Tests of *CPT* conservation in the neutral kaon system

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We make use of high-precision measurements in the neutral kaon system to analyze how well *CPT* is conserved. Although present data provide rather stringent tests of *CPT*, actually these measurements provide bounds only on differences of *CPT*-violating parameters. Although *CP*-violating phenomena in the $\Delta S=2$ sector are dominantly *CPT* conserving, present data do not allow a similar unambiguous statement to be made for the $\Delta S=1$ sector.

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The *CPT* theorem [1] is based on very general principles, such as Lorentz invariance and the spin-statistics connection, which are difficult to doubt. However, this theorem also uses the idea of local interactions which, although perfectly reasonable, could conceivably break down [2]. Thus, it is important to test the *CPT* theorem experimentally as well as one can. The purpose of this work is to present an analysis of *CPT* conservation in the neutral kaon system, based on recent precision data on *CP*-violating phenomena [3,4] in this complex.

Our analysis brings forth a number of interesting features, which can be summarized as follows: (i) All existing *CPT* tests in the neutral kaon system only give bounds on differences in *CPT*-violating parameters; (ii) barring cancellations among parameters, *CPT*-violating effects are subdominant in the $\Delta S=2$ sector and the ratio of *CPT*-violating to *CPT*-conserving amplitudes is, at most, of order 10^{-4}; (iii) present data on the phase difference of the *CP*-violating parameters η_{+-} and η_{00} in $K^0 \to \pi\pi$ do not exclude a *CPT*-violating contribution to $\mathrm{Re}(\epsilon'/\epsilon)$ at the level of 10^{-3}; and (iv) one can, in principle, disentangle all the *CPT*-violating parameters in the neutral kaon system by measurements in a high luminosity ϕ factory [5].

If one does not assume *CPT* conservation, there are two sources of *CPT* violation that can arise in the neutral kaon system. On the one hand, without assuming *CPT* conservation, the (diagonal) masses and the total widths of the flavor eigenstates K^0 and $\bar{K}^0$ will not necessarily be equal. This will cause the physical eigenstates of the system not to have the same proportion of K^0 and $\bar{K}^0$ states in them. If M and Γ are the 2×2 Hermitian [6] mass and decay matrices of the K^0-$\bar{K}^0$ complex, the physical eigenstates are easily found to be

$$|K_S\rangle = \frac{1}{\sqrt{2(1+|\epsilon_K+\delta_K|^2)}} \times\{(1+\epsilon_K+\delta_K)|K^0\rangle+(1-\epsilon_K-\delta_K)|\bar{K}^0\rangle\} , \tag{1a}$$

$$|K_L\rangle = \frac{1}{\sqrt{2(1+|\epsilon_K-\delta_K|^2)}} \times\{(1+\epsilon_K-\delta_K)|K^0\rangle-(1-\epsilon_K+\delta_K)|\bar{K}^0\rangle\} , \tag{1b}$$

where

$$\epsilon_K \simeq e^{i\phi_{sw}}\frac{[-\mathrm{Im}M_{12}+i\,\mathrm{Im}\Gamma_{12}/2]}{\sqrt{2}\Delta m} \tag{2}$$

is a *CP*-violating but *CPT*-conserving parameter, while

$$\delta_K \simeq e^{i\phi_{sw}}\frac{i(M_{11}-M_{22})+(\Gamma_{11}-\Gamma_{22})/2}{2\sqrt{2}\Delta m} \tag{3}$$

is a *CP*- and *CPT*-violating parameter. Here, $\Delta m = m_L - m_S$ is the K_L-K_S mass difference and

$$\phi_{sw}=\arctan[2\Delta m/(\Gamma_S-\Gamma_L)]=(43.73\pm0.14)^\circ$$

is the so-called superweak phase.

If *CPT* is violated, there is no longer any relation between the amplitudes for $K^0\to f$ and those for $\bar{K}^0\to\bar{f}$. Thus, *CPT* violation will introduce, in addition to the "mixing" parameter δ_K, further parameters associated with *CPT* breaking in the decay amplitudes. For our analysis we shall need both the 2π and the K_{l3} semileptonic amplitudes of the neutral kaons, and it is useful to adopt a notation where *CPT* violation is manifest. For these purposes, following Barmin *et al.* [7], we write

