PHYSICAL REVIEW D VOLUME 46, NUMBER 6 15 SEPTEMBER 1992

Emergence of radiation from gravitational potential wells: The absence of ωM effects

Richard H. Price and Jorge Pullin
Department of Physics, University of Utah, Salt Lake City, Utah 84112
(Received 07 February 1992)

We consider a source of gravitational waves of frequency ω, located near the center of a massive galaxy of mass M and radius R, with $\omega \gg R^{-1}$. The case of odd-parity gravitational waves propagating through a perfect-fluid galaxy is particularly simple; for this case we find that, in addition to the expected redshift of the radiation emerging from the galaxy, there is a small amount of backscatter, of order $M/\omega^2 R^3$. We show that there is no suppression of radiative power by the factor $1+\omega^2 M^2/4$ as has been recently predicted by Kundu. The origin of Kundu's suppression lies in the interpretation of a term in the expansion of the exterior field of the galaxy in inverse powers of radius. It is shown why that term is not related to the source strength or to the strength of the emerging radiation.

PACS number(s): 4.30.+x

I. INTRODUCTION

If a source of gravitational radiation is located in or near a massive body the curvature of spacetime caused by that body may influence the generation and propagation of the radiation produced by the source. If, for example, a quadrupole oscillator is located at the center of a galaxy of mass M and radius R, we might guess that the effect of the surrounding galaxy on the radiation produced inside it is of the same order, M/R, as the characteristic Newtonian potential, at least in the case, $M/R \ll 1$, that the galaxy is nearly Newtonian. (Here and throughout we use units in which G and c are unity.) In this paper we analyze just what these effects really are for high-frequency waves.

To do this we consider a fairly specific astrophysical configuration. We suppose that there is a source of gravitational radiation emitting waves at frequency ω, and confined to a central source region $r < r_S$ of the galaxy. We require that the source be small compared to the radius of the galaxy ($r_S \ll R$). We also require the source frequency to be high enough so that there is a region outside the source ($r > r_S$) which is deep inside the galaxy ($r \ll R$) and in the wave zone of the source ($r \gg \omega^{-1}$). In this region it is meaningful to talk about the gravitational-wave flux well inside the galaxy. For a typical galaxy $R \approx 10^{23}$ cm and $M \approx 10^{16}$ cm, and for a kilohertz gravitational wave $\omega \approx 10^{-7}$ cm^{-1}. For these values the region we require is the range of radii satisfying $\max(r_S, 10^7 \text{cm}) \ll r \ll 10^{20}$ cm.

For such a configuration the standard analysis tells us that as the high-frequency waves propagate outward there are two effects of spacetime curvature that affect their passage. First the frequency of the waves is redshifted so that the frequency observed far outside the galaxy is reduced from that at the source roughly by the factor $(1+\Phi_0)$, where Φ_0, the central potential, is of order M/R. The second effect is associated with the meaning of the radial coordinate. If r is the usual (i.e., Schwarzschild) radial coordinate, then for waves radially propagating outward, the rate dr/dt is slightly less than unity, and there is an attendant gradual phase shift of the waves, of order ωM, as viewed in the r coordinate. (For a discussion of propagation of gravitational waves, and the distinction between generation and propagation for an "isolated" source, see Thorne [1]).

We consider what other effects influence the propagation of gravitational waves, and find that there are interactions between the spacetime curvature and the waves which are interesting as points of principle, if not of astrophysical importance. There is, however, a significant additional motivation for such a calculation, and a major motivation for this paper. Kundu [2, 3] has recently argued that gravitational-wave energy propagating out of a gravitational potential well will be reduced in intensity by the factor $(1+M^2\omega^2/4)^{-1}$. Because ωM can be large (of order 10^9 for the typical numbers given above), such a reduction of kilohertz gravitational-wave signals originating in other galaxies would make detection of signals impossible and would be of crucial importance in connection with the detection of gravitational waves by instruments now being developed.

The remainder of the paper is organized as follows. We start in Sec. II by outlining the mathematical origin of Kundu's argument that gravitational radiation is suppressed. We then describe the argument against suppression given by Kozameh, Newman, and Rovelli [4], and its relationship to the present work. In Sec. III we derive the necessary connections between the Newman-Penrose (NP) formalism, used by Kundu, and the formalism of metric perturbations. We show, in the Schwarzschild exterior, how the NP projection Ψ_0 is related to the the Zerilli function [5] in the case of even-parity perturbations, and in the odd-parity case to the function solving the Regge-Wheeler equation [6]. For outgoing solutions of both parities, suppression factors arise in the relationship between the terms describing the strength of radiation, and the terms describing the apparent quadrupole moment. The subsequent analysis then takes advantage of the considerable simplicity possible in the odd-parity case. A model problem is defined with a central source of odd-parity waves which propagate outward through a perfect-fluid galaxy. In Sec. IV a Green-function solution to the odd-parity gravitational-wave problem is constructed which shows clearly the relationships among the source strength of the waves, the intensity of the outgoing radiation, and the various terms that can be identified as the quadrupole moment. Section V takes up the problem of the extent to which the galaxy is transparent to (odd-parity) radiation. Numerical results are then presented which show that even for strong gravitational fields, the effect of gravitational potential wells on the propagation of high-frequency radiation is negligible (except, of course, for the well known redshift effect). A summary and discussion of conclusions is given in Sec. VI.

II. THE KUNDU SUPPRESSION AND THE KNR MODEL

Kundu's arguments are framed in the Newman-Penrose [7] (NP) formalism and are based on the Weyl projection Ψ_0 in that formalism. For an outgoing solution Ψ_0 takes the form

$$\Psi_0 = \psi_0^0(u,\theta,\phi)r^{-5} + O(r^{-6}), \tag{1}$$

where u is retarded time. Due to its r^{-5} falloff at large r, the quantity Ψ_0 is not usually viewed as a direct measure of radiation intensity for the outgoing solution, but rather as encoding information about the multipole moments of the source in the near zone (i.e., at distances from the source small compared to a wavelength). The shear

$$\sigma = \sigma_0(u,\theta,\phi)r^{-2} + O(r^{-4}) \tag{2}$$

is well accepted as carrying the information about gravitational-wave energy density, specifically in the Bondi news function [8] $d\sigma_0/du$.

