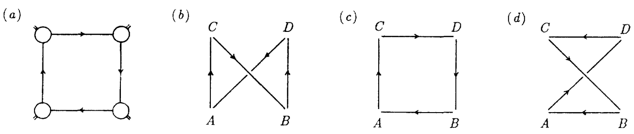

FIG. 2. Schematic representation of the lowest-order amplitude (3.24) of the meson-meson scattering (a). Each line denotes the stationary solution $\tilde{S}^{(0)}$ of the full quark propagator. Mesonic wave functions A, B, C, and D are then projected out from the $\Delta\langle q\bar{q}\rangle^{(1)}$'s in (a) to get the constituent rearrangement diagrams shown in (b)–(d).

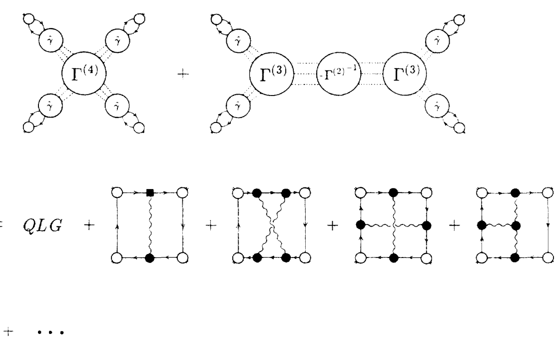

FIG. 3. The general form of the meson-meson scattering amplitude (3.22) and some correction diagrams for the quark line graph (QLG) in Fig. 2(a). A set of three dotted lines indicates the contraction of the variables in the meson-glueball channel and is explicitly given by the possible combinations of $\tilde{S}^{(0)}$ and $\tilde{D}^{(0)}$. A three-point vertex and a wavy line denote the vacuum solutions $C_3^{(0)}$ and $\tilde{D}^{(0)}$, respectively. The mixing matrix $\hat{\gamma}$ is defined in (3.21) and the solid square represents $v_3^0(\bar{q},A,q)+(i/2)C_3^{(0)}(\bar{q},A,q)$. The other notation is the same as in Fig. 2.

of the CRD or quark rearrangement diagram (QRD), respectively.

In the same way, the higher-order corrections to the quark-line graphs are obtained form (3.22) by taking further into account the remaining terms of the effective action (3.17). The total form of (3.22) is graphically shown in Fig. 3. $\Gamma^{(4)}_{\psi_X,\psi_Y,\psi_Z,\psi_W}$ and $\Gamma^{(3)}_{\psi_X,\psi_Y,\psi_Z}$ parts of the diagram do not have a set of internal propagators which connects their adjacent external lines. These corrections have already been included in the second step of the formalism as the interactions to make up the bound state. Some of them are restored in the diagram through the mixing matrix $\hat{\gamma}_{X,\tilde{S}}$ to have the proper mesonic wave functions in the external legs. By the effects of such channel mixings, not only the quark-line diagrams with internal corrections but also other types of graphs such as gluon loop diagrams appear in the result. Of course, it can also be checked that the result recovers a series of perturbative diagrams if we *perturbatively* use the stationary condition (3.2) of the effective action.

2. *Baryon-baryon scattering*

Next we examine the baryon-baryon scattering diagrams. Here we take $[G_1,\tilde{G}_2,\tilde{G}_3]$ as the independent set of the variables of Γ. The baryonic modes are determined by the variations $\Delta\langle qqq\rangle^{(1)}=\Delta\tilde{G}_3(q,q,q)$ and $\Delta\langle\bar{q}\,\bar{q}\,\bar{q}\rangle^{(1)}=\Delta\tilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})$ of (3.3). The indices specifying each baryonic mode are to be projected out from these on-shell variations. As in the previous example, consider the terms proportional to $(\Delta\langle qqq\rangle^{(1)})^2(\Delta\langle\bar{q}\,\bar{q}\,\bar{q}\rangle^{(1)})^2$. Using the notation of De Dominicis and Martin, they are extracted by the operations

$$\Bigg[\frac{6}{4!}\Gamma\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(q,q,q)}\Delta\tilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(q,q,q)}\Delta\tilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\tilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta G_3^{(1)}(\bar{q},\bar{q},\bar{q})$$
$$+\frac{3!}{3!}\Gamma\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(q,q,q)}\Delta\tilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\tilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})\frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(2)}+\frac{1}{2!}\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(2)}\frac{\overleftarrow{\delta}}{\delta\psi_Y}\Delta\psi_Y^{(2)}\Bigg]_0 . \quad (3.26)$$

