Quantum emission from two-dimensional black holes

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We investigate Hawking radiation from two-dimensional dilatonic black holes using standard quantization techniques. In the background of a collapsing black-hole solution the Bogoliubov coefficients can be exactly determined. In the regime after the black hole has settled down to an "equilibrium" state but before the back reaction becomes important these give the known result of a thermal distribution of Hawking radiation at a temperature $\lambda/2\pi$. The density matrix is computed in this regime and shown to be purely thermal. Similar techniques can be used to derive the stress tensor. The resulting expression agrees with the derivation based on the conformal anomaly and can be used to incorporate the back reaction. Corrections to the thermal density matrix are also examined, and it is argued that to leading order in perturbation theory the effect of the back reaction is to modify the Bogoliubov transformation, but not in a way that restores information lost to the black hole.

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I. INTRODUCTION

The discovery of Hawking radiation [1] has raised a long-standing puzzle: What happens to black holes once they are done evaporating? There are at least two reasons why this problem is interesting. The first is general: The final stages of black-hole evaporation typically involve physics near the Planck scale, where quantum gravity is expected to become important. Black holes provide a theoretical laboratory where one can attempt to develop one's understanding of this physics. The second reason stems from the problem of black-hole information.[1] One may form a black hole from a pure quantum state; however, in Hawking's calculation the outgoing radiation is not in a pure state—it appears that information is lost to the black hole. Attempts to explain how the information is restored once the black hole disappears run into serious difficulties. It has even been conjectured that physics is fundamentally nonunitary [5]. Perhaps this problem is giving us a deep clue about the nature of quantum gravity.

Recently, black holes in two-dimensional gravity have received considerable attention [6–10] following Witten's identification of a black hole in string theory. In particular, in [11], Callan *et al.* investigated a toy model for black-hole formation and evaporation. This model is two-dimensional dilaton gravity coupled to free scalar fields and is both renormalizable[2] and classically soluble. This toy model has the virtue of greatly simplifying the physics without discarding many of the essential issues. In particular, [11] found "collapsing" black-hole solutions, and a simple technique for treating Hawking radiation and its back reaction on the geometry was investigated. It was argued that in the limit where the number N of matter fields is large, the back reaction removes the classical black-hole singularity; however, in [13,14], a new type of singularity was found. Subsequent study [15,16] has uncovered singular static solutions of the back-reaction-corrected equations and has clarified the nature of the final configuration of the evaporation process.

This model has numerous issues that have not been completely addressed. One of these is the physical interpretation of the singularities of [13,14]. It appears that a proper quantum treatment of the theory will be required to say anything further about these. In particular, one would like to understand the physics outside the large-N approximation. A second is the information problem. It may be difficult to resolve this without contending with the singularities. However, it is conceivable that aspects of proposed resolutions to the problem can be investigated. For example, Refs. [17,3] have advocated the possibility that, at least in theories without global symmetries, proper treatment of the back reaction might reveal that information is extracted from infalling matter and appears in the outgoing corrected Hawking radiation. This is partly motivated by the desire to believe that the entropy versus area relationship (which in the two-dimensional context is modified to $S \propto M$) is a true indicator of the amount of information stored by a black hole of mass M. Such questions are more tractable in this toy model.

The present paper takes steps toward answering some of these questions. In particular, it is clear that a full accounting of Hawking radiation and its back reaction requires more than just knowledge of the expectation value of the stress tensor, as in [11]. A finer description requires computation of states and correlation functions, etc., by means such as the Bogoliubov transformation. There are other motivations for investigating this model under the precepts of quantum field theory in curved space-time. One is to elucidate the connection between the conventional treatment of Hawking radiation and that in [11]. Another is that, as we will see, the present model is a very simplified arena in which to apply the corresponding machinery; this has pedagogical value.

In outline, this paper first reviews the collapsing black-hole solutions of [11]. We then recall the general procedure of computing Hawking radiation using the Bo-

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[1]For other discussions of this, see [2–4].

[2]For more on this issue, see [12].

goliubov coefficients and derive these coefficients for the two-dimensional black hole in Sec. III. Next is a discussion of the late-time thermal behavior of Hawking radiation, including derivation of the late-time density matrix. This is followed by a direct computation of the stress tensor of the Hawking radiation; we then discuss the issue of coupling it to gravity to incorporate the back reaction, corroborating the approach of [11]. Finally, we investigate the corrections to the thermal density matrix. These arise from including both the early-time transitory behavior and back reaction. It is argued that neither of these is likely to restore information lost to the black hole.

II. REVIEW

We first review some salient aspects of two-dimensional dilaton gravity. This theory is described by the action

$$S=\frac{1}{2\pi}\int d^2x\sqrt{-g}\left[e^{-2\phi}[R+4(\nabla\phi)^2+4\lambda^2]-\frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right], \tag{2.1}$$

where ϕ is the dilaton field, λ^2 is a cosmological constant, and f_i are N matter fields. It is most easily investigated in conformal coordinates $x^{\pm}=x^0\pm x^1$, where the metric takes the form

$$ds^2=-e^{2\rho}dx^+dx^- . \tag{2.2}$$

The classical solutions for the matter fields are then

$$f_i=f_{i+}(x^+)+f_{i-}(x^-) . \tag{2.3}$$

For given functions f_{i+}, f_{i-}, one may explicitly find the corresponding solution for ϕ and ρ as in [11]. Particular cases are the vacuum solutions [6,7]

$$ds^2=-\frac{dx^+dx^-}{M/\lambda-\lambda^2x^+x^-},\quad e^{-2\phi}=\frac{M}{\lambda}-\lambda^2x^+x^- , \tag{2.4}$$

which correspond to black holes of mass M. The $M=0$ solution is the linear dilaton vacuum, which is the classical ground state.

Sending a pulse of f matter into the linear dilaton vacuum produces a black hole. In particular, one may take a limit of smooth configurations which corresponds to a sharp left-moving pulse:

$$T^f_{++}=\frac{1}{2}(\partial_+f)^2=\frac{M}{\lambda x_0^+}\delta(x^+-x_0^+) . \tag{2.5}$$

This gives the solution

$$\begin{aligned} ds^2&=-\frac{dx^+dx^-}{-\lambda^2x^+x^--(M/\lambda x_0^+)(x^+-x_0^+)\Theta(x^+-x_0^+)} ,\\ e^{-2\phi}&=-\lambda^2x^+x^--\frac{M}{\lambda x_0^+}(x^+-x_0^+)\Theta(x^+-x_0^+) . \end{aligned} \tag{2.6}$$

Before the pulse, this is the linear dilaton vacuum; after, it is a black hole of mass M. It has a singularity along the line where the denominator vanishes and a horizon at $x^-=-M/\lambda^3x_0^+$.

More generally, we may take an arbitrary pulse of left-moving matter which turns on and then off again between times x_i^+ and x_f^+. On shell one may always choose coordinates so that $\rho=\phi$, and the general solution of [11] then becomes

$$e^{-2\rho}=e^{-2\phi}=-\lambda^2x^+x^--\int dx^+\int dx^+T^f_{++} . \tag{2.7}$$

For $x^+>x_f^+$ the last term reduces to

$$-\int dx^+\int dx^+T^f_{++}=\frac{M}{\lambda}-\lambda^2x^+\Delta , \tag{2.8}$$

where M and Δ are constants. After x_f^+, the metric therefore takes the form

$$ds^2=-\frac{dx^+dx^-}{M/\lambda-\lambda^2x^+(x^-+\Delta)} . \tag{2.9}$$

This is a black hole of mass M with horizon at $x^-=-\Delta$; the solution (2.6) corresponds to $\Delta=M/\lambda^3x_0^+$. The Penrose diagram for the general solution is shown in Fig. 1.

