Current quark mass and chiral-symmetry breaking in QCD at finite temperature in a mean-field approximation

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The effects of current quark masses on chiral-symmetry breaking in QCD at finite temperature are examined using a mean-field approximation. The current quark mass plays a role analogous to that of an external magnetic field in the ferromagnetic transition, as it explicitly violates the chiral symmetry whose restoration characterizes the phase transition. For small enough masses, the order parameter, related to the quark condensate, maintains the features of the massless case up to the critical temperature, and then it approaches a constant value depending on the current mass term itself. The consequences on the behavior of the pion mass and decay constant are also studied. Our calculations are done with a variational composite operator approach. We also discuss the critical exponents (in temperature, mass, and for susceptibility) that follow in general from assuming the possible validity of a Landau mean-field theory.

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I. INTRODUCTION

Strongly interacting matter is expected to exhibit deconfinement and chiral-symmetry restoration at high temperatures and/or densities. This seems to be true at least in two limiting situations: namely, for a pure gauge theory (deconfinement), from lattice simulations, and from model calculations with massless quarks.

We shall refer here to previous attempts based on phenomenological chiral models, indicating the possible phase structure in temperature [1].

However, in the realistic case of quarks with finite current masses, chiral symmetry is already broken explicitly and therefore the phase transition suppressed, as happens in a ferromagnetic case in the presence of an external magnetic field. We have thus to better specify to what extent one can still speak of phase transition.

In this paper we extend the analysis performed in a QCD model [1] considered in the chiral limit to the case of massive quarks. First we analyze in a general way the consequences which arise if we postulate the validity of a mean-field approach in QCD. In particular, we summarize the derivation of various critical exponents describing how quantities such as the chiral condensate approach the critical point. In the context of a previously proposed specific approximation to QCD [2], we check that the conditions for the validity of the mean-field approach are satisfied. Specifically we verify the absence of infrared divergences in the coefficients of the expansion of the effective potential around the critical point. This model calculation will allow us to get definite values for the coefficients of the expansion of the effective potential.

In the framework of the model we study the extrema of the effective potential with respect to a dimensionless field (order parameter) simply related to the quark-antiquark condensate. We find that the order parameter as a function of T shows the same main features as in the massless case up to the critical point, then it approaches a constant value depending on the current quark mass.

We also study the gap in the value of the quark-antiquark condensate between the region of low temperatures and the region of high temperatures for increasing masses. It is evident that the phenomenon of dynamical chiral-symmetry breaking concerns only light quarks, because for the heavy ones the dynamically generated mass is much smaller than the current mass (see, for instance the discussion in Ref. [3]).

Finally we examine how a current mass influences the properties of the light mesons, such as decay constant and mass.

II. MEAN-FIELD EFFECTIVE POTENTIAL

In previous work [1] we have shown how the mechanism of dynamical chiral-symmetry breaking, in massless QCD, and its restoration at some finite temperature, can be described by an effective potential approach. In particular we found that, in such a massless case, the order parameter of the theory (essentially the quark-antiquark condensate) and some other related physical quantities [such as $f_\pi(T)$] show the typical behavior coming from a Landau mean-field theory near the critical point for a second-order phase transition [4].

It is therefore interesting to consider a more realistic model in which a mass term is present, which breaks explicitly the original chiral symmetry of the Lagrangian, and to look for the behavior of various physical parameters around the critical point. As expected, also in this case our model is equivalent to a mean-field theory. In this section we will summarize the general results concerning the critical exponents that can be derived in such a framework. For simplicity we will for the moment consider the case of a single flavor. The discussion can be easily extended to the more general case.

With massive quarks the situation is very similar to that of a ferromagnetic system in presence of a small external magnetic field $\mathcal{H}$. The correspondence is

$$\langle \bar{\psi}\psi \rangle_T \leftrightarrow \mathcal{M}(T), \quad m_0 \leftrightarrow \mathcal{H} , \tag{2.1}$$

where $\langle \bar{\psi}\psi \rangle_T$ is the quark-antiquark condensate, $\mathcal{M}$ is the magnetization, and m_0 is the current quark mass.

Let us assume that there exists an effective potential describing the broken phase of QCD at $T=0$, and let us extend this approach to $T\neq 0$. The effective potential will be a function of the order parameter

$$\chi(T) \sim \langle \bar{\psi}\psi \rangle_T$$

and of the explicit symmetry-breaking term m_0.

Let us suppose that our effective potential admits a mean-field expansion à la Landau. This means that the effective potential, for small values of m_0, is such that [5] (i) it can be expanded in powers of χ around $\chi=0$, (ii) the coefficients of the expansion are regular functions of the temperature for T close to T_c, the temperature at which the coefficient of χ^2 vanishes, and (iii) the coefficient of χ^4 in the expansion is positive.

However as discussed in Ref. [5] the mean-field approximation is valid only if we can safely take the limit $T\to T_c$ in the coefficients of the expansion of the effective potential.

As a rule, for $D>4$ (D = dimension of space-time) one obtains the universal predictions of the mean-field theory; however, for $D<4$ the infrared divergences which affect the coefficients of the expansion show that the predictions of the mean-field theory cannot be correct in general.

Because we are interested in the case $D=4$, we have to be very careful in this kind of approximation. The hypothesis that the effective potential for QCD can be expanded à la Landau near the critical point will be safe only if the coefficients of the expansion are not plagued by infrared divergences.