As in (3.18), the subscripts X and Y represent the variables introduced for the meson-glueball channel. By using (3.19) with $\tilde{G}_3$ instead of C_3, the general form of the corresponding connected S-matrix element is then derived from (3.26) as follows:

$$\Bigg[\frac{6}{4!}\Gamma\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(q,q,q)}\Delta\tilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(q,q,q)}\Delta\tilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\tilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\tilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})$$
$$+\frac{1}{2!}\left[\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(q,q,q)}\Delta G_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\tilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})\right]\left[\frac{\overrightarrow{\delta}}{\delta J_Y}\frac{\overrightarrow{\delta}W}{\delta J_X}\right]$$
$$\times\left[\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_Y}\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(q,q,q)}\Delta\tilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\tilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})\right]\Bigg]_0 . \quad (3.27)$$

The nonzero contribution of the first term of (3.27) comes only from $-i\kappa$ part of (3.17) but it does not contribute to the conventional quark-line graphs. The lowest-order contributions are obtained, on the other hand, from the second term of (3.27)

$$\frac{1}{2!}\left[\Gamma\frac{\overleftarrow{\delta}}{\delta\tilde{S}}\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(q,q,q)}\Delta\tilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\tilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})\right]_0\left[-\Gamma\frac{\overleftarrow{\delta}}{\delta\tilde{S}}\frac{\overleftarrow{\delta}}{\delta\tilde{S}'}\right]_0^{-1}$$
$$\times\left[\Gamma\frac{\overleftarrow{\delta}}{\delta\tilde{S}'}\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(q,q,q)}\Delta\tilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\tilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\tilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})\right]_0 , \tag{3.28}$$

where the following part of (3.17) is considered as the effective action Γ:

$$\Gamma\simeq i\,\mathrm{Tr}\ln\tilde{S}+\frac{i}{3!}C_3(q_i,q_j,q_k)\tilde{S}_{ii'}\tilde{S}_{jj'}\tilde{S}_{kk'}C_3(\bar{q}_{i'},\bar{q}_{j'},\bar{q}_{k'})$$
$$=i\,\mathrm{Tr}\ln\tilde{S}-\frac{i}{3!}\tilde{G}_3(\bar{q}_i,\bar{q}_j,\bar{q}_k)\tilde{S}_{ii'}^{-1}\tilde{S}_{jj'}^{-1}\tilde{S}_{kk'}^{-1}\tilde{G}_3(q_{i'},q_{j'},q_{k'}) . \tag{3.29}$$

Combining (3.28) with (3.29), we then obtain the lowest-order contribution

$$\frac{i}{8}\tilde{S}_{jj'}^{(0)\;-1}\tilde{S}_{kk'}^{(0)\;-1}\tilde{S}_{ri'}^{(0)\;-(1)}\tilde{S}_{ir'}^{(0)\;-1}\tilde{S}_{ss'}^{(0)\;-1}\tilde{S}_{tt'}^{(0)\;-1}\Delta\langle\bar{q}_i\bar{q}_j\bar{q}_k\rangle^{(1)}\Delta\langle q_{i'}q_{j'}q_{k'}\rangle^{(1)}\Delta\langle\bar{q}_r\bar{q}_s\bar{q}_t\rangle^{(1)}\Delta\langle q_{r'}q_{s'}q_{t'}\rangle^{(1)} . \tag{3.30}$$

