Approaching the quantum limit with optically instrumented multimode gravitational-wave bar detectors

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A laser-illuminated Fabry-Perot cavity with a finesse of 83000 and power dissipation of a few μW is tested at room temperature. The sensitivity to changes in the length of the cavity is 5.6×10^{-17} m/$\sqrt{\text{Hz}}$. The sensitivity of multimode gravitational-wave bar detectors operated between 0.03 and 4.2 K and instrumented with a Fabry-Perot transducer of similar characteristics is calculated. The sensitivity projected for a three-mode 2400-kg Al 5056 antenna operating at 800 Hz and cooled to 0.03 K is $\delta h = 6.8\times10^{-21}$ and the noise temperature of the system is a factor of 16 above the standard quantum limit. Such a sensitivity would permit observations of astrophysical significance.

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I. INTRODUCTION

Observations of the spin down of PSR 1913+16 by Taylor and his colleagues have now been carried out for many years [1,2]. They make a very convincing case for the emission of gravitational radiation in the amount predicted by the theory of general relativity in its weak-field approximation. The production of gravitational waves in the strong-field case has been extensively studied and is thought to be well understood. The detection of such waves could give us new information about the Universe which may be inaccessible in any other way. At present, the largest signals are likely to be generated in violent astrophysical events lasting a few milliseconds such as collapses of stellar cores and collisions of neutron stars or black holes. At the surface of the Earth, signal amplitudes in h at the level of 10^{-18} may be produced by events occurring in our Galaxy [3,4]. Such events may be rare. Signal amplitudes of the order of 10^{-20} to 10^{-22} may be produced more frequently by sources further away [5,6]. In this paper, general remarks are made on bar detectors and multimode transducers in Secs. II and III. In Sec. IV, previous results related to the development of a cryogenic optical transducer are reviewed briefly. In Sec. V, room-temperature measurements of the sensitivity to changes in length of a high-finesse 820-μm Fabry-Perot cavity are reported. In Secs. VI and VII, projections are made for the sensitivity of three-, four-, and five-mode gravitational-wave bar detectors instrumented with a Fabry-Perot transducer and operated between 0.03 and 4.2 K.

II. SENSITIVITY AND BANDWIDTH OF BAR DETECTORS

The sensitivity of a resonant bar antenna to short pulses of gravitational radiation is conveniently expressed in terms of its noise temperature T_p in the following way [7]:

$$\delta h = \frac{\pi}{4}\left[\frac{2k_B T_p}{m_1\omega_0^2 L^2}\right]^{1/2}, \tag{1}$$

where k_B is Boltzmann's constant, m_1 and L are the dynamic mass and the length of the bar antenna and $\omega_0 = 2\pi f_0$ where f_0 is the frequency of the fundamental longitudinal mode of the antenna. For a cylindrical antenna, the operating frequency is determined by the length of the antenna and the speed of sound. The sensitivity can be improved by increasing the mass and reducing T_p. T_p is itself limited to a value of the order of T_n where T_n is the noise temperature of the transducer system. Without back-action evasion, T_n has a "standard" quantum limited value of $T_{\rm QL} = \hbar\omega_0/k_B \ln 2$. At 800 Hz, $T_{\rm QL} = 5.5\times10^{-8}$ K and the sensitivity of a 2400-kg 3-m-long Al bar such as is currently in use could reach $\delta h \cong 2.3\times10^{-21}$. Such a sensitivity would be in the range of interest indicated above. A bar or a spherical [8] antenna with larger cross section would permit a higher sensitivity.

Multimode detectors have been proposed to achieve high sensitivity and provide increased bandwidth [9,10]. They consist of n coupled resonant harmonic oscillators with decreasing effective masses. The largest of these is the bar antenna providing the coupling to the gravitational field. The mass of the last resonator is chosen to optimize the operation of the transducer selected. In a multimode system, the contribution $T_{p\rm th}$ to the noise temperature from the thermal noise originating in a bar antenna and the mechanical resonators coupled to it is given by [11]

$$T_{p\rm th} = \frac{\pi f_0 T_d}{\Delta f}\left[\frac{1}{Q_1} + \sum_{i=2}^{n}\frac{Y_i}{Q_i}\right] \tag{2}$$

where T_d is the detector temperature, Δf is the detector effective bandwidth. Q_1 is the bar mechanical quality factor, Q_i is the quality factor of the ith resonator and Y_i is the transfer function from the ith resonator to the bar antenna integrated over the bandwidth Δf. In a multimode system where the ratio of two successive masses is a constant $\mu = m_{i+1}/m_i$, a useful limit on Δf is

$$\Delta f \cong 2f_0\sqrt{\mu}\ . \tag{3}$$

Large bandwidths are possible with multimode detectors since μ can be made arbitrarily close to unity by increasing the number of resonators. More specifically, bandwidths of the order of 40% of the detector center frequency could be achieved providing for a frequency coverage from 500 to 2000 Hz with a few detectors [10].