For the time being let us assume that we can safely expand à la Landau the effective potential. We will verify *a posteriori* that in our model all the hypothesis for the expansion are fulfilled. For our purposes, it will be enough to keep the first order in m_0 and the fourth order in χ in the expansion of the effective potential around $\chi=0$ ($T=T_c$) and $m_0=0$. We then get

$$V(\chi,m_0,T)|_{m_0\to 0,\,T\to T_c} \simeq a_0(T)+a_2(T)\chi^2 + a_4(T)\chi^4 + b_1(T)m_0\chi + b_3(T)m_0\chi^3 + \cdots \tag{2.2}$$

The critical exponents are defined as follows. The β exponent by the behavior of the order parameter at $m_0=0$ and $T\to T_c$:

$$\chi(T) \sim \left[1-\frac{T}{T_c}\right]^{\beta} .$$

The δ exponent by the behavior of the order parameter at $T=T_c$ and $m_0\to 0$:

$$\chi(T_c) \sim m_0^{1/\delta} .$$

The γ exponent by looking at the derivative of the order parameter with respect to the mass (magnetic susceptibility) at $m_0=0$ and $T\to T_c$:

$$\left.\frac{\partial\chi(T)}{\partial m_0}\right|_{m_0=0} \sim \left[1-\frac{T}{T_c}\right]^{-\gamma} .$$

The critical exponents can be easily obtained from the effective potential (2.2). For instance, the critical temperature is determined by the vanishing of the coefficient $a_2(T)$ at $m_0=0$,

$$a_2(T_c)=0 , \tag{2.3}$$

and, since our system is in a broken phase for $T<T_c$, we have

$$a_2(T)\le 0, \quad T\le T_c \tag{2.4}$$

and, furthermore,

$$a_4(T)>0, \quad T\simeq T_c . \tag{2.5}$$

By expanding $a_2(T)$ around the critical temperature

$$a_2(T) \simeq A(T_c)\left[1-\frac{T}{T_c}\right] \tag{2.6}$$

we can determine the minimum in the $T\to T_c$ limit by using Eq. (2.2) for the effective potential. We get (for $m_0=0$)

$$\chi(T) = \begin{cases} a_\chi(T_c)\left[1-\frac{T}{T_c}\right]^{1/2}, & T\le T_c, \\ 0, \quad T\ge T_c , \end{cases} \tag{2.7}$$

with

$$a_\chi(T_c) = -\left[\frac{-A(T_c)}{2a_4(T_c)}\right]^{1/2} \tag{2.8}$$

which leads to $\beta=1/2$.

Again, from the extremum condition,

$$2a_2(T)\chi+4a_4(T)\chi^3+b_1(T)m_0+3b_3(T)m_0\chi^2=0 \quad (2.9)$$

and, for $T=T_c$, we find, at the leading order in m_0,

$$\chi(T_c)=-\left[\frac{b_1(T_c)}{4a_4(T_c)}\right]^{1/3} m_0^{1/3} \quad (2.10)$$

giving $\delta=3$ for the critical exponent.

Finally, by evaluating the derivative with respect to m_0, at $m_0=0$, of Eq. (2.9), and in the limit $T\to T_c$, we get

$$\left.\frac{\partial\chi(T)}{\partial m_0}\right|_{m_0=0}=-\frac{b_1(T_c)}{8a_4(T_c)a_\chi^2(T_c)}\left[1-\frac{T}{T_c}\right]^{-1}, \quad T<T_c \quad (2.11)$$

and

$$\left.\frac{\partial\chi(T)}{\partial m_0}\right|_{m_0=0}=-\frac{b_1(T_c)}{4a_4(T_c)a_\chi^2(T_c)}\left[\frac{T}{T_c}-1\right]^{-1}, \quad T>T_c \quad (2.12)$$

which gives for the susceptibility the critical exponent $\gamma=1$. Also, the susceptibility is discontinuous at $T=T_c$ with a prescribed ratio of 2 between the coefficients of the term in temperature after and before the critical temperature.

Notice that the critical exponents derived from the mean-field approximation satisfy the Widom law $\gamma=\beta(\delta-1)$ [6].

The previous results are rather general as they rely on the assumption that the coefficients of the mean-field expansion are free of infrared divergences. This infrared safety depends obviously on the model. We will see in the next section that in our model the coefficients are finite in the limit $T\to T_c$, and thus the previous discussion applies indeed to our effective potential. For the coefficients a_2 and a_4 this has been already observed in Ref. [4].

III. THE EFFECTIVE POTENTIAL

To explicitly evaluate the effective potential we refer to the study performed in the massless quark case [1] (trying to stress the differences due to the presence of the mass term) and to Ref. [2], where all the details of the model in the massive case and at zero temperature can be found.

Calculations are carried out in the imaginary-time formalism, with Euclidean metrics. Here are the main steps. One starts from the QCD Lagrangian with a mass term for the quarks and considers the effective action for composite operators as a functional of the fermion self-energy. We stress that this approach, developed from a method of Cornwall, Jackiw, and Tomboulis [7], is based on a variational method and not on a perturbative approximation. Following Ref. [2] the zero temperature Euclidean effective action for a QCD-like gauge theory is

$$\Gamma[S]=-\Gamma_2[S]+\mathrm{Tr}\left[S\frac{\delta\Gamma_2}{\delta S}\right]-\mathrm{Tr}\ln\left[S_0^{-1}+\frac{\delta\Gamma_2}{\delta S}\right], \quad (3.1)$$

where S and S_0 are the full and the free quark propagator, respectively. S_0 is given by

$$S_0(p)=(i\not{p}-m)^{-1} \quad (3.2)$$

where m is the quark mass matrix. Γ_2 is given by the two-particle-irreducible vacuum diagrams, and Σ is the dynamical variable of the theory defined by the equation

$$\Sigma=-\frac{\delta\Gamma_2}{\delta S}. \quad (3.3)$$

At the minimum of the action, Σ is nothing but the fermion self-energy.