Each set of three external points obtained by the operation $\vec{\delta}/\delta\tilde{G}_3(q_i,q_j,q_k)$ or $\vec{\delta}/\delta\tilde{G}_3(\bar{q}_i,\bar{q}_j,\bar{q}_k)$ in (3.28) is accompanied by the antisymmetric unit tensor

$$\delta^{\mathrm{as}}_{ijk,i'j'k'}\equiv(1/3!)(\delta_{ii'}\delta_{jj'}\delta_{kk'}-\delta_{ij'}\delta_{ji'}\delta_{kk'}+\cdots) ,$$

which has been absorbed into the wave-function part in (3.30):

$$\delta^{\mathrm{as}}_{ijk,i'j'k'}\Delta\langle q_{i'}q_{j'}q_{k'}\rangle^{(1)}=\Delta\langle q_iq_jq_k\rangle^{(1)} , \tag{3.31a}$$

$$\delta^{\mathrm{as}}_{ijk,i'j'k'}\Delta\langle\bar{q}_{i'}\bar{q}_{j'}\bar{q}_{k'}\rangle^{(1)}=\Delta\langle\bar{q}_i\bar{q}_j\bar{q}_k\rangle^{(1)} . \tag{3.31b}$$

Graphically the result (3.30) is shown in Fig. 4(a). The lowest-order variations in (3.30) satisfy

$$\Delta\langle qqq\rangle^{(1)}=\langle\bar{B}|qqq|0\rangle+\langle 0|qqq|B\rangle$$

and

$$\Delta\langle\overline{qqq}\rangle^{(1)}=\langle B|\bar{q}\,\bar{q}\,\bar{q}|0\rangle+\langle 0|\bar{q}\,\bar{q}\,\bar{q}|\bar{B}\rangle$$

with an appropriate baryonic (antibaryonic) state $|B\rangle$ ($|\bar{B}\rangle$). So $\Delta\langle qqq\rangle^{1}$ and $\Delta\langle\bar{q}\,\bar{q}\,\bar{q}\rangle^{(1)}$ can directly be replaced by the initial and the final baryonic wave functions, respectively. For instance, the S-matrix element for the two-body baryonic scattering $AB\to CD$ is given by

$$\frac{i}{4}\tilde{S}_{jj'}^{(0)\;-1}\tilde{S}_{kk'}^{(0)\;-1}\tilde{S}_{ri'}^{(0)\;-1}\tilde{S}_{ir'}^{(0)\;-1}\tilde{S}_{ss'}^{(0)\;-1}\tilde{S}_{tt'}^{(0)\;-1}(C_{ijk}A_{i'j'k'}D_{rst}B_{r's't'}-D_{ijk}A_{i'j'k'}C_{rst}B_{r's't'}) , \tag{3.32}$$

where A, B, C, and D represent the appropriate wave functions of the corresponding baryonic modes. The result corresponds to X and X_d types of CRD given in Figs. 4(b) and 4(c), respectively. Of course, baryon-antibaryon scattering diagrams can also be derived from (3.30) in the form of H and H_d types of CRD as shown in Figs. 4(d) and 4(e).

The inclusion of the higher-order terms is now systematic by using (3.27) (see Fig. 5). The stationary condition of the effective action is assumed for each variable in the result. By the definition of the three-particle irreducibility, the internal corrections for the legs of $\Delta\tilde{G}_3(q,q,q)$ or $\Delta\tilde{G}_3(\bar{q},\bar{q},\bar{q})$ appear when we graphically write down each term of (3.27). The topological property of the diagram is different from the previous two-body scattering case; however, the on-shell expansion scheme again assures a methodical inclusions of the interactions which are responsible for the necessary information of the observable quantity.

3. Baryon-meson scattering

Finally we consider the baryon-meson scattering. In the same way as in the above two examples, we concentrate on the

$$(\Delta\langle q\bar{q}\rangle^{(1)})^2\Delta\langle qqq\rangle^{(1)}\Delta\langle\bar{q}\,\bar{q}\,\bar{q}\rangle^{(1)}$$