$$\langle 2\pi;I|T|K^0\rangle=(A_I+B_I)e^{i\delta_I} , \tag{4a}$$

$$\langle 2\pi;I|T|\bar{K}^0\rangle=(A_I^*-B_I^*)e^{i\delta_I} , \tag{4b}$$

$$\langle \pi^0 l^+\nu_l|T|K^0\rangle=a+b , \tag{4c}$$

$$\langle \pi^+ l^-\bar{\nu}_l|T|\bar{K}^0\rangle=a^*-b^* . \tag{4d}$$

In the above, the B_I and b amplitudes are *CPT* violating. At the same time, in the phase convention in which $CP\,K^0(\bar{K}^0)=\bar{K}^0(K^0)$, *CP* conservation would require that the A_I and a amplitudes be real. In Eqs. (4a) and (4b) δ_I are the strong $\pi\pi$ scattering phase shifts for states of total isospin I.

Armed with Eqs. (1) and (4), it is straightforward to deduce formulas for the five *CP*-violating observables measured to date in the kaon system:

$$\eta_{+-}\equiv\frac{\langle\pi^+\pi^-|T|K_L\rangle}{\langle\pi^+\pi^-|T|K_S\rangle}=|\eta_{+-}|e^{i\phi_{+-}}\approx\epsilon+\epsilon' , \tag{5a}$$

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0|T|K_L\rangle}{\langle \pi^0\pi^0|T|K_S\rangle} = |\eta_{00}|e^{i\phi_{00}} \approx \epsilon - 2\epsilon' \ , \tag{5b}$$

$$\mathcal{A}_{K_L}(l^+,l^-) \equiv \frac{\Gamma(K_L\to\pi^-l^+\nu_l)-\Gamma(K_L\to\pi^+l^-\bar{\nu}_l)}{\Gamma(K_L\to\pi^-l^+\nu_l)+\Gamma(K_L\to\pi^+l^-\bar{\nu}_l)} . \tag{5c}$$

One finds, expanding to first order in small quantities,

$$\epsilon = \left[\epsilon_K + i\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right] + \left\{\frac{\mathrm{Re}B_0}{\mathrm{Re}A_0} - \delta_K\right\}$$
$$\approx \frac{1}{\sqrt{2}}\left[\frac{-\mathrm{Im}M_{12}}{\Delta m} + \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\frac{2\Delta m}{\Delta\Gamma}\right]e^{i\phi_{\mathrm{sw}}}$$
$$+\frac{1}{\sqrt{2}}\left\{\frac{M_{22}-M_{11}}{2\Delta m} - \frac{\mathrm{Re}B_0}{\mathrm{Re}A_0}\right\}ie^{i\phi_{\mathrm{sw}}} , \tag{6a}$$

$$\epsilon = \frac{1}{\sqrt{2}}e^{i(\delta_2-\delta_0+\pi/2)}\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}$$
$$\times\left[\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right] + i\left\{\frac{\mathrm{Re}B_0}{\mathrm{Re}A_0} - \frac{\mathrm{Re}B_2}{\mathrm{Re}A_2}\right\}\right] , \tag{6b}$$

$$\mathcal{A}_{K_L}(l^+,l^-) = 2\,\mathrm{Re}\epsilon_K + \left\{2\frac{\mathrm{Re}b}{\mathrm{Re}a} - 2\,\mathrm{Re}\delta_K\right\} . \tag{6c}$$

In the above, the *CPT*-violating parameters are those that appear in curly brackets. Furthermore, the approximate form for ϵ given in the second line of Eq. (6a) follows from assuming that the decay matrix Γ of the neutral kaon system is saturated by the $\pi\pi(I=0)$ states. Notice that, using this approximation, both for ϵ and for ϵ', the *CPT*-conserving and *CPT*-violating terms appear at 90° to each other [7,8].

There are three distinct *CPT* tests that follow from present experimental results on the parameters of Eq. (5). The first *CPT* test compares $\mathrm{Re}\epsilon$ with $\mathcal{A}_{K_L}(l^+,l^-)$. Using the values for $\eta_{\pm}$, η_{00}, and $\mathcal{A}_{K_L}(l^+,l^-)$ summarized in the Particle Data Group compilation [9], and the expression for $\epsilon = \frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00}$ deduced from Eqs. (5a) and (5b) [10], one finds that

$$\frac{\mathrm{Re}B_0}{\mathrm{Re}A_0} - \frac{\mathrm{Re}b}{\mathrm{Re}a} = \mathrm{Re}\epsilon - \tfrac{1}{2}\mathcal{A}_{K_L}(l^+,l^-)$$
$$= (-0.6\pm0.7)\times10^{-4} . \tag{7}$$