To illustrate the level of sensitivity which can be achieved, we can consider a case where the resonators are made of a material with a Q higher than the Q of the antenna and the transducer noise is less than the antenna thermal noise. Assuming $Q_1=10^7$ at 800 Hz for a 2400-kg Al5056 bar antenna at 4.2 K, and a μ ratio corresponding to a bandwidth of 150 Hz, then $T_p \cong T_{p\text{th}}=7.0$ μK. The corresponding sensitivity in h is $\delta h=2.1\times10^{-20}$. This is a high sensitivity which, in principle, can be reached without cooling to dilution refrigerator temperatures. However, transducers have to be improved before such sensitivities can be reached.

III. MULTIMODE DETECTORS AND TRANSDUCERS

To the present time, two transducers schemes have been particularly successful: the inductance modulation transducer used at Stanford [12,13] and Louisiana State University (LSU) [14] and the resonant capacitor transducer initially developed at Maryland [15,16] and further developed for the Rome antenna at CERN [17]. Both transducers are coupled to a superconducting quantum interference device (SQUID). With both these transducers, it has been found useful to use tuned mechanical resonators attached to the end of the bar antenna to improve matching between the bar and the SQUID and reduce the relative importance of the SQUID wideband noise. Two-mode systems have been used up to now. Three-mode systems are being considered for the Stanford [18] and LSU [19] detectors with projected sensitivity in h beyond 10^{-18}. Three- and five-mode systems have been tested at Maryland [11,20].

As a different approach to improve sensitivity, we proposed recently to instrument the multimode detector with a Fabry-Perot transducer [21,22]. In principle, the optical transducer can reach the standard quantum limit. If such a transducer is incorporated in a three or more mode detector with last resonator mass of the order of one gram, the optical power absorbed by the mirrors can be limited to a few μW.

IV. THE FABRY-PEROT TRANSDUCER

In a multimode bar detector instrumented with an optical transducer, two mirrors are used. One is a "reference" mirror attached to the bar or to one of the other resonators. The second "moving" mirror is attached to the last resonator. As shown schematically in Fig. 1, a rf phase modulated laser beam, stabilized in power and frequency illuminates the Fabry-Perot transducer. The demodulated rf signal drives a feedback system which maintains the length of the cavity at the resonance condition with capacitance forces. The signal at the bar frequency is extracted from the demodulated rf signal. A low-finesse ($F=117$) 14-mm-long prototype Fabry-Perot transducer incorporating feedback stabilization of the cavity length has been tested on an optical table at room temperatures. In that test, a sensitivity of 3.7×10^{-15} m/$\sqrt{\text{Hz}}$ was measured [23].

In order to minimize the optical power dissipated in

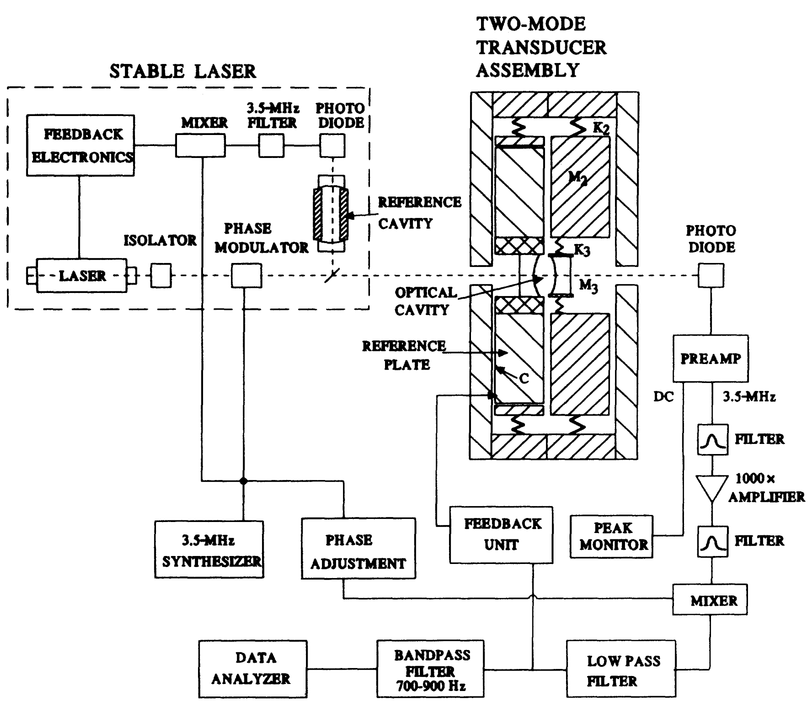

FIG. 1. Layout for the optical transducer.

the optics, it is essential to use a Fabry-Perot transducer with the highest finesse at the cryogenic temperatures envisioned. In an earlier experiment [24] with "superpolished" and "supercoated" mirrors, the finesse was measured at 24 000 at room temperature and remained constant at 19 000 from approximately 80 down to 4.2 K. The initial room-temperature finesse was recovered at warm-up and no damage to the optics could be detected. This experiment suggests the highest finesse achieved at room temperature is likely to suffer only modest degradation at cryogenic temperatures.

