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Soft-pion emission in high-energy heavy-ion collisions

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We revive Heisenberg's approach to multiparticle production in the context of soft-pion emission in heavy-ion collisions. Adopting appropriate boundary conditions, we find a general analytic solution of the classical equations of motion for the nonlinear σ model (for soft pions this model is an approximation to QCD). The solution is used to discuss various features of soft pion production in nuclear collisions.

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I. INTRODUCTION

The behavior of hadronic matter at unusually high density is interesting for a variety of well known reasons. Such dense systems can be created in the laboratory during heavy-ion collisions, and new experimental facilities will soon be put into operation or are being designed to study such collisions. On the theory side, much remains to be done. Ideally, one would like a space-time description of the collision process rooted in the microscopic theory (QCD). The aim of this paper is to describe an attempt in this direction.

It is widely believed that much of the low-energy physics of QCD can be represented by an effective Lagrangian [1], describing the interactions of colorless excitations of the physical QCD vacuum. This Lagrangian has the form of an infinite series of terms involving an increasingly large number of derivatives and has to be truncated for all practical purposes. If one keeps only the first term, the one with two derivatives, one gets the so-called nonlinear σ model, which describes fairly well the low-energy pion dynamics. We propose to use the effective Lagrangian and the classical approximation to describe soft-pion emission in heavy-ion collisions. Notice that the number of soft pions produced at high energy in such a collision can be quite large, which makes plausible the validity of the classical approximation. In this exploratory work we limit ourselves to the nonlinear σ model. With this approximation the discussion will be fully analytic and, we hope, a maximum clarity will be achieved. Obviously, improvements are possible but will require more computational effort.

The idea to use the classical approximation to field theory in a study of multiparticle production brings us back to some old papers by Heisenberg [2]. The founding father of quantum field theory had realized that quantization of meson fields implies the existence of multiple-meson production, and he extensively used the classical approximation to develop his shock-wave picture of production processes. He argued that meson spectra expected in renormalizable and in nonrenormalizable theories, respectively, differ qualitatively and he apparently hoped that the study of particle production will eventually yield evidence for the existence of a fundamental length. With this motivation, Heisenberg did not engage in a down-to-earth phenomenological study and sometimes insisted on those of his predictions which were untenable from the empirical point of view. His ideas were abandoned during the late 1950's and almost completely forgotten. Although we do not share his dreams, we found his work inspiring and we have borrowed from him, as will be noticed by the reader.

The plan of this paper is as follows. In Sec. II we derive the most general solution of the classical equations of motion for the nonlinear σ model, assuming Heisenberg's idealized boundary conditions. In Sec. III we use these solutions to calculate the canonical energy-momentum tensor. Remarkably enough, this tensor is formally that of an expanding relativistic fluid. We also calculate the one-particle inclusive spectrum. In Sec. IV we discuss the physical picture underlying our model, we say a few words about the predicted long-range rapidity correlations, we give an estimate of the formation time of a meson, we discuss the neutral/charged particle ratio which is expected to exhibit rather remarkable fluctuations, we make a couple of remarks on the relation of our theory to the hydrodynamical model, and we use the data to give an estimate of various physical quantities. We conclude in Sec. V. The attention of the reader should be called to related papers by Anselm and Ryskin [3] and Bjorken [3].

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II. SOLVING THE CLASSICAL FIELD EQUATIONS

The Lagrangian of the nonlinear σ model can be written in the form

$$L = \frac{f_\pi^2}{2}[(\partial\sigma)^2 + (\partial\boldsymbol{\pi})^2], \tag{1}$$

where $f_\pi = 93$ MeV is the pion decay constant and the fields satisfy the constraint

$$\sigma^2 + \boldsymbol{\pi}^2 = 1 \ . \tag{2}$$

For convenience of notation the pion field $\boldsymbol{\pi}$ is defined to be dimensionless. It is, of course, an isovector. The Euler-Lagrange equations are readily found to be equivalent to the following current-conservation equations:

$$\partial \cdot \mathbf{V} = 0 \ , \tag{3}$$

$$\partial \cdot \mathbf{A} = 0 \ , \tag{4}$$

where

$$\mathbf{V}_\mu = \boldsymbol{\pi} \times \partial_\mu \boldsymbol{\pi} \ , \tag{5}$$

$$\mathbf{A}_\mu = \boldsymbol{\pi}\, \partial_\mu \sigma - \sigma\, \partial_\mu \boldsymbol{\pi} \tag{6}$$

are the Noether isovector and iso-axial-vector currents associated with the global SU(2)×SU(2) symmetry of the theory.

