# Implications of the CERN LEP results for SO(10) grand unification

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We consider the breaking of the grand unification group SO(10) to the standard model gauge group through several chains containing one intermediate breaking scale. Using the values of the gauge coupling constants at the scale $M_Z$ derived from recent data from the CERN $e^+e^-$ collider LEP, we determine the intermediate and the unification scales using two-loop renormalization group equations with appropriate matching conditions. Some chains are ruled out from experimental constraints. For the allowed ones, the intermediate scale is high, in the range of $10^9$–$10^{11}$ GeV.

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Recently it has been emphasized [1, 2] that the precision of data from the CERN $e^+e^-$ collider LEP allows the extraction of the three coupling constants of the standard model very accurately, and this enables an extrapolation to high energies with very small errors. In these papers it was shown that if one extrapolates the couplings without change of particle content (i.e., with three families of fermions and one doublet of Higgs boson) or the group structure in the intervening energy scale, these couplings do not come together at a single point. Although this result is not unexpected, the statistical significance has been greatly improved. For example, LEP data imply [3]

$$\begin{aligned} \alpha_1(M_Z) &= 0.016\,887 \pm 0.000\,040, \\ \alpha_2(M_Z) &= 0.033\,22 \pm 0.000\,25, \\ \alpha_3(M_Z) &= 0.120 \pm 0.007\,, \end{aligned} \tag{1}$$

where $\alpha_i = g_i^2/4\pi$, and $g_1$, $g_2$, $g_3$ denote the normalized gauge coupling constants for the $\mathrm{U}(1)_Y$, $\mathrm{SU}(2)_L$, and $\mathrm{SU}(3)_c$ factors, respectively. Now, if we take the central values for $\alpha_1$ and $\alpha_2$, we need $\alpha_s(M_Z) \simeq 0.07$ in order to achieve unification. This is more than seven standard deviations removed from the mean value.

In Refs. [1] and [2] it was further pointed out that the minimal supersymmetric model, with the supersymmetry-breaking scale $M_{\mathrm{SUSY}} \simeq 1\,\mathrm{TeV}$, gives the best fit for all couplings to evolve to a single unification point, with the grand unification scale $M_{\mathrm{GUT}} \simeq 10^{16}\,\mathrm{GeV}$. This scale of grand unification is quite consistent with limits on proton decay. Other modifications considered [1] which allow for unification to occur, include the introduction of six doublets of Higgs bosons in the nonsupersymmetric version, or of three additional Higgs boson doublets in the supersymmetric model with higher scale for $M_{\mathrm{SUSY}}$. Here, we wish to consider unification in the context of a bigger gauge group than SU(5), namely, SO(10), which has enjoyed considerable popularity.

The group SO(10) for grand unification has a long history [4]. Among its many attractive features are that the fermions of one family occur in one irreducible representation, and it permits neutrinos to develop small masses through the seesaw mechanism. Since the group is larger than SU(5), it is possible to have many different chains of symmetry breaking down to the standard model gauge group. Some of these include several intermediate scales. In this paper, we will consider chains with only one intermediate scale. The recent LEP data and the assumption of unification can determine the intermediate scale in such cases.

It is, of course, useless to discuss SO(10) breaking through SU(5). The chain involving flipped SU(5) has been discussed recently in light of the LEP data [5]. We consider chains where the group $\mathrm{SU}(2)_R$ or the neutral component of it, $\mathrm{U}(1)_R$, appears intact at intermediate energy. If $\mathrm{SU}(2)_R$ is part of the gauge symmetry, the discrete parity symmetry ($P$) is either good at the intermediate energy [i.e., the gauge coupling constants for $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ are equal] or is broken, depending on the choice of Higgs representation for various breakings. At low energies, we have the standard model with only one doublet of Higgs boson. Some, but not all, of these chains have been considered recently [6, 7] in the light of LEP data, but only at the one-loop level. We include the Higgs boson effects, and also use two-loop renormalization group equations (RGE's) with matching conditions appropriate to a two-loop analysis [8]. This makes significant difference to the results obtained by previous authors, eliminating certain chains.