part of $\Gamma[\psi+\Delta\psi]$, which now appears in the terms

$$\Bigg[\frac{12}{4!}\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_Y}\Delta\psi_Y^{(1)}\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(q,q,q,)}\Delta\widetilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\widetilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})$$
$$+\frac{3}{3!}\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_Y}\Delta\psi_Y^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_Z}\Delta\psi_Z^{(2)}+\frac{3!}{3!}\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(1)}\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(q,q,q)}\Delta\widetilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\widetilde{G}_3^{(2)}(\bar{q},\bar{q},\bar{q})$$
$$+\frac{3!}{3!}\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(1)}\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(q,q,q)}\Delta\widetilde{G}_3^{(2)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\widetilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})$$
$$+\frac{3!}{3!}\Gamma\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(q,q,q)}\Delta\widetilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\widetilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})\frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(2)}$$
$$+\frac{2}{2!}\Gamma\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(q,q,q)}\Delta\widetilde{G}_3^{(2)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\widetilde{G}_3^{(2)}(\bar{q},\bar{q},\bar{q})+\frac{1}{2!}\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(2)}\frac{\overleftarrow{\delta}}{\delta\psi_Y}\Delta\psi_Y^{(2)}\Bigg]_0 . \quad (3.33)$$

The same notation X, Y, and Z is employed for meson-glueball variables. We then use (3.19) and the explicit forms of $\Delta\widetilde{G}_3^{(2)}(q,q,q)$ and $\Delta\widetilde{G}_3^{(2)}(\bar{q},\bar{q},\bar{q})$:

$$\Delta\widetilde{G}_3^{(2)}(q,q,q)=\frac{1}{2!}\left[\frac{\overrightarrow{\delta}}{\delta J_{\bar{q}\bar{q}\bar{q}}}\frac{\overrightarrow{\delta}W}{\delta J_{qqq}}\right]_0\left[2\Gamma\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(\bar{q},\bar{q},\bar{q})}\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(q,q,q)}\Delta\widetilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(1)}+\cdots\right]_0 , \quad (3.34a)$$

$$\Delta\widetilde{G}_3^{(2)}(\bar{q},\bar{q},\bar{q})=\frac{1}{2!}\left[\frac{\overrightarrow{\delta}}{\delta J_{qqq}}\frac{\overrightarrow{\delta}W}{\delta J_{\bar{q}\bar{q}\bar{q}}}\right]_0\left[2\Gamma\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(q,q,q)}\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\widetilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})\frac{\overleftarrow{\delta}}{\delta\psi_X}\Delta\psi_X^{(1)}+\cdots\right]_0 . \quad (3.34b)$$

The symbolic notation J_{qqq} and $J_{\bar{q}\bar{q}\bar{q}}$ is explicitly given as $J_3(q,q,q)/3!$ and $J_3(\bar{q},\bar{q},\bar{q})/3!$ of (3.9), respectively. The full-order connected S-matrix element of the baryon-meson scattering is now derived from (3.33) as

$$\Bigg[\frac{1}{2}\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\hat{\gamma}_{X,\bar{S}}\Delta\widetilde{S}^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_Y}\hat{\gamma}_{Y,\bar{S}}\Delta\widetilde{S}^{(1)}\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(q,q,q)}\Delta\widetilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\widetilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})$$
$$+\frac{1}{2}\left[\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_X}\frac{\overleftarrow{\delta}}{\delta\psi_Z}\hat{\gamma}_{Z,\bar{S}}\Delta\widetilde{S}^{(1)}\frac{\overleftarrow{\delta}}{\delta\psi_W}\hat{\gamma}_{W,\bar{S}}\Delta\widetilde{S}^{(1)}\right]\left[\frac{\overrightarrow{\delta}}{\delta J_Y}\frac{\overrightarrow{\delta}W}{\delta J_X}\right]$$
$$\times\left[\Gamma\frac{\overleftarrow{\delta}}{\delta\psi_Y}\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(q,q,q)}\Delta\widetilde{G}_3^{(1)}(q,q,q)\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\widetilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})\right]$$
$$+\left[\Gamma\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(\bar{q},\bar{q},\bar{q})}\frac{\overleftarrow{\delta}}{\delta\psi_X}\hat{\gamma}_{X,\bar{S}}\Delta\widetilde{S}^{(1)}\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(q,q,q)}\Delta\widetilde{G}_3^{(1)}(q,q,q)\right]\left[\frac{\overrightarrow{\delta}}{\delta J_{qqq}}\frac{\overrightarrow{\delta}W}{\delta J_{\bar{q}\bar{q}\bar{q}}}\right]$$
$$\times\left[\Gamma\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(q,q,q)}\frac{\overleftarrow{\delta}}{\delta\psi_Y}\hat{\gamma}_{Y,\bar{S}}\Delta\widetilde{S}^{(1)}\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(\bar{q},\bar{q},\bar{q})}\Delta\widetilde{G}_3^{(1)}(\bar{q},\bar{q},\bar{q})\right]\Bigg]_0 , \quad (3.35)$$

where $\hat{\gamma}$ is the mixing matrix defined in (3.21).