The second *CPT* test compares the phase of ϵ expected from Eq. (6a) to its experimental value $\phi_\epsilon \simeq \frac{2}{3}\phi_{+-} + \frac{1}{3}\phi_{00}$ (see Fig. 1). Let us write the second line of Eq. (6a) as

$$\epsilon = \epsilon_1 e^{i\phi_{\mathrm{sw}}} + i\epsilon_2 e^{i\phi_{\mathrm{sw}}} . \tag{8}$$

Then, using the experimental value [9] for $\phi_{+-} = (46\pm1.2)°$ and the weighted average of the results of the NA31 and E731 Collaborations [11] $\phi_{+-} - \phi_{00} = (-0.3\pm2.0)°$, one finds, for the ratio of *CPT*-violating to *CPT*-conserving components,

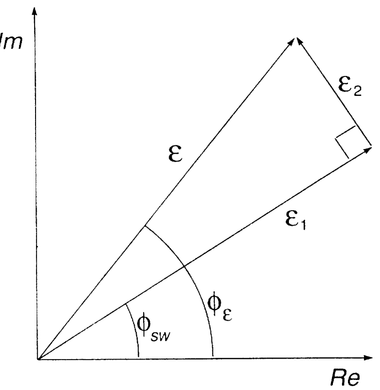

FIG. 1. Vector diagram in the complex plane of ϵ, its *CPT*-conserving component ϵ_1, and its *CPT*-violating component ϵ_2. Notice that the phase of ϵ_1 is ϕ_{sw}, and that ϵ_1 and ϵ_2 are at 90° from each other.

$$\frac{\epsilon_2}{\epsilon_1} = \tan(\phi_\epsilon - \phi_{\mathrm{sw}}) = (3.8\pm2.5)\times10^{-2} . \tag{9}$$

Since $\phi_{\mathrm{sw}} \approx \pi/4$, the above gives a value for both $\{(\mathrm{Re}B_0/\mathrm{Re}A_0) - \mathrm{Re}\delta_K\}$ and $\mathrm{Im}\delta_K$. Using the result of Eq. (9), one finds

$$\frac{\mathrm{Re}B_0}{\mathrm{Re}A_0} - \mathrm{Re}\delta_K \simeq \mathrm{Im}\delta_K \simeq \frac{M_{11}-M_{22}+(\Gamma_{11}-\Gamma_{22})/2}{4\Delta m}$$
$$= (0.64\pm0.40)\times10^{-4} . \tag{10}$$

We note that, as indicated earlier, both the *CPT* tests (7) and (10) involve *differences* in *CPT*-violating parameters. Barring cancellations, they bound the ratio of *CPT*-violating to *CPT*-conserving amplitudes to 10^{-4}. Furthermore, even though Eqs. (9) and (10) give nearly a 2σ *CPT*-violating effect, one should properly consider the size of the error in Eq. (10) as a bound on possible *CPT* violation rather than as a positive signal. Indeed, the very recent E731 value for ϕ_{+-} [4] of $(43.2\pm1.6)°$ is perfectly consistent with ϕ_{sw}, but the error gives results comparable to those in Eqs. (9) and (10) [12].

The third *CPT* test involves ϵ' and uses the fact that the overall phase of ϵ' is rather close to ϕ_{sw}. Using the most recent analysis of the $\pi\pi$ scattering phase shifts [13], one has that

$$\Delta \equiv \delta_2 - \delta_0 + \pi/2 - \phi_{\mathrm{sw}} = (1.3\pm6)° . \tag{11}$$

Denoting the *CPT*-conserving and *CPT*-violating terms in Eq. (6b) by

$$\epsilon_1' \equiv [(\mathrm{Im}A_2/\mathrm{Re}A_2) - (\mathrm{Im}A_0/\mathrm{Re}A_0)]$$

and

$$\epsilon_2' \equiv \{(\mathrm{Re}B_2/\mathrm{Re}A_0) - (\mathrm{Re}B_2/\mathrm{Re}A_2)\} ,$$

respectively, one has

$$\epsilon' = e^{i(\delta_2-\delta_0+\pi/2)}\frac{1}{\sqrt{2}}\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}(\epsilon_1' + i\epsilon_2') . \tag{12}$$