The metric (2.7) is asymptotically flat in the black-hole region $x^+>x_f^+$. This is explicitly seen in the coordinates $\sigma^{\pm}$, where

$$e^{\lambda\sigma^+}=\lambda x^+,\quad e^{-\lambda\sigma^-}=-\lambda(x^-+\Delta) , \tag{2.10}$$

and $-\infty<\sigma^{\pm}<\infty$. In these coordinates the metric is

$$ds^2=\begin{cases}\dfrac{-d\sigma^+d\sigma^-}{1+\Delta\lambda e^{\lambda\sigma^-}} & \text{if } \sigma^+<\sigma_i^+ ,\\[2ex] \dfrac{-d\sigma^+d\sigma^-}{1+(M/\lambda)e^{\lambda(\sigma^--\sigma^+)}} & \text{if } \sigma^+>\sigma_f^+ ,\end{cases} \tag{2.11}$$

where $\lambda x_{i,f}^+=e^{\lambda\sigma_{i,f}^+}$. This clearly asymptotes to the flat metric at both $\mathcal{I}_R^+$ ($\sigma^+\to\infty$) and $\mathcal{I}_R^-$ ($\sigma^-\to-\infty$). Likewise, it is useful to introduce flat coordinates $y^{\pm}$ for the dilaton vacuum region; these are defined by

$$x^+=\frac{1}{\lambda}e^{\lambda y^+},\quad x^-=-\Delta e^{-\lambda y^-} . \tag{2.12}$$

In this region the metric is then $ds^2=-dy^+dy^-$ and the horizon is the line $y^-=0$.

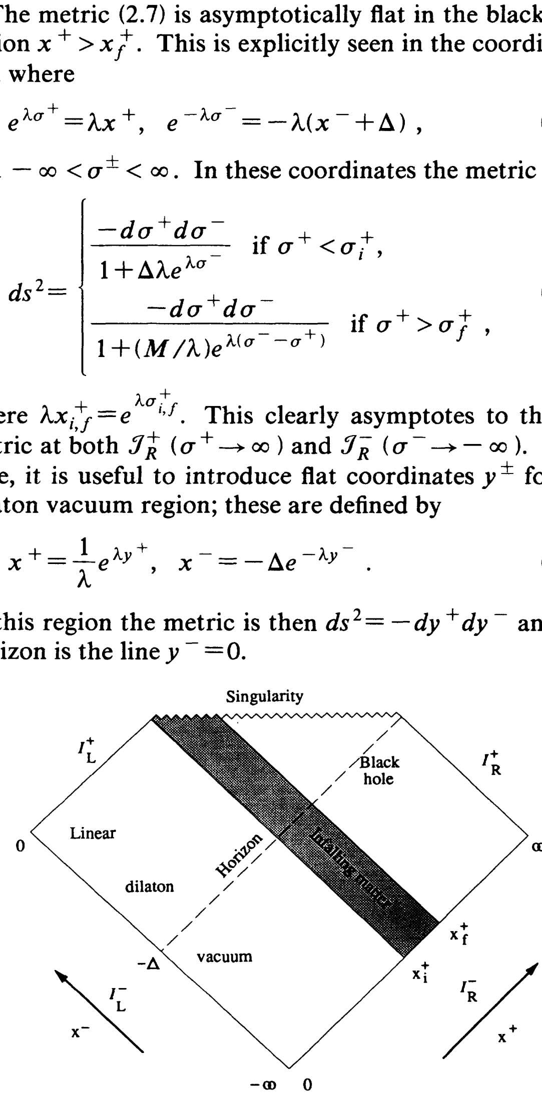

FIG. 1. Shown is the Penrose diagram for a black hole formed from an arbitrary distribution of collapsing matter concentrated between times x_i^+ and x_f^+.

III. BOGOLIUBOV TRANSFORMATION

In this section and the following, we will study the Hawking radiation of one of the fields f_i in the background solutions (2.6) and (2.7). Although one would, of course, like to study Hawking radiation including effects of the back reaction, that is a more complicated problem whose details are postponed for future work. We will focus on the two asymptotically flat regions $\mathcal{I}_L^-$ and $\mathcal{I}_R^+$, which we also call the "in" and "out" regions. In these two regions we imagine observers stationed, carrying out measurements on the quantum field f, and we calculate the relation between their observations. The result is the Boboliubov transformation, which encodes the detailed structure of the Hawking radiation.

Let us first recall the general framework.[3] For the purposes of this paper, we will use the decomposition (2.3) and ignore the left-moving modes since the right movers transmit the Hawking radiation. The (right-moving part of the) field f can be expanded in terms of mode functions and annihilation/creation operators either appropriate to the in region near $\mathcal{I}_L^-$ or to the out region near $\mathcal{I}_R^+$. Convenient bases of modes are

$$\begin{aligned} u_\omega &= \frac{1}{\sqrt{2\omega}} e^{-i\omega y^-} \quad (\text{in}) , \\ v_\omega &= \frac{1}{\sqrt{2\omega}} e^{-i\omega\sigma^-} \Theta(y^-) \quad (\text{out}) ; \end{aligned} \tag{3.1}$$

here $\omega > 0$ and Θ is the usual step function. Note that the v_ω have support only outside the horizon—the out basis must therefore be complemented by a set of modes $\hat{v}_\omega$ for the region internal to the black hole. There is no canonical definition of particles inside the black hole since this region is not asymptotically flat. Therefore the choice of such a basis is rather arbitrary. In practice, states inside the black hole are not observed and instead are traced over, and so this arbitrariness does not affect physical results.

The mode expansions are

$$\begin{aligned} f_- &= \int_0^\infty d\omega [a_\omega u_\omega + a_\omega^\dagger u_\omega^*] \quad (\text{in}) \\ &= \int_0^\infty d\omega [b_\omega v_\omega + b_\omega^\dagger v_\omega^* + \hat{b}_\omega \hat{v}_\omega + \hat{b}_\omega^\dagger \hat{v}_\omega^*] \\ &\qquad (\text{out}+\text{internal}) . \end{aligned} \tag{3.2}$$

The operators $a_\omega^\dagger$ are the creation operators appropriate to the in region, and $b_\omega^\dagger$ and $\hat{b}_\omega^\dagger$ are similarly used for the out region and for particles falling into the singularity. Annihilation and creation operators multiply positive- and negative-frequency modes, respectively.