Here we will neglect the mixing between different flavors originating, for instance, by terms such as the 't Hooft fermion determinant [8]. It follows that only the flavor-diagonal elements of the fermion self-energy can be different from zero at the minimum [2]. With vanishing off-diagonal terms, the effective potential decomposes into the sum of n_f contributions (n_f=number of flavors), one for each flavor. Therefore, to study the minima of the effective potential, it is formally sufficient to consider a single contribution. Of course, the choice of a given flavor number is reflected in the particular parameters assumed. In the present paper, as in Ref. [2], we will take $n_f=3$ and a number of color $N=3$. The values of the parameters will be specified later on.

In QCD the operator product expansion suggests (neglecting logarithmic corrections) to take for Σ a momentum behavior as $1/p^2$ for large p^2. We have chosen as a variational ansatz [1]

$$\Sigma(p)=\chi\frac{M^3}{M^2+p^2}, \quad (3.4)$$

where $M\simeq 280$ MeV (see Ref. [1]) is a mass scale and χ (order parameter) is a dimensionless field. The effective potential, defined as $V(\chi)=\Gamma(\chi)/\Omega$, with Ω the space-time volume, will be minimized with respect to χ. The value of χ at the minimum, $\bar{\chi}(T)$, is related to the chiral condensate, renormalized at the scale M (see later), by

$$\langle\bar{\psi}\psi\rangle_T=\frac{3M^3}{g^2(T)}\bar{\chi}(T). \quad (3.5)$$

In Eq. (3.5) we have taken the QCD coupling $g(T)$ as [1]

$$\frac{g^2(T)}{2\pi^2}\equiv\frac{1}{c(T)}=\frac{1}{c_0+(\pi^2/b)\ln(1+\xi T^2/M^2)}, \quad (3.6)$$

where $b=24\pi^2/(11N-2n_f)$, $c_0=0.554$ (see Ref. [1]), and ξ is a parameter which has to be determined phenomenologically. By comparing our model in the low-T regime with the results of Ref. [9], we have found $\xi=0.44(n_f^2-1)/n_f$, which gives $\xi\simeq 1$ for $n_f=3$.

In Eq. (3.1) we have evaluated Γ_2 at the two-loop level. The final expression of our effective potential is

$$V=\frac{NM^4}{4\pi^2}\sum_{i=1}^{n_f}\bar{V}(\chi_i,\alpha_i), \quad (3.7)$$

where $\alpha_i=m_i/M$ and

$$\bar{V}(\chi,\alpha)=\frac{c(T)}{3}\chi^2-\frac{1}{2}\int_0^1 dy\left[\frac{1-y}{y^3}\ln\left[1+\frac{\chi^2y^3+2\alpha\chi y^2}{1-y+\alpha^2y}\right]-\frac{2\alpha\chi}{y}\right]-8\frac{T}{M}\sum_{k=1}^{3}\int_0^\infty dy\, y^2\ln(1+e^{-\beta M\sqrt{y^2+z_k}})+\alpha\chi z(T) \quad (3.8)$$

with $\beta=1/T$ and $\alpha=m_0/M$. The right-hand side (RHS) of Eq. (3.8) is composed of five terms: the first one comes from Γ_2, whereas the second is the contribution of the one-loop term at $T=0$, which is ultraviolet divergent for $\alpha\neq0$ and it is regularized by the third term. The fourth term is the part explicitly dependent on T of the one-loop term, with z_k satisfying

$$x^3+(2+\alpha^2)x^2+(1+2\alpha^2+2\alpha\chi)x+(\alpha+\chi)^2=\prod_{k=1}^{3}(x+z_k)\ . \quad (3.9)$$

The last term

$$z(T)=2c(T)-\int_0^1 dt\frac{\hat{\chi}^2(T)t^2}{1-t+\hat{\chi}^2(T)t^3} \quad (3.10)$$

is a finite counterterm added in order to satisfy the renormalization condition discussed below. In Eq. (3.10) $\hat{\chi}$ is the minimum of the effective potential in the case $\alpha=0$, evaluated by neglecting the fourth term in Eq. (3.8). Let us comment about this point.

At $T\neq0$ the effective potential does not acquire any extra divergence with respect to the $T=0$ case. The theory at $T=0$ is renormalized by requiring that the derivative of the effective potential with respect to the chiral-symmetry-breaking term, evaluated at the minimum, in the small mass limit satisfies, for each flavor, the condition [2]

$$\lim_{m_0\to0}\left.\frac{\partial V}{\partial(m_0\langle\bar{\psi}\psi\rangle_0)}\right|_{\min}=1 \quad (3.11)$$

which is equivalent to the Adler-Dashen formula

$$m_\pi^2f_\pi^2=-2m_0\langle\bar{\psi}\psi\rangle_0\ . \quad (3.12)$$

Therefore for $T\neq0$ we have to use a renormalization condition which reduces to (3.11) in the limit $T\to0$; that is, it should (in the limit) reproduce the Adler-Dashen relation.

However, this last equality holds only in the soft pion limit, which means temperatures well below the critical temperature, where the pion mass is expected to increase (see later). As a consequence of these considerations we can neglect the fourth term in the potential, since it goes to zero exponentially when $T\to0$. Therefore we write the generalization of the normalization condition (3.11) at $T\neq0$ as

$$\lim_{m_0\to0}\left.\frac{\partial\tilde{V}}{\partial(m_0\langle\bar{\psi}\psi\rangle_T)}\right|_{\min}=1\ , \quad (3.13)$$

where $\tilde{V}$ is the effective potential (3.7) with the fourth term [see Eq. (3.8)] omitted.