Using the numerical value for $(\mathrm{Re}A_2/\mathrm{Re}A_0)/$

$(\sqrt{2}|\epsilon|)\approx 14$, one can write

$$\mathrm{Re}(\epsilon'/\epsilon)\simeq 14[\epsilon_1'\cos\Delta-\epsilon_2'\sin\Delta] \tag{13a}$$

$$\phi_{+-}-\phi_{00}\simeq 3\,\mathrm{Im}(\epsilon'/\epsilon)\approx\left[\frac{\epsilon_1'\sin\Delta+\epsilon_2'\cos\Delta}{4.2\times 10^{-4}}\right]^\circ . \tag{13b}$$

Although there is experimental controversy at the moment regarding the value of $\mathrm{Re}(\epsilon'/\epsilon)$,

$$\mathrm{Re}(\epsilon'/\epsilon)=\begin{cases}(23\pm 7)\times 10^{-4} & \text{NA31 [3]},\\ (6\pm 7)\times 10^{-4} & \text{E731 [4]},\end{cases} \tag{14}$$

even if one assumed that the *CPT*-conserving contribution ϵ_1' dominates $\mathrm{Re}(\epsilon'/\epsilon)$, it would still give a negligible contribution to the phase difference [cf. Eq. (13b)]. Consequently, one can use the present values of the phase difference [11] $\phi_{+-}-\phi_{00}=(-0.3\pm 2.0)^\circ$ to bound directly the *CPT*-violating parameter ϵ_2'. One finds that

$$\epsilon_2'=\frac{\mathrm{Re}B_0}{\mathrm{Re}A_0}-\frac{\mathrm{Re}B_2}{\mathrm{Re}A_2}\approx(-1.3\pm 8.4)\times 10^{-4} . \tag{15}$$

We note that such a value for ϵ_2' by itself is compatible with the range of measurements for $\mathrm{Re}(\epsilon'/\epsilon)$ given in Eq. (14), even in the absence of any *CPT*-conserving component of ϵ'. So, unless one can sharpen the bound on the phase difference $\phi_{+-}-\phi_{00}$ and on Δ, one cannot exclude the possibility that a purely *CPT*-violating contribution is giving the apparent $\Delta S=1$ *CP*-violating signal.

We note also that this third *CPT* test involves differences of *CPT*-violating parameters. Although there is no "theory" of *CPT* violation, one could imagine circumstances in which the ratio of *CPT*-violating to *CPT*-conserving amplitudes would be universal. Then, even though *CPT* is violated, the *CPT* tests of Eqs. (7) and (15) would not give rise to any measurable results. If, in addition, the mass difference between the K^0 and the $\bar{K}^0$ states were to be in the same relation to their total widths (i.e., $M_{11}-M_{22}\approx[\Gamma_{22}-\Gamma_{11}]/2$), as it happens to be for the physical states K_L and K_S (i.e., $\Delta m\approx[\Gamma_S-\Gamma_L]/2$), then the *CPT* test of Eq. (10) would also give a negative result. In this manner, the present measurements could accidentally hide a large violation of *CPT*. Although we have no theoretical reason to argue for this peculiar circumstance, it would be highly desirable if one could disentangle all the *CPT*-violating parameters from each other. This can be done at a high luminosity ϕ factory, where a direct measurement of $\mathrm{Re}\delta_K$ itself is possible at the level of 10^{-4}. Measuring $\mathrm{Re}\delta_K$ independently to this accuracy, the result of the analysis given here corresponds to the measurement of the $K^0-\bar{K}^0$ mass difference to one part in 10^{18}, which is a significant result. However, without an independent measurement of $\mathrm{Re}\delta_K$, no such unambiguous statement can be made.

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[3] The most recent summary of NA31 results is found in G. Barr, in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics,* Geneva, Switzerland, 1991, edited by S. Hegarty, K. Potter, and E. Quercigh (World Scientific, Singapore, 1992).

[4] The most recent summary of E731 results is found in B. Winstein, *Proceedings of Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics* [3].

[5] For a discussion, see, for example, C. Buchanan *et al.*, Phys. Rev. D **45**, 4088 (1992).

[6] Although we allow for *CPT* to be broken, we still assume that the dispersive and absorptive matrices M and Γ are Hermitian.

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[10] Notice that, because ϵ'/ϵ is small, to the same order of approximation one can simply use $\epsilon\approx\eta_{+-}$.

[11] NA31 Collaboration, R. Carosi *et al.*, Phys. Lett. B **237**, 303 (1990); E731 Collaboration, M. Karlsson *et al.*, Phys. Rev. Lett. **64**, 2976 (1990).

[12] We must also note that the contribution to the kaon width from states other than the $\pi\pi(I=0)$ state give a contribution to ϵ_2 which is much below the error in Eq. (9).

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