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# Cosmological consequences of a time-dependent Λ term

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The phenomenological approach to investigate the decay of the effective cosmological constant, as recently proposed by Chen and Wu, is generalized to include a term proportional to $H^2$ on the time dependence of Λ, where $H$ is the Hubble parameter. This new term can modify some features of the standard Friedmann-Robertson-Walker model and its free parameter may be adjusted in accordance with nucleosynthesis constraints. The model also allows a deceleration parameter $q_0$ assuming negative values so that the density parameter $\Omega_0$ is smaller than $\frac{2}{3}$ and the age of the Universe is always bigger than $H_0^{-1}$. In these cases, the usual matter creation rate appearing in models with a decaying vacuum energy is smaller than the one present in the steady-state model.

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## I. INTRODUCTION

The smallness of the effective cosmological constant observed today ($\Lambda_0 \leq 10^{-56}$ cm$^{-2}$) is one of the most difficult problems involving cosmology and elementary particle physics theory. In order to explain the striking cancellation between the "bare" cosmological constant and the ordinary vacuum energy contributions of the quantum fields, several mechanisms have been proposed in the last few years [1]. Phenomenologically, the simplest of them is to assume that the effective cosmological "constant" is a variable dynamic degree of freedom so that in an expanding universe it relaxes to its present value [2–6]. In other words, the "effective cosmological constant" is quite small today because the Universe is too old. From a macroscopic point of view, the problem reduces to determine the right dependence of Λ on the universal scale factor $R$ and eventually in its first derivatives, taking into account the proper cosmological constraints.

Recently, Chen and Wu argued in favor of an $R^{-2}$ dependence of Λ, based on a dimensional argument in line with quantum cosmology [6]. They also showed that such a behavior may alleviate some problems in reconciling the observational data with the inflationary scenario. However, it is easy to see that the "ansatz" of the authors does not fix $\Lambda=\alpha R^{-2}$, $\alpha$ constant, as the only possible decaying law. For example, one may assume, for the sake of simplicity,

$$\Lambda \propto \frac{1}{l_{\mathrm{Pl}}^2}\left[\frac{t_{\mathrm{Pl}}}{t_H}\right]^n$$

where $l_{\mathrm{Pl}}$ and $t_{\mathrm{Pl}}$ are respectively the Planck length and time, $n$ is an integer number, and $t_H \simeq H^{-1}$ is the Hubble time. So, recalling that general relativity is a classical theory, in order to get rid of the $\hbar$ dependence of Λ one needs to put $n=2$. Thus, since the Hubble parameter $H$ is given by $\dot{R}/R$, there is also the possibility of Λ scaling with $(\dot{R}/R)^2$. As will be seen later, this new term can play a fundamental role in solving some cosmological puzzles. In what follows, we extend the cosmological scenario discussed in [6] by considering a more general Λ term of the form

$$\Lambda = 3\beta\left[\frac{\dot{R}}{R}\right]^2 + \frac{3\alpha}{R^2}, \tag{1}$$

where $\alpha$ and $\beta$ are dimensionless numbers of the order of unity, with the factor 3 being introduced for mathematical convenience.

## II. THE FIELD EQUATIONS

Let us consider the Friedmann-Robertson-Walker (FRW) line element ($c=1$)

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right], \tag{2}$$

where $k=0,\pm1$ is the curvature parameter. For a comoving perfect fluid, the nontrivial Einstein's field equations with a Λ term given by (1) can be cast in the form

$$\frac{8\pi G\rho}{3} = (1-\beta)\frac{\dot{R}^2}{R^2} + \frac{k-\alpha}{R^2}, \tag{3}$$

$$8\pi G p = -2\frac{\ddot{R}}{R} - (1-3\beta)\frac{\dot{R}^2}{R^2} - \frac{k-3\alpha}{R^2}, \tag{4}$$

where $\rho$ and $p$ are respectively the energy density and pressure of the cosmic fluid.

By considering the "$\gamma$-law" equation of state $p=(\gamma-1)\rho$, it is easy to see that the scale factor is governed by the second-order differential equation

$$R\ddot{R} + \Delta_1\dot{R}^2 + \Delta_2 = 0, \tag{5}$$

the first integral of which is

$$\dot{R}^2 = AR^{-2\Delta_1} - \frac{\Delta_2}{\Delta_1} \quad (\Delta_1 \neq 0) \tag{6a}$$

and

$$\dot{R}^2 = A - 2\Delta_2 \ln R \quad (\Delta_1 = 0) . \tag{6b}$$

Here,

$$\Delta_1 = \frac{3\gamma(1-\beta)-2}{2}, \quad \Delta_2 = \frac{3\gamma(k-\alpha)-2k}{2}$$

and $A(\gamma)$ is an arbitrary constant. Using Eqs. (1), (3), and (6) one obtains for the radiation- ($\gamma = \frac{4}{3}$) (RD) and matter- ($\gamma = 1$) dominated (MD) phases the following results:

(i) RD phase ($p = \frac{1}{3}\rho$, $\beta \neq \frac{1}{2}$)

$$\dot{R}^2 = A_1 R^{-2+4\beta} + \frac{2\alpha - k}{1-2\beta} , \tag{7}$$

$$\frac{8\pi G \rho_r}{3} = (1-\beta) A_1 R^{-4+4\beta} + \frac{\alpha - \beta k}{1-2\beta} R^{-2} , \tag{8}$$

$$\frac{8\pi G \rho_v}{3} = \beta A_1 R^{-4+4\beta} + \frac{\alpha - \beta k}{1-2\beta} R^{-2} . \tag{9}$$

(ii) MD phase ($p = 0$, $\beta \neq \frac{1}{3}$)

$$\dot{R}^2 = A_2 R^{-1+3\beta} + \frac{3\alpha - k}{1-3\beta} , \tag{10}$$

$$\frac{8\pi G \rho_m}{3} = (1-\beta) A_2 R^{-3+3\beta} + \frac{2\alpha - 2\beta k}{1-3\beta} R^{-2} , \tag{11}$$

$$\frac{8\pi G \rho_v}{3} = \beta A_2 R^{-3+3\beta} + \frac{\alpha - \beta k}{1-3\beta} R^{-2} . \tag{12}$$

In the above expressions it was implicitly assumed that the vacuum couples only with the dominant component in each phase.

If $\beta = 0$ and the constants $A_1$, $A_2$, and $\alpha$ are taken greater than zero, the cosmological scenario obtained by Chen and Wu [6] is recovered. If $\beta = 0$, $A_1 < 0$, and $\alpha = k = 1$, the above equations reproduce the nonsingular model of Ozer and Taha [2]. For $\alpha = 0$ and $k = 0$, one can see that the parameter defined for any epoch $x = \rho_v / (\rho + \rho_v)$ is equal to the constant $\beta$. This case comprises a concrete example of the scenario discussed by Freese *et al.* [3].

## III. SOME COSMOLOGICAL CONSEQUENCES

We now examine the consequences of the $\beta$ term upon cosmic evolution. In general, the physical features of the model are strongly dependent on the values of the parameters $\alpha$ and $\beta$ as well as on the sign of the integration constants $A_1$ and $A_2$. As in Ref. [6], the constant $\alpha$ plays the role of a curvature parameter with a reversed sign. However, because of the existence of the $\beta$ term, two new effects are present: (i) unlike in the Chen and Wu paper, if $\beta$ is large enough, the curvature contribution from the radiation energy density cannot be neglected in early times [see Eq. (8)]; (ii) the $\beta$ parameter also contributes, on one hand, to increase the age of the Universe with respect to the values computed in the standard model and, on the other hand, to diminish the particle production rate as compared with the result obtained by Chen and Wu. We shall see later that $\beta$ can also be held in accordance with the nucleosynthesis constraints.

From now on we suppose $A_1 > 0$ and $\beta < 1$ so that the cosmic scenario starts from a singularity as in the standard model. In order to compute the age $t_0$ of the Universe one must integrate the first integral of $R$ for the matter-dominated phase [Eq. (10)]. In this case, defining the present time quantities

$$q_0 = -\left[\frac{R\ddot{R}}{\dot{R}^2}\right]_{t_0} \text{ and } H_0 = \left[\frac{\dot{R}}{R}\right]_{t_0} ,$$

it is straightforward to obtain $t_0$ from Eqs. (5) and (10):

$$t_0 = H_0^{-1} \int_0^1 \left[1 - \frac{2q_0}{1-3\beta} + \frac{2q_0}{1-3\beta} \frac{1}{x^{1-3\beta}}\right]^{-1/2} dx . \tag{13}$$

Note that if $\beta = 0$ the formal result of the standard model is recovered [7]. We observe that, since the deceleration parameter obeys the relation

$$\frac{k-3\alpha}{(1-3\beta)R_0^2} = \left[\frac{2q_0}{1-3\beta} - 1\right] H_0^2 , \tag{14}$$

$\alpha$ also affects, although indirectly, the value of the age. However, for $\beta = 0$, the age continues restricted to the interval $\frac{2}{3} \leq H_0 t_0 \leq 1$, as remarked by Chen and Wu. An important effect of the $\alpha$ parameter is to make a flat model ($k = 0$) simulate the dynamic behavior of an open FRW model with the age approaching the maximum value $H_0^{-1}$.