Finally, in writing the effective potential (3.8), one has neglected a constant (dependent on T) coming from the normalization of the one-loop term, which has a thermodynamical meaning, but it is irrelevant for the study of the symmetry breaking. In fact, we remind that the effective potential at the minimum is minus the pressure.

IV. BEHAVIOR OF THE ORDER PARAMETER FOR $T\neq0$

The behavior of the condensate for $T\neq0$ can be studied by searching for the absolute minimum of the effective potential given in (3.8) at various temperatures. By taking the chiral limit $\alpha\to0$, one recovers the situation studied in Ref. [1], where the effective potential is a function of χ^2 and there are two symmetrical absolute minima at $T=0$, which smoothly approach the origin for increasing temperatures. The result is that the condensate vanishes without discontinuities and there is a second-order phase transition.

For $\alpha\neq0$, the effective potential is no longer symmetrical. In this case one finds a relative minimum (for $\chi>0$) which decreases and disappears for growing temperature, whereas the absolute one (for $\chi<0$) approaches continuously a finite value depending on α. This result is not surprising since the symmetry is explicitly broken from the beginning by the mass term, and so no phase transition is expected (of course, the situation could be different when considering the possible running of the current mass with temperature).

As already said, our numerical analysis will be performed for $n_f=N=3$, and we have that, for $\xi\simeq1, T_c\simeq103$ MeV (see Ref. [1]). In Fig. 1 we show the

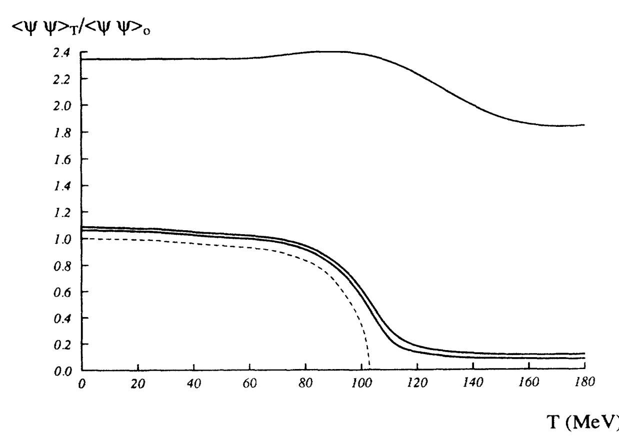

FIG. 1. Fermion condensate vs temperature for $m_u=8$ MeV, $m_d=11$ MeV, and $m_s=181.5$ MeV (from lowest to highest curves, respectively), normalized to $\langle\bar{\psi}\psi\rangle_0\simeq-(197\ \text{MeV})^3$, the value at $T=0$ and $m_0=0$. The dashed line is the normalized condensate in the massless limit.

behavior of the condensate (normalized to its value at $T=0$ and $m_0=0$) vs the temperature, for $m_0=\alpha M=8$, 11 and 181.5 MeV, which are the values for the masses of the up, down, and strange quarks, respectively, obtained by a fit at $T=0$ in SU(3) [2]. It is evident from this figure that as the mass increases, the gap in the quark condensate, when crossing the critical temperature, decreases and tends to vanish for sufficiently high m_0. This can be seen by plotting versus m_0 the difference between the values of the condensate at $T=0$ and at $T=180$ MeV (when it starts to stabilize), normalized to its value at $T=0$ (see Fig. 2).

We can comment on the result qualitatively as follows. When the current mass is zero (chiral limit), the condensate (and thus the fermion mass) is entirely dynamical. For small masses the $T=0$ value of the condensate is not very much affected by the current mass insertion, whereas above the critical temperature it is entirely due to the presence of the mass term, since in this regime it is zero in the massless case. Thus we can roughly say that the region of low temperatures is dominated by the phenomenon of dynamical symmetry breaking, and the high-temperature regime by the explicit symmetry breaking.

In this sense the jump of the condensate that we have plotted in Fig. 2 determines, although only qualitatively, to what extent we can retain the notion of phase transition if a mass term is inserted in the model. For small masses the condensate behaves almost as in the chiral limit: $\langle\bar{\psi}\psi\rangle_T$ steeply decreases at the critical temperature and then it takes a value proportional to the current mass. Thus, for $\alpha \ll 1$, we can still speak of critical phenomena since the changes in the thermodynamical properties of the system will be rather abrupt around T_c due to this steep jump of the condensate. The same does not happen for $\alpha \sim 1$, since in this case the relative variation of the condensate between the low-T and the high-T regions is small.

Similar conclusions have been proposed by other authors [3], who at the same time stress that $\Sigma \sim 1/p^2$ for high momenta can be chose only if the fermion mass is not too large. However in our case we take at most $\alpha \sim 1$.