In order to derive the quark-line diagrams, (3.29) is used for the effective action Γ again. In this case, we find

$$\left[\frac{\overrightarrow{\delta}}{\delta J_{q_iq_jq_k}}\frac{\overrightarrow{\delta}W}{\delta J_{\bar{q}_{i'}\bar{q}_{j'}\bar{q}_{k'}}}\right]_0=-i\,3!\widetilde{S}_{ii''}^{(0)}\widetilde{S}_{jj''}^{(0)}\widetilde{S}_{kk''}^{(0)}\delta^{\rm as}_{i''j''k'',i'j'k'} , \quad (3.36)$$

which is defined as the inverse of

$$\left[-\Gamma\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(q_i,q_j,q_k)}\frac{\overleftarrow{\delta}}{\delta\widetilde{G}_3(\bar{q}_{i'},\bar{q}_{j'},\bar{q}_{k'})}\right]_0=\frac{i}{3!}\widetilde{S}_{ii''}^{(0)\,-1}\widetilde{S}_{jj''}^{(0)\,-1}\widetilde{S}_{kk''}^{(0)\,-1}\delta^{\rm as}_{i''j'',k'',i'j'k'} . \quad (3.37)$$

After a short calculation, the following simple result is obtained from (3.35):

$$\frac{i}{2}(\widetilde{S}_{in}^{(0)\,-1}\widetilde{S}_{ol}^{(0)\,-1}\widetilde{S}_{mi'}^{(0)\,-1}\widetilde{S}_{jj'}^{(0)\,-1}\widetilde{S}_{kk'}^{(0)\,-1}+\widetilde{S}_{il}^{(0)\,-1}\widetilde{S}_{mi'}^{(0)\,-1}\widetilde{S}_{jn}^{(0)-1}\widetilde{S}_{oj'}^{(0)\,-1}\widetilde{S}_{kk'}^{(0)\,-1})$$
$$\times\Delta\langle\bar{q}_i\bar{q}_j\bar{q}_k\rangle^{(1)}\Delta\langle q_{i'}q_{j'}q_{k'}\rangle^{(1)}\Delta\langle q_l\bar{q}_m\rangle^{(1)}\Delta\langle q_n\bar{q}_o\rangle^{(1)} , \quad (3.38)$$

which is graphically represented in Fig. 6(a). Consider the baryon-meson scattering $B_1M_1\rightarrow B_2M_2$, for example. Then the connected S matrix element is symbolically written as in the previous cases as

$$\frac{i}{2}\widetilde{S}_{in}^{(0)\,-1}\widetilde{S}_{ol}^{(0)\,-1}\widetilde{S}_{mi'}^{(0)\,-1}\widetilde{S}_{jj'}^{(0)\,-1}\widetilde{S}_{kk'}^{(0)\,-1}(B_2)_{ijk}(B_1)_{i'j'k'}\{(M_1)_{lm}(M_2)_{no}+(M_2)_{lm}(M_1)_{no}\}$$
$$+i\widetilde{S}_{il}^{(0)\,-1}\widetilde{S}_{mi'}^{(0)\,-1}\widetilde{S}_{jn}^{(0)\,-1}\widetilde{S}_{oj'}^{(0)\,-1}\widetilde{S}_{kk'}^{(0)\,-1}(B_2)_{ijk}(B_1)_{i'j'k'}(M_1)_{lm}(M_2)_{no} . \quad (3.39)$$