In our model if, for instance, $k = 3\alpha$, then $2q_0 = 1 - 3\beta$ and from the field equations we obtain

$$R = R_0 \left[\frac{2}{3(1-\beta)} H_0 t\right]^{2/3(1-\beta)} .$$

Thus, the age obtained is

$$t_0 = \frac{2}{3(1-\beta)} H_0^{-1} ,$$

as one can check from (13). Observe that if $\frac{1}{3} \leq \beta \leq \frac{1}{2}$ it follows that $1 \leq H_0 t_0 \leq \frac{4}{3}$ in accordance with the observational limits ($0.6 \leq H_0 t_0 \leq 1.4$) claimed by several authors [8–10]. Since $\alpha$ may assume negative, null, or positive values, such a result holds for open, flat, or closed models obeying the constraint $k = 3\alpha$. In general, the age of the Universe will depend on the values taken by the parameters $\beta$ and $q_0$, which for $k = 0$ will fix the sign of $\alpha$. The behavior of $H_0 t_0$ as a function of $\beta$ and $q_0$ is shown in Fig. 1 for three values of $\beta$.

The dynamical critical behavior of the models with respect to a recollapse in the future is now established by the effective curvature parameter $\bar{k} = (k - 3\alpha)/(1 - 3\beta)$. If $\bar{k} > 0$, a recollapse will happen regardless of the sign of the spatial curvature parameter $k$.

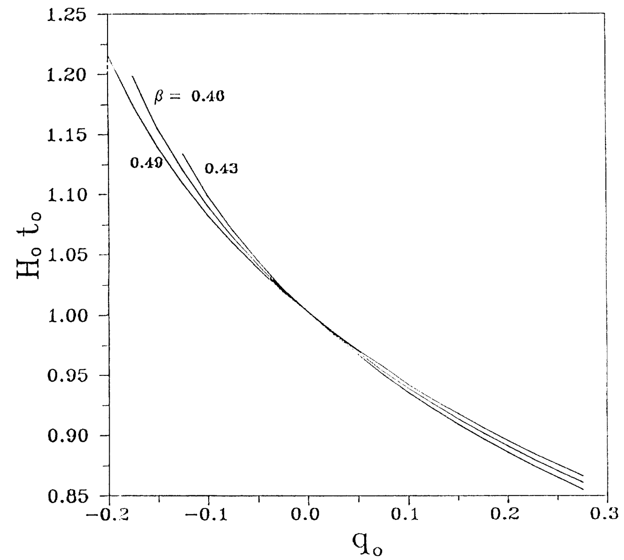


FIG. 1. The age of the Universe as a function of $q_0$ calculated from Eq. (13). The curves are for three values of $\beta$ as indicated.

Defining, as usual, the critical density by $\rho_c = 3H^2/8\pi G$ and the density parameter by $\Omega = \rho/\rho_c$, one can deduce for the present values of the cosmological constant and the density parameter the expressions

$$\frac{\Lambda_0}{H_0^2} = 1 - 2q_0 + \frac{k}{R_0^2 H_0^2} , \tag{15}$$

$$\Omega_0 = 1 + \frac{k}{R_0^2 H_0^2} - \frac{\Lambda_0}{3H_0^2} . \tag{16}$$

Thus, for $k=0$ (in agreement with inflation) one obtains $\Omega_0 = \frac{2}{3}q_0 + \frac{2}{3}$. Note that this expression holds regardless of the time dependence of $\Lambda$. Moreover, $\Omega_0 < \frac{2}{3}$ is possible only if $q_0 < 0$. We remark that dynamical estimates suggest $\Omega_0 = 0.2 \pm 0.1$ [11] and a negative value of $q_0$ is not ruled out by the observations [12,13]. However, the model of Chen and Wu does not comprise $q_0 < 0$, and this explains why it only alleviates the density parameter problem. In our scenario, regardless of the value of $k$, we have along the MD phase that $q_0 H_0^2 = [(1-3\beta)/2] A_2 R_0^{3\beta - 1}$. Thus, if $\beta > \frac{1}{3}$ and $A_2 > 0$, one has $q_0 < 0$ and it is possible to accommodate a low energy density universe.