We employ the following notation for the different scales:

$$\mathrm{SO}(10) \overset{M_U}{\to} G_I \overset{M_I}{\to} \{1_Y 2_L 3_c\} \overset{M_Z}{\to} \{1_Q 3_c\}\,, \tag{2}$$

where $G_I$ is the gauge group for the intermediate mass range, and the symbol $\{1_Y 2_L 3_c\}$, e.g., stands for the group $\mathrm{U}(1)_Y \times \mathrm{SU}(2)_L \times \mathrm{SU}(3)_c$. Throughout the paper, we will use such notation for denoting different groups. The different chains that we consider are now specified by giving the the intermediate range gauge group in each case. We call the chains

$$\begin{aligned} &\text{chain 1a}: G_I = \{2_L 2_R 4_C\}, \\ &\text{chain 1b}: G_I = \{2_L 2_R 4_C \times P\}, \\ &\text{chain 2a}: G_I = \{2_L 2_R 1_X 3_c\}, \\ &\text{chain 2b}: G_I = \{2_L 2_R 1_X 3_c \times P\}, \\ &\text{chain 3}: G_I = \{2_L 1_R 4_C\}, \\ &\text{chain 4}: G_I = \{2_L 1_R 1_X 3_c\}\,, \end{aligned} \tag{3}$$

 

where $X = (B - L)/2$ and the factor $P$ in chains 1b and 2b denotes an unbroken parity symmetry which requires the gauge coupling constants of $\{2_L\}$ and $\{2_R\}$ to be equal.

For each chain, we employ the following procedure to find the intermediate scale. We use two-loop RGE's to evolve the couplings from a scale of energy $M_Z$ to an intermediate scale $M_I$. At this scale the coupling constants are matched to the couplings of the intermediate gauge group using appropriate matching conditions which will be discussed later. The couplings in $G_I$ are then evolved, again using two-loop RGE's. The value of $M_I$ is varied so that, if one starts with the central values of $\alpha_1$, $\alpha_2$, and $\alpha_3$ given in Eq. (1), the various gauge coupling constants of $G_I$ satisfy the unification criterion at some scale larger than $M_I$. This higher scale is then identified as $M_U$ or the unification scale. Once a value of $M_I$ is determined this way, we use the $1\sigma$ errors of Eq. (1) to find the uncertainties in the unification scale. We realize, of course, that the intermediate scale also has some uncertainty because of the same errors in the coupling constants at the scale $M_Z$, which can make the real uncertainty in $M_U$ larger. This issue will be discussed in detail in a longer paper [9].

The two-loop RGE's for the gauge coupling constants can be written down as follows:

$$\frac{d\omega_i(\mu)}{d\ln\mu} = -\frac{a_i}{2\pi} - \sum_j \frac{b_{ij}}{8\pi^2\omega_j}\,, \tag{4}$$

where $i, j$ index different subgroups of the gauge group at the energy scale $\mu$, and

$$\omega_i = \alpha_i^{-1} = 4\pi/g_i^2 \tag{5}$$

for any subgroup. Our U(1) factors are always normalized the same way as the non-Abelian gauge couplings. The quantities $a_i$ and $b_{ij}$ are constants, whose values depend on the number and representations of the fermions and scalars lighter than $\mu$.

As for fermions, we assume that there are three generations and that all of them are lighter than $M_Z$. Although it is now known that the $t$ quark can be a little heavier than the $Z$, and the right-handed neutrinos could, in principle, be heavier as well, the difference is negligible for the purpose of our calculations. As for Higgs bosons, those which have masses of order $M_U$ do not enter the renormalization group analysis. To determine which Higgs bosons are lighter than $M_U$, we use the hypothesis of minimal fine-tuning [10]. Thus, at the level above $M_Z$ where the unbroken gauge symmetry is $\{1_Y 2_L 3_c\}$, we have only one Higgs doublet. Using the formulas given by Jones [11], we then find

$$a = \begin{pmatrix} \frac{41}{10} \\ -\frac{19}{6} \\ -7 \end{pmatrix}, \quad b = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix}, \tag{6}$$

where the rows and columns are arranged in the order in which the different factors are given in the gauge group, i.e., the first row or column corresponds to the $1_Y$ factor, the second to the $2_L$ factor, and the third to $3_c$.

