Consequences of a neutrino magnetic moment in inelastic neutrino reactions

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For any inelastic neutrino interaction $\nu+A\to\nu+X$, the presence of a neutrino magnetic moment adds to the normal cross section an amount that is calculable in terms of the virtual photon process $\gamma^*+A\to X$. In particular, there is a correction to the neutrino forward-scattering theorem expressible in terms of the neutrino magnetic moment and the photoproduction cross section $\gamma+A\to X$. There is an effect on the neutral current to charged current ratio at high energies. Effects also occur in the interaction of low-energy neutrinos with atoms; this is illustrated for the coherent process $\nu+\text{atom}\to\nu+\text{atom}+\gamma$.

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It has been noted that an anomalously large neutrino magnetic moment [1] can have an impact on the elastic neutrino-electron scattering process $\nu+e\to\nu+e$ [2]. In particular, the differential cross section in the kinetic energy of the recoil electron obtains an additive correction

$$\frac{d\sigma}{dT_e}=\left[\frac{d\sigma}{dT_e}\right]_{\text{normal}}+\pi r_e^2\left[\frac{\mu_\nu}{\mu_B}\right]^2\left[\frac{1}{T_e}-\frac{1}{E}\right], \quad (1)$$

where $r_e=\alpha/m_e$, μ_ν=neutrino magnetic moment, $\mu_B=e/(2m_e)$ is the Bohr magneton, and E is the incident neutrino energy.

The purpose of this Brief Report is to present a general result that gives the corresponding modification of the neutrino cross section for any inelastic process $\nu+A\to\nu+X$. Denoting by k, k', and p the momenta of the initial neutrino, final neutrino, and the target particle A, and introducing the variables $q=k-k'$, $Q^2=-q^2$, $y=p\cdot q/p\cdot k=\nu/E$, and $x=Q^2/2p\cdot q$, the correction due to a neutrino magnetic moment is

$$\left[\frac{d\sigma}{dQ^2d\nu}\right]_{\nu+A\to\nu+X}=\left[\frac{d\sigma}{dQ^2d\nu}\right]_{\text{normal}}+\frac{\mu_\nu^2}{4\pi^2}\frac{1}{\sqrt{\nu^2+Q^2}}\left\{\left[1-y-\frac{Q^2}{4E_\nu^2}\right]\sigma_T+(1-\tfrac{1}{2}y)^2\sigma_L\right\}. \quad (2)$$

Here σ_T and σ_L denote the transverse and longitudinal cross sections for the virtual photon process $\gamma^*+A\to X$, which are measurable in the corresponding electroproduction reaction $e+A\to e+X$:

$$\left[\frac{d\sigma}{dQ^2d\nu}\right]_{e+A\to e+X}=\frac{\alpha}{\pi}\frac{1}{Q^2}\frac{1}{\sqrt{\nu^2+Q^2}}\left[\left[1-y+\tfrac{1}{2}y^2+\frac{Q^2}{4E^2}\right]\sigma_T+\left[1-y-\frac{Q^2}{4E^2}\right]\sigma_L\right]. \quad (3)$$

The derivation of the result (2) is as follows. The squared matrix element that describes the reaction $\nu+A\to\nu+X$, induced by a neutrino magnetic moment μ_ν, can be written as

$$|\overline{M}|^2=\frac{e^2\mu_\nu^2}{Q^4}N^{\mu\nu}J_{\mu\nu},$$

where

$$N^{\mu\nu}=\tfrac{1}{2}\sum_{\text{spins}}[\bar{u}(k')\sigma^{\mu\alpha}q_\alpha u(k)][\bar{u}(k)\sigma^{\nu\beta}q_\beta u(k')]$$
$$=\tfrac{1}{2}\text{Tr}[\not{k}'\sigma^{\mu\alpha}\not{k}\sigma^{\nu\beta}]q_\alpha q_\beta$$
$$=2k\cdot k'(k+k')_\mu(k+k')_\nu$$

and

$$J_{\mu\nu}=\frac{1}{2S_A+1}\sum_{\text{spin }A}\langle A|J_\mu^{\text{em}}|X\rangle\langle X|J_\nu^{\text{em}}|A\rangle. \quad (4)$$

The tensor $J_{\mu\nu}$ is identical to the tensor which describes the electroproduction process $e+A\to e+X$. Writing $J_{\mu\nu}$ in terms of σ_T and σ_L in the usual way [3] and contracting with the neutrino tensor $N^{\mu\nu}$, we obtain the result (2).