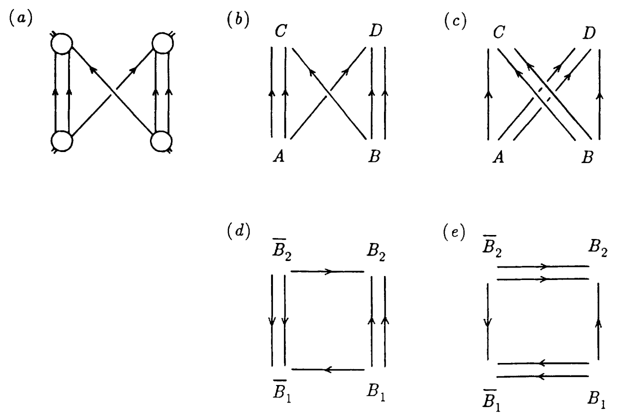

FIG. 4. Graphical representation of the lowest-order amplitude (3.30) of the two-body baryonic scattering [graph (a)]. Similar notations have been employed as in Fig. 2. The quark-line graphs for baryon-baryon and baryon-antibaryon scattering amplitudes are schematically shown in (b), (c) and (d), (e), respectively.

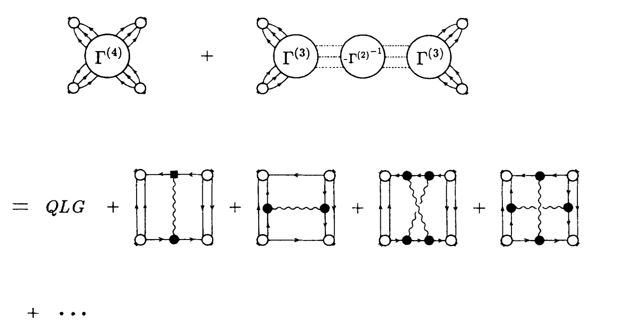

FIG. 5. The general form of the two-body baryonic scattering amplitude (3.27) and some correction diagrams for the quark-line graph (QLG) in Fig. 4(a). The notations are the same as in Figs. 3 and 4.

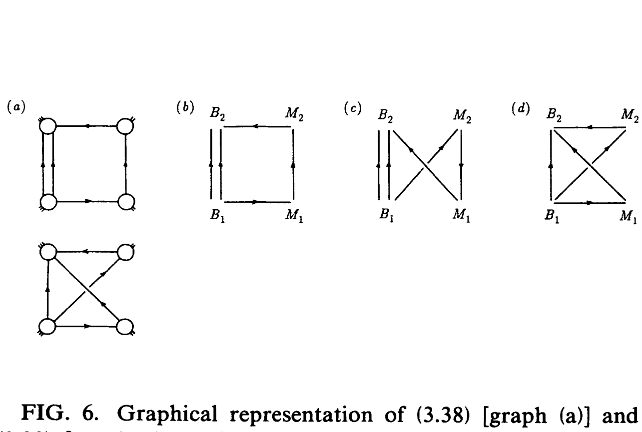

FIG. 6. Graphical representation of (3.38) [graph (a)] and (3.39) [graphs (b)–(d)] for the baryon-meson scattering case. The same notation is employed as in the previous examples.

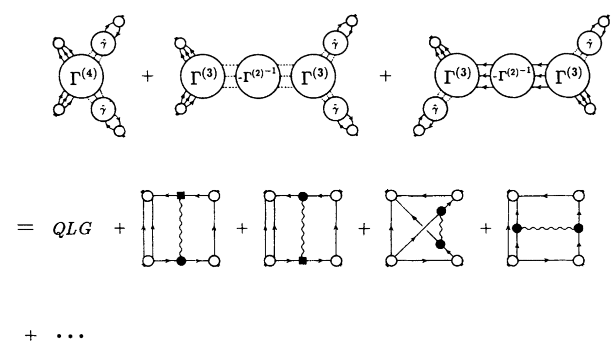

FIG. 7. Schematical representation of the baryon-meson scattering amplitude (3.35) and some correction diagrams for the quark-line graphs in Fig. 6(a). The notation is the same as in the previous cases.

The first term of (3.39) is graphically shown in Figs. 6(b) and 6(c) and the second term in Fig. 6(d). The scattering diagrams in Figs. 6(b)–6(d) correspond to H, X, and Z types of CRD, respectively. Here again the higher-order terms can be derived step by step from (3.35). Some of the diagrams are schematically shown in Fig. 7.