To proceed further we adopt Heisenberg's idealized boundary conditions. The two impinging Lorentz-contracted nuclei overlap at some initial time t=0. At this moment the whole energy of the collision is localized within a thin slab. Following Heisenberg we assume that this slab has an infinite transverse extent and is infinitesimally thin, which means that we reduce the problem to a $(1+1)$-dimensional one. Furthermore, the symmetry of the problem implies that the pion field depends on the invariant $s = t^2 - x^2$ only.

The $(1+1)$-dimensional theory is described by the Lagrangian (1), where the space-time indices are restricted to values 0 and 1 ($x^0 \equiv t$, $x^1 \equiv x$) and the dimensionful constant f_π^2 is replaced by a dimensionless constant $f_\pi^2 S$. The parameter S has the dimension of an area and will be interpreted as the total transverse area later on, when we shall establish a contact with the $(1+3)$-dimensional world.

Now, the currents have the form $f(s)x_\mu$ and a current conservation equation reads

$$sf' + f = 0, \tag{7}$$

where the prime denotes differentiation with respect to s. Thus, f equals s^{-1} up to a multiplicative integration constant. Equations (3) and (4) yield

$$\boldsymbol{\pi} \times \boldsymbol{\pi}' = \frac{\mathbf{a}}{s} \tag{8}$$

and

$$\boldsymbol{\pi}\sigma' - \sigma\boldsymbol{\pi}' = \frac{\mathbf{b}}{s}, \tag{9}$$

where $\mathbf{a}$ and $\mathbf{b}$ are integration constants. Equations (8) and (9) imply that these constants are not independent: $\mathbf{a} \cdot \mathbf{b} = 0$. We now decompose $\boldsymbol{\pi}$ along the axes of the triad $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c} = \mathbf{a} \times \mathbf{b}$. After some algebra one finds

$$\pi_a = 0 \ , \tag{10}$$

$$\pi_b = \frac{s}{b}\sigma' \ , \tag{11}$$

$$\pi_c = \frac{a}{b}\sigma \ . \tag{12}$$

Differentiating the constraint equation (2) and using (10)–(12) one finally gets

$$\sigma'[s(s\sigma')' + \kappa^2\sigma] = 0, \tag{13}$$

where $\kappa^2 = a^2 + b^2$. Hence, disregarding the uninteresting case σ=const we conclude that σ satisfies a linear differential equation. The corresponding real solution is

$$\sigma = C \cos\left(\kappa \ln \frac{s}{s_0}\right) , \tag{14}$$

where C is a constant and s_0 a scale parameter. The pion field is easily found from (10)–(12). It is evident that it satisfies the same linear equation as σ, viz.

$$(s\boldsymbol{\pi}')' + \frac{\kappa^2}{s}\boldsymbol{\pi} = 0 \ . \tag{15}$$

Notice that although the equations are linear the normalization of the solution is not arbitrary, but fixed by the constraint (2). It is here that the nonlinear character of the problem shows up. Thus, one easily finds that $C = b\kappa^{-1}$. Finally

$$\pi_a = 0 \ , \tag{16}$$

$$\pi_b = -\sin\left(\kappa \ln \frac{s}{s_0}\right) \ , \tag{17}$$

$$\pi_c = \frac{a}{\kappa}\cos\left(\kappa \ln \frac{s}{s_0}\right) \ . \tag{18}$$

This solution to the field equations depends on the initial conditions via the two orthogonal isovectors $\mathbf{a}$ and $\mathbf{b}$ which specify the orientation in isospace of the vector and the axial-vector currents, respectively, and the scale parameter s_0. The violent oscillations near $s = 0$ do not appear in the field theories considered by Heisenberg, although he has worked with the same boundary conditions. They do not result from our neglect of the pion mass: assuming PCAC (partial conservation of axial-vector current) and treating the pion mass as a perturbation we do not generate terms indicating a qualitative modification of the solution at small values of s. In fact, these violent oscillations are contained in a well defined region of the space-time diagram, namely between the light cone $x = \pm t$ and the hyperbola $s = s_0$. At any fixed t, such a region contains an infinite amount of energy. Since the amplitude of the pion field is bounded, because of the constraint (2), the only way to store a large amount of energy in the field is by producing these violent oscillations. It is tempting to speculate that these oscillations are taking place in a region where the pion field strongly interacts with its source; in such a region, the higher-order derivative terms in the effective Lagrangians should be taken into account, but presumably a better description would be in terms of quark degrees of freedom. Following this reasoning, we are led to give to the parameter $\sqrt{s_0}$ the significance of the length/time scale below which the model cannot be trusted. Notice that the soft-pion spectrum calculated in the next section is independent of the value of s_0.