V. FLUCTUATIONS IN A HIGH-FINESSE FABRY-PEROT CAVITY

In this section, we report measurements performed on the output in transmission of a high-finesse "rigid" cavity to determine the level of optical wideband noise in a Fabry-Perot transducer. In addition, the back-action force associated with such a transducer is calculated. These permit a projection of the transducer noise temperature and performance on an actual bar detector. In the experiment, the optical cavity was operated at room temperature. The bandwidth of interest was centered at 1660 Hz corresponding to the lowest fundamental longitudinal mode of a 1200-*kg* 1.5-m-long Al bar antenna. The measurement bandwidth was 1000 Hz. First, a few definitions are recalled.

A cavity supports resonant optical Gaussian modes of the type TEM_{ijk}. The indices i and j denote different spatial modes and are related to the number of intensity nodes in the transverse directions. The longitudinal mode index k is essentially one less than the number of half-wavelengths in standing between the cavity mirrors. The cavity free spectral range (FSR), is the frequency interval between two modes (i,j,k) and $(i,j,k\pm1)$. This is given by $c/2l$ where c is the speed of light and l is the distance between the mirrors. Spatial modes of the same value of $(i+j)$ are degenerate in a cavity with cylindrical symmetry, and the spatial mode spacing (SMS) is the frequency interval between two modes $[(i+j),k]$ and $[(i+j)\pm1,k]$.

The rigid cavity tested is shown in Fig. 2. It holds two

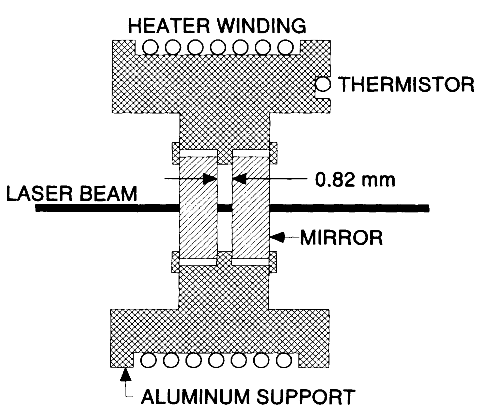

FIG. 2. "Rigid" high-finesse cavity with temperature control.

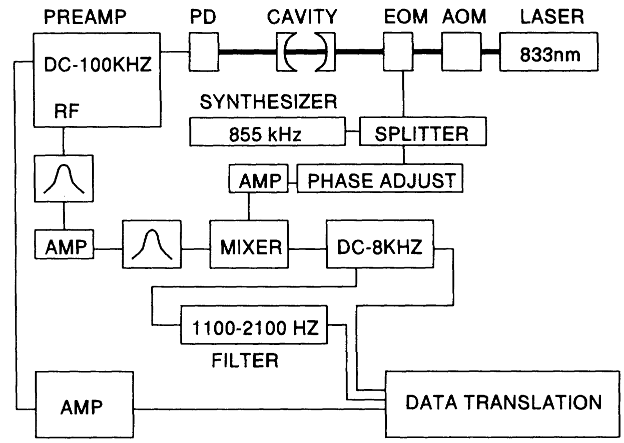

FIG. 3. Electronic layout for the high-finesse cavity.

General Optics/OJAI Research high finesse mirrors, one flat, one with 0.7 m radius of curvature at a distance 820 μm. It has a heater winding which permits changing its temperature and length. The cavity FSR is determined by its length and is 183 GHz. It is covered by a change of temperature which was calculated and observed to be $\cong$24 K. The SMS, calculated from the cavity parameters, was predicted to be 1992 MHz and was observed at 1900 ± 36 MHz.

The setup used for the finesse and noise measurements is shown on Fig. 3. The rf is 855 kHz, an appropriate value to match the cavity bandwidth. The laser used is a Melles-Griot HeNe 1.0-mW model 0.5-LHP-900 which operates at 632.8 nm, corresponding to a frequency [25] $\nu=4.7\times10^{14}$ Hz. The laser free spectral range (LFSR) is 883 MHz. The laser is soft mounted in a sealed aluminum chamber with walls 1.27 cm thick. Sound absorbing material has been added in the chamber. The laser beam exits through an optical window with antireflective coating. A highly regulated power supply from Glassman with appropriate additional resistive load is used to drive the laser. The laser frequency fluctuations are $\cong2.2$ Hz/$\sqrt{\text{Hz}}$ or less and the laser power fluctuations are $1.0\times10^{-6}/\sqrt{\text{Hz}}$ in the frequency band 1100 to 2100 Hz.

The finesse of the cavity was determined by introducing a small offset in the current setting of its heater to produce a slow drift in its length. The dc optical output of the cavity was sampled by a Data Precision model 6100 wave-form analyzer as the temperature and the length of the cavity were changing. Figure 4 shows the cavity SMS and the LFSR in units of time. The known values of SMS and LFSR are used to evaluate the calibrating ratio Hz/s. Figure 5 is an enlarged portion of the same record (revealing a high level of noise at atmospheric pressure). The figure permits evaluating the width of the cavity resonance in units of time and then Hz: $\Gamma=2.2$ MHz and the finesse $F=\text{FSR}/\Gamma=83\,000$. This is the highest finesse we have measured with such mirrors. Its high value may be due to the shortness of the cavity and its small 52-μm spot size.