Recall that the above-obtained results are all represented by the nontrivial variables defined through the Legendre transformation—the full propagators and the dressed three-point vertices in this case. The reconstruction of the scattering diagrams in terms of these variables leads to the important topological property of two- and three-particle irreducibility and this is essential for the study of the nonperturbative aspects of QCD. Only by using the intuitive graphical approach, may it be difficult to include the nonperturbative quantities into the scattering diagrams. Compared with the conventional diagrammatical approach, the present method is suitable to clarify the problem since the nontrivial changes of the variables are completed at the first stage of formalism, i.e., by the Legendre transformation. It is particularly convenient when we examine various channels through many different variables in general. By using the on-shell expansion of the effective action, we can count up the interactions responsible for the necessary information unambiguously.

IV. DISCUSSIONS: CONFINEMENT AND STRING PICTURE

We have discussed in a formal way how to get the correct two-body scattering amplitudes of hadronic system based on QCD theory. Since, however, QCD is not solved at present, all the essential nonperturbative parameters are left undetermined, or they can be regarded as phenomenological parameters and are fitted by using the experimental data.

The essential ingredient of QCD is the color confinement where any colored state, such as a single quark state, is not observed as a physical entity. In our paper, we took the fact as granted and the color-singlet channels are discussed. But the quark confinement itself is discussed in our formalism as follows.

We recover in (3.16) the elementary quark field as G_1 and calculate Γ as a function of $G_1(q)=\langle q\rangle$ and $G_1(\bar{q})=\langle\bar{q}\rangle$. Then the excitation spectrum in the quark channel is given by solving the on-shell equation of the form

$$\left[\Gamma\frac{\overleftarrow{\delta}}{\delta\langle\bar{q}_i\rangle}\frac{\overleftarrow{\delta}}{\delta\langle q_j\rangle}\right]_0 \Delta\langle q_j\rangle^{(1)}=0 \ . \tag{4.1}$$

Now there are two possibilities. If we take the perturbative vacuum and evaluate $(\cdots)_0$ of (4.1) in terms of this state, then the perturbative quark state appears as a solution to (4.1). If, however, the correct condensed vacuum is chosen, a confining solution may be obtained. From experiences with two-dimensional QCD in the large-N limit [22,23], there are two cases for confinement. The first case is that (4.1) has only a trivial solution $\Delta\langle q\rangle^{(1)}=0$ which corresponds to the case of a direct infrared cutoff [22] leading to an infinite quark mass. The second one [23] is that Eq. (4.1) has a nontrivial solution but the scattering involving quarks is completely separated from hadronic scattering; quarks are not produced by scattering among hadrons. This happens when one takes the principal part regularization for infrared singularity. In both cases we are allowed to leave aside all the quark sectors as has been done in the previous sections.

If, further, confinement is correctly described by the vortex of a color-electric field, then the meson wave function $\Delta\langle q\bar{q}\rangle^{(1)}$, for example, has a shape corresponding to this vortex structure. The scattering amplitudes which are the coefficients of $(\Delta\langle q\bar{q}\rangle^{(1)})^4$ in our on-shell expansion have the characteristic form due to the presence of a vortex. In terms of the action functional, this is idealized as a sheet which represents the space-time evolution of the scattering process—the string picture. All the above stories are not proved in the continuum QCD up to now but our formalism provides rigorous expressions that have to be used for the hadronic scattering realized upon the condensed vacuum.

The application of our formalism to two-dimensional QCD in the large-N limit will be an interesting subject. Here explicit calculations are possible and we can see how our on-shell expansion scheme works and can construct a concrete expression for the generating functional of the "hadronic" observable quantities. The subject is under investigation and the results will be published in a separate paper.

[1] For a recent review, see R. W. Haymaker, Riv. Nuovo Cimento **14**(8) (1991).

[2] A. Jevicki and C. K. Lee, Phys. Rev. D **37**, 1485 (1988), and the references therein.

[3] R. Fukuda, Prog. Theor. Phys. **78**, 1487 (1987).