Kundu considers linear perturbations about a Schwarzschild background of mass M and shows that there is a simple relationship between the quantity ψ_0^0 that carries information about multipole moments, and the quantity σ_0 that carries information about radiation. To express this relationship it is convenient to define the "despun" [9] equivalents $\hat{\Psi}_0$ and $\hat{\sigma}$, of the spin-weight +2 quantities Ψ_0 and σ, by

$$\hat{\Psi}_0 \equiv (1/2)\bar{\eth}\bar{\eth}\Psi_0, \quad \hat{\sigma} \equiv (1/2)\bar{\eth}\bar{\eth}\sigma \ , \tag{3}$$

where, on spin-weight +2 quantities,

$$\bar{\eth}\bar{\eth} \equiv \left(\frac{\partial}{\partial\theta} + \cot\theta - \frac{i}{\sin\theta}\frac{\partial}{\partial\phi}\right) \times \left(\frac{\partial}{\partial\theta} + 2\cot\theta - \frac{i}{\sin\theta}\frac{\partial}{\partial\phi}\right) \ . \tag{4}$$

For a multipole mode, of index ℓ, in terms of despun quantities, we find

$$\frac{d^3\hat{\psi}_0^0}{du^3} = -\frac{1}{4}\frac{(\ell+2)!}{(\ell-2)!}\frac{d\bar{\hat{\sigma}}_0}{du} - 3M\frac{d^2\hat{\sigma}_0}{du^2} \ , \tag{5}$$

in which the overbar over $\hat{\sigma}_0$, in the first term on the right, indicates complex conjugation. A useful feature of despun quantities is that their real and imaginary parts correspond, respectively, to even- and odd-parity modes, so that, for $\ell = 2$, we have from the real and imaginary parts of (5)

$$\frac{d^3\left(\hat{\psi}_0^0\right)_{\rm even}}{du^3} = -\frac{6}{r^2}\left[\frac{d\left(\hat{\sigma}_0\right)_{\rm even}}{du} + \frac{M}{2}\frac{d^2\left(\hat{\sigma}_0\right)_{\rm even}}{du^2}\right], \tag{6}$$

$$\frac{d^3\left(\hat{\psi}_0^0\right)_{\rm odd}}{du^3} = \frac{6}{r^2}\left[\frac{d\left(\hat{\sigma}_0\right)_{\rm odd}}{du} - \frac{M}{2}\frac{d^2\left(\hat{\sigma}_0\right)_{\rm odd}}{du^2}\right] \ . \tag{7}$$

When time dependence $e^{i\omega t}$ is assumed, the result becomes

$$\hat{\sigma}_0 = \pm\frac{\omega^2}{6(1 \pm i\omega M/2)}\hat{\psi}_0^0 \ , \tag{8}$$

with the + signs applying for even-parity perturbations, and the − signs for odd.

Kundu interprets this equation as telling us that the radiation amplitude, for a quadrupole source, is reduced due to the mass of the Schwarzschild background by the factor $(1 \pm i\omega M/2)^{-1}$, so that the radiation power flux (proportional to $|d\sigma_0/du|^2$) is reduced by his suppression factor $(1 + \omega^2M^2/4)^{-1}$.

Kundu's arguments depend crucially on his interpretation of ψ_0^0 as the quadrupole moment of the source (aside from multiplicative factors). There are two types of justification given by Kundu for this identification. First, he argues [2] that this identification is valid in linearized theory [10], and is valid in the full nonlinear theory [11] for stationary spacetimes. Furthermore, in the time dependent case ψ_0^0 has the required transformation behavior for the quadrupole moment. As a second and distinct justification, Kundu considers a gravitational-wave source in a massive galaxy and argues that the source integral for the quadrupole moment will be affected by the galaxy only to order M/R, and therefore the quadrupole moment will be negligibly different from that if the source were in flat spacetime.

A model problem has recently been published which suggests that Kundu's suppression factor is a mathematical artifact, and not of physical importance. Kozameh, Newman, and Rovelli [4] (hereafter KNR) use a simple model of a perturbative scalar field Φ, and show that the same apparent suppression applies to quadrupole radiation in this model as in the case of gravitational waves. They consider the equations for $\ell = 2$ scalar perturbations in a Schwarzschild background of mass M and define ψ with $\Phi = \psi(r,u)Y_{2m}(\theta,\phi)$, where r is the Schwarzschild radial coordinate, u is retarded time, and Y_{2m} is a standard (spin-weight 0) $\ell = 2$ spherical harmonic. An outgoing solution will then have the form

$$\psi = \frac{\psi_0}{r} + \frac{\psi_1}{r^2} + \frac{\psi_2}{r^3} + \frac{\psi_3}{r^4} + O(r^{-5}) \ , \tag{9}$$

in which the ψ_k are functions only of u. For a stationary solution ψ_0 and ψ_1 would vanish, and ψ_2 would be the quadrupole moment. To emphasize that the situation may be more ambiguous for dynamical solutions in curved spacetime, KNR refer to ψ_2 as the "field quadrupole," and designate it by Q^f. The equations for the scalar field in the Schwarzschild background then show that ψ_0, the radiative part of ψ is related to Q^f by

$$\psi_0 = -\frac{\omega^2}{3}\,\frac{Q^f}{1+i\omega M/6}. \tag{10}$$

For a given time-varying quadrupole, therefore, the radiated power is reduced by the factor $(1+\omega^2M^2/36)^{-1}$ from what it would be in flat spacetime. KNR assume that this suppression is analogous to the suppression of gravitational radiation found by Kundu, and that the scalar example provides a simple model for understanding the suppression. KNR then proceed to use the scalar model to investigate the question of whether the field quadrupole Q^f is really what is usually considered the "quadrupole moment" of a gravitational-wave source, that is, whether Q^f is the same as the "source quadrupole moment" Q.

To address this question they consider a very simple model: a source inside a massive spherical shell of radius R. Since the spacetime inside the shell is flat, ψ has a simple closed-form outgoing solution for $r < R$. In the interior of the shell Q^f can easily be shown to be identical to the source quadrupole Q. The interior and exterior solutions are then matched at $r = R$ with the condition that the scalar field is continuous across the shell. A consequence of this is to require that Q^f be *discontinuous* across the shell; it increases across the shell by the factor $1 + i\omega M/6$. This enhancement factor cancels the suppression factor and one concludes that the relationship between the scalar radiation, and the scalar source quadrupole (aside from negligible factors of order M/R) is the same as in flat spacetime.

The KNR model is very suggestive of the root of the problem, that the (exterior) "field quadrupole" differs from the source quadrupole. But one might ask what details of the model could be changed to make the argument more convincing. Two details would seem to deserve the most attention. First, the model involves scalar fields and it is difficult to be certain that the lesson of scalar fields applies to gravitational perturbations. A second detail of the KNR model is more important. In the KNR model the assumption that ψ is continuous is tantamount to assuming that the shell is transparent to scalar radiation. But for a shell which cannot absorb or reflect radiation, in a time-invariant background, we know *a priori* that the radiation outside must be related to the source in the same way as the radiation inside. The matching condition, then, eliminates at the outset any possibility of a Kundu effect. A more convincing calculation would model the interaction of the waves and galaxy to allow for interactions, in particular for backscatter.