The total wideband noise was obtained by producing again a very low temperature drift and analyzing the dc

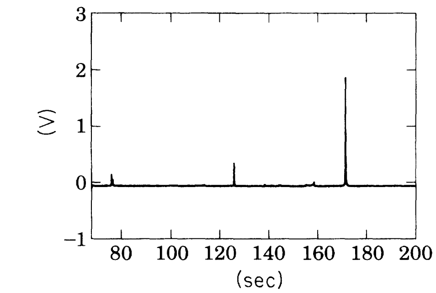

FIG. 4. Finesse measurement: transmitted intensity as the optical cavity expands after heater power offset. The conversion factor is 19.5 MHz/s or 3.4×10^{-11} m/s.

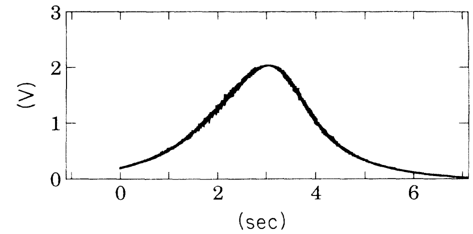

FIG. 6. Noise measurement: dc component of the transmitted intensity. The full width at half height corresponds to 3.8×10^{-12} m. The conversion factor is 0.98 MHz/s or 1.7×10^{-12} m/s.

and rf portions of the optical signal. In a first run, these were recorded as the cavity went through a resonance of its TEM_{00k} mode. These data shown on Figs. 6 and 7 provided the calibrating rate of change in the amplitude of the rf output per unit of length change of the cavity. In addition, many recordings providing approximately 5000 measurements at interval 200 μs were taken very near the peak of the same TEM_{00k} resonance. The recordings were short enough for the gain of the discriminating cavity to be approximately a constant equal to its value at the resonance. In a typical plot of the spectral density of the rf amplitude (Fig. 8), the total wideband noise was at the level of $\sigma_{x\mathrm{WB}}=5.6\times10^{-17}$ m/$\sqrt{\mathrm{Hz}}$. The electronic noise floor measured with the optical beam blocked is at 4.2×10^{-18} m/$\sqrt{\mathrm{Hz}}$ and the noise contributed by the fluctuations in the laser frequency is $l\sigma_\nu/\nu=3.8\times10^{-18}$ m/$\sqrt{\mathrm{Hz}}$ or less. The source of the excess noise above the electronic and laser frequency induced noise has not been determined. Since the noise measurement was performed as the cavity length was changing because of a drifting temperature, it may be that the noise associated with such expansion cannot be neglected at this level of sensitivity and accounts for the relatively small noise observed in this room-temperature experiment.

In the instrumentation of a gravitational-wave bar detector, both the wideband noise discussed in the previous paragraph and the back-action noise force acting on the moving mirror are relevant. For the experiment discussed here, the classical component of the back action is larger than the quantum one. The maximum power output of the cavity was 5 μW. If the transmission and absorption at each mirror are assumed to be equal, the power incident on each mirror was 0.23 W and the noise force on each mirror due to laser power fluctuations at the level of $10^{-6}/\sqrt{\mathrm{Hz}}$ was $F_{\mathrm{BA}}=1.6\times10^{-15}$ N/$\sqrt{\mathrm{Hz}}$. Such a back action force would be less than the thermal noise force acting on a 1-g 1600-Hz resonator for mechanical Q values which satisfy $Q/T_d<2.2\times10^{8}$ and mechanical Q values up to 1.1×10^{7} at 50 mK. In such cases, the thermodynamic temperature of the detector normal modes would not raise significantly above the detector temperature.

An equivalent transducer noise temperature at 800 Hz can be calculated from the wideband noise and the back action noise force assuming values similar to the ones measured at 1100 Hz:

$$T_n=\omega_0\sigma_{x\mathrm{WB}}F_{\mathrm{BA}}/k_B=34\ \mu\mathrm{K}\ . \tag{4}$$

If the actual noise floor for $\sigma_{x\mathrm{WB}}$ is the electronic one at cryogenic temperatures, then the transducer noise temperature would be 2.5 μK. In a 2400-kg 800-Hz multimode Al bar detector optimally matched to and noise limited by a 2.5 μK transducer, the sensitivity in h would be 1.2×10^{-20}. Since optimum matching is difficult to achieve in practical systems, detailed sensitivity projections are derived in the next section.