The magnetic-moment-induced term in Eq. (2) has the feature that it does not vanish in the limit $Q^2=0$. This has implications for neutrino scattering in the forward direction. The normal neutrino cross section for forward scattering is given by Adler's PCAC (partially conserved axial-vector current) theorem [4]:

$$\left[\frac{d\sigma}{dQ^2 d\nu}\right]_{\text{normal, } Q^2=0} = \frac{G^2}{2\pi^2}\frac{1}{\nu}(1-y) f_{\pi^0}^2 \sigma_{\pi^0}(\nu) \ . \qquad (5a)$$

Here $\sigma_{\pi^0}(\nu)$ is the cross section for $\pi^0 + A \to X$ at pion energy $E_\pi = \nu$, and $f_{\pi^0} = 93$ MeV. This theorem applies to semileptonic processes, i.e., when A and X are hadronic systems. In cases where the process $\nu + A \to \nu + X$ is governed by the purely leptonic weak interaction, the equivalent result is formally obtained [5,6] by replacing the π^0 by a (hypothetical) axion and f_{π^0} by v [the axion-lepton interaction being defined by $(m_l/v) a^0 \bar{l}\gamma_5 l$]:

$$\left[\frac{d\sigma}{dQ^2 d\nu}\right]_{\text{normal, } Q^2=0} = \frac{G^2}{2\pi^2}\frac{1}{\nu}(1-y) v^2 \sigma_{a^0}(\nu) \ . \qquad (5b)$$

Our general result (2) states that, in the presence of a neutrino magnetic moment, the forward-scattering theorem [in the form (5a) or (5b)] receives a correction given by

$$\left[\frac{d\sigma}{dQ^2 d\nu}\right]_{\text{magnetic, } Q^2=0} = \frac{\mu_\nu^2}{4\pi^2}\frac{1}{\nu}(1-y)\sigma_\gamma(\nu) \ , \qquad (6)$$

$\sigma_\gamma(\nu)$ being the cross section of the photoproduction process $\gamma + A \to X$ at energy $E_\gamma = \nu$.

In inclusive neutrino-nucleon scattering $\nu + N \to \nu +$ anything at high energy, the normal and magnetic-moment-induced cross sections are [7]

$$\left[\frac{d\sigma}{dx\, dy}\right]_{\text{normal}} = \frac{G^2 ME}{\pi}[g_L^2\{q(x)+\bar{q}(1-y)^2\} + g_R^2\{\bar{q}(x)+q(x)(1-y)^2\}] \ ,$$

$$\left[\frac{d\sigma}{dx\, dy}\right]_{\text{magnetic}} = \frac{\mu_\nu^2 \alpha}{Q^2} ME(1-y)\tfrac{5}{9}[q(x)+\bar{q}(x)] \ . \qquad (7)$$

Thus the correction relative to the normal neutrino cross section is of order $\mu_\nu^2 \alpha\pi / G^2 Q^2$. In the limit $Q^2 \to 0$, the ratio of the magnetic and normal terms is determined by Eqs. (5a) and (6) to be $(\mu_\nu^2 / G^2 f_{\pi^0}^2)(\sigma_{\gamma N}/\sigma_{\pi N})$. The correction to the neutral current cross section given in Eq. (7) implies that the neutral current to charged current ratios R_ν and $R_{\bar{\nu}}$ receive corrections proportional to μ_ν^2. A cursory examination of the semileptonic neutrino data suggests an upper limit for the magnetic moment of ν_μ of about $10^{-8}\mu_B$. This is weaker than the limit derived from $\nu_\mu + e \to \nu_\mu + e$ [8].

Reactions that are especially sensitive to the effects of a neutrino magnetic moment are low-energy neutrino interactions with atomic electrons. These include the neutrino-induced photoeffect ($\nu + A \to \nu + e^- + A^+$), coherent radiative scattering ($\nu + A \to \nu + A + \gamma$), and incoherent Compton scattering ($\nu + A \to \nu + A^* + \gamma$). In all of these cases, the presence of a neutrino magnetic moment leads to an enhancement of the cross section which can be calculated using Eq. (2). As an example, we consider the coherent radiative process

$$\nu + A \to \nu + A + \gamma \ , \qquad (8)$$

where the atom A recoils without excitation or ionization. This reaction is a significant energy-loss mechanism for neutrinos whose wavelength is comparable to atomic dimensions ($E \sim R_{\text{at}}^{-1}$). The normal weak cross section for the process involves the vector coupling constant C_V of the neutrinos with electrons, which has the value $\frac{1}{2} + 2\sin^2\Theta_w$ for ν_e and $-\frac{1}{2} + 2\sin^2\Theta_w$ for ν_μ or ν_τ. The differential cross section (neglecting the longitudinal term) is [6]

$$\left[\frac{d\sigma(\nu + A \to \nu + A + \gamma)}{dQ^2 d\nu}\right]_{\text{normal}} = \frac{G^2}{2\pi^2}\frac{Q^2}{\sqrt{Q^2+\nu^2}}\left\{1 - y + \frac{1}{2}y^2 + \frac{Q^2}{4E^2}\right\} \times \frac{C_V^2}{4\pi\alpha}\sigma(\gamma^* A \to \gamma A) \ , \qquad (9a)$$

where $\sigma(\gamma^* A \to \gamma A)$ is the Rayleigh scattering cross section for a transverse virtual photon of mass q^2. If the neutrino possesses a magnetic moment, there is a correction to the above result given by

$$\left[\frac{d\sigma(\nu + A \to \nu + A + \gamma)}{dQ^2 d\nu}\right]_{\text{magnetic}} = \frac{\mu_\nu^2}{4\pi^2}\frac{1}{\sqrt{Q^2+\nu^2}}\left\{1 - y - \frac{Q^2}{4E^2}\right\}\sigma(\gamma^* A \to \gamma A) \ . \qquad (9b)$$