The equations of motion imply the existence of the conserved Klein-Gordon inner product:

$$(f,g) = -i \int_\Sigma d\Sigma^\mu f \overleftrightarrow{\nabla}_\mu g^* , \tag{3.3}$$

for an arbitrary Cauchy surface Σ. The modes (3.1) have been normalized so that

$$\begin{aligned} (u_\omega, u_{\omega'}) &= (v_\omega, v_{\omega'}) = 2\pi\delta(\omega-\omega') , \\ (u_\omega, u_{\omega'}^*) &= (v_\omega, v_{\omega'}^*) = 0 , \\ (u_\omega^*, u_{\omega'}^*) &= (v_\omega^*, v_{\omega'}^*) = -2\pi\delta(\omega-\omega') , \end{aligned} \tag{3.4}$$

and we assume a similar normalization for the $\hat{v}_\omega$. Furthermore, the inner products between the modes v_ω and $\hat{v}_\omega$ all vanish since these have support in different regions. Equations (3.2) and (3.4), together with the canonical commutation relation

$$\begin{aligned} [f_-(x), \partial_0 f_-(x')]_{x^0=x'^0} &= \tfrac{1}{2}[f(x), \partial_0 f(x')]_{x^0=x'^0} \\ &= \pi i \delta(x^1 - x'^1) , \end{aligned} \tag{3.5}$$

imply that the operators a_ω satisfy the usual commutators

$$[a_\omega, a_{\omega'}^\dagger] = \delta(\omega-\omega'), \quad [a_\omega, a_{\omega'}] = 0, \quad [a_\omega^\dagger, a_{\omega'}^\dagger] = 0 , \tag{3.6}$$

and similarly for b_ω and $\hat{b}_\omega$. Finally, the in and out vacua are defined by

$$a_\omega |0\rangle_{\text{in}} = 0, \quad b_\omega |0\rangle_{\text{out}} = 0 , \tag{3.7}$$

for all $\omega > 0$. One can also define an internal "vacuum" by

$$\hat{b}_\omega |0\rangle_{\text{int}} = 0 ; \tag{3.8}$$

this definition is, however, rather arbitrary.

Although the in and out regions are flat, their natural timelike coordinates are related in such a way that a field mode which has positive frequency according to observers in one region inevitably becomes a mixture of positive and negative frequencies according to observers in the other regions. This mixing is interpreted as particle creation. To study it we define coefficients $a_{\omega\omega'}$ and $\beta_{\omega\omega'}$ by

$$v_\omega = \int_0^\infty d\omega' [\alpha_{\omega\omega'} u_{\omega'} + \beta_{\omega\omega'} u_{\omega'}^*] . \tag{3.9}$$

These coefficients are called Bogoliubov coefficients, and they may be calculated using (3.4) and (3.9):

$$\alpha_{\omega\omega'} = \frac{1}{2\pi}(v_\omega, u_{\omega'}), \quad \beta_{\omega\omega'} = -\frac{1}{2\pi}(v_\omega, u_{\omega'}^*) . \tag{3.10}$$

The Bogoliubov coefficients $\hat{\alpha}_{\omega\omega'}$ and $\hat{\beta}_{\omega\omega'}$ for the internal modes are defined similarly.

Equivalence of the expansions (3.2) gives the relation between the field operators in the in and out regions:

$$\begin{aligned} a_\omega &= \int_0^\infty d\omega' [b_{\omega'} \alpha_{\omega'\omega} + b_{\omega'}^\dagger \beta_{\omega'\omega}^* + \hat{b}_{\omega'} \hat{\alpha}_{\omega'\omega} + \hat{b}_{\omega'}^\dagger \hat{\beta}_{\omega'\omega}^*] , \\ b_\omega &= \int_0^\infty d\omega' [\alpha_{\omega\omega'}^* a_{\omega'} - \beta_{\omega\omega'}^* a_{\omega'}^\dagger] , \\ \hat{b}_\omega &= \int_0^\infty d\omega' [\hat{\alpha}_{\omega\omega'}^* a_{\omega'} - \hat{\beta}_{\omega\omega'}^* a_{\omega'}^\dagger] . \end{aligned} \tag{3.11}$$

If $\beta_{\omega\omega'} \neq 0$, then the in vacuum is not considered vacuous by the out observer; particle creation has occurred. Indeed, it follows from (3.11) that

$${}_{\text{in}}\langle 0 | N_\omega^{\text{out}} | 0 \rangle_{\text{in}} = \int_0^\infty d\omega' |\beta_{\omega\omega'}|^2 , \tag{3.12}$$

[3] For a more complete review, see [18].

where $N^{\rm out}_\omega = b^\dagger_\omega b_\omega$ is the number operator for out modes of frequency ω. Using matrix notation and introducing the "square" matrices

$$A=\begin{bmatrix}\alpha_{\omega\omega'}\\ \hat{\alpha}_{\omega\omega'}\end{bmatrix},\quad B=\begin{bmatrix}\beta_{\omega\omega'}\\ \hat{\beta}_{\omega\omega'}\end{bmatrix}, \tag{3.13}$$

the in vacuum can be written as

$$|0\rangle_{\rm in}\propto \exp\left\{-\tfrac{1}{2}(b^\dagger \hat{b}^\dagger)B^*A^{-1}\begin{bmatrix}b^\dagger\\ \hat{b}^\dagger\end{bmatrix}\right\}|0\rangle_{\rm out}|0\rangle_{\rm int}\ , \tag{3.14}$$

in the combined out/internal Fock space.

We now calculate the Bogoliubov coefficients for this model. They are found using the relation between the coordinates

$$\sigma^- = -\frac{1}{\lambda}\ln[\lambda\Delta(e^{-\lambda y^-}-1)]\ , \tag{3.15}$$

so that

$$v_\omega=\frac{1}{\sqrt{2\omega}}\exp\left\{\frac{i\omega}{\lambda}\ln[\lambda\Delta(e^{-\lambda y^-}-1)]\right\}\Theta(y^-)\ . \tag{3.16}$$

The inner products (3.10) can then be computed at the null surface $\mathcal{I}_L^-$:

$$\begin{aligned}\alpha_{\omega\omega'}&=-\frac{i}{\pi}\int_{-\infty}^0 dy^- v_\omega \partial_- u^*_{\omega'}\\ &=\frac{1}{2\pi}\left[\frac{\omega'}{\omega}\right]^{1/2}\int_{-\infty}^0 dy^-\exp\left\{\frac{i\omega}{\lambda}\ln[\lambda\Delta(e^{-\lambda y^-}-1)]+i\omega' y^-\right\}\ ,\\ \beta_{\omega\omega'}&=\frac{i}{\pi}\int_{-\infty}^0 dy^- v_\omega \partial_- u_{\omega'}\\ &=\frac{1}{2\pi}\left[\frac{\omega'}{\omega}\right]^{1/2}\int_{-\infty}^0 dy^-\exp\left\{\frac{i\omega}{\lambda}\ln[\lambda\Delta(e^{-\lambda y^-}-1)]-i\omega' y^-\right\}\ .\end{aligned} \tag{3.17}$$

With the substitution $x=e^{\lambda y^-}$, $\alpha_{\omega\omega'}$ becomes

$$\frac{1}{2\pi\lambda}\left[\frac{\omega'}{\omega}\right]^{1/2}(\lambda\Delta)^{i\omega/\lambda}\int_0^1 dx\,(1-x)^{i\omega/\lambda}x^{-1+i(\omega'-\omega)/\lambda}\ ; \tag{3.18}$$

the integral is a β function. $\beta_{\omega\omega'}$ is computed similarly, and altogether one has

$$\begin{aligned}\alpha_{\omega\omega'}&=\frac{1}{2\pi\lambda}\left[\frac{\omega'}{\omega-i\epsilon}\right]^{1/2}(\lambda\Delta)^{i\omega/\lambda}B\left[-\frac{i\omega}{\lambda}+\frac{i\omega'}{\lambda}+\epsilon,1+\frac{i\omega}{\lambda}\right]\ ,\\ \beta_{\omega\omega'}&=\frac{1}{2\pi\lambda}\left[\frac{\omega'}{\omega-i\epsilon}\right]^{1/2}(\lambda\Delta)^{i\omega/\lambda}B\left[-\frac{i\omega}{\lambda}-\frac{i\omega'}{\lambda}+\epsilon,1+\frac{i\omega}{\lambda}\right]\ .\end{aligned} \tag{3.19}$$

The pole prescriptions are necessary to completely define these quantities; they are chosen so that the expansion (3.9) and the inverse expansion of u_ω in terms of v_ω actually hold. [Note that the derivation of (3.10) was actually somewhat formal.] With the pole prescriptions as given above, one may verify that this Bogoliubov transformation satisfies the necessary "completeness" identifies; for example,

$$\int_0^\infty d\omega'[\alpha_{\omega\omega'}\alpha^*_{\omega''\omega'}-\beta_{\omega\omega'}\beta^*_{\omega''\omega'}]=\delta(\omega-\omega'')\ . \tag{3.20}$$

The Bogoliubov coefficients given in (3.19) are central to the study of the Hawking radiation. Note that they depend only on Δ, not on M or on other details of the collapsing black hole.