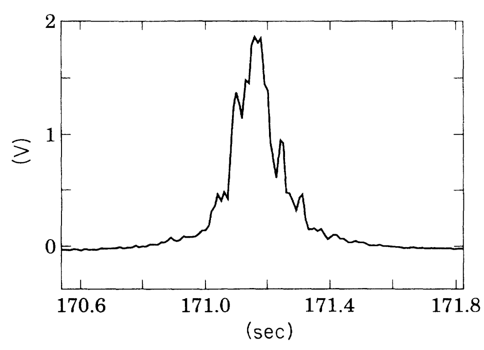

FIG. 5. Finesse measurement: expanded region of the data shown in Fig. 4 showing the fundamental $i=j=0$ mode of the cavity. The full width at half height corresponds to 3.8×10^{-12} m. The conversion factor is 19.5 MHz/s or 3.4×10^{-11} m/s .

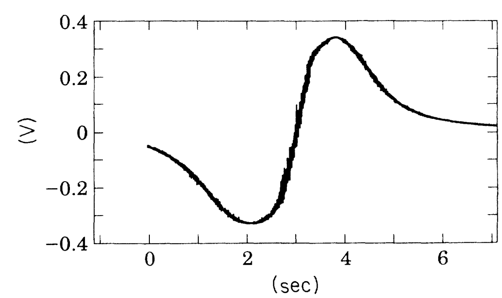

FIG. 7. Noise measurement: rf component of the transmitted intensity. The conversion factor is 0.98 MHz/s or 1.7×10^{-12} m/s.

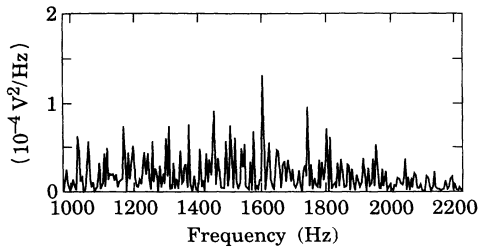

FIG. 8. Noise measurement: Power spectrum of the observed noise.

VI. SENSITIVITY PROJECTIONS FOR A 0.03-K DETECTOR

In this section, we project the noise temperature and the sensitivity of a 30-mK three-mode bar detector instrumented with a Fabry-Perot transducer with parameters very similar to those of the prototype 820-μm cavity discussed above. First, we indicate the parameters assumed for the antenna and the two-mode transducer assembly of the detector incorporating the optical transducer. Next, we discuss the parameters selected for the optical system. Then, the noise of the three-mode detector is analyzed and the sensitivity and bandwidth are evaluated.

A. Parameters assumed for the 3-mode detector

Here, we assume the antenna to be an Al5056 2400-kg cylindrical bar, 3 meters long, with fundamental longitudinal mode at $f_0 = 800$ Hz. It is assumed to be cooled to 30 mK and to have a mechanical Q of 20×10^6. It is instrumented as a three-mode detector with 2nd and 3rd masses at 1.09 kg and 1.0 g respectively and with same Q's as the bar. The third mass of the system includes a 0.4-g supermirror and part of an associated niobium cantilever suspension [26]. An additional mass holding the second (reference) mirror of the optical cavity is of the order of 0.5 kg and is attached to the bar through a low-frequency suspension of adequately high Q [27]. The Q figures selected for this 0.030 K system may be optimistic by a factor of 2 or 3 but are so chosen to better illustrate the potential of the optical transducer.

As was done in a previous experiment with a low finesse transducer, the length of the optical cavity is assumed to be maintained at the resonance condition with a capacitive force which, because of the parameters of the system, can be designed to introduce no significant excess noise [28].

It is planned to maintain the laser power absorbed by the mirrors below or at the level of 2 μW. Such power could be conducted away by a thin (a few microns) coating of copper on the back of the mirror and its suspension. For operation of an antenna at 4.2 K, the optical power required (2 μW or less) could be conducted away by residual helium gas in the bar environment.

B. Parameters of the optical transducer

It is assumed the 0.5-mW Melles-Griot laser discussed above is used. Its frequency stability when operated open loop at frequencies of the order of 1 kHz is 2.2 Hz/$\sqrt{\text{Hz}}$. In feedback operation, some additional noise is introduced for a total of 3 Hz/$\sqrt{\text{Hz}}$. Planned minor improvements to the power supply and feedback unit are expected and assumed to bring the frequency stability to [29] $\sigma_\nu = 2$ Hz/$\sqrt{\text{Hz}}$. Fiber-optic links between the laser system and the transducer are not expected to add significant excess frequency noise [30].

The present fractional stability of the optical power produced by the Melles-Griot laser when powered by a stabilized Glassman power supply is $\sigma_P/P = 1.0\times10^{-6}/\sqrt{\text{Hz}}$. We retain this figure in the present discussion.

In the experiment reported in Sec. V, the length of the cavity was 820 μm. In the projected transducer, the length is reduced to $l = 200$ μm. This will not change in any significant way the spot size on the mirrors and should have a negligible effect on the finesse. The shortening of the cavity in itself will be straightforward.

In three experiments performed at cryogenic temperatures [24,31,32], we have measured values of finesse which were within 10% to 20% of the values measured at room temperature for a finesse of 300, 1700, and 24 000 in cavities 15 cm long. In the prototype 820-μm cavity, we have measured a finesse of 83 000 at room temperature. We assume a finesse $F = 75\,000$ for the 200-μm cavity at 4.2 K and below [33].