In the following sections we attempt to fill in some of these details. We study gravitational waves produced by a central source and propagating outward through a "galaxy." We include all effects of interaction and show that a nearly Newtonian galaxy is indeed transparent to the propagation of waves. We also show explicitly why the r^{-2} term in the outgoing solution *does* correspond to the quadrupole moment near the source at the center of the galaxy, but not outside the galaxy.

III. RADIATION SUPPRESSION IN THE ZERILLI AND REGGE-WHEELER EQUATIONS

Although Kundu's analysis of gravitational radiation is carried out in the NP formalism, it turns out to be convenient, as well as instructive, to look at the problem in terms of metric perturbations. We start by showing the relation of the suppression factor in the two formalisms. Both inside and outside the galaxy we take the form of the background metric to be

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{11}$$

with ν and λ functions of r only. We define the radial variable r_* by

$$dr/dr_* \equiv e^{(\nu-\lambda)/2} \equiv e^{\alpha(r)}, \tag{12}$$

and the retarded time u by

$$u \equiv t - r_*. \tag{13}$$

In the Schwarzschild geometry, even-parity perturbations for a particular multipole moment (with $\ell \geq 2$) are conveniently described by the Zerilli [5] function $Z^{(+)}$ which satisfies a simple potential type equation.

$$\left(\frac{\partial^2}{\partial r_*^2} - \frac{\partial^2}{\partial t^2}\right) Z^{(+)} = V^+ Z^{(+)}, \tag{14}$$

where, for $\ell = 2$,

$$V^{(+)} = \frac{(1-2M/r)}{r^2(1+3M/2r)^2}\left[6 + \frac{6M}{r} + \frac{9M^2}{r^2} + \frac{9M^3}{2r^3}\right]. \tag{15}$$

(We use here the notation of Eq. (62), Sec. 24, of Chandrasekhar [12].) From this Zerilli equation one infers that for outgoing radiation $Z^{(+)}$ has the form

$$\begin{aligned} Z^{(+)} &= z_0^{(+)}(u) + z_1^{(+)}(u)r^{-1} + z_2^{(+)}(u)r^{-2} \\ &\quad + z_3^{(+)}(u)r^{-1} + \cdots, \end{aligned} \tag{16}$$

and that, for $\ell = 2$,

$$\begin{aligned} dz_1^{(+)}/du &= 3z_0^{(+)}, \\ dz_2^{(+)}/du &= z_1^{(+)} - 3Mz_0^{(+)}, \\ dz_3^{(+)}/du &= -Mz_1^{(+)} + (21/4)M^2 z_0^{(+)}. \end{aligned} \tag{17}$$

These equations show that $z_0^{(+)}$ and $z_1^{(+)}$ vanish for stationary $\ell = 2$ perturbations. For stationary solutions the $z_2^{(+)}$ term is the first nonvanishing term, and is considered to carry information about the source quadrupole moment. For nonstationary solutions it is the function $z_0^{(+)}$ that carries information about gravitational radiation, since the gravitational wave-power is proportional to $|dz_0^{(+)}/du|^2$. From Eqs. (17) we can then deduce a relationship, $d^2z_2^{(+)}/du^2 = 3(z_0^{(+)} - Mdz_0^{(+)}/du)$, between the radiation quantity $z_0^{(+)}$, and the "quadrupole moment" $z_2^{(+)}$. This relationship implies a suppression factor $|1 - i\omega M|^{-2}$, which is different from the factor found by Kundu. The difference arises from the difference in the choice of the "quadrupole moment" one infers from Ψ_0 and from $Z^{(+)}$, and demonstrates the importance of that choice in the inference of suppression of radiation.

For even-parity perturbations, the relationship between $\mathrm{Re}(\Psi_0)$ and $Z^{(+)}$ is given in Sec. 31, Eq. (352) of Chandrasekhar [12] as

$$-\frac{\mathrm{Re}(\Psi_0)\sin^2\theta}{C_{\ell+2}^{-3/2}}\left(1-\frac{2M}{r}\right) = \frac{3+3M/r+9M^2/2r^2+9M^3/4r^3}{r^3(1+3M/2r)^2}Z^{(+)} + \left[\frac{1}{r}\frac{\partial}{\partial u} + \frac{1-3M/r-3M^2/2r^2}{r^2(1+3M/2r)}\right]\frac{\partial}{\partial r}Z^{(+)}\,. \tag{18}$$

The following should be noted about our use of that result here. (i) The Zerilli functions defined by different authors differ by multiplicative factors, but overall multiplicative factors will not affect the frequency-dependent suppression factor. (ii) The Weyl projection Ψ_0 is invariant with respect to infinitesimal tetrad rotations and infinitesimal coordinate changes. We therefore need not be concerned, for example, that Chandrasekhar employs a nonstandard coordinate gauge. (iii) Chandrasekhar assumes azimuthal symmetry for Ψ_0 and angular dependence $C_{\ell+2}^{-3/2}(\theta)\csc^2\theta$, which is proportional to the spin-weight 2 spherical harmonic ${}_2Y_{\ell m}$, for $m = 0$. For this case the angular functions are pure real, and the real and imaginary parts of Ψ_0 describe, respectively, even- and odd-parity perturbations.

When the outgoing form for $Z^{(+)}$ in Eq. (16) is substituted on the right-hand side of Eq. (18), and Eqs. (17) are used, we find

$$-\frac{\mathrm{Re}(\Psi_0)\sin^2\theta}{C_{\ell+2}^{-3/2}} = \frac{q(u)}{r^5} + O(r^6). \tag{19}$$

Here the quadrupole moment $q(u)$ is given by

$$q(u) \equiv z_2^{(+)}(u) + (3M/2)z_1^{(+)}(u). \tag{20}$$

Aside from numerical multiplicative factors, $q(u)$ is equal to ψ_0^0, and is what Kundu interprets as the source quadrupole moment. We note that Eqs. (17) and (20) give us $d^2q/du^2 = 3[z_0^{(+)} + (M/2)dz_0^{(+)}/du]$, and hence the even-parity Kundu suppression factor $(1+i\omega M/2)^{-1}$.