It will be convenient to have a specific basis for the interior region as well; a useful choice is

$$\hat{v}_\omega(y^-)=v^*_\omega(-y^-)\ . \tag{3.21}$$

The Bogoliubov coefficients of these modes are found to be

$$\begin{aligned}\hat{\alpha}_{\omega\omega'}&=\alpha^*_{\omega\omega'}\ ,\\ \hat{\beta}_{\omega\omega'}&=\beta^*_{\omega\omega'}\ .\end{aligned} \tag{3.22}$$

Finally, we note that in the presence of the dilaton there is an ambiguity in the metric used to compute the Hawking radiation. In the present case, one could have, for example, taken the metric to be $\hat{g}=e^{-2\phi}g$. From (2.6) one sees that this is the flat metric. Therefore, if this is used as the background reference metric, the Bogoliubov transformation is trivial and there is no Hawking radiation. In particular, if the Faddeev-Popov ghosts from gauge fixing of general coordinate invariance are defined with respect to the metric g, then one concludes that the black hole is unstable with respect to thermal absorption of ghosts. As has been suggested in [10,19], this problem is solved if the ghosts are instead coupled to $\hat{g}$.

IV. HAWKING RADIATION AT LATE TIMES

As a first application of the Bogoliubov transformation (3.19), we investigate the late-time Hawking radiation along the lines of [1] and verify that it is indeed thermal.

We begin by computing the expected occupation numbers of the out modes, using (3.12). Following [1], the late-time Bogoliubov transformation is found by replacing the integrand in (3.17) by its approximate value near the horizon, $y^-=0$. This gives

$$\alpha_{\omega\omega'} \simeq \frac{1}{2\pi}\left[\frac{\omega'}{\omega}\right]^{1/2} \int_{-\infty}^{0} dy^- \exp\left\{\frac{i\omega}{\lambda}\ln(-\lambda^2\Delta y^-) + i\omega' y^-\right\} . \quad (4.1)$$

Note that $\beta_{\omega\omega'}$ differs from this only by the sign of ω' in the integrand. Deforming the contour in (4.1) to the positive y^- axis and changing variables $y^- \to -y^-$ flips this sign and gives the crucial relation

$$\alpha_{\omega\omega'} \simeq -e^{\pi\omega/\lambda}\beta_{\omega\omega'} . \quad (4.2)$$

Finally, setting $\omega=\omega''$ in relation (3.20) implies

$$\int_0^\infty d\omega' [|\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2] = t . \quad (4.3)$$

Here we have replaced the infinite quantity $\delta(0)$ by a large time cutoff t; this identification arises from considering the Fourier transform of δ. Combining this with (4.2) and (3.12) gives

$$ {}_{\rm in}\langle 0|N_\omega^{\rm out}|0\rangle_{\rm in} = \int_0^\infty d\omega' |\beta_{\omega\omega'}|^2 \simeq t\frac{e^{-2\pi\omega/\lambda}}{1-e^{-2\pi\omega/\lambda}} . \quad (4.4)$$

Thus the modes are thermally populated at a temperature $T_H=\lambda/2\pi$.

We now proceed further to show that the late-time density matrix is purely thermal (if one neglects the back reaction); i.e., it has no hidden correlations that would correspond to information escape from the black hole. For performing such physical calculations in the out region, it is useful to have a set of normalizable modes that are also localized. Following Hawking [1], we introduce the complete orthonormal set of wave-packet modes:

$$v_{jn} = \epsilon^{-1/2}\int_{j\epsilon}^{(j+1)\epsilon} d\omega\, e^{2\pi i\omega n/\epsilon} v_\omega , \quad (4.5)$$

with integers j,n and $j\geq 0$. These wave packets have frequency $\omega\simeq\omega_j$, with $\omega_j\equiv j\epsilon$, and they are peaked about $\sigma^-=2\pi n/\epsilon$ with width ϵ^{-1}; an example is pictured in Fig. 2. The Bogoliubov coefficients in this basis are easily found to be

$$\begin{aligned} \alpha_{jn\omega'} &= \epsilon^{-1/2}\int_{j\epsilon}^{(j+1)\epsilon} d\omega\, e^{2\pi i\omega n/\epsilon}\alpha_{\omega\omega'} , \\ \beta_{jn\omega'} &= \epsilon^{-1/2}\int_{j\epsilon}^{(j+1)\epsilon} d\omega\, e^{2\pi i\omega n/\epsilon}\beta_{\omega\omega'} . \end{aligned} \quad (4.6)$$

For the wave-packet modes (4.5), "late time" means large $2\pi n/\epsilon$. We will also take ϵ small so that the modes are narrowly peaked in frequency; this, of course, broadens them in position. Combining expressions (3.17) and (4.6) gives

$$\alpha_{jn\omega'} = \frac{1}{2\pi\sqrt{\epsilon}}\int_{j\epsilon}^{(j+1)\epsilon} d\omega \left[\frac{\omega'}{\omega}\right]^{1/2} e^{2\pi i\omega n/\epsilon}\int_{-\infty}^{0} dy^- \exp\left\{\frac{i\omega}{\lambda}\ln[\lambda\Delta(e^{-\lambda y^-}-1)] + i\omega' y^-\right\} . \quad (4.7)$$

For large values of $2\pi n/\epsilon$, the double integral receives contributions mainly from the vicinity of the horizon, $y^-\simeq 0$, so that the integrand may be approximated as in (4.1). Deforming the contour and changing variables now gives the result

$$\begin{aligned} \alpha_{jn\omega'} &\simeq -\frac{1}{2\pi\sqrt{\epsilon}}\int_{j\epsilon}^{(j+1)\epsilon} d\omega \left[\frac{\omega'}{\omega}\right]^{1/2} e^{2\pi i\omega n/\epsilon} e^{\pi\omega/\lambda}\int_{-\infty}^{0} dy^- \exp\left\{\frac{i\omega}{\lambda}\ln(-\lambda^2\Delta y^-) - i\omega' y^-\right\} \\ &\simeq -\frac{e^{\pi\omega_j/\lambda}}{2\pi\sqrt{\epsilon}}\int_{j\epsilon}^{(j+1)\epsilon} d\omega \left[\frac{\omega'}{\omega}\right]^{1/2} e^{2\pi i\omega n/\epsilon}\int_{-\infty}^{0} dy^- \exp\left\{\frac{i\omega}{\lambda}\ln(-\lambda^2\Delta y^-) - i\omega' y^-\right\} , \end{aligned} \quad (4.8)$$

where the assumption of small ϵ was used in the second line. In the latter expression we recognize the approximation of $\beta_{jn\omega}$, and so

$$\beta_{jn\omega} \simeq -e^{-\pi\omega_j/\lambda}\alpha_{jn\omega} . \quad (4.9)$$