As remarked before, the agreement between our models and the nucleosynthesis predictions can put more definite limits on the parameters $\alpha$ and $\beta$. According to Freese *et al.* [3], element abundances from primordial nucleosynthesis require the ratio $x = \rho_v/(\rho_r + \rho_v) \leq 0.1$ during nucleosynthesis. In fact, this upper bound for $x$ was established by considering $x =$const during the RD phase. Such a ratio for our spatially flat models ($k=0$) is given by

$$x = \beta + \frac{\alpha}{R^2 H^2} , \tag{17}$$

which is explicitly time dependent. Note that only if $\alpha = 0$ or, more generally, if $\alpha/R^2H^2 \ll \beta$ at the time of nucleosynthesis, does one obtain the stringent limit $\beta \leq 0.1$. In this case the solution of the age problem is rather prejudiced since $t_0$ will only increase about 10% with respect to the values computed in the standard model. We remark that for larger values of $\beta$, as for example, $\beta$ in the interval $(\frac{1}{3}, \frac{1}{2})$, the $\alpha$ term may be important for the radiation energy during the nucleosynthesis epoch. To show this, we now roughly estimate the ratio $y = 8\pi G \rho_r / |\alpha| R^{-2}$ at the nucleosynthesis, where $\rho_r \simeq \rho_0 (R_0/R)^{4-4\beta}$ with $\rho_0 = (\pi^2/15) T_0^4$ being the radiation energy today and $R_0 \simeq 5 \times 10^{41}$ GeV$^{-1}$. By substituting $G \simeq 6.7 \times 10^{-39}$ GeV$^{-2}$ and using the fact that $TR^{1-\beta}$ is nearly constant it follows that

$$|\alpha| y \simeq 2.2 \times 10^{-4} [T_N/T_0]^{(2-4\beta)/(1-\beta)} ,$$

where $T_N$ stands for the nucleosynthesis temperature. Thus, for $T_N \simeq 10^{-3}$ GeV, $T_0 \simeq 3 \times 10^{-13}$ GeV, and $\beta \simeq 0.43$ one obtains $y \simeq 10|\alpha|^{-1}$. This shows that, in fact, the $\alpha$ term can be important at the nucleosynthesis epoch. So, if $\alpha$ is negative the nucleosynthesis constraints can in principle be satisfied for a larger value of $\beta$ increasing, consequently, the age $t_0$.

Models with a time varying $\Lambda$ are usually endowed with matter creation at the expense of the vacuum energy decay. From conservation of the total energy-momentum tensor one has a "balance equation" for the material component:

$$\dot{\rho} + 3H(\rho + p) = -\frac{\dot{\Lambda}}{8\pi G} . \tag{18}$$

In the present case, as a function of $\Omega_0$, the above expression takes the form

$$\frac{1}{R_0^3} \left[ \frac{d}{dt}(\rho R^3) \right]_0 = \frac{2(1-\Omega_0)}{\Omega_0} \rho_0 H_0 \left[ 1 + \beta \frac{3\Omega_0 - 2}{2(1-\Omega_0)} \right] .$$

If $\beta = 0$ this equation reduces to the result obtained in Ref. [6] properly rewritten in terms of $\Omega_0$. We see that if $\Omega_0 < \frac{2}{3}$, the $\beta$ term contributes effectively to diminish the rate of matter creation. In particular, if $\Omega_0 \approx 0.4$ it follows that

$$\frac{1}{R_0^3} \left[ \frac{d}{dt}(\rho R^3) \right]_0 = 3\rho_0 H_0 [1 - \tfrac{2}{3}\beta] ,$$

which is smaller than the creation rate $3\rho_0 H_0$ of the steady-state model [7,14].

## IV. CONCLUSIONS

Finally we observe that, although many authors have argued against the introduction of a nonvanishing cosmological constant, there is some observational evidence supporting this hypothesis [15–17]. In fact, if more accurate observational data confirm $\Omega_0 \simeq 0.1-0.3$ and $H_0 t_0 \geq 0.6$, a $\Lambda \neq 0$ will be required to save a $k=0$ model as predicted by inflation. However, since the observed value of $\Lambda$ is unusually small, although capable of solving the low energy density problem, we have introduced a more attractive scenario in which, as the Universe evolves, the effective cosmological constant decreases toward its natural value $\Lambda=0$. We explored this hypothesis by assuming a $\Lambda$ term varying as $\alpha R^{-2}+\beta H^2$. This behavior is justified by using the same kind of simple and general arguments used by Chen and Wu. We have discussed how these new terms modify some features of both the standard FRW and Chen-Wu models, especially in regard to the age and the low energy density problems.

We have assumed throughout this paper that the vacuum couples to radiation and matter in the same way; that is, $\alpha$ and $\beta$ are both the same value during the radiation- and matter-dominated epochs. If we relax this hypothesis it will be much easier to satisfy the nucleosynthesis constraints and to solve the age and the density problems in a $k=0$ universe. A natural extension of this work would be to explore different scenarios obtained by making $A_1<0$ and for which the models are nonsingular. Some other important aspects of the model, such as, for example, the classical cosmological tests and the growth of density perturbations, were not analyzed here. Further investigations are going in this direction.

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