The power incident on the photodiode with quantum efficiency 0.8 is assumed to be 2.5 times less than measured with the prototype discussed in Sec. V: $P = 2$ μW. The absorption of the coating is assumed to be of the order of the measured transmissivity $T = 21\times10^{-6}$, a factor of 2 better than the manufacturer's specification. The corresponding power absorbed by each mirror is 2 μW and the maximum power density within the spot is less than endured in the prototype.

The width of the resonance of a 200-μm-long cavity with a finesse $F = 75\,000$ is $\Gamma = 10$ MHz. The optimum phase modulation frequency at a depth of 1.1 rad is of the order of $0.35\times\Gamma$ or 3.5 MHz for a cavity used in transmission [34,35]. We have developed a low-noise preamplifier with noise level of $\sigma_{ia} \cong 1.0$ pA/$\sqrt{\text{Hz}}$ at that frequency. That figure is expected to improve by a factor three after field-effect transistor selection and other minor modifications. Thus, the figure $\sigma_{ia} \cong 0.3$ pA/$\sqrt{\text{Hz}}$ is used for noise projections.

C. Sensitivity projection for a 0.03-K detector

In this section, we use approximate but intuitive formulas to project the performance of the bar detector and optical transducer described in Secs. VI A and VI B. This will be followed by results obtained by numerical integration of expressions based on the optimum filter theory of signal detection. Both results are in close agreement. We will consider the mechanical system thermal noise, the transducer back action and white noise and the total noise of the detector in succession.

For the three-mode detector considered, the ratio of the masses of two successive resonators is $\mu = 9.12\times10^{-4}$. The maximum practical bandwidth for the system is the "bandwidth of the mechanical system" [Eq. (3)]: $\Delta f \cong 2f_0\sqrt{\mu} \cong 48$ Hz. In the absence of transducer noise, the noise temperature for the detection of short pulses of gravitational radiation would be limited by the thermal noise originating in the antenna and the resonators [Eq. (2)]:

$$T_{pth} \cong \frac{nT_d\omega_0}{2Q_1\Delta f} = 2.3\times10^{-7}\ \text{K} . \tag{5}$$

To this number, we must add contributions from the classical and quantum components of the back action of the transducer. The quantum back action is associated with fluctuations in the number of photons traveling back and forth in the cavity and is given by

$$F_{\text{QBA}} = \left[\frac{4\pi\hbar P}{T^2 c\lambda}\right]^{1/2} = 1.8\times10^{-16}\ \text{N}/\sqrt{\text{Hz}} , \tag{6}$$

where $\hbar$ is Planck's constant. The classical back action is associated with the fluctuations in the power of the laser at the signal frequency (800 Hz) which produce fluctuations in the dc pressure on the mirrors of the cavity. The average power on each mirror is $P_m = P/T$ and the dc force is $2P_m/c$. The classical back action is

$$F_{\text{CBA}} = \frac{2P_m}{c}\frac{\sigma_P}{P} = 6.37\times10^{-16}\ \text{N}/\sqrt{\text{Hz}} . \tag{7}$$

The relative importance of the back action components depends on the magnitude of the thermal noise force acting on the last resonator m_r given by

$$F_{\text{th}} = \left[\frac{4k_B T_d\omega_0 m_r}{Q}\right]^{1/2} = 6.45\times10^{-16}\ \text{N}/\sqrt{\text{Hz}} . \tag{8}$$

In this case, the back-action noise is of the order of the thermal noise introduced by one of the n resonators [36]. It will raise the noise temperature by a factor

$$T_{\text{mult}} = 1 + \frac{F_{\text{QBA}}^2 + F_{\text{CBA}}^2}{nF_{\text{th}}^2} = 1.35 \tag{9}$$

above T_{pth}. This brings the noise temperature for the total narrow band noise at $T_{p\text{NB}} = 3.1\times10^{-7}$ K. It is now possible to evaluate the total rms displacement of the last resonator associated with such a noise temperature:

$$x_{\text{NB}} = \left[\frac{k_B T_{p\text{NB}}}{\omega_0^2 m_r}\right]^{1/2} = 1.3\times10^{-17}\ \text{m} . \tag{10}$$

Two components of the wideband noise associated with the optical sensor will now be evaluated and compared to x_{NB} for the evaluation of their contribution to the noise temperature of the detector.