For a quantitative comparison, we have integrated the cross sections (9a) and (9b) over the variables Q^2 and ν. The Rayleigh cross section $\gamma^* A \to \gamma A$ was calculated using a realistic atomic form factor, with allowance for the virtuality of the initial photon [6]. The resulting cross

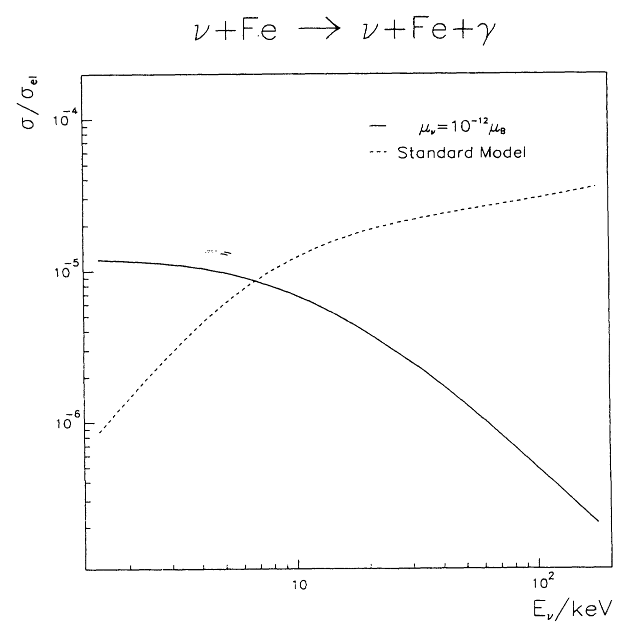

FIG. 1. Cross section of $\nu_e + \text{Fe} \to \nu_e + \text{Fe} + \gamma$ in units of $\sigma(\nu_e + e \to \nu_e + e)$.

section for radiative ν_e scattering off Fe atoms is shown in Fig. 1. We observe that the contribution from a neutrino magnetic moment $\mu_{\nu e}=10^{-12}\mu_B$ exceeds the normal cross section for $E_\nu \lesssim 10$ keV.

In the case of ν_μ or ν_τ scattering, the weak vector coupling $C_V=-\frac{1}{2}+2\sin^2\Theta_w=-0.04$ is strongly suppressed compared to that of ν_e. Thus the contribution from a neutrino magnetic moment has an enhanced importance in the coherent scattering of ν_μ and ν_τ from atomic electrons.

For high-energy neutrinos ($E_\nu \gg 100$ keV) the dominant contribution to coherent radiative scattering is from scattering on the nucleus, $\nu+\mathcal{N}\to\nu+\mathcal{N}+\gamma$. The standard model calculation of this process was given in Ref. [9]. The magnetic-moment correction may be obtained from Eq. (9b) by inserting the Compton cross section $\sigma(\gamma^*\mathcal{N}\to\gamma\mathcal{N})$. In the intermediate energy domain $E_\nu \sim 100$ keV, there will be interference of nuclear scattering with scattering from the electron cloud. (For the case of *elastic* neutrino scattering from atoms, such interference effects were studied in Ref. [10].)

[1] M. B. Voloshin, M. I. Vysotskii, and L. B. Okun, Zh. Eksp. Teor. Fiz. **91**, 754 (1986) [Sov. Phys. JETP **64**, 446 (1986)].

[2] A. V. Kyuldjien, Nucl. Phys. **B243**, 387 (1984); A. O. Barut, Z. Z. Aydin, and I. H. Duru, Phys. Rev. D **26**, 1794 (1982).

[3] See, e.g., F. Halzen and A. D. Martin, *Quarks and Leptons: An Introductory Course in Modern Particle Physics* (Wiley, New York, 1984).

[4] S. L. Adler, Phys. Rev. **135**, B963 (1964).

[5] L. M. Sehgal, Phys. Rev. D **38**, 2750 (1988).

[6] A. Weber and L. M. Sehgal, Nucl. Phys. **B359**, 262 (1991).

[7] Here g_L^2 and g_R^2 are neutral current couplings given by $g_L^2=\frac{1}{2}-\sin^2\Theta_w+\frac{5}{9}\sin^4\Theta_w$, $g_R^2=\frac{5}{9}\sin^4\Theta_w$, and $q,\bar{q}$ are related to quark distributions by $q(x)=x[u(x)+d(x)]$, $\bar{q}(x)=x[\bar{u}(x)+\bar{d}(x)]$. [For $\bar{\nu}$ scattering, interchange g_L and g_R in Eq. (7).]

[8] See, e.g., D. A. Krakauer *et al.*, Phys. Lett. B **252**, 177 (1990).

[9] D. Rein and L. M. Sehgal, Phys. Lett. **104B**, 394 (1981).

[10] L. M. Sehgal and M. Wanninger, Phys. Lett. B **178**, 313 (1986).