In order to know the Higgs boson content at other scales in various chains, we need to know which SO(10) multiplet of Higgs bosons is used to perform the symmetry breaking in any given chain. For chains where there is no discrete parity symmetry at the intermediate scale, this has been discussed in detail by previous authors [12, 13]. In chains where parity symmetry is intact at the intermediate scale, one just needs to enlarge the light Higgs boson sector in order to make it left-right symmetric. For example, in chain 1a, the vacuum expectation value (VEV) that breaks $\{2_L 2_R 4_C\} \to \{1_Y 2_L 3_c\}$ comes from a (1,3,10) submultiplet of the 126-dimensional multiplet of SO(10). Thus, this submultiplet is expected to have a mass around $M_I$ and is therefore relevant for the evolution of coupling constants between $M_I$ and $M_U$, and the hypothesis of minimal fine-tuning tells us that all the other components of 126 are superheavy. However, when we discuss chain 1b, we must have a left-

TABLE I. Renormalization group coefficients for the intermediate groups. The factor $P$ denotes the discrete parity symmetry. The quantum number $X$ appearing in various chains equals $(B - L)/2$. We have assumed that the only light particles are the three generations of fermions, the appropriate gauge bosons, and the Higgs multiplets given in the second column, identified by their transformation under $G_I$ and with SO(10) representations given as subscripts.

| Group $G_I$ | Higgs content | $a$ | $b$ |
|---|---|---|---|
| $\{2_L 2_R 4_C\}$ | $(2,2,1)_{10}$<br>$(1,3,10)_{126}$ | $\begin{pmatrix} -3 \\ \frac{11}{3} \\ -\frac{23}{3} \end{pmatrix}$ | $\begin{pmatrix} 8 & 3 & \frac{45}{2} \\ 3 & \frac{584}{3} & \frac{765}{2} \\ \frac{9}{2} & \frac{153}{2} & \frac{643}{6} \end{pmatrix}$ |
| $\{2_L 2_R 4_C \times P\}$ | $(2,2,1)_{10}$<br>$(1,3,10)_{126}$<br>$(3,1,10)_{126}$ | $\begin{pmatrix} \frac{11}{3} \\ \frac{11}{3} \\ -\frac{14}{3} \end{pmatrix}$ | $\begin{pmatrix} \frac{584}{3} & 3 & \frac{765}{2} \\ 3 & \frac{584}{3} & \frac{765}{2} \\ \frac{153}{2} & \frac{153}{2} & \frac{1759}{6} \end{pmatrix}$ |
| $\{2_L 2_R 1_X 3_c\}$ | $(2,2,0,1)_{10}$<br>$(1,3,1,1)_{126}$ | $\begin{pmatrix} -3 \\ -\frac{7}{3} \\ \frac{11}{2} \\ -7 \end{pmatrix}$ | $\begin{pmatrix} 8 & 3 & \frac{3}{2} & 12 \\ 3 & \frac{80}{3} & \frac{27}{2} & 12 \\ \frac{9}{2} & \frac{81}{2} & \frac{61}{2} & 4 \\ \frac{9}{2} & \frac{9}{2} & \frac{1}{2} & -26 \end{pmatrix}$ |
| $\{2_L 2_R 1_X 3_c \times P\}$ | $(2,2,0,1)_{10}$<br>$(1,3,1,1)_{126}$<br>$(3,1,1,1)_{126}$ | $\begin{pmatrix} -\frac{7}{3} \\ -\frac{7}{3} \\ 7 \\ -7 \end{pmatrix}$ | $\begin{pmatrix} \frac{80}{3} & 3 & \frac{27}{2} & 12 \\ 3 & \frac{80}{3} & \frac{27}{2} & 12 \\ \frac{81}{2} & \frac{81}{2} & \frac{115}{2} & 4 \\ \frac{9}{2} & \frac{9}{2} & \frac{1}{2} & -26 \end{pmatrix}$ |
| $\{2_L 1_R 4_C\}$ | $(2,\frac{1}{2},1)_{10}$<br>$(1,1,10)_{126}$ | $\begin{pmatrix} -\frac{19}{6} \\ \frac{15}{2} \\ -\frac{29}{3} \end{pmatrix}$ | $\begin{pmatrix} \frac{35}{6} & \frac{1}{2} & \frac{45}{2} \\ \frac{3}{2} & \frac{87}{2} & \frac{405}{2} \\ \frac{9}{2} & \frac{27}{2} & -\frac{101}{6} \end{pmatrix}$ |
| $\{2_L 1_R 1_X 3_c\}$ | $(2,\frac{1}{2},0,1)_{10}$<br>$(1,1,-1,1)_{126}$ | $\begin{pmatrix} -\frac{19}{6} \\ \frac{9}{2} \\ \frac{9}{2} \\ -7 \end{pmatrix}$ | $\begin{pmatrix} \frac{35}{6} & \frac{1}{2} & \frac{3}{2} & 12 \\ \frac{3}{2} & \frac{15}{2} & \frac{15}{2} & 12 \\ \frac{9}{2} & \frac{15}{2} & \frac{25}{2} & 4 \\ \frac{9}{2} & \frac{3}{2} & \frac{1}{2} & -26 \end{pmatrix}$ |