III. PION RADIATION

The canonical, traceless energy-momentum tensor is

$$T_{\mu\nu} = f_\pi^2 S\{\partial_\mu\sigma\,\partial_\nu\sigma + \partial_\mu\boldsymbol{\pi}\cdot\partial_\nu\boldsymbol{\pi} - \tfrac{1}{2}g_{\mu\nu}[(\partial\sigma)^2 + (\partial\boldsymbol{\pi})^2]\},\tag{19}$$

and, using Eqs. (16)–(18) one finds

$$T_{\mu\nu} = \frac{2f_\pi^2\kappa^2 S}{s^2}(2x_\mu x_\nu - s g_{\mu\nu})\ .\tag{20}$$

The singularity at $s = 0$ is nonintegrable. Again, this results from the idealized boundary conditions, which can only be realized if infinite energy is stored in the initial pancake. Such a singularity would go away once a more realistic initial state is used. However, a finite energy-momentum tensor for the initial source would break the symmetry of the problem, the fields would not depend on s alone, and we would lose the analytical simplicity of our solution. Such an extra complication is not needed at this exploratory stage; in addition, we shall soon focus on soft meson production which is insensitive to this singularity.

The ratio $u_\mu = x_\mu/\sqrt{s}$ can be regarded as the "two-velocity" of a covolume element with coordinate x_μ. Hence

$$T_{\mu\nu} = \frac{2f_\pi^2\kappa^2 S}{s}(2u_\mu u_\nu - g_{\mu\nu}).\tag{21}$$

This is the form of the energy-momentum tensor of a $(1+1)$-dimensional fluid with energy density ϵS, where

$$\epsilon = \frac{2f_\pi^2\kappa^2}{s},\tag{22}$$

The pressure is equal to the energy density, a fact which in $1+1$ dimensions just reflects the tracelessness of the energy-momentum tensor. Notice also, that the energy density of a $(1+1)$-dimensional fluid must fall like s^{-1} in a conformally invariant theory such as the one we are considering. This is a simple consequence of $T^\mu_\mu = 0$ together with $\partial^\mu T_{\mu\nu} = 0$. However, the fluid analogy which emerges so naturally does not imply that the pion fluid is in local equilibrium once one goes to $1+3$ dimensions, and indeed it is not (see Sec. IV E).

Next, we calculate the spectrum of pion radiation. To this end we Fourier transform the fields entering Eq. (19), at fixed t, using the formula [4]

$$\int_{-t}^{+t} dx\, e^{ikx}(t^2 - x^2)^{\pm i\kappa} = \sqrt{\pi}[2t/\omega]^{\frac{1}{2}\pm i\kappa}\Gamma(1 \pm i\kappa)J_{\frac{1}{2}\pm i\kappa}(\omega t)\tag{23}$$

where $\omega = \mid k \mid$ is the energy of the mode k. The energy radiated at time t, viz. $\int dx\, T_{00}(x,t)$ can be represented as a sum over k (which diverges) of the energy dE radiated in mode k. Using the asymptotic expansion for the Bessel function in (23), one finds the following simple expression for dE:

$$dE = Sh\, dk,\tag{24}$$

where

$$h = \frac{f_\pi^2\kappa}{\tanh(\pi\kappa)}.\tag{25}$$

The one-particle inclusive spectrum is therefore

$$dN = Sh\frac{dk}{\omega}\ .\tag{26}$$

The right-hand side of (26) becomes independent of κ for $\kappa \ll 1$. It is so because for any finite κ there exists an invariant interval where the pion field oscillates dramatically. However, for physical reasons, the size of this interval cannot be arbitrarily small (see the end of Sec. II and the next section).