Fluctuations in the frequency of the laser contribute fluctuations in the measurement of the position of the last resonator given by the simple formula

$$\sigma_{x\text{LFN}} = l\sigma_\nu/\nu = 8.4\times10^{-19}\ \text{m}/\sqrt{\text{Hz}} . \tag{11}$$

In addition, the optical readout exhibits shot noise associated with quantum fluctuations in the number of photons and the finite value of the charge of the electron. First, we can make a very approximate evaluation of the shot noise in the following way. With a finesse of 75 000, the output of the Fabry-Perot varies by an amount of the order of its maximum value for a displacement of one of the mirrors of the order of $\lambda/2F = 4.2\times10^{-12}$ m. If the power incident on the photodiode is 2 μW, the current generated is $i = 0.8$ μA with shot noise $\sigma_{i\text{SN}} = \sqrt{2ei} = 0.5$ pA/$\sqrt{\text{Hz}}$, which, in principle, would permit a resolution at the level of 2.6×10^{-18} m/$\sqrt{\text{Hz}}$.

A more appropriate evaluation of the shot noise takes into account the method used here whereby the phase of the laser signal is modulated at rf frequency and the laser frequency is centered on a cavity resonance [34]. As assumed earlier, the modulation depth is 1.1 rad and the modulating frequency is 0.35Γ. In such a case, the shot noise is

$$\sigma_{x\text{SN}} \cong \frac{\lambda}{2F}\frac{\sigma_i}{1.6i} = 1.9\times10^{-18}\ \text{m}/\sqrt{\text{Hz}} , \tag{12}$$

where $\sigma_i = 0.6$ pA/$\sqrt{\text{Hz}}$ is the quadrature sum of the preamplifier noise and the shot noise associated with the total current of 0.8 μA in the photodiode. This value of the shot noise is close to the one obtained in the previous paragraph.

The total wideband noise is obtained by quadrature of its two components,

$$\sigma_{x\text{WB}} = \sqrt{\sigma_{x\text{SN}}^2 + \sigma_{x\text{LFN}}^2} = 2.1\times10^{-18}\ \text{m}/\sqrt{\text{Hz}} , \tag{13}$$

and corresponds to rms displacements in the 48-Hz bandwidth of interest:

$$x_{\text{WB}} = \sigma_{x\text{WB}}\sqrt{\Delta f} = 1.4\times10^{-17}\ \text{m} . \tag{14}$$

This wideband noise will further increase the noise temperature of the detector to

$$T_p = T_{p\text{NB}}[1 + (x_{\text{WB}}/x_{\text{NB}})^2] = 7.0\times10^{-7}\ \text{K} . \tag{15}$$

It is interesting to compare this temperature with the transducer noise temperature:

$$T_n = \omega_0\sigma_{x\text{WB}}F_{\text{BA}}/k_B = 4.5\times10^{-7}\ \text{K} , \tag{16}$$

where F_{BA} is the quadrature sum of the classical and quantum back action given in Eqs. (6) and (7). The noise temperature T_p of the detector is slightly above the transducer noise temperature T_n reflecting the fact that the noise contribution from the high-Q–low-temperature mechanical system is less than the one from the optical transducer. The detector noise temperature corresponds to a sensitivity in h equal to

$$\delta h = \frac{\pi}{4}\left[\frac{2k_B T_p}{m_1\omega_0^2 L^2}\right]^{1/2} = 5.0\times10^{-21} . \tag{17}$$

A more precise projection of the sensitivity of the detector assuming optimum filtering of the data can be obtained from a numerical integration of the ratio of the signal power to the noise power over the usable bandwidth [37,38]. In that analysis, the wideband noise and the back-action force used are those calculated above. For the three-mode detector considered, the results are

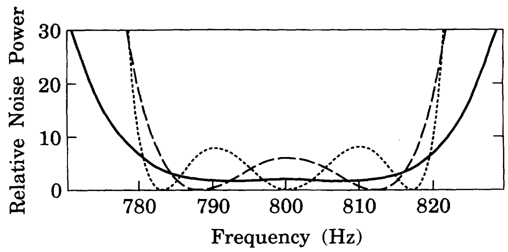

FIG. 9. Thermal noise from the antenna and resonators (solid line), laser back action (long dash), and shot noise (short dash). Unity corresponds to a noise force density of 4.16×10^{-31} N^2/Hz.

$$T_{p\mathrm{OF}}=8.9\times10^{-7}\ \mathrm{K} \quad \text{and} \quad \delta h_{\mathrm{OF}}=7.0\times10^{-21} \tag{18}$$

in close agreement with the values quoted in the previous paragraph. $T_{p\mathrm{OF}}$ is a factor 16 above the standard quantum limit T_{QL} given in Sec. II. The relative contributions of the thermal noise, the laser back action, and the laser wideband noise are shown in Fig. 9 and show a reasonably good match of the optical components and a relatively uniform sensitivity over the bandwidth of interest.