right-symmetric Higgs boson spectrum, so we must assume that the (3,1,10) submultiplet of 126 also has mass around $M_I$. The one- and two-loop coefficients of the RG equations that appear in Eq. (4) are then given by the entries in Table I.

These coefficients determine the equations, but in order to solve them we also need the boundary conditions. At the scale $M_Z$, these are given in Eq. (1). We discuss below in some detail how to put the matching conditions at the scales $M_I$ and $M_U$.

We start with $M_U$, where, in general, a grand group $\mathcal{G}$ [which is SO(10) in this paper] breaks into several factors: $\mathcal{G} \to \prod_i \mathcal{G}_i$. If one solves one-loop RGE's, one demands that the coupling constant of each $\mathcal{G}_i$ should equal that of the grand group $\mathcal{G}$ at the scale $M_U$. This is no more the correct condition when one solves two-loop RGE's because of virtual effects of gauge bosons and other particles which acquire masses at the scale $M_U$. The correct matching conditions have been derived by Hall [14]. Neglecting terms involving logarithms of various mass ratios which are expected to be small, we can write these conditions as [13]

$$\omega_{\mathcal{G}} - \frac{C_{\mathcal{G}}}{12\pi} = \omega_{\mathcal{G}_i} - \frac{C_{\mathcal{G}_i}}{12\pi}\,, \qquad (7)$$

where $C_{\mathcal{G}}$, e.g., stands for the quadratic Casimir invariant for the group $\mathcal{G}$.

One can use Eq. (7) even at the scale $M_I$ if some factor of the gauge group below $M_I$ (i.e., $\{1_Y 2_L 3_c\}$) comes entirely from one factor of $G_I$. For example, in chains 1a and 1b, the factor $\{3_c\}$ comes entirely from the group $\{4_C\}$ above $M_I$. In these chains, then, the matching of $\omega_{3c}$ and $\omega_{4C}$ at the scale $M_I$ is governed by Eq. (7).

However, in all the chains we discuss, the group $\{1_Y\}$ comes from two different factors in $G_I$. In this case, the matching conditions are different. Following again Hall's procedure [14] and neglecting logarithmic terms, we find

$$\begin{aligned}
\text{chains 1a, 1b}: \omega_{1Y} &= \frac{3}{5}\left(\omega_{2R} - \frac{C_2}{12\pi}\right) \\
&\quad + \frac{2}{5}\left(\omega_{4C} - \frac{C_4}{12\pi}\right), \\
\text{chains 2a, 2b}: \omega_{1Y} &= \frac{3}{5}\left(\omega_{2R} - \frac{C_2}{12\pi}\right) + \frac{2}{5}\omega_{1X}\,, \\
\text{chain 3}: \omega_{1Y} &= \frac{3}{5}\omega_{1R} + \frac{2}{5}\left(\omega_{4C} - \frac{C_4}{12\pi}\right), \\
\text{chain 4}: \omega_{1Y} &= \frac{3}{5}\omega_{1R} + \frac{2}{5}\omega_{1X}\,. 
\end{aligned} \qquad (8)$$

Here, $C_2$ and $C_4$ are Casimir invariants for the groups SU(2) and SU(4), which are 2 and 4 respectively. Notice that if all the Casimir invariants are omitted, these equations reduce to the 1-loop matching conditions.