IV. DISCUSSION

A. Physical picture

The global picture of the collision process that we have in mind is close to that reviewed in the well known paper by Bjorken [5], but stripped from the hydrodynamical considerations. As already mentioned, the model is unreliable for invariant distances $s \le s_0$. Thus, in the center-of-mass frame, there are two Lorentz-contracted regions, where most of the collision energy is concentrated, which recede fast from each other and where our theory does not apply. For physical reasons we expect s_0 to be of the order of 1 fm^2, but strictly speaking this is merely a guess. Our model describes, if at all, what happens in the region between the two receding pancakes. We assume that in this region, say 1 fm/c after the beginning of the collision, the isovector and iso-axial-vector currents are, in a sense, frozen in some classical configurations and specified by the two vectors $\mathbf{a}$ and $\mathbf{b}$. The detailed mechanisms operating at the initial stage of the collision are outside the scope of the model and can at best be parametrized by a probability density $\rho(\mathbf{a},\mathbf{b})$. Given $\mathbf{a}$ and $\mathbf{b}$, the further evolution is entirely determined by the classical equations of motion. The behavior of physical observables reflects the coherence in the propagation of the pion field and also the disorder effects described by the initial probability density $\rho(\mathbf{a},\mathbf{b})$.

Equation (26) indicates that not-too-energetic pions will populate a rapidity plateau (for massive pions we replace $dk/\omega \to dy$, y denoting the rapidity). The infinite extent of this plateau is clearly an artifact of the assumption that at $t = 0$ the system is infinitesimally thin. However, a plateau of finite extent also presents an apparent problem: strictly speaking, the nonlinear σ model is an approximation to QCD provided one considers soft pions only, and in this context "soft" means "with momentum of the order of f_π," the only scale in the effective theory. One can wonder whether the validity of (26) is not limited to these soft quanta. The following symmetry argument should make things clear.

The center-of-mass frame has nothing special in it. The physical picture of the collision is exactly the same in a variety of collinear frames and our result (26) holds in each of these frames. The energetic quanta which seem to appear in the spectrum are to be considered as soft quanta boosted to a different frame. Said differently, pions populating a small rapidity interval are soft in an appropriate frame and everything one needs in order to find the height of the rapidity plateau is a theory of soft-pion emission.

An extra complication arises as one goes to 1+3 dimensions. It is likely that the heretofore neglected terms in the Lagrangian produce a broadening of the transverse-momentum spectrum. Indeed, the observed value of $\langle k_\perp \rangle \approx 300$ MeV/c can be considered rather large compared to f_π. Consequently, phenomenological applications of Eq. (26) require some care, as will be discussed in Sec. IV F.

In three dimensions the invariant energy density is given by Eq. (22). The relation between κ and the energy density ϵ_0 of the dense system appearing at the early stage of the collision between the two receding pancakes (cf. Ref. [5]) is therefore

$$\epsilon_0 = \frac{2f_\pi^2\kappa^2}{s_0} \ . \tag{27}$$

B. Correlations

A classical solution of field equations corresponds to a coherent state. Thus, as long as one does not average over the initial conditions, particles are produced independently, and follow Poisson distributions. The averaging over κ generates correlations. For example, the normalized two-body rapidity correlation is proportional to the variance of κ:

$$R(y_1, y_2) \equiv \frac{I_2(y_1, y_2)}{I_1(y_1)I_1(y_2)} - 1 = \frac{\mathrm{Var}(\kappa)}{\langle\kappa\rangle^2} \ . \tag{28}$$

In this formula, the n-particle inclusive rapidity spectrum is denoted by $I_n(y_1, ..., y_n)$. We have assumed that κ is large enough to replace $\tanh(\pi\kappa)$ by unity. Notice, that this correlation is rapidity independent (on the plateau). In view of (27), its value measures the variance of the initial energy density, since

$$\mathrm{Var}(\kappa) \approx \frac{s_0}{8f_\pi^2\langle\epsilon_0\rangle}\mathrm{Var}(\epsilon_0) \ . \tag{29}$$

This result is similar to that obtained in the framework of the conventional hadron theory, viz. in the Reggeon calculus [6], where the long-range rapidity correlation is a measure of the variance of the number of radiating "strings."

In the present approximation the short-range rapidity correlations do not appear. At least part of these short-range correlations is due to resonance production. However, terms involving pion resonances are of higher order in the expansion of the effective Lagrangian and have been neglected. Also, the discussion of the Bose-Einstein correlations is beyond the scope of this paper; that would require a more detailed description of the initial state.