Finally, we can evaluate analytically the minimum for $T \to \infty$, and for $\alpha \to 0$, by observing that for $T > T_c$ the chiral-symmetry solution is $\bar{\chi}=0$, and therefore we can expand around this point. From Eq. (3.8), at the leading order in α, one finds that the minimum, for $\alpha \ll 1$ and $T \geq T_c$, is

$$\bar{\chi}(T) \simeq -\alpha \left[\frac{\partial/\partial\chi[\partial\bar{V}/\partial\alpha]}{\partial^2\bar{V}/\partial\chi^2} \right]_{\alpha=\chi=0} + O(\alpha^2) \ . \qquad (4.1)$$

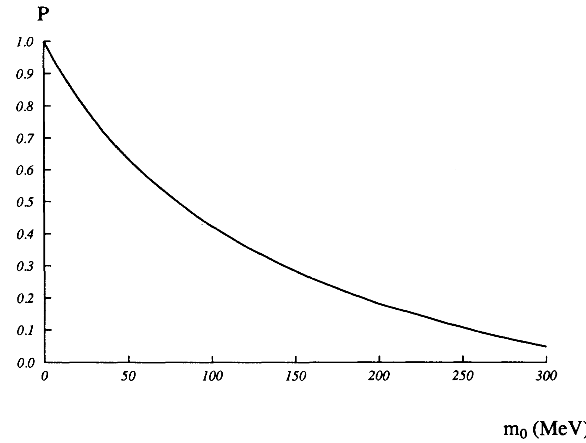

FIG. 2. Jump of the fermion condensate from the low-T to the high-T regions, relative to the $T=0$ value, for increasing masses. Here $P=1-\langle\bar{\psi}\psi\rangle(m_0,T=180\ \text{MeV})/\langle\bar{\psi}\psi\rangle(m_0,T=0)$.

By expanding this expression for high temperatures with standard techniques (see, for instance, Ref. [10]) it turns out that the asymptotic value of the absolute minimum is

$$\bar{\chi}(T \to \infty) = -\frac{13}{3}\alpha + O(\alpha^2) \ .$$

V. PION DECAY CONSTANT AND MASS AT $T \neq 0$

We want to make some comments about the influence of the mass term on the behavior of the pion mass and decay constant at finite temperature, and to compare it with the study performed in the chiral case [4].

To evaluate $f_\pi(T)$ we will consider the matrix element of the zero component of the axial-vector current between the vacuum and the one-pion state:

$$\langle 0|J_0^{5i}|\pi^j(q)\rangle_\beta = i f_\pi(T) m_\pi(T) \delta^{ij} \ . \qquad (5.1)$$

The evaluation of this matrix element has been done in Ref. [4], for the massless case. The only difference is in the quark propagator, which in the massive case is given by

$$S^{-1}(p) = i\hat{p} - \Sigma(p) - m_0 \ , \qquad (5.2)$$

where, for the sake of simplicity, we have assumed that the up and down quarks are degenerate in mass and $m_0=(m_u+m_d)/2=9.5$ MeV.

In the limit $q \to 0$ we find

$$f_\pi^2(T) = 12 \sum_{n=-\infty}^{\infty} (-)^n \int \frac{d^4p}{(2\pi)^4} \frac{\Sigma(p)\left[[\Sigma(p)+m_0] - 2p_0^2 \frac{\partial\Sigma(p)}{\partial p^2} \right]}{[p^2+(\Sigma(p)+m_0)^2]^2} e^{in\beta p_0} \ . \qquad (5.3)$$

To explicitly evaluate $f_\pi(T)$ it is convenient to separate in (5.3) the $n=0$ contribution $\bar{f}_\pi(T)$ and write

$$f_\pi^2(T)=\bar{f}_\pi^2(T)+\hat{f}_\pi^2(T)\ , \tag{5.4}$$

where

$$\hat{f}_\pi^2(T)=\frac{6}{\pi^2}M^2\chi^2(T)\frac{d}{d\chi^2}\left[\sum_{k=1}^{3}\frac{dz_k}{d\chi^2}(1-z_k)\int_0^\infty dy y^2\frac{1-2y^2-3z_k}{\sqrt{y^2+z_k}}\frac{1}{1+\exp(\beta M\sqrt{y^2+z_k})}\right] + \frac{6}{\pi^2}M^2\alpha\chi\frac{d}{d\chi^2}\left[\sum_{k=1}^{3}\frac{dz_k}{d\chi^2}(1-z_k)^3\int_0^\infty dy y^2\frac{1}{\sqrt{y^2+z_k}}\frac{1}{1+\exp(\beta M\sqrt{y^2+z_k})}\right]\ , \tag{5.5}$$

where z_k are given by Eq. (3.9), and the expression must be evaluated at the absolute minimum $\chi=\bar{\chi}(T)$ of the effective potential.

In Fig. 3 we plot $f_\pi(T)$ as a function of T. It shows the same behavior as the condensate and the discussion we have done with regard to $\langle\bar{\psi}\psi\rangle_T$ can be also applied to $f_\pi(T)$.

In order to evaluate the pion mass $m_\pi(T)$ we have to extend the previous discussion on the effective potential by including a pseudoscalar field π, the chiral partner of χ. A complete discussion on this aspect can be found in Ref. [2]. As a consequence the effective potential is modified only through the substitution

$$\chi^2\to\chi^2+\pi^2\ .$$

To compute the mass of the pseudoscalar meson one has to take the second derivative with respect to the field π, evaluated at the minimum. The actual value of the mass will be obtained by multiplying the second derivative by the appropriate factor a that relates the physical pseudoscalar field φ_π to π according to $\varphi_\pi=a\pi$. This factor can be obtained in terms of the decay constant f_π through standard arguments of current algebra. One gets

$$a=-\frac{f_\pi}{\sqrt{2}\chi}\ ,$$

$$m_\pi^2=\left.\frac{\partial^2V}{\partial\varphi_\pi^2}\right|_{\min}=\left.\frac{1}{a^2}\frac{\partial^2V}{\partial\pi^2}\right|_{\min}=\left.\frac{2\chi^2}{f_\pi^2}\frac{\partial^2V}{\partial\pi^2}\right|_{\min}\ .$$