[4] R. Fukuda, M. Komachiya, and M. Ukita, Phys. Rev. D **38**, 3747 (1988).

[5] M. Komachiya, M. Ukita, and R. Fukuda, Phys. Rev. D **42**, 2792 (1990); M. Ukita, in *Proceedings of 1989 Workshop on Dynamical Symmetry Breaking*, Nagoya, Japan, edited by T. Muta and K. Yamawaki (Nagoya University, Nagoya, 1990), p. 204.

[6] R. Fukuda, in *Proceedings of 1991 Nagoya Spring School of Dynamical Symmetry Breaking*, Nagoya, Japan, edited by K. Yamawaki (World Scientific, Singapore, 1991), p. 223.

[7] M. Komachiya, M. Ukita, and R. Fukuda, Phys. Rev. D **40**, 2654 (1989).

[8] K. Higashijima, Prog. Theor. Phys. Suppl. **104**, 1 (1991). See also Section X of Ref. [1].

[9] D. W. McKay and H. J. Munczek, Phys. Rev. D **40**, 4151 (1989); C. J. Burden, R. T. Cahill, and J. Praschifka, Aust. J. Phys. **42**, 147 (1989); R. T. Cahill, J. Praschifka, and C. J. Burden, *ibid.* **42**, 161 (1989); R. T. Cahill, *ibid.* **42**, 171 (1989); M. Lutz and J. Praschifka, University of Regensburg Report No. TRP-90-47, 1990 (unpublished).

[10] M. Bando, T. Kugo, N. Maekawa, N. Sasakura, Y. Watanabe, and K. Suehiro, Phys. Lett. B **246**, 466 (1990); K.-I. Aoki, T. Kugo, and M. G. Mitchard, *ibid.* **266**, 467 (1991).

[11] A. N. Vasil'ev and A. K. Kazanskii, Theor. Math. Phys. **12**, 875 (1972); **14**, 215 (1973); Yu. M. Pis'mak, *ibid.* **18**, 211 (1974); A. N. Vasil'ev, A. K. Kazanskii, and Yu. M. Pis'mak, *ibid.* **19**, 443 (1974); **20**, 754 (1974).

[12] M. Combescure and F. Dunlop, Ann. Phys. (N.Y.) **122**, 102 (1979).

[13] A. Cooper, J. Feldman, and L. Rosen, Ann. Phys. (N.Y.) **137**, 146 (1981); **137**, 213 (1981); J. Math. Phys. **23**, 846 (1982); Phys. Rev. D **25**, 1565 (1982).

[14] J. L. Rosner, Phys. Rev. Lett. **21**, 950 (1968); **22**, 689 (1969); H. Harari, *ibid.* **22**, 562 (1969); S. Okubo, Prog. Theor. Phys. Suppl. **63**, 1 (1978); Y. Igarashi, M. Imachi, T. Matsuoka, K. Ninomiya, S. Otsuki, S. Sawada, and F. Toyoda, *ibid.* **63**, 49 (1978).

[15] M. Komachiya and R. Fukuda, Int. J. Mod. Phys. A (to be published); M. Komachiya, in *Proceedings of 1989 Workshop on Dynamical Symmetry Breaking* [5], p. 221.

[16] K. B. Efetov, Adv. Phys. **32**, 53 (1983).

[17] H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento **1**, 205 (1955).

[18] S. Weinberg, Phys. Rev. D **11**, 3583 (1975); R. J. Crewther, Riv. Nuovo Cimento **2**, 63 (1979).

[19] R. Jackiw, Phys. Rev. D **9**, 1686 (1974).

[20] J. M. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. D **10**, 2428 (1974).

[21] C. De Dominicis and P. C. Martin, J. Math. Phys. **5**, 14 (1964); **5**, 31 (1964).

[22] G. 't Hooft, Nucl. Phys. **B72**, 461 (1974); **B75**, 461 (1974).

[23] C. G. Callan, N. Coote, and D. J. Gross, Phys. Rev. D **13**, 1649 (1976); M. B. Einhorn, *ibid.* **14**, 3451 (1976); E. Witten, Nucl. Phys. **B160**, 57 (1979).