C. Formation time

As the proper time of a volume element increases, the energy density ϵ given by (22) decreases to zero. For some value of s this energy density becomes comparable to the mass of a pion divided by the cube of its Compton wavelength. In this manner one obtains an estimate of the proper formation time τ:

$$\tau \approx \frac{\sqrt{2}f_\pi\kappa}{m_\pi^2} \ . \tag{30}$$

A numerically similar estimate is obtained comparing Eq. (15) to the Klein-Gordon equation written assuming that the solution depends on s only. One then finds $m_\pi^2/4$ in the place of κ^2/s. Clearly, it does not make much sense to talk about free-pion propagation until the latter quantity becomes smaller than the former one. Hence

$$\tau \approx \frac{2\kappa}{m_\pi} \ . \tag{31}$$

In $1+1$ dimensions the picture is self-consistent. It is not quite obvious how to carry these estimates to the $(1+3)$-dimensional world. It seems reasonable to take Eq. (31) and to replace the pion mass which appears there by the pion's transverse mass.

Note that in both estimates $\tau \propto \kappa$, that is the formation time grows with multiplicity and can indeed become quite large (see Sec. IV F). On the other hand, τ cannot be too small, so that κ is bounded from below. For example, taking $1/m_\pi$ as the minimum value for τ, one gets from (31) $\kappa > \frac{1}{2}$.

D. Neutral-to-charged ratio

Assume that there is no privileged direction in isospace. Although our classical solution of the field equations corresponds to a specific choice of the isovectors $\mathbf{a}, \mathbf{b}$, all the orientations of these vectors compatible with the constraint $\mathbf{a}\cdot\mathbf{b} = 0$ are *a priori* equally probable. We have argued that these isovectors characterize the state of the initial dense system. Therefore, we expect that the neutral/charged particle ratio fluctuates strongly from event to event. Since all isospace orientations of the pion field are equally likely, the probability that neutral pions constitute a fraction r of all soft pions produced in the collision equals the probability that a random vector in isospace has its third component equal to $\sqrt{r}$:

$$dP(r) = \frac{1}{2\sqrt{r}}dr \ . \tag{32}$$

Such large fluctuations of the neutral/charged ratio have also been predicted in Ref. [3].

Unfortunately, the nice prediction (32) can be put in jeopardy by the following argument: the full translational symmetry in transverse coordinates is a far-reaching idealization. It is likely that regions of the initial dense system that are sufficiently well separated in the transverse direction do not act coherently. An excess of π^0's emitted in one place could then be compensated by a deficit of π^0's emitted elsewhere. With such a scenario, extra model assumptions are needed, in particular concerning the dependence of $\rho(\mathbf{a}, \mathbf{b})$ on transverse coordinates, to get a quantitative prediction. This problem deserves further study.

E. Relation to the hydrodynamical model

As already mentioned, Eq. (21) describes a (1 + 1)-dimensional fluid. As a direct consequence of the idealization made at the very beginning, where we assumed following Heisenberg that the problem is one dimensional, there is no transverse pressure whatsoever and the energy density is equal to a pressure involving longitudinal motion only. There is a basic difference between the approach advocated here (and earlier by Heisenberg) and the hydrodynamical model: although we introduce a probability density to describe the initial dense system, for given **a** and **b** the propagation is coherent and controlled by the equations of motion. In Landau's hydrodynamical model [7] the system is assumed to be in local equilibrium during the whole expansion process. That means that, in a local rest frame, the distribution of particle momenta is isotropic, so that, for massless particles, the pressure and the energy density are related by $p = \epsilon/3$. It results then from the equations of motion, $\partial_\mu T^{\mu\nu} = 0$, that the energy density goes as $s^{-2/3}$, to be contrasted with the s^{-1} dependence in Eq. (22). These time dependences ($s = t^2$ when $x = 0$) are easy to understand. In either case, the system is undergoing a uniform expansion, so that a covolume element centered at $x = 0$ grows as t. In hydrodynamics, the total energy contained in the covolume decreases because of the work the covolume has to do against its neighbors in order to expand. In the case of the classical pion field, one may understand the decrease of the energy as a result of the decrease of the effective energy of each of the normal modes in the expanding covolume [see Eq. (15)]. One can also understand in this way the origin of the various powers of κ in the expression of the energy density, Eq. (22), and the particle multiplicity, Eq. (26). The number of quanta in a given covolume is constant and proportional to κ, as expressed by Eq. (26). The energy in the covolume is proportional to the number of quanta, and to the energy ($\propto \kappa$) of each quantum, hence the factor κ^2 in Eq. (22).