VII. SENSITIVITY PROJECTIONS FOR OTHER DETECTORS

In Sec. VI, we selected a bar detector with low enough temperature and high enough Q's for the sensitivity to be limited in the most part by the Fabry-Perot transducer. This was done to illustrate the capability of such a transducer. The optimum filter analysis method has been used to project the sensitivity of other detectors close to existing ones. The results are shown in Table I. Here again, the antenna is assumed to be a 2400-kg Al5056 bar resonating at 800 Hz. The antenna and resonator Q's are assumed to be the same. The finesse of the Fabry-Perot transducer is assumed to be 75 000. The detector temperature, Q, and number of modes are specified for each of the eight detectors considered. The first detector shown is the one discussed in detail in the previous section. Detectors 2 to 4 are projections for detectors under current development to operate with dilution refrigerators at temperatures of the order of 0.05 K. Three- or four-mode instrumentation is assumed and the sensitivity projections for the detection of short pulses are at the level of $\delta h \cong 1.0\times10^{-20}$. In these systems, the optical transducer noise is of the order of the detector thermal noise. Detectors 6 to 8 are projections for current detectors operating at liquid-helium temperature but modified to operate as three-, four-, or five-mode detectors. Here, the thermal noise from the mechanical system is dominant and the power incident on the photodiode can be reduced for the three- and four-mode systems without significant impact on the sensitivity. The sensitivity range is $\delta h \cong (3.9-7.3)\times10^{-20}$.

VII. CONCLUSION

We have reported the measurement of a displacement sensitivity of 5.6×10^{-17} m/$\sqrt{\mathrm{Hz}}$ for a high-finesse optical Fabry-Perot cavity at room temperature. Following this, a detailed intuitive analysis substantiated by a numerical one has been used to project the sensitivity of multimode bar detectors instrumented with a Fabry-Perot transducer essentially identical to the one tested. In the first case, a 0.030-K detector similar to ones being developed has been analyzed and a sensitivity limited in the most part by the transducer noise has been projected. The noise temperature is 8.9×10^{-7} K, a factor of 16 above the standard quantum limit. The sensitivity in h is

TABLE I. Projections for noise components and performance of various multimode detectors instrumented with an optical transducer with finesse equal to 75 000. The thermal noise shown is generated by the damping mechanisms in the bar and associated mechanical resonators. The transducer noise includes the classical and quantum components of the back action, the shot noise, and the noise associated with fluctuations in the laser frequency. Detectors 1 to 4 require ^{3}He dilution refrigeration. Detector 1 is discussed in detail in Sec. VI. Detectors' 2, 3, and 4 temperatures and Q's are projections for detectors currently under development. Detector 5 would operate with a ^{3}He refrigerator. Detectors 6 to 8 are projections of the current generation of detectors operating at 4.2 K.

Detector label		1	2	3	4	5	6	7	8
Bar temperature	(K)	0.03	0.05	0.1	0.1	0.4	4.2	4.2	4.2
Bar mechanical quality factor	(10^6)	20	10	7	7	7	7	7	7
Number of modes		3	3	3	4	3	3	4	5
Effective bandwidth	(Hz)	48	48	48	155	48	48	155	280
Power on photodiode	(μW)	2	2	2	2	2	0.25	1.0	2.0
Transducer noise temperature	(μK)	0.45	0.45	0.45	0.45	0.45	0.33	0.40	0.45
Thermal noise	(μK)	0.19	0.65	1.86	0.77	7.4	78	32	22
Transducer noise	(μK)	0.70	0.82	0.95	1.57	1.1	17	6.0	4.8
Transducer noise contribution		78%	56%	34%	67%	13%	18%	16%	17%
Detector noise temperature	(μK)	0.89	1.5	2.8	2.3	8.5	95	38	27
Factor above quantum limit		16	27	51	43	150	1700	690	490
δh	(10^{-20})	0.68	0.91	1.2	1.1	2.2	7.3	4.6	3.9

7.0×10^{-21}. When detectors operating at 4.2 K are considered, it is found that the sensitivity ranges between 3.9×10^{-20} and 7.3×10^{-20}. These results show that optically instrumented multimode gravitational-wave bar detectors operated at 4.2 K or lower temperature have the potential to play a significant role in the search for gravitational radiation.

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[25] The development and commercial availability of monolithic, unidirectional single-mode Nd:YAG laser operating at 1.06 μm is also of great interest. Very narrow line widths have been obtained with them and good power stabilization can be achieved. In addition, supermirrors with losses at the level of 30 ppm are now available on a custom basis for operation at that wavelength.

[26] We have obtained Q's of 5.6×10^{6} at 4.2 K with such suspension and a resonator mass of 4 g. The 0.4-g mirrors have been acquired.

[27] The Q requirement is less stringent here since the resonant frequency is away from the signal frequency and the reference mass may be attached to the antenna.

[28] The development of an appropriate feedback system is nontrivial. The dynamic range and slew rate should be large enough to prevent loss of lock in the presence of low-frequency antenna motion. The feedback system used in the test of a low-finesse transducer (Ref. [23]) is being improved. This task is eased with common mode reduction achieved by matching the displacements of the last resonator and the reference mass at low frequencies.

[29] Preliminary measurements on our improved system indicate that the current stability is 1.0 $\mathrm{Hz}/\sqrt{\mathrm{Hz}}$.

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(2 μW) is well within the dynamic range of the photodiode. For these reasons, because the implementation of the transmission method is easier and because a very high sensitivity can be achieved in that mode, the Fabry-Perot transducer is presently used in that way. Operation in reflection will be considered later.

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