But at the minimum one has

$$\left.\frac{\partial^2V}{\partial\pi^2}\right|_{\min}=-\left.\frac{1}{\chi}\frac{\partial V}{\partial\chi}\right|_{\min}$$

and therefore

$$m_\pi^2=-\left.\frac{2\chi}{f_\pi^2}\frac{\partial V}{\partial\chi}\right|_{\min} \tag{5.6}$$

which is nothing but the Goldstone theorem. Finally, in our case we get, for m_π,

$$m_\pi^2=-\frac{m_0}{f_\pi^2}\frac{NM^3}{2\pi^2}\chi\left[\int_0^1 dy\frac{(\alpha+\chi y)^2}{(1-y)+y(\alpha+\chi y)^2}+\left[2c(T)-\int_0^1 dy\frac{\hat{\chi}^2(T)y^2}{1-y+\hat{\chi}^2(T)y^3}\right] + 4\frac{T}{M}\sum_{k=1}^{3}\frac{\partial z_k}{\partial(\alpha\chi)}\int_0^\infty dy\ln(1+e^{-\beta M\sqrt{y^2+z_k}})\right] \tag{5.7}$$

and again the expression must be evaluated at $\chi=\bar{\chi}(T)$.

In Fig. 4 we plot the pion mass vs temperature for a quark mass $m_0=9.5$ MeV (solid line), compared to the curves obtained for $m_0=1$ MeV and $m_0=5$ MeV (dashed lines). The value of $m_\pi(T)$ is dominated by the current quark mass for temperatures below the critical value, whereas it becomes independent of m_0 at the critical temperature or above it. This should be expected, because, for $T\simeq T_c$, the pion becomes an ordinary resonance and its mass is dominated by $\Lambda_{\rm QCD}$ and not by m_0. This is a clear signal that the pion loses its Goldstone nature once the approximate chiral symmetry is restored.

This tendency of the pion mass to grow and to become independent of the quark mass when $T\to T_c$ can be also seen, for instance, by the Adler-Dashen relation (3.12). This formula allows us, for small masses, to evaluate the pion mass by taking the values for the fermion condensate and for the pion decay constant in the chiral limit,

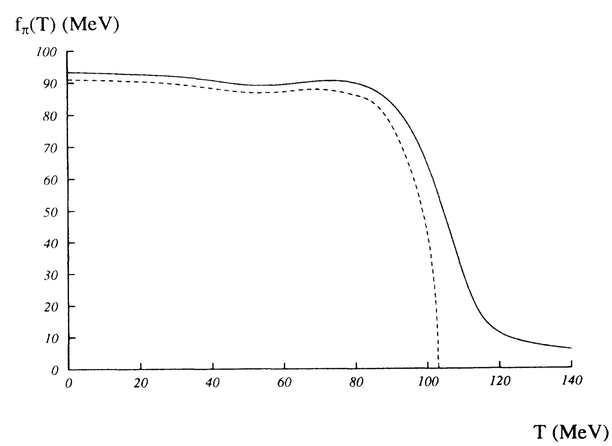

FIG. 3. Plot of f_π vs temperature in the massless limit (dashed line) and for a mass $m_0=(m_u+m_d)/2=9.5$ MeV (solid line).

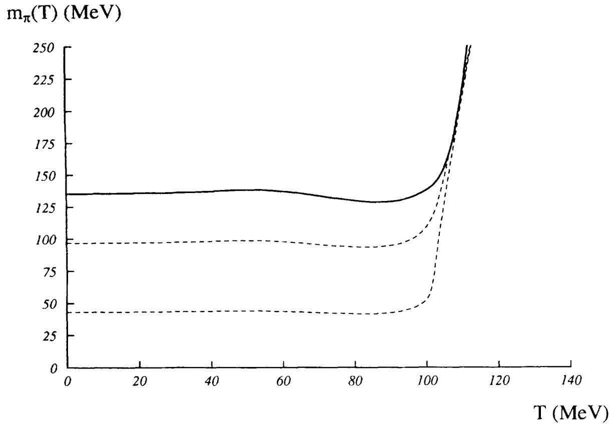

FIG. 4. Pion mass vs temperature for $m_0=(m_u+m_d)/2=9.5$ MeV (solid line), compared to the cases of smaller masses (dashed lines) $m_0=1$ and 5 MeV, from lower to higher curve. The pion mass becomes independent of the quark mass around the critical temperature (here $T_c=103$ MeV).

and so, since we know that the critical exponents which describe the behavior of $\langle\bar\psi\psi\rangle$ and f_π around T_c are both 0.5 [4], the pion mass diverges as $(1-T/T_c)^{-1/4}$. Furthermore, for $T\geq T_c$, for small quark masses, both the condensate and the pion decay constant are proportional to the quark mass and so the pion mass is independent of the quark mass at a temperature slightly above T_c. However we are certainly not able to say what happens at temperatures well above T_c because once the pion mass starts to grow up the soft pion theorems are no longer applicable.

VI. BEHAVIOR FOR $T\to T_c$ AND SMALL MASSES

In the small mass limit and around the critical point we can expand $\bar V$ [Eq. (3.7)] in terms of α and χ, as was done in Eq. (2.2):

$$\bar V(\chi,\alpha,T)|_{\alpha\to 0,T\to T_c}\simeq a_2(T)\chi^2+a_4(T)\chi^4+b_1(T)\alpha\chi+b_3(T)\alpha\chi^3+\cdots\ . \tag{6.1}$$

We have explicitly checked that the coefficients are free of infrared divergences and therefore the assumptions about the existence of a mean-field theory within our model are satisfied. It should be noticed that this is not quite trivial, because χ is proportional to the mass gap parameter, and, therefore, in the limit $\chi\to 0(T\to T_c)$, and $m_0\to 0$, the fermions become massless and infrared divergences could show up. However, as we have shown in Ref. [4] for the chiral case, it is just the thermal average which gives rise to the cancellation of the infrared divergences which are present in the $T=m_0=0$ case for $\chi\to 0$. This is indeed consistent with recent analysis showing that the generalization of the Kinoshita-Lee-Nauemberg theorem holds at finite temperature in QCD up to the two-loop level [11].