F. Some numerical estimates

The spectrum given by (26), and derived in a (1 + 1)-dimensional model for soft pions, should be interpreted as the rapidity spectrum after integration over transverse momentum. But the latter should also be soft, in the sense of this paper: $k_\perp \leq \lambda$, with λ of order f_π. Thus, comparing (26) to actual data one should multiply the observed plateau height by a correction factor ζ which, assuming a Gaussian shape of the $k_\perp$ distribution, is given by

$$\zeta = 1 - \exp[-(\lambda^2/\langle k_\perp^2\rangle)] \approx (\lambda/\langle k_\perp\rangle)^2 \ . \tag{33}$$

This is a phenomenological artifice, which will become unnecessary once the proper extension of this theory to 1 + 3 dimensions is worked out. Unfortunately, for the time being, our predictive power is poor since ζ is quite sensitive to the exact value of the cutoff λ.

Setting $S = \pi R^2$ and $R = 1.2A^{1/3}$ fm we obtain, from (26),

$$\kappa \approx \frac{\zeta}{A^{\frac{2}{3}}}\frac{dN}{dy} \ . \tag{34}$$

Strictly speaking, the quantity given above is $\langle\kappa\rangle$, but we write just κ for the sake of simplicity. Thus, κ varies like A^α, with α presumably in the range $\frac{1}{3}$ to $\frac{2}{3}$. So does the formation time τ. The initial energy density varies like $A^{2\alpha}$.

Consider now some data. Central interactions of ^{16}O with ^{197}Au at 200 GeV/nucleon have been studied by the WA80 Collaboration [8]. They observe about 100 charged secondaries per unit rapidity in the central rapidity region. In a different publication [9] the same collaboration finds that the transverse-energy pseudorapidity distribution is larger by a factor 1.5 when a ^{32}S projectile is used instead of ^{16}O. Hence, the rapidity density of all pions produced in collisions of ^{32}S with ^{197}Au at 200 GeV/nucleon should be roughly 225 .

The cutoff λ should be somewhere between f_π and $\langle k_\perp\rangle$ (cf. the discussion in Sec. IV A). Assuming tentatively that $\lambda = 150$ MeV, halfway between the above limits, we get $\zeta \approx \frac{1}{4}$ and $\kappa \approx 5.6$. This yields the estimate $\epsilon_0 \approx 2.8$ GeV/fm^3. This estimate is close to that obtained with Bjorken's formula. One should remember, however, that the qualitative trend is different, since we expect a quadratic dependence of ϵ_0 on $A^{-\frac{2}{3}}\,dN/dy$, while in Bjorken's formula this relation is linear. Finally, taking a value of 300 MeV for the transverse mass, and using this in Eq. (31), one gets $\tau \approx 8$ fm/c. This large value reflects the coherence of the source. It also points to a limitation of the model in its present form: since this time is larger than the tranverse size of the system under consideration, i.e. ^{32}S, clearly a better description of the transverse degrees of freedom is called for.

V. CONCLUSION

Our results are summarized in the subsections of the preceding section and need not be repeated here. Hence, let us end with a few general remarks.

Inspired by Heisenberg's approach to multiparticle production we have attempted to apply it in the context of soft-pion emission in heavy-ion collisions. In this case the effective theory for the pion field can be regarded as an approximation to QCD (because the pions are soft) and the classical approximation may not be unreasonable (in view of the large number of emitted quanta). We work with Heisenberg's idealized boundary conditions, reducing in this way the problem to 1 + 1 dimensions. Similar conditions have been used in the context of nuclear collisions many times, in particular in hydrodynamical calculations. This enables us to proceed analytically.

Among the problems for the future, let us mention corrections coming from the next terms in the effective Lagrangian (those with up to four derivatives) and the transverse flow. The difficulties are non-negligible, but mostly technical. On the other hand, the theory of Bose-Einstein correlations seems to present a conceptual challenge. It appears that the description of incoherent phenomena demands a better understanding of the physics governing the behavior of the initial dense system.

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