We now integrate the RG equations of Eq. (4) numerically for different chains to find the evolution of the coupling constants. The results are presented in Figs. 1–3 for all chains where it was possible to find a solution for the intermediate scale. For one chain, viz., chain 4, no solution can be obtained with the spectrum given in Table I. This can be inferred from the earlier work of Gipson

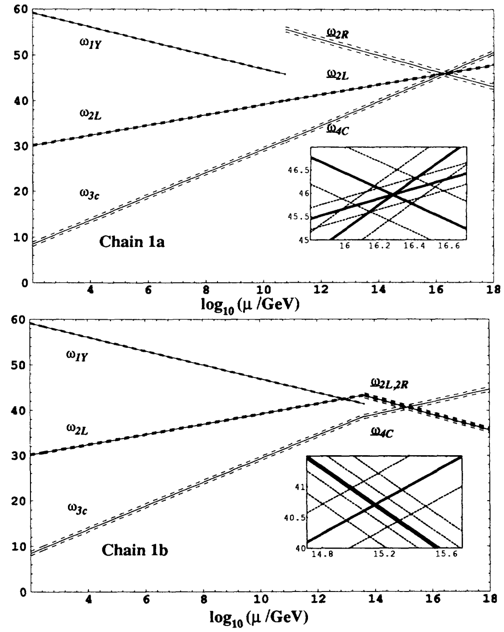


FIG. 1. Evolution of coupling constants for chains 1a and 1b. $\omega$ is defined in Eq. (5). Above the intermediate scale, we plot $\underline{\omega}_i$, which equals $\omega_i - (C_i/12\pi)$ for each subgroup, where $C_i$ is the quadratic Casimir invariant for the same. The reason for this is explained in the text.

and Marshak [12], who found the value of $\sin^2\theta_W(M_W)$ to be definitely smaller than 0.215 for this chain, which contradicts Eq. (1).

For the other chains, the results can be read from the graphs in Figs. 1–3. We list the values of the intermediate scale and the range of values of the unification scale in Table II for the sake of convenience. To show explicitly

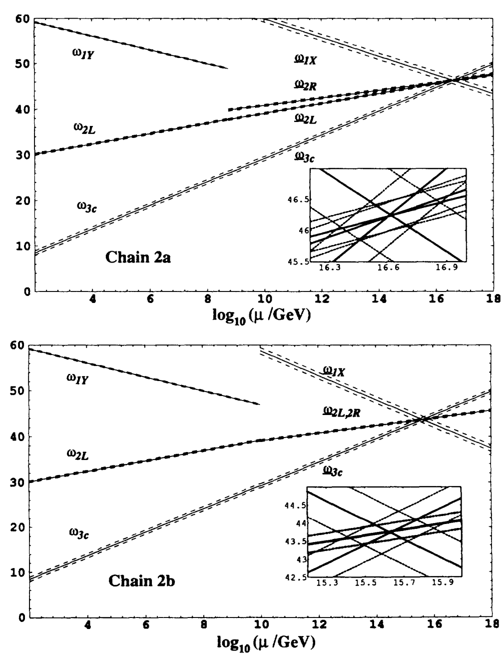


FIG. 2. Evolution of coupling constants for chains 2a and 2b. Notations are defined in Fig. 1.