We can now give the numerical values for the coefficients appearing in Eqs. (6.1) and (2.6) which are relevant for the evaluation of the critical behaviors $(T\to T_c)$:

$$a_2(T)\sim A(T_c)\left[1-\frac{T}{T_c}\right],\quad A(T_c)=-0.35\ ,$$
$$a_4(T_c)=0.0033,\quad b_1(T_c)=1.25\ . \tag{6.2}$$

By using Eqs. (2.7), (3.5), and (3.6) we find, for $m_0=0$, [4]

$$\frac{\langle\bar\psi\psi\rangle_T}{\langle\bar\psi\psi\rangle_0}\simeq a_\psi(T_c)\left[1-\frac{T}{T_c}\right]^{1/2}, \tag{6.3}$$

where

$$a_\psi(T_c)=\frac{a_\chi(T_{c)}}{c_0\chi_0}\left\{c_0+\frac{9}{8}\ln\left[1+\left[\frac{T_c}{M}\right]^2\right]\right\} \tag{6.4}$$

with a_χ defined in Eq. (2.8), and where $\chi_0=-4.06$ is the value of the order parameter at $T=m_0=0$. One then gets

$$a_\chi(T_c)=-7.28,\quad a_\psi(T_c)=2.25\ . \tag{6.5}$$

Analogously, from Eq. (2.10),

$$\frac{\langle\bar\psi\psi\rangle_{T_c,m_0}}{\langle\bar\psi\psi\rangle_{0,0}}\simeq b_\psi(T_c)\left[\frac{m_0}{M}\right]^{1/3},\quad b_\psi(T_c)=1.41\ . \tag{6.6}$$

Finally from Eqs. (2.11) and (2.12)

$$\frac{\partial}{\partial m_0}\frac{\langle\bar\psi\psi\rangle_{T,m_0}}{\langle\bar\psi\psi\rangle_{0,0}}\Bigg|_{m_0=0}\simeq\begin{cases}\dfrac{1}{M}d_\psi(T_c)\left[1-\dfrac{T}{T_c}\right]^{-1}, & T<T_c,\quad (6.7)\\ -\dfrac{2}{M}d_\psi(T_c)\left[1-\dfrac{T}{T_c}\right]^{-1}, & T>T_c\ ,\quad (6.8)\end{cases}$$

where

$$d_\psi(T_c)=0.28\ . \tag{6.9}$$

The same analysis can be repeated for $f_\pi(T,m_0)$. From Eq. (5.3) we can see that

$$\frac{f_\pi(T,m_0)}{f_0}=|\chi(T,m_0)|G(T,\chi(T,m_0),m_0)\ , \tag{6.10}$$

where $f_0\simeq 91$ MeV is the pion decay constant in the chiral limit. One can check that $G(T_c)$ is free of infrared divergences in the limit $\chi\to 0$ and $m_0\to 0$. It follows that the behavior of $f_\pi(T,m_0)$ around T_c and for $m_0\to 0$ is the same as for the condensate (see also Ref. [4])

$$\frac{f_\pi(T)}{f_0}\simeq a_f(T_c)\left[1-\frac{T}{T_c}\right]^{1/2},$$
$$a_f(T_c)=3.13,\quad T\leq T_c \tag{6.11}$$

with

$$a_f(T_c)=|a_\chi(T_c)|G(T_c)\ . \tag{6.12}$$

Again we find

$$\frac{f_\pi(T_c,m_0)}{f_0}\simeq b_f(T_c)\left[\frac{m_0}{M}\right]^{1/3},\quad b_f(T_c)=1.96 \tag{6.13}$$

and

$$\left.\frac{\partial}{\partial m_0}\frac{f_\pi(T,m_0)}{f_0}\right|_{m_0=0}\simeq\begin{cases}\dfrac{1}{M}d_f(T_c)\left[1-\dfrac{T}{T_c}\right]^{-1}, & T<T_c, \quad (6.14)\\[2ex] -\dfrac{2}{M}d_f(T_c)\left[1-\dfrac{T}{T_c}\right]^{-1}, & T>T_c\ , \quad (6.15)\end{cases}$$

where

$$d_f(T_c)=0.38\ . \tag{6.16}$$

Thus, in addition to verifying that in our model the hypotheses for the validity of a mean-field expansion are satisfied, we have also explicitly evaluated the coefficients of the scaling laws.

VII. OUTLOOK AND CONCLUSIONS

We have studied the role of quark masses in finite temperature QCD. Quark masses explicitly break chiral symmetry and thus they affect the notion of chiral restoration at high temperatures. Their role is similar to that of an external magnetic field in a ferromagnetic phase transition.

On the assumption of validity of a Landau mean-field theory near the chiral transition one can discuss the critical coefficients governing the behavior of the quark condensate at the transition point: an exponent β for the T behavior in the massless case; an exponent δ for the behavior for small quark mass at the critical point; and an exponent γ for the derivative of the order parameter with respect to the quark mass (an equivalent of the magnetic susceptibility of the ferromagnetic case).

The mean-field expansion in the manner of Landau is known to be marginally valid for four space-time dimensions. We verify that it is valid in the composite-operator method that we quantitatively employ, for which we verify in particular the absence of infrared divergences (infrared safety) which would destroy a basic assumption for the Landau expansion.

The composite operator method we use is the same we have previously applied to the massless case. It is a variational approach, and it is quite distinct from perturbation theory. As it satisfies the assumptions for a mean-field expansion it automatically leads to the classical critical exponents.

We have also discussed the temperature dependence of the pion decay constant. Its critical exponents are the same as for the quark condensate.

Our discussion and model are appropriate to study the chiral transition. On the other hand, it must be stressed that the overall physical picture might be changed in a more complete description of QCD, including, for instance, additional gluonic effects, and we do not know whether, in that case, a mean-field theory would still remain as a valid approximation around the critical point. We recall that for full QCD one does not know a definite set of order parameters, or even better, a single order parameter, to describe its phase behaviors. For an infinite quark mass the thermally averaged Polyakov loop is appropriate to the description of the deconfinement transition. However in the presence of quarks of finite (rather than infinite) mass it loses part of its significance. For vanishing quark masses, the quark bilinears are certainly the appropriate order parameters to describe the chiral transition from the broken to the chiral restored phase. The underlying symmetry, chiral symmetry, is well defined in the massless limit. For full QCD, with quarks of different finite masses, no obvious symmetry or set of symmetries suggests itself to formulate the problem of phase transitions in the conventional way, of different phases related to different symmetries. The prevailing opinion of a unique phase transition is still not explicitly demonstrated and in any case studies with a number of would-be order parameters are inevitable, and one may expect that such different order parameters affect each other in their variations. The study of the general features of a multiorder parameter approach to phase transitions constitutes by itself an interesting thermodynamical problem.

As we have said, chiral symmetry is already broken in the Lagrangian when there are quark masses. What we can retain, in this case, of the notion of phase transition, may be quantitatively described by the relative variation of the quark condensate in passing through the original (i.e., of the massless case) critical region. We find that such an "effective" order parameter decreases with the quark mass, at first linearly for small masses. Numerically, it clearly appears that only for the order parameters related to u and d one can continue to usefully speak of an approximate phase transition, whereas such a notion becomes less evident for the s quark. The temperature evolution for the chiral s order parameter is quite smooth on large intervals of T. The presence of the massive s quark will of course indirectly affect the behavior of the u and d condensates through a number of mechanisms, one being via the fermionic ('t Hooft) determinant in flavor SU(3), effectively arising from the anomaly. The occurrence of the determinant will produce a mixing among flavors and the effective potential will not decompose into independent flavor contributions.

Chiral symmetry being already broken from quark masses, we can still, for small masses at least, choose as a possible mark of the region where the condensate has a rapid variation that particular temperature at which the condensate value is one-half of its value at zero temperature. For a small quark mass such a temperature may already be looked at as a kind of vestige of the chiral critical temperature for massless quarks. Of course, such a temperature will no more exist for a sufficiently large quark mass, as the condensate, beyond a certain mass, will always remain larger than one-half of its $T=0$ value (at least this happens in our model, where such a mass comes out to be around 80 MeV). We find that the surrogate critical temperature we have just defined in the presence of a massive quark increases linearly when the quark mass increases (from about 100 MeV for zero quark mass to about 110 MeV for a quark mass of 30 MeV).

An increase of the critical temperature with the quark mass is intuitively expected, as in the massive case one expects that the melting of the condensate requires larger temperatures. Gerber and Leutwyler [12] suggest a similar result, although, as they say, the critical temperature is beyond the range of validity of their formulas, which are based on the first terms of a low-temperature expansion. The critical temperatures they give, based on projecting such a low-T calculation, are rather larger than ours.

The approach of Gerber and Leutwyler is appropriate to the low-T region and we can try to compare our results with theirs in such a limit. For the massless case the comparison has already been done, and in fact it was used to fix the value of our parameter ξ. To compare in the presence of small quark masses, always at low temperature, it is enough to look at the variation of the pion decay constant with a pion mass, in the zero temperature limit. We both find a correction linear in the quark mass. Since one fits the experimental pion decay constant f_π at the physical pion mass, the mass variation finally depends on the value of f_π in the massless case, which is for us is about 91 MeV and for them a few MeV smaller. There are however rather large uncertainties, coming from fitting the pion-pion scattering data, in the numerical estimate of their fourth-order chiral expansion, bringing in the correction of f_π an admitted uncertainty of 20%. We therefore consider that, even quantitatively, the agreement is satisfactory, given the experimental data and the fact that our model is not supposed to be as rigorous in the low-T region as the chiral expansion.

We have also calculated the temperature-dependent pion mass. Below the critical temperature such a mass is controlled by the current quark mass. Near and above the critical temperature, for some temperature interval where one may like to consider the pion as a remnant degree of freedom, the pion mass becomes independent of the quark mass. This shows that the pion becomes in that region an ordinary resonance and loses its Goldstone character.

Another interesting point on which we may comment is the mechanism of chiral symmetry restoration. The χ field appearing in our model is essentially the σ field, as in a linear sigma model, by suitable normalization. Our effective potential around the critical temperature, after introducing the pion field through a substitution of the type $\chi^2 \to \chi^2+\pi^2$ (see Sec. V), is that of a linear σ model, with an additional explicit symmetry-breaking term proportional to the χ field and to the quark mass. The coefficient of the quadratic term changes sign, from negative to positive, when moving through the critical temperature, corresponding to chiral-symmetry restoration (apart from the explicit symmetry-breaking term). The pion and the σ are then degenerate in mass (apart the explicit symmetry breaking). It therefore appears that for a limited range of temperatures slightly beyond the critical temperature, our model is equivalent to a nonspontaneously broken linear σ model, thus suggesting almost degenerate pions and σ as