The function $Z^{(-)}$ (in the notation of Chandrasekhar), which describes odd-parity metric perturbations in the Schwarzschild geometry, satisfies the potential-type "Regge-Wheeler" [6] equation

$$\left(\frac{\partial^2}{\partial r_*^2} - \frac{\partial^2}{\partial t^2}\right)Z^{(-)} = V^{(-)}Z^{(-)}, \tag{21}$$

where

$$V^{(-)} = \frac{(1-2M/r)}{r^2}\left[\ell(\ell+1) - 6M/r\right]. \tag{22}$$

For a solution of the form

$$Z^{(-)} = z_0^{(-)} + z_1^{(-)}/r + z_2^{(-)}/r^2 + z_3^{(-)}/r^3 + \cdots, \tag{23}$$

the Regge-Wheeler equation tells us, for $\ell = 2$ multipoles, that

$$\begin{aligned} &dz_1^{(-)}/du = 3z_0^{(-)},\\ &dz_2^{(-)}/du = z_1^{(-)} - (3M/2)z_0^{(-)},\\ &dz_3^{(-)}/du = 0. \end{aligned} \tag{24}$$

The relationship of Ψ_0 and $Z^{(-)}$, for odd-parity perturbations, is given in Sec. 31, Eq. (345) of Chandrasekhar [12] as

$$-\frac{\mathrm{Im}(\partial\Psi_0/\partial u)\sin^2\theta}{C_{\ell+2}^{-3/2}}\left(1-\frac{2M}{r}\right) = \frac{1}{2r^3}\left[\ell(\ell+1) - \frac{6M}{r}\right]Z^{(-)} + \left[\frac{1}{r^2}\left(1-\frac{3M}{r}+r\frac{\partial}{\partial u}\right)\right]\frac{\partial}{\partial r}Z^{(-)}\,. \tag{25}$$

When the expansion in (24), for an outgoing $\ell = 2$ mode, is put into (25) we find

$$-\frac{\mathrm{Im}(d\Psi_0/du)\sin^2\theta}{C_{\ell+2}^{-3/2}}\left(1-\frac{2M}{r}\right) = \frac{z_2^{(-)}}{r^5} + O(r^{-6}), \tag{26}$$

so that, aside from multiplicative constants, $z_2^{(-)}$ is equal to $d\psi_0^0/du$ and is the derivative of what Kundu identifies as the quadrupole moment. From Eqs. (24) we have that $d^2z_2^{(-)}/du^2 = 3[z_0^{(-)} - (M/2)dz_0^{(-)}/du]$ and hence the odd-parity Kundu suppression factor $(1 - i\omega M/2)$.

The mathematics of the odd-parity modes can be much simpler than that for even-parity modes since there is no odd-parity degree of freedom for the motions of a perfect fluid. The issue of suppression is the same for both parities, so we choose to take advantage of the opportunity for simplicity and we consider below only odd-parity perturbations.

In the standard formalism for metric perturbations, odd-parity motions are described as deviations of the metric in (11). We follow here the notation of Thorne and Campolattaro [13], in which the Regge-Wheeler [6] gauge is used and azimuthal symmetry is assumed. For odd-parity perturbations of multipole index ℓ, the only nonvanishing metric perturbations are, in this notation,

$$\delta g_{t\phi} = h_0(r,t)\sin\theta\partial P_\ell(\cos\theta)/\partial\theta, \tag{27}$$

$$\delta g_{r\phi} = h_1(r,t)\sin\theta\partial P_\ell(\cos\theta)/\partial\theta \ , \tag{28}$$

where P_ℓ indicates the Legendre polynomial of index ℓ. In terms of the notation of Thorne and Campolattaro, the Chandrasekhar function $Z^{(-)}$ is

$$Z^{(-)} = e^{\alpha}h_1/r \ . \tag{29}$$

The Schwarzschild perturbation function $Z^{(-)}$ is, of course, only defined in the exterior vacuum of the galaxy, where $e^\alpha = e^\nu = e^{-\lambda} = 1 - 2M/r$, but (29) allows us to extend the definition of $Z^{(-)}$ to the interior. We consider the stress-energy tensor to be decomposable as $T_{\mu\nu} = T^{\rm perf}_{\mu\nu} + T^{\rm source}_{\mu\nu}$. Here $T^{\rm perf}_{\mu\nu}$ is the stress-energy tensor of the perfect fluid; its odd-parity perturbations can be expressed in terms of $Z^{(-)}$ (See Appendix B of Ref. [13]). The second term, $T^{\rm source}_{\mu\nu}$, is the perturbative stress-energy tensor due to the source of the gravitational perturbations. From the odd-parity field equations in Ref. [13], the interior generalization of (21) is found to be

$$\left(\frac{\partial^2}{\partial r_*^2} - \frac{\partial^2}{\partial t^2}\right) Z^{(-)}$$
$$-\frac{1}{r^2}\left[e^\nu \ell(\ell+1) - r\frac{d\alpha}{dr}e^{2\alpha} + 2\left(e^{2\alpha} - e^\nu\right)\right] Z^{(-)} = \mathcal{S}\,, \tag{30}$$

where the source term $\mathcal{S}$ is defined by

$$\frac{16\pi r e^\alpha}{\sin^2\theta}\left[\frac{\partial}{\partial r}\left(\frac{e^\nu T^{\rm source}_{\theta\phi}}{r^2}\right) - \frac{e^\nu \sin^2\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{T^{\rm source}_{r\phi}}{\sin^2\theta}\right)\right]$$
$$= -\mathcal{S}\frac{\partial}{\partial\theta}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}P_\ell(\cos\theta)\right] \ . \tag{31}$$

We apply (30) to the following model. At the center of a massive galaxy, of mass M and radius R there is a source of odd-parity gravitational waves at frequency ω. The source is confined to the region for r less than some source radius r_S. The source, e.g., an oscillating neutron star, must of course not consist of a perfect fluid, since perfect-fluid motions cannot generate the odd-parity waves. [The source term in (30) vanishes.] For this reason we take the matter of the galaxy to be a perfect fluid, so that there can be a clean separation between the generation and the propagation of the gravitational waves, a separation that is not possible for even-parity waves.

It is worth noting here that the resulting mathematical formulation differs very little from that for a scalar field of the type considered by KNR. Let scalar field Φ have a source density Σ so that

$$\Phi^{;\mu}_{,\mu} = \Sigma \ . \tag{32}$$

In the spacetime of (11), for a multipole of index ℓ, this equation reads

$$\left(\frac{\partial^2}{\partial r_*^2} - \frac{\partial^2}{\partial t^2}\right)(r\Phi)$$
$$-\frac{1}{r^2}\left[e^\nu\ell(\ell+1) + e^{2\alpha}r\frac{d\alpha}{dr}\right](r\Phi) = re^\nu\Sigma \ . \tag{33}$$

The difference between the form of this equation for $r\Phi$ and (30) for $Z^{(-)}$ is only in the details of terms of order M/R. We will show, in the next section, that these terms affect detailed numerical results but, for a nearly Newtonian galaxy, cannot cause significant suppression of radiation. Other features of the scalar and the odd-parity problems are parallel. In particular, for both cases we can consider a compact central source (no radiation originating from the bulk of the galaxy) and the matching conditions at the surface of the galaxy are that the fields and their radial derivatives are continuous.

IV. ANALYSIS OF OUTGOING WAVES

To investigate the nature of outgoing solutions we take the time dependence of the source, and of $Z^{(-)}$ to be $e^{i\omega t}$, and we write

$$Z^{(-)} = \psi(r)e^{i\omega u}, \quad \mathcal{S} = e^{i\omega t}S(r) \ .$$

The equation for odd-parity waves then takes the form

$$\psi'' + (\alpha' - 2i\omega e^{-\alpha})\psi'$$
$$-\frac{1}{r^2}[e^\lambda\ell(\ell+1) - r\alpha' + 2(1-e^\lambda)]\psi$$
$$= e^{-2\alpha}e^{i\omega r_*}S(r), \tag{34}$$

where a prime denotes differentiation with respect to r.

It is straightforward, in principle, to construct a Green-function solution to Eq. (34) from two homogeneous solutions. We define a "central" solution, ψ_c, as the homogeneous solution which is well behaved at $r \to 0$, with the limit

$$\psi_c(r) \stackrel{r\to 0}{\longrightarrow} r^{\ell+1}. \tag{35}$$

The second solution is taken to be the "wave" solution ψ_w defined by the condition that it represents outgoing waves at large radii. The mathematical condition on this asymptotically outgoing solution is

$$\psi_w(r) \stackrel{r\to\infty}{\longrightarrow} 1 + O(1/\omega r). \tag{36}$$

We define $W \equiv W(\psi_c, \psi_w) = \psi_w'\psi_c - \psi_c'\psi_w$ to be the Wronskian of these two solutions, and we note that W must have the form

$$W = e^{2i\omega r_*} e^{-\alpha(r)}/K \tag{37}$$

in which K is a constant.

If the source is confined to the region inside some radius r_S, then for $r > r_S$ the Green-function solution takes the form

$$\psi(r) = K\psi_w(r) \int_0^{r_S} \psi_c(r) S(r) e^{-\alpha} e^{-i\omega r_*}\, dr \equiv K\psi_w(r) I_\omega. \tag{38}$$

For $r_S \ll R$, and in the long wavelength ($\omega r_S \ll 1$) limit, the source integral I_ω has the approximate value

$$I_\omega \approx e^{-\nu_0/2} \int_0^{r_S} r^{\ell+1} S(r) dr.$$

Note that the absence of a conical singularity requires $\lambda \to 0$ at $r \to 0$, but $\nu(r=0) \equiv \nu_0$ will in general be of order M/R. It should also be noted that, aside from multiplicative numerical constants, the integral above is the usual integral for the ℓth multipole moment of the source.

The function $\psi_w(r)$ corresponds to the solution that is asymptotically outgoing, but, due to backscatter, at small radius ($r \ll R$), it does not in general have the appearance of a *locally* outgoing solution. We define a locally outgoing solution by the high-frequency expansion

$$\psi_{\rm out}(r) = 1 + i\frac{a(r)}{\omega_0} + \frac{b(r)}{\omega_0^2} + i\frac{c(r)}{\omega_0^3} + \cdots. \tag{39}$$

Here

$$\omega_0 \equiv \omega e^{-\nu_0/2}$$

is the blueshifted frequency in the central region of the galaxy. This frequency governs the wavelength ($\lambda = 2\pi c/\omega_0$ for $r \ll R$) in the central region and is therefore the appropriate parameter to simplify (39). By solving the homogeneous wave equation (34) to various orders in ω_0 we find, for example, that $a(r)$ and $b(r)$ must satisfy

$$a' = \frac{1}{2} e^{\alpha} e^{-\nu_0/2} \left[e^{\lambda} \frac{\ell(\ell+1)}{r^2} - \frac{\alpha'}{r} + \frac{2}{r^2}\left(1 - e^{\lambda}\right) \right], \tag{40}$$

$$b' = \frac{1}{2} e^{\alpha} e^{-\nu_0/2} \left[a'' + a'\alpha' - a\left\{ e^{\lambda} \frac{\ell(\ell+1)}{r^2} - \frac{\alpha'}{r} + \frac{2}{r^2}\left(1 - e^{\lambda}\right) \right\} \right]. \tag{41}$$

The functions $a, b, c, d, \ldots$ are determined only after the metric functions ν, λ are specified, but we can state some general conclusions. For $r \ll R$ the metric coefficients can be expanded in powers of r and, for a geometry nonsingular at $r = 0$, we have $\nu' = 0$ and $\lambda' = 0$ at $r = 0$. As a result, the solutions for $a, b, \ldots$ take the form

$$a = -\frac{1}{2r}\frac{(\ell+1)!}{(\ell-1)!} + \frac{M}{R^2}\left[a_1 \frac{r}{R} + a_2 \frac{r^2}{R^2} + \cdots \right], \tag{42}$$

$$b = -\frac{1}{8r^2}\frac{(\ell+2)!}{(\ell-2)!} + \frac{M}{R^3}\left[b_0 \ln\left(\frac{r}{R}\right) + b_1 \frac{r}{R} + \cdots \right], \tag{43}$$

$$c = \frac{1}{48r^3}\frac{(\ell+3)!}{(\ell-3)!} + \frac{M}{R^4}\left[\tilde{c}_{-1} \frac{R}{r} \ln\left(\frac{r}{R}\right) + c_{-1}\frac{R}{r} + \cdots \right], \tag{44}$$

and so forth. Here the coefficients $a_k, b_k, \ldots$ are numerical constants aside from corrections of order M/R. More precisely they are functions of the parameters of the interior geometry which have finite limits as $M/R \to 0$.

In the original homogeneous equation, Eq. (21) with the source set to zero, there is symmetry with respect to $t \to -t$ and complex conjugation. From this symmetry we get a second, ingoing, solution

$$\psi_{\rm in} = e^{2i\omega r_*} \bar{\psi}_{\rm out}\ , \tag{45}$$

in which the overbar denotes complex conjugation.

The asymptotically outgoing solution $\psi_w(r)$ must be some combination of $\psi_{\rm out}$ and $\psi_{\rm in}$, which we write as

$$\psi_w(r) = \mathcal{T}\psi_{\rm out}(r) + \mathcal{R}\psi_{\rm in}(r), \tag{46}$$

where the constants $\mathcal{T}$ and $\mathcal{R}$ can be considered transmission and reflection coefficients. The value of $|\mathcal{T}|^2 - |\mathcal{R}|^2$ is computed by considering the Wronskian of $Z_w^{(-)} \equiv e^{i\omega u}\psi_w$ and its complex conjugate, and is found to be equal to unity aside from small corrections. (The value of $|\mathcal{T}|^2 - |\mathcal{R}|^2$ can be made precisely unity by a small correction in the normalization of ψ_w.)

For $r \ll R$ the solution $\psi_w(r)$ in (46) can be expanded in powers of r. For $\ell = 2$ this gives

$$\psi_w = \mathcal{T}\left\{ 1 + \frac{i}{\omega_0}\left[-\frac{3}{r} + O(Mr/R^3) \right] + \frac{1}{\omega_0^2}\left[-\frac{3}{r^2} + O\left(\frac{M}{R^3}\ln\left(\frac{r}{R}\right) \right) \right] + \frac{i}{\omega_0^3} O\left(\frac{M}{R^3 r}\ln\frac{r}{R} \right) + \cdots \right\}$$
$$+ \mathcal{R} e^{2i\omega r_*}\left\{ 1 - \frac{i}{\omega_0}\left[-\frac{3}{r} + O(Mr/R^3) \right] + \frac{1}{\omega_0^2}\left[-\frac{3}{r^2} + O\left(\frac{M}{R^3}\ln\left(\frac{r}{R}\right) \right) \right] - \frac{i}{\omega_0^3} O\left(\frac{M}{R^3 r}\ln\frac{r}{R} \right) + \cdots \right\}. \tag{47}$$

For $r \ll R$ then, the term in $\psi_{\rm w}$ that goes as r^{-2} is

$$\frac{\mathcal{T}+\mathcal{R}}{\omega_0^2}\left\{-3+O\left(\frac{M}{\omega^2R^3}\right)\right\}. \tag{48}$$

Thus, aside from corrections which are small for high-frequency ($\omega R \gg 1$) sources, the r^{-2} term in ψ is

$$(\mathcal{T}+\mathcal{R})(-3/\omega_0^2)KI_\omega. \tag{49}$$

If, as expected, backscatter is insignificant, then $|\mathcal{T}| \approx 1$ and $|\mathcal{R}| \ll 1$ so that the r^{-2} term is approximately

$$(-3/\omega_0^2)KI_\omega. \tag{50}$$

The r^{-2} term therefore gives a direct measure of the quadrupole source integral. From Eqs. (36) and (38) it follows that $\psi \to KI_\omega$ as $r \to \infty$, and that the r^{-2} term in the deep interior also gives a measure of the intensity of the outgoing radiation.

For the exterior solution, very different conclusions follow. Here it is possible to expand ψ_w in inverse powers of r as

$$\begin{aligned}\psi_w = 1 &+ \frac{i}{\omega}\left[-\frac{3}{r}+A_1\frac{M}{r^2}+A_2\frac{M^2}{r^3}+\cdots\right]\\ &+\frac{1}{\omega^2}\left[-\frac{3}{r^2}+B_1\frac{M}{r^3}+\cdots\right]\\ &+\frac{1}{\omega^3}\left[C_1\frac{M}{r^4}+\cdots\right]+\cdots,\end{aligned} \tag{51}$$

in which the numerical constants, $A_1 = 3/2, A_2 = 0, B_1 = 0$, etc., are easily evaluated from the Schwarzschild metric functions. In the exterior solution then the r^{-2} term in ψ is

$$(-3/\omega^2 + 3iM/2\omega)KI_\omega \tag{52}$$

and is larger than the interior r^{-2} term by the (possibly large) factor $(1 - i\omega M/2)$. But now the r^{-2} coefficient no longer gives the intensity of the outgoing radiation, or a measure of the source integral. If the coefficient is used to denote (in the notation of KNR) the "field quadrupole," then it must be understood that this field quadrupole is larger, by the factor $(1 - i\omega M/2)$, than the quadrupole moment which measures the source integral, which governs the intensity of outgoing radiation at infinity, or which governs the locally outgoing radiation deep inside the galaxy.

It is clear mathematically why the field quadrupole and the physical quadrupole are so different: the $ia(r)/\omega_0$ term in (47) lacks a term that goes as r^{-2}. From (40) we see that the presence of such a term would require a galaxy spacetime that tends to a singularity as $r \to 0$. The absence of such a term is why, for a high-frequency source, the r^{-2} term for $r \ll R$ can be given the same meaning—that of the quadrupole moment—as in a flat spacetime background. In (51) the $ia(r)/\omega$ term *does* have a r^{-2} term. This is possible because the expansion in (51) cannot be extended to small r. But the interpretation of the r^{-2} term as the quadrupole moment, in some expansion for ψ, is justifiable only if that expansion can be extended to small radii. The field quadrupole is, then, a formal construct and its use as a physical quadrupole moment is the reason that gravitational radiation appears to be suppressed.

This insight gives the answer to an interesting question. Let us denote the coefficient of the r^{-2} term as Q_f, both in the deep interior and in the exterior. When Q_f is computed in the exterior we find a different value than in the interior, and than we would find in the absence of the galaxy. How can the "source integral" for Q_f, when it is computed in the exterior, have large non-Newtonian contributions from the galaxy, especially in the case that the galaxy is nearly Newtonian? The answer is that the "source integral," both in the exterior and in the deep interior, is the same. It is I_ω of (38). But the way in which this source integral enters into the expression for the coefficient of the r^{-2} term, and hence into the value inferred for Q_f, is different in the exterior and the interior.

In the next section we consider just what the magnitude is of the influence of the galaxy spacetime on the outgoing radiation.

V. REFLECTION AND TRANSMISSION OF OUTGOING WAVES

We take up here the question of what the actual influence is of the curved spacetime of the galaxy on the propagation of gravitational waves (specifically, of odd-parity gravitational waves). One obvious influence, of course, is the redshift which is built into the expressions for the radiation. The net power in terms of coordinate time t must be independent of the distance from the source. The locally measured proper time differs from coordinate time by $e^{\nu/2}$ and hence the locally measured power (proportional to the square of the time derivative of ψ) will differ from that far outside the source by the redshift factor e^ν.

The question of other influences on the radiation is much less obvious, and there are several ways in which it can be asked. One approach is to look at the relation of the outgoing radiation and the source as embodied in (31) and (38). This approach is most transparent if the source is taken to be compact, i.e., $r_S \ll 1/\omega$ as well as $r_S \ll R$. In this case, the radial derivative of ν will be smaller (by r_S/R) than the radial derivative of the Ricci components, so we can approximate

$$16\pi r e^{3\nu_0/2}\left[\frac{d}{dr}\left(T^{\rm source}_{\hat\theta\hat\phi}\right)-\frac{\sin\theta}{r}\frac{d}{d\theta}\left(\frac{T^{\rm source}_{\hat r\hat\phi}}{\sin\theta}\right)\right]$$

$$= -\mathcal{S}\sin\theta\frac{\partial}{\partial\theta}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}P_\ell(\cos\theta)\right]. \tag{53}$$

Here the terms $T^{\rm source}_{\hat\theta\hat\phi}$ and $T^{\rm source}_{\hat r\hat\phi}$ are the perturbations of the source stress-energy projected on an orthonormal tetrad, and are the quantities that would be computed (e.g., for a neutron star) by a nearby observer. We therefore write

$$\mathcal{S} = e^{3\nu_0/2}\mathcal{S}_{\rm local} \tag{54}$$

to indicate the relation of the source term referred to the coordinates of (11) and the source term measured by a local observer.

For the compact source, with corrections of order $r_S M/R^2$ and $1/r_S\omega$ ignored, (38) can be written

$$\psi(r) = Ke^{\nu_0} I_{\rm local}\psi_{\rm w}, \tag{55}$$

in which

$$I_{\rm local} = \int_0^{r_S} r^{\ell+1}\mathcal{S}_{\rm local}\, dr \tag{56}$$

is the source term that would be computed by a local observer. In the case of flat spacetime K defined by (37) is easily shown to be $-(i)^\ell\omega^\ell[(2\ell+1)!!]^{-1}$ so that, finally, the relation of source and field can be written as

$$\psi(r) = -(i)^\ell\omega^\ell[(2\ell+1)!!]^{-1} I_{\rm local}\psi_{\rm w} e^{\nu_0}\kappa_{\rm corr} \tag{57}$$

with

$$\kappa_{\rm corr} \equiv -(2\ell+1)!!(-i)^\ell\omega^{-\ell}K. \tag{58}$$

In this equation the influence of the galaxy is contained in the factor $e^{\nu_0}\kappa_{\rm corr}$.

There is another, rather different, way in which the influence of the galaxy can be viewed. One can ask what the relationship is between the outgoing radiation far outside the galaxy, and the radiation in the deep interior of the galaxy. In (46) this relationship is contained in the constants $\mathcal{T}$ and $\mathcal{R}$, in which $\mathcal{R}$ describes, approximately, the fraction of the radiation reflected back towards the source, due to the galaxy's spacetime curvature. Roughly speaking, the magnitude of $|\mathcal{R}|$, or of $|\mathcal{T}|-1$, is a measure of the extent to which the galaxy is not perfectly transparent to gravitational radiation. It is only an approximate measure because there is, at the outset, a limit to the precision to which an observer can measure radiation as if in flat spacetime. The metric for a "flat" coordinate system over a region of size L will deviate from the Minkowski metric by corrections of order $(L/R_c)^2$, where R_c is the spacetime radius of curvature. For the galaxy spacetime $R_c \sim (R^3/M)^{1/2}$, so that over one wavelength there will be metric corrections of order M/ω^2R^3. One manifestation of this is that we have, from the Wronskian of $\psi_{\rm w}$ and $e^{2i\omega r_*}\overline{\psi_{\rm w}}$, and the expressions in (47) and (51), that

$$|\mathcal{T}|^2 - |\mathcal{R}|^2 = 1 + O(M/\omega^2R^3). \tag{59}$$

The $O(M/\omega^2R^3)$ correction factor is simple to compute, once ν and λ are specified, from the forms for $a, b, \ldots$. The correction factor M/ω^2R^3 will, in any case, be negligible (of order 10^{-39} for kilohertz waves and ordinary galaxies).

There is a close relationship between the two viewpoints above for looking at the influence of the galaxy. The Wronskian in (37) can be written

$$W(\psi_c,\psi_w) = \mathcal{T}W(\psi_c,\psi_{\rm out}) + \mathcal{R}W(\psi_c,\psi_{\rm in}), \tag{60}$$

and the Wronskians on the right-hand side can be evaluated to give, for $\ell = 2$,

$$W(\psi_c,\psi_w) = (\mathcal{T}+\mathcal{R})e^{2i\omega r_*}e^{-\alpha}15\ \omega_0^{-2}e^{\nu_0/2} \times \left[1 + O(M/\omega^2R^3)\right] . \tag{61}$$

As in (59), the $O(M/\omega^2R^3)$ correction term is easily evaluated from (39) and the small-radius forms of $a(r), b(r), \ldots$, once the the metric functions ν and λ are specified.

When we combine (61) with (37) and (58), for $\ell = 2$, we have

$$\kappa_{\rm corr} = e^{-3\nu_0/2}(\mathcal{T}+\mathcal{R})^{-1}[1 + O(M/\omega^2R^3)] . \tag{62}$$

The effect of the galaxy is then contained in two types of terms. There are terms of order M/ω^2R^3 [e.g., in (62] and (59)) that are "local" in the sense that they can be computed from the small-radius solutions for ψ. The second influence of the galaxy is through the coefficient $\mathcal{R}$, and is not local. If there is any way in which a nearly Newtonian galaxy can have a significant influence on the propagation of high-frequency waves, it is through the possibility that $|\mathcal{R}|$ is not small.

That possibility can, in fact, be ruled out with a WKB argument, but such an argument cannot easily tell us how small $|\mathcal{R}|$ really is. To find this out we have numerically integrated the equation for ψ starting in the exterior, at large r, with the expansion in (47). The integration to small radius was done with the method of GEAR [14], suitable to the stiff differential equation for ψ. At the surface $r = R$, the field equations require that ψ and ψ' be continuous. The $\mathcal{R}$ coefficient was extracted from the numerically computed solution ψ_w, by using the flat spacetime solutions $\psi_{\rm out}^{\rm flat} \equiv 1 - 3i/r\omega_0 - 3/r^2\omega_0^2$, and $\psi_{\rm in}^{\rm flat} \equiv e^{2i\omega r_*}\overline{\psi_{\rm out}^{\rm flat}}$, and the computed quantity

$$\mathcal{R}_{\rm index} \equiv W(\psi_{\rm out}^{\rm flat},\psi_{\rm w})/W(\psi_{\rm out}^{\rm flat},\psi_{\rm in}^{\rm flat}) . \tag{63}$$

This can be evaluated at small r with the expansion in (39) to give

$$\mathcal{R}_{\rm index} = \left[\mathcal{R} + \frac{a_1 M}{2\omega_0^2R^3}e^{-2i\omega r_*}\mathcal{T}\right] \times \left[1 + \mathcal{O}\left(\frac{1}{\omega_0 r}\right) + \mathcal{O}\left(\frac{r}{R}\right)\right] , \tag{64}$$

where a_1 is the coefficient defined in (42). Note that (64) does not assume $M \ll R$; it can be used for galaxies with relativistically strong gravity.

For a constant-density interior (Schwarzschild interior) the value of a_1 is easily shown to be

$$a_1 = 2 + e^{-\nu_0/2} = 2 + \left[\frac{3}{2}\left(1 - \frac{2M}{R}\right)^{1/2} - \frac{1}{2}\right]^{-1}$$

and we apply (64) to the results for $\mathcal{R}_{\rm index}$ found with (63) from the numerically computed values of $\psi_{\rm w}$. In Fig. 1 we show the real part of $\mathcal{R}_{\rm index}$ as a function of r, for the parameters $M/R = 0.1$ and $\omega R = 2000$, and we compare it to the predicted expression in (64)

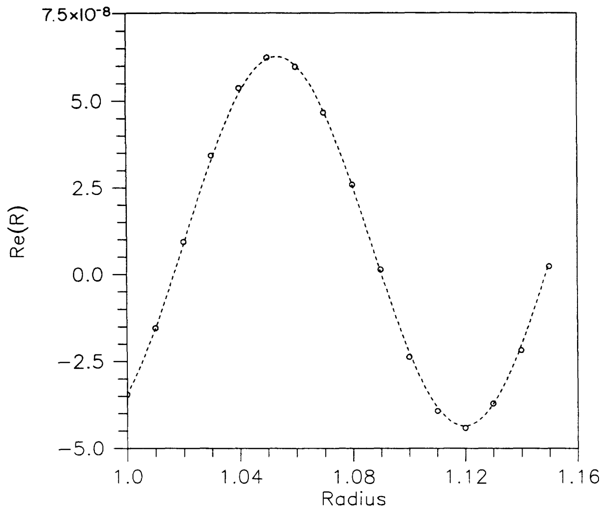

FIG. 1. $\mathcal{R}_{\rm index}$ is plotted for the numerical runs (circles) and the value given by the analytical estimate Eq. (68) (curve). The parameters of the run are $R = 100$, $M = 10$, $\omega = 20$. The error bars on the numerical points are of the order of the size of the circles.

for the best-fit values $\mathcal{R} = (0.29 + 1.40i)10^{-8}$ and $\mathcal{T} = e^{-0.4\pi i}$. Numerical runs with different parameters show that $|\mathcal{R}|$ is proportional to M/ω^2R^3 (aside from higher-order corrections in ωR), so that the $\mathcal{R}$ and $\mathcal{T}$ terms on the right-hand side of (64) are of the same order.

Figure 2 gives $|\mathcal{R}|$ as a function of M/R, for different values of ωR. The plots clearly indicate that $|\mathcal{R}| \approx \frac{2M}{3\omega^2R^3}$ as long as $M/R \ll 1$ and $\omega R \gg 1$. When M/R is no longer small, it remains true that $|\mathcal{R}| \propto \omega^{-2}R^{-2}$, but the dependence on M/R must be read from the figure. When ω is not large compared to R^{-1}, the assumptions used in deriving (64) fail as does much of the meaning of "reflection."

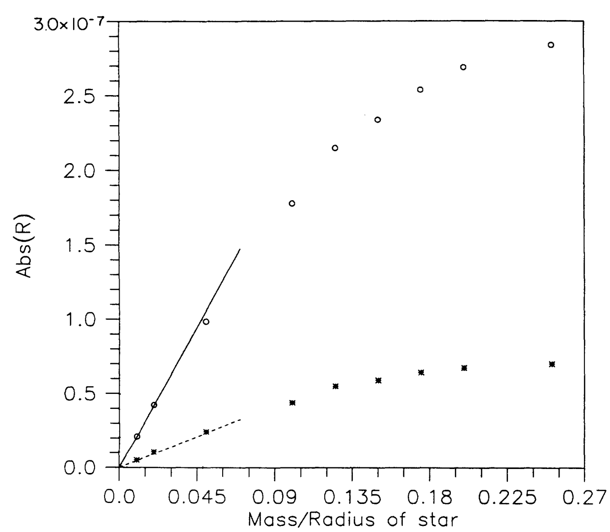

FIG. 2. The absolute value of $\mathcal{R}_{\rm index}$ plotted as a function of M/R for two values of ωR. The circles are for $\omega R = 1000$ and the stars for $\omega R = 2000$. The continuous curves represent the approximation for small M/R given by $|R| \approx 2M/3\omega^2R^3$.

VI. SUMMARY AND CONCLUSIONS

We have studied a configuration in which waves propagate on a spherical static curved spacetime background. In the geometric-optics limit, the limit of infinite frequency, the only physical influence of the background is the familiar redshift of the waves. We have found effects for high but finite frequency ω, for propagation in a background of mass M and radius R. Most notably, we have found that the radiation reaching arbitrary distances is different from that emitted, by a fractional correction of order M/ω^2R^3. The computed reflection coefficients in Sec. V may be considered the first corrections to the geometric-optics limit.

Some details of Secs. IV and V are specific to odd-parity gravitational waves, but with very minor modifications apply also to massless, minimally coupled, scalar fields. The generalization to even-parity gravitational waves is not immediate. For even-parity waves propagating through a perfect-fluid galaxy, or for waves of either parity propagating through a region with more complex material properties, the matter will, in general, oscillate in response to the passage of the wave, and will retard and absorb radiation much as a dielectric material interacts with an electromagnetic wave. (The computation for even-parity waves through a perfect-fluid galaxy would be relatively straightforward to carry out with formalisms in which fluid perturbations do not explicitly appear [15], but the problem is made difficult by its four degrees of freedom.) These interactions, however, can be estimated reliably [16] and except for contrived circumstances will be very small.

What do these results imply for the possibility of suppression of gravitational radiation, as predicted by Kundu [2, 3]? The use of a model calculation specific to odd-parity waves is irrelevant. The suppression is inferred by Kundu from the external Schwarzschild geometry, in which the mathematics of even- and odd-parity waves is essentially the same. Our analysis is rather specific to a particular configuration: a compact central wave source embedded in a massive spherical background. One might ask whether the suppression might apply to very different configurations, such as a source near a massive black hole. It would be strange, of course, if the suppression —inferred only from the Schwarzschild background—applied for one wave source and not another within that background. Barring that possibility, the general lessons of our spherical configurations should apply insofar as well defined questions can be asked about suppression. In particular, for a compact source, in the geometrical-optics limit (wavelength$\ll$ all other length scales), the effect of background curvature should be only the standard redshift and the bending of the null geodesics (absent in the spherically symmetric case). For finite frequency we would expect the first corrections from the geometric-optics limit to be of order $(\omega R_c)^{-2}$, where R_c is the characteristic spacetime radius of curvature.

The conclusions based on the configuration considered in this paper should then give the generally correct picture of the relationship of radiation and quadrupole moment. In that picture we are able to distinguish a number of different "quadrupole moments": (i) a quadrupole moment given by an integral over the source, (ii) the interior "field quadrupole" inferred from the coefficient of the r^{-5} term in the NP Weyl projection Ψ_0 near the source, and (iii) the "field quadrupole" from the r^{-5} term far from the source of background curvature. We have shown that the quadrupole moments of types (i) and (ii) are the same (aside from corrections of order $M/\omega^2 R^3$) and that they govern the outgoing radiation produced by the source, both deep within the galaxy and outside the galaxy. The exterior "field quadrupole," however, differs significantly from the other quadrupole moments. Its interpretation as a quadrupole moment is based on an expansion in inverse powers of r, and an identification of the expansion coefficients with those of similar expressions for $r \ll R$. But the expansion in the exterior cannot be extended inward, so that the expansion coefficients do not have their usual physical meaning. In particular the "field quadrupole" is not the quadrupole moment of the source, and does not govern the radiation produced by the source.

ACKNOWLEDGMENTS

We wish to thank Kip Thorne for discussions of this work. Support for the research reported here was provided by the National Science Foundation under Grant No. PHY-8907937. We thank Prasun Kundu for pointing out an error in a preprint of this paper.

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