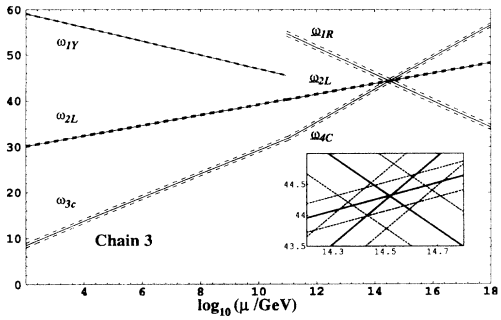


FIG. 3. Evolution of coupling constants for chain 3. Notations are defined in Fig. 1.

that the unification criterion, Eq. (7), has been satisfied with these choices of scales, we have plotted the value of $\omega - (C/12\pi)$ for each subgroup above $M_I$ instead of $\omega$. According to Eq. (7), these values should be equal for all subgroups at the unification scale.

We now discuss the constraints on the unification scale that comes from proton decay. The present experimental limit is

$$\tau(p \to e^+\pi^0) > 3 \times 10^{32}\,\mathrm{yr}\,. \tag{9}$$

The theoretical estimate for the lifetime is

$$\tau^{-1} \simeq \frac{m_p^5}{\omega_U^2\, M_U^4}\,. \tag{10}$$

Using Eq. (9), we then obtain the constraint

$$\left(\frac{\omega_U}{40}\right)\left(\frac{M_U}{10^{15}\,\mathrm{GeV}}\right)^2 > 2.5\,. \tag{11}$$

From this constraint, we see that the chain 3 is definitely ruled out. Chain 1b is barely acceptable, but any improvement in proton lifetime would rule it out. Chains 1a, 2a, and 2b are thus preferred from the present data. For chain 1a, the ratio $\omega_{2R}/\omega_{2L} = 1.4$ whereas for chain 2a, $\omega_{2R}/\omega_{2L} = 1.055$ at the intermediate scale.

TABLE II. Intermediate scale ($M_I$) and unification scale ($M_U$) obtained by solving the renormalization group equations.

| Chain | $G_I$ | $\log_{10}(M/1\ \mathrm{GeV})$ | | $\omega_U$ |
|---|---|---|---|---|
| | | $M_I$ | $M_U$ | |
| 1a | $2_L 2_R 4_C$ | 10.75 | $16.3 \pm 0.3$ | $45.9 \pm 0.6$ |
| 1b | $2_L 2_R 4_C \times P$ | 13.65 | $15.1 \pm 0.4$ | $40.7 \pm 0.5$ |
| 2a | $2_L 2_R 1_X 3_c$ | 8.7 | $16.6 \pm 0.3$ | $46.2 \pm 0.4$ |
| 2b | $2_L 2_R 1_X 3_c \times P$ | 10.0 | $15.6 \pm 0.3$ | $43.7 \pm 0.4$ |
| 3 | $2_L 1_R 4_C$ | 11.0 | $14.5 \pm 0.2$ | $44.3 \pm 0.4$ |

Thus, we have identified the chains that are viable with one intermediate scale. We note that in the allowed chains, intermediate scales are roughly in the range of $10^9$–$10^{11}$ GeV. This range is interesting for its predictions regarding neutrino masses. In each case, the generator of $\{1_R\}$ breaks at the scale $M_I$, so this is the scale which enters the seesaw relation [15]. If we assume that the Yukawa couplings generating the masses are of the same order in any generation, we have $m_{\nu_e} \sim \frac{1}{3} m_u M_W / M_I$ and similarly for other neutrino masses. If we require $m_{\nu_\tau} \sim 30$ eV, we obtain $M_I \sim 10^{11}$ GeV using $m_t \sim 100$ GeV. This value of $M_I$ then implies $m_{\nu_\mu} \sim 10^{-3}$ eV and $m_{\nu_e} \sim 10^{-7}$ eV. Note that the mass-square differences between the $\nu_\mu$ and the $\nu_e$ are in the range necessary for the solution of the solar neutrino problem via the Mikheyev-Smirnov-Wolfenstein mechanism.

In this paper, we have considered SO(10) breaking via one intermediate scale. We plan to consider multistage breaking in a forthcoming article. We expect, in such a scenario, one breaking scale close to $M_I$ and the possibility of having another scale in the TeV range, for example, an additional $Z$ boson.

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