We can similarly approximate the modes $\hat{v}_\omega$, which were defined in the previous section. Their Bogoliubov coefficients are also found to satisfy

$$\hat{\beta}_{jn\omega} \simeq -e^{-\pi\omega_j/\lambda}\hat{\alpha}_{jn\omega} . \quad (4.10)$$

These two relations are crucial because they allow one to form a new orthonormal mode basis, which is simply related to the old one and which is purely positive frequency in the in region, as follows (cf. [20,21]):

$$\begin{aligned} v_{jn}^1 &= (1-\gamma_j^2)^{-1/2}[v_{jn} + \gamma_j \hat{v}_{jn}^*] , \\ v_{jn}^2 &= (1-\gamma_j^2)^{-1/2}[\hat{v}_{jn} + \gamma_j v_{jn}^*] , \end{aligned} \quad (4.11)$$

where $\gamma_j = e^{-\pi\omega_j/\lambda}$. One can easily see that

$$\beta_{jn\omega}^1 = \beta_{jn\omega}^2 = 0 , \quad (4.12)$$

verifying positivity in the in region.

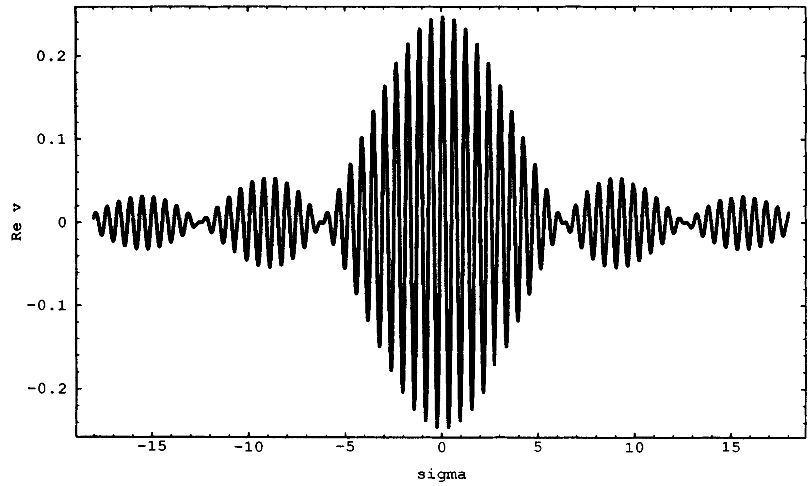

FIG. 2. Plotted is the wave-packet mode $v_{jn}(\sigma^-)$ with $\epsilon=1$, $n=0$, and $j=10$.

Since these modes are positive frequency at $\mathcal{I}_L^-$, the incoming state may be completely characterized using their associated annihilation operators by

$$0=a_{jn}^1|0\rangle_{\rm in}=a_{jn}^2|0\rangle_{\rm in} . \tag{4.13}$$

However, from the transformation between $v_{jn},\hat{v}_{jn}$ and v_{jn}^1,v_{jn}^2, we can derive

$$\begin{aligned} a_{jn}^1&=(1-\gamma_j^2)^{-1/2}[b_{jn}-\gamma_j\hat{b}_{jn}^\dagger] , \\ a_{jn}^2&=(1-\gamma_j^2)^{-1/2}[\hat{b}_{jn}-\gamma_j b_{jn}^\dagger] , \end{aligned} \tag{4.14}$$

so that $|0\rangle_{\rm in}$ is characterized in terms of the out operators by

$$\begin{aligned} (b_{jn}-\gamma_j\hat{b}_{jn}^\dagger)|0\rangle_{\rm in}&=0 , \\ (\hat{b}_{jn}-\gamma_j b_{jn}^\dagger)|0\rangle_{\rm in}&=0 . \end{aligned} \tag{4.15}$$

A particularly useful combination of Eqs. (4.13) and (4.14) is

$$\begin{aligned} 0&=[a_{jn}^{1\dagger}a_{jn}^1-a_{jn}^{2\dagger}a_{jn}^2]|0\rangle_{\rm in} \\ &=[b_{jn}^\dagger b_{jn}-\hat{b}_{jn}^\dagger\hat{b}_{jn}]|0\rangle_{\rm in} \\ &=[N_{jn}-\hat{N}_{jn}]|0\rangle_{\rm in} , \end{aligned} \tag{4.16}$$

where $N_{jn},\hat{N}_{jn}$ are the particle number operators corresponding to $v_{jn},\hat{v}_{jn}$, respectively. Although the notion of "particle" is somewhat ambiguous inside the black hole, we see that with the present definition hatted and unhatted particles occur in pairs in the outgoing state. This corresponds to the common statement that Hawking radiation proceeds by creation of particle pairs, with one particle inside the horizon and one outside.

Now we will use Eq. (4.15) to express $|0\rangle_{\rm in}$ in terms of out particle states. Using $N_{jn}=\hat{N}_{jn}$, we can already write [22]

$$|0\rangle_{\rm in}=\sum_{\{n_{jn}\}} c(\{n_{jn}\})|\{\hat{n}_{jn}\}\rangle|\{n_{jn}\}\rangle , \tag{4.17}$$

where the n_{jn} are sets of occupation numbers for the modes jn and the coefficients $c(\{n_{jn}\})$ are to be determined. Focusing on a single mode $j'n'$, we see that Eq. (4.15) implies

$$c(\{n_{jn}\})=\exp\{-\pi\omega_{j'}/\lambda\}c(\{n_{jn}-\delta_{jj'}\delta_{nn'}\}) , \tag{4.18}$$

which gives altogether

$$c(\{n_{jn}\})=c(\{0\})\exp\left\{-\frac{\pi}{\lambda}\left[\sum_{nj} n_{jn}\omega_j\right]\right\} . \tag{4.19}$$

[This can equivalently be found from (3.14).] Here $c(\{0\})$ is an overall normalization, which is infinite unless we restrict to finite time as in (4.3). In actuality, the black hole cannot evaporate for infinite time; the above result is invalid once the back reaction becomes relevant.

To predict what is seen by observers at $\mathcal{I}_R^+$, we must trace over the internal (hatted) states to produce a density matrix dependent only on the external particle states. In other words,

$$\begin{aligned} \rho^{\rm out}_{\{n_{jn}\}\{n'_{jn}\}}&\equiv\sum_{\{\hat{n}_{jn}\}}\langle\{n_{jn}\}|\langle\{\hat{n}_{jn}\}|0\rangle_{\rm in\,in}\langle 0|\{\hat{n}_{jn}\}\rangle|\{n'_{jn}\}\rangle \\ &=|c(\{n_{jn}\})|^2\delta_{\{n_{jn}\}\{n'_{jn}\}} \\ &=|c(\{0\})|^2\delta_{\{n_{jn}\}\{n'_{jn}\}}\exp\left\{-\frac{2\pi}{\lambda}\left[\sum_{jn} n_{jn}\omega_j\right]\right\} . \end{aligned} \tag{4.20}$$

This is a completely thermal density matrix. Note that it is totally independent of the details of the collapsing matter.

We emphasize that the formula (4.20) for the density matrix is an approximate expression valid only at late times and then only to the extent that the back reaction can be neglected. The former condition is

$$0<-y^-\ll\frac{1}{\lambda} \tag{4.21}$$

or, equivalently, from (3.15),

$$e^{-\lambda\sigma^-}\ll\lambda\Delta . \tag{4.22}$$

To understand the latter condition, one must understand what effect the outgoing Hawking radiation has on the geometry; this is the subject of the next section.

V. STRESS TENSOR FOR HAWKING RADIATION

A long-standing issue in black-hole physics is that of incorporating the back reaction of Hawking radiation on the black-hole geometry. In [11] this problem was investigated for the semiclassical limit of dilaton gravity. In this case one can calculate the quantum stress tensor $\langle T^f_{\mu\nu}\rangle$ in the background of the classical solution by starting with the known conformal anomaly and integrating the conservation equation [23]. The stress tensor is then determined up to boundary conditions reflecting the choice of incoming quantum state. The expectation value of this stress tensor is appended to Einstein's equations to incorporate the effect of the back reaction.

This section will investigate some details of this procedure and confirm its validity. In particular, we will show asymptotic equivalence of the stress tensor calculated from the conformal approach with the stress tensor of Hawking radiation described above. We will also comment on the issue of why coupling this stress tensor to gravity gives an accurate representation of the effects of the back reaction.

The preceding sections have shown that the quantum state representing vacuum in the in region, which we refer to as $|0\rangle_{\rm in}$, is not the same as the state $|0\rangle_{\rm out}$, which represents vacuum in the out region. In the out region the state $|0\rangle_{\rm in}$ includes the outgoing particles of the Hawking radiation. We have shown by one method that this radiation has a thermal spectrum, and we will now check this, as well as the treatment of [11], by directly computing ${}_{\rm in}\langle 0|T^f_{\mu\nu}|0\rangle_{\rm in}$ asymptotically in the out region.

The latter expression is given by

$$\langle T^f_{\mu\nu}\rangle_{\rm in}={}_{\rm in}\langle 0|\tfrac{1}{2}\left[\partial_\mu f\,\partial_\nu f-\tfrac{1}{2}g_{\mu\nu}g^{\lambda\sigma}\partial_\lambda f\,\partial_\sigma f\right]|0\rangle_{\rm in}\ . \tag{5.1}$$

To begin with, note that $\langle T^f_{++}\rangle=\langle T^f_{+-}\rangle=0$, the first because the Bogoliubov transformation is trivial for left-moving modes (since $\sigma^+=y^+$) and the second because the trace anomaly is zero in the asymptotic region from vanishing of the curvature. Our focus is therefore on

$${}_{\rm in}\langle 0|T^f_{--}(\sigma^-)|0\rangle_{\rm in}={}_{\rm in}\langle 0|\tfrac{1}{2}\partial_-f(\sigma^-)\partial_-f(\sigma^-)|0\rangle_{\rm in}\ . \tag{5.2}$$

Since T^f_{--} is a product of operators at the same point, it must be carefully defined. It is required that ${}_{\rm out}\langle 0|T^f_{--}|0\rangle_{\rm out}=0$ (at $\mathcal{I}^+_R$) so that one should expand and normal order T^f_{--} with respect to $b_\omega, b^\dagger_\omega$ and then evaluate its expectation value in $|0\rangle_{\rm in}$. This procedure can be streamlined by using point splitting.

We start with the coordinate transformation inverse to (3.15), namely,

$$y^-=-\frac{1}{\lambda}\ln\left[\frac{1}{\lambda\Delta}e^{-\lambda\sigma^-}+1\right]\ . \tag{5.3}$$

The f_- field is given by

$$\begin{aligned} f_-&=\int_0^\infty\frac{d\omega}{\sqrt{2\omega}}[a_\omega e^{-i\omega y^-}+a^\dagger_\omega e^{i\omega y^-}]\\ &=\int_0^\infty\frac{d\omega}{\sqrt{2\omega}}\left\{a_\omega\exp\left[\frac{i\omega}{\lambda}\ln\left[\frac{1}{\lambda\Delta}e^{-\lambda\sigma^-}+1\right]\right]+\text{H.c.}\right\}\ . \end{aligned} \tag{5.4}$$

Now, in T^f_{--}, we shift the coordinate of one of the ∂_-f factors from σ^- to $\sigma^-+\delta$, where δ is a small number, and compute the point-split value

$$\begin{aligned} \langle T^f_{--}\rangle_{\rm in}&=\tfrac{1}{4}\int_0^\infty\omega\,d\omega\frac{\exp\{(i\omega/\lambda)\ln[(1/\lambda\Delta)e^{-\lambda\sigma^-}+1]\}}{1+\lambda\Delta e^{\lambda\sigma^-}}\,\frac{\exp\{(i\omega/\lambda)\ln[(1/\lambda\Delta)e^{-\lambda(\sigma^-+\delta)}+1]\}}{1+\lambda\Delta e^{\lambda(\sigma^-+\delta)}}\\ &=-\frac{\lambda^2}{4}\,\frac{\{\ln[(1/\lambda\Delta)e^{-\lambda\sigma^-}+1]-\ln[(1/\lambda\Delta)e^{-\lambda(\sigma^-+\delta)}+1]\}^{-2}}{(1+\lambda\Delta e^{\lambda\sigma^-})(1+\lambda\Delta e^{\lambda(\sigma^-+\delta)})}\ , \end{aligned} \tag{5.5}$$

where the integration was performed with a large-ω convergence factor. From this we will subtract the out vacuum value

$${}_{\rm out}\langle 0|T^f_{--}|0\rangle_{\rm out}=-\frac{1}{4\delta^2}\ , \tag{5.6}$$

before taking the limit $\delta\to 0$. This subtraction produces an expression normal ordered with respect to the out vacuum.

The remainder of the computation consists of expanding (5.5) in powers of δ, with the renormalized result

$${}_{\rm in}\langle 0|T^f_{--}|0\rangle_{\rm in}=\frac{\lambda^2}{48}\left[1-\frac{1}{(1+\lambda\Delta e^{\lambda\sigma^-})^2}\right]\ , \tag{5.7}$$

which is identical to that of [11] in the out region. Note that here the thermal value is achieved at

$$e^{-\lambda\sigma^-}\lesssim\lambda\Delta\ , \tag{5.8}$$

which agrees with (4.22).

Next, we comment on the issue of coupling the stress tensor $\langle T_{\mu\nu}\rangle$ to gravity to represent the back reaction.

Within the relativity literature there has been much debate on the issues of whether $\langle T_{\mu\nu}\rangle$ is the appropriate quantity to place on the right-hand side of Einstein's equations[4] and how to compute the correct value for $\langle T_{\mu\nu}\rangle$. The second of these issues is generally resolved by appealing to a result of Wald: all computation techniques which satisfy four physically reasonable conditions known as Wald conditions will produce the same answer, up to well-defined ambiguities. The only such ambiguity in two dimensions is the cosmological constant.

Within the present context both issues are addressed by considering quantization via the functional integral:

$$Z=\int \mathcal{D}g\,\mathcal{D}\phi\, e^{iS_G}\int \mathcal{D}_g f_i \exp\left\{-\frac{i}{4\pi}\int d^2x\sqrt{-g}\sum_{i=1}^{N}(\nabla f_i)^2\right\}, \tag{5.9}$$

where S_G is the purely gravitational/dilatonic part of the action (2.1) and where the g dependence of the f_i measure is explicitly indicated. The functional integral over f_i is one that has been studied extensively in the string literature and elsewhere. If regulated in a generally covariant manner, it yields

$$\begin{aligned} e^{iS_{\rm PL}} &= \int \mathcal{D}_g f_i \exp\left\{-\frac{i}{4\pi}\int d^2x\sqrt{-g}\sum_{i=1}^{N}(\nabla f_i)^2\right\} \\ &= \exp\left\{-\frac{iN}{96\pi}\int\sqrt{-g(x)}d^2x\int\sqrt{-g(x')}d^2x'\,R(x)G(x,x')R(x')\right\}, \end{aligned} \tag{5.10}$$

where $G(x,x')$ is the Green's function for the d'Alembertian,

$$\Box_x G(x,x')=\frac{\delta^2(x-x')}{\sqrt{-g(x)}}\ . \tag{5.11}$$

Equation (5.10) is unique up to local counterterms and up to the boundary conditions needed to define the Green's function. If one assumes that ϕ does not couple to f_i, then the only counterterm is the cosmological constant which may be fine-tuned to zero. The boundary conditions are fixed as in [11] by the demand that $\langle T^f_{\mu\nu}\rangle$ have the correct form in the in region.

The resulting classical equations

$$\frac{2\pi}{\sqrt{-g}}\,\frac{\delta S_G}{\delta g^{\mu\nu}}=\langle T_{\mu\nu}\rangle \tag{5.12}$$

accurately describe evolution in regions where the coupling e^{ϕ} is small.[5] As was argued in [11], the evaporation of the black hole can be arranged to take place purely within the weak-coupling region by taking the number N of matter fields to be large. A discussion of the resulting solutions of these equations was given in [3,13–16], where it was argued that the black hole settles down to a final state of the linear dilaton vacuum terminated in the region where

$$e^{2\phi}\simeq\frac{12}{N}\ . \tag{5.13}$$

In this region the evaluation becomes singular and the classical equations are invalid. Alternatively, one could go beyond to investigate the quantum dynamics of the theory; this is described by including the term $S_{\rm PL}$ in the remaining functional integral over g and ϕ. The latter term incorporates the full quantum effect of the back reaction from Hawking emission of matter.

VI. BEYOND THE THERMAL LIMIT

The density matrix (4.20) describes a mixed state of thermal radiation. In the four-dimensional context, this has been taken as strong evidence that an initially pure state can evolve into a mixed state in the course of black-hole formation and evaporation. One should be cautious in drawing this conclusion, however, since, as we have stated, (4.20) is only approximately correct; it is (barely) conceivable that once corrections are taken into account the missing information will be restored.

To investigate the importance of modifications to (4.20), let us first determine its domain of validity. First, as was indicated at the end of Sec. IV, (4.20) is only valid at late times as given by (4.21) or (4.22). Next, the derivation neglected the effect of the back reaction. A very crude estimate of when this becomes important is found by asking when the integrated energy in the Hawking radiation equals the initial mass of the black hole. This can be determined by integrating the asymptotic value of the stress tensor (5.7) along $\mathcal{I}_L^-$ as in [11].[6] The amount of mass radiated up to the time given by

$$e^{-\lambda\sigma^-}\sim\lambda\Delta \tag{6.1}$$

is easily estimated to be

[4]See, e.g., [18], pp. 214–224.

[5]Actually, this is not precisely true, as has been argued in [3,13–16]; the weak-coupling expansion breaks down because of the vanishing of an eigenvalue of the kinetic term at the singularities described in these papers.

[6]Note, however, that one should not make the assumption of small mass.

$$M_{\rm rad} \sim \frac{N\lambda}{48} \ . \tag{6.2}$$

Equation (4.20) will be valid over a nonvanishing domain only if by the time the thermal approximation (4.22) holds the radiated mass is negligible compared with the initial mass:

$$M \gg \frac{N\lambda}{48} \ . \tag{6.3}$$

Note that from (4.4) the typical energy of an emitted particle is λ; thus, the condition (6.3) is the statement that the black hole be capable of emitting a large number of particles of each type. Once the radiation becomes thermal, $T_{--} \simeq N\lambda^2/48$, and so the black hole evaporates over a time of order

$$\Delta\sigma^- \sim \frac{48M}{N\lambda^2} \ . \tag{6.4}$$

The density matrix (4.20) is therefore approximately correct in the range

$$-\frac{1}{\lambda}\ln(\lambda\Delta) + \frac{48M}{N\lambda^2} \gg \sigma^- \gg -\frac{1}{\lambda}\ln(\lambda\Delta) \ . \tag{6.5}$$

To investigate the question of whether corrections to (4.20) solve the information problem, the early-time transitory behavior and the back reaction must be incorporated. There are two possible approaches to determining to what extent the outgoing state is still mixed. One is to calculate the density matrix directly as in Sec. IV, now including these effects. However, finding the density matrix, even taking into account the transitory behavior, is rather complicated, and an alternative approach is to investigate the behavior of correlation functions of the form

$$\langle b^\dagger_{jn} \cdots b_{j'n'} \cdots \rangle_{\rm in} \ , \tag{6.6}$$

with an arbitrary number of creation and annihilation operators. All details of the outgoing state are encoded in such correlators.

Using the exact form of the Bogoliubov transformation, these correlation functions are in fact exactly calculable in the "early-time" limit where one includes the transitory behavior, but neglects the back reaction. Indeed, given the Bogoliubov coefficients (4.6) and the relations

$$\begin{aligned} b_{jn} &= \int d\omega [\alpha^*_{jn\omega} a_\omega - \beta^*_{jn\omega} a^\dagger_\omega] \ , \\ b^\dagger_{jn} &= \int d\omega [\alpha_{jn\omega} a^\dagger_\omega - \beta_{jn\omega} a_\omega] \ , \end{aligned} \tag{6.7}$$

one may calculate the expectation value of any operator built from the b_{jn} and $b^\dagger_{jn}$. For example, it is easy to see that the two-point correlator is given by

$$\langle b^\dagger_{jn} b_{j'n'} \rangle_{\rm in} = \int_0^\infty d\omega\, \beta_{jn\omega} \beta^*_{j'n'\omega} \ , \tag{6.8}$$

which is in principle exactly calculable using (4.6) and (3.19).

Although potentially instructive, such calculations are not expected to address directly the information problem. The reason for this is that the information, if it escapes the black hole at all, is expected to emerge in the evaporation of the black hole, not in the initial transitory behavior. Note also that (3.19) implies that the correlators depend on the infalling matter distribution only through the single quantity Δ; the transitory behavior is not even dependent on the details of the collapse.

To actually answer the question of whether or not enough information escapes in black-hole evaporation to solve the information problem, one must include the effects of the back reaction. A complete treatment of this seems to require describing the initial configuration as a quantum state and studying the quantum evolution of the system. Although we will not work this out in detail in the present paper, one can see the resulting modifications on a qualitative level using the semiclassical approximation.[7]

Consider the situation where a black hole is formed from a pure quantum state with left-moving energy momentum concentrated between times x_i^+ and x_f^+, as in Sec. II. In the weak-coupling region, we may work to leading order in e^ϕ, and this state again produces a geometry such as that of Fig. 1 if the back reaction is neglected. The geometry that arises when the back reaction is included was discussed in [3,13–16] and is shown in Figs. 3 and 4. The infalling matter gives rise to a new "quantum singularity" that is hidden behind an apparent horizon. As the black hole loses mass, both the singularity and apparent horizon asymptote to the global horizon. The final sate is the linear dilaton vacuum to the right of the region where

$$e^{2\phi} \simeq \frac{12}{N} \ ; \tag{6.9}$$

beyond this, the semiclassical equations are not be trusted.

The outgoing state will again be described in the natural asymptotically flat coordinates y^- at $\mathcal{I}^+_R$. Now we are not able to write down explicitly the coordinate transformation from σ^- to y^- because of insufficient knowledge of the back-reaction-corrected geometry; therefore, the precise form of the Bogoliubov transformation has not been determined. However, for large M, we know that it agrees with (4.6) and (3.19) throughout the range (6.5). As the back reaction becomes important, the Hawking radiation turns off; correspondingly, the Bogoliubov coefficients $\beta_{\omega\omega'}$ and $\beta_{jn\omega}$ should die off. To leading order in e^ϕ, the only effect of the left-moving matter is to produce this nontrivial Bogoliubov transformation for the right movers. The in vacuum can be rewritten in the out/internal Fock space as in (3.14); equivalently, we may write

$$|0\rangle_{\rm in} = \sum_{\{n_{jn}\},\alpha} c(\{n_{jn}\},\alpha)|\hat{\alpha}\rangle|\{n_{jn}\}\rangle \ , \tag{6.10}$$

where now we have adopted an arbitrary basis $|\hat{\alpha}\rangle$ for the states to the left of the global horizon that fall into

[7]Investigation of the information problem in the semiclassical limit has also been advocated by Russo, Susskind, and Thorlacious [13,3,24].

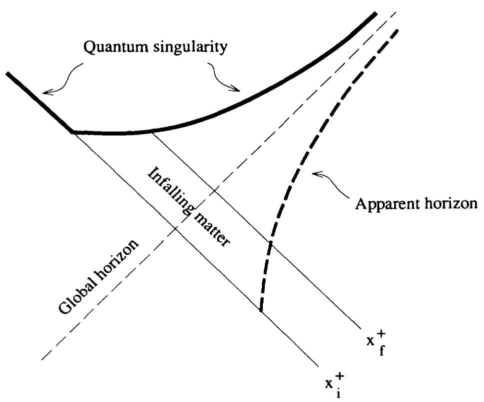

FIG. 3. Kruskal geometry for the back-reaction-corrected gravitational collapse of a matter distribution. An apparent horizon forms; behind it is the "quantum singularity" where the semiclassical equations break down. Both of these asymptote to a global horizon.

the singularity. The asymptotic density matrix is derived from this by tracing over these internal states. This density matrix would be pure if one could rewrite it in the form

$$|0\rangle_{\rm in}=|\Psi\rangle_{\rm int}|\Psi\rangle_{\rm out}\ , \tag{6.11}$$

for some states $|\Psi\rangle_{\rm int}$ and $|\Psi\rangle_{\rm out}$ in the internal and out Hilbert spaces. However, leading-order agreement with (4.17) and the general fact that the Bogoliubov transformation sets up correlations between internal and external states makes it appear very unlikely that this could be the case. Together with the fact that modes can fall into the singularity without escaping, this indicates that information can indeed be lost to the quantum singularity and that the entropy of the outgoing density matrix should consequently be nonzero.

These statements may, of course, be invalidated once higher-order quantum corrections are taken into account. However, these corrections are expected to be unimportant until the weak-coupling approximation breaks down. This only happens in the final stages of the black-hole evaporation. The above arguments therefore strongly suggest that within the present model information does not escape until the black hole is very small. Making these rigorous will therefore rule out one suggested resolution of the black-hole information problem, namely, that the information escapes over the course of black-hole evaporation if the effects of the back reaction are included. Other possibilities are described in [4].

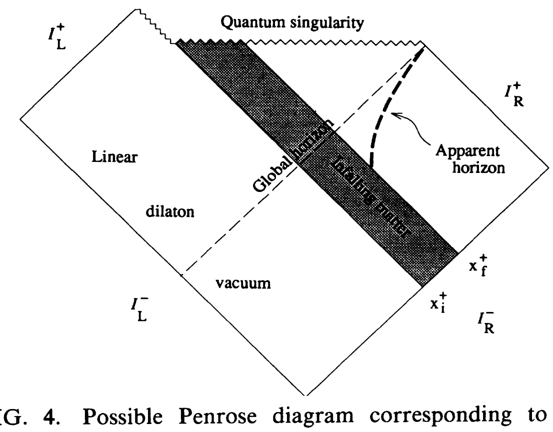

FIG. 4. Possible Penrose diagram corresponding to the Kruskal geometry of Fig. 3.

VII. CONCLUSIONS

The two-dimensional process of black-hole formation and evaporation studied in [11] is a simplified arena for investigation of physical issues relevant to higher dimensions. We have shown that, in particular, the Bogoliubov transformation is exactly calculable if the back reaction is neglected. This, in principle, allows exact determination of all correlation functions and of the density matrix describing the outgoing Hawking radiation. After a transitory period the Hawking radiation has the expected thermal behavior with temperature $\lambda/2\pi$. (In contrast with the four-dimensional case, even the transitory period is exactly describable.)

For large black holes, $M \gg \lambda N/48$, the thermal density matrix is an accurate descriptor of the outgoing state for the time after the falloff of the transitory behavior, but before the black hole has lost a substantial fraction of its mass to Hawking evaporation. As in the four-dimensional case, this suggests that a pure initial state evolves into a mixed final state. However, a conclusive statement to this effect cannot be made while neglecting the back reaction. We have argued that to leading order in the weak-coupling expansion the effect of the back reaction is to modify the Bogoliubov transformation, but not in such a way as to restore the information lost to the black hole. However, a definitive proof that information is lost even in the presence of the back reaction is beyond the scope of this paper.

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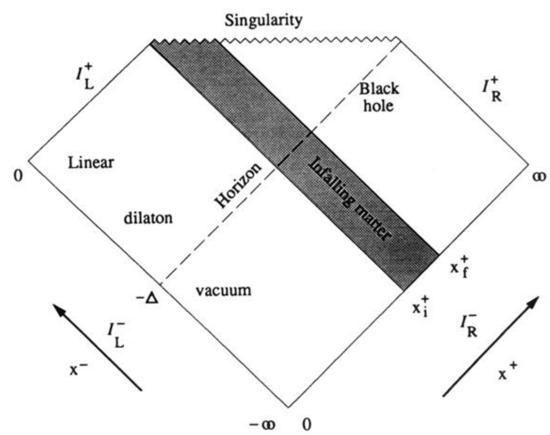

FIG. 1. Shown is the Penrose diagram for a black hole formed from an arbitrary distribution of collapsing matter concentrated between times x_i^+ and x_f^+.

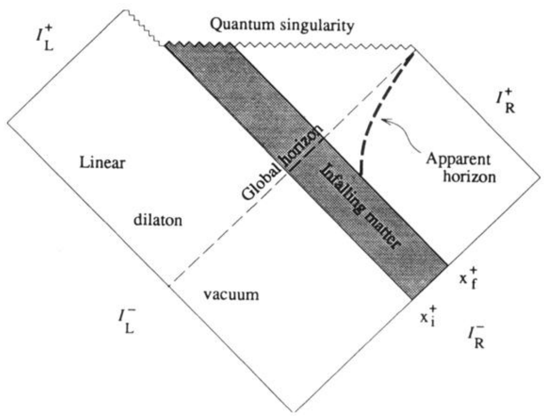

FIG. 4. Possible Penrose diagram corresponding to the Kruskal geometry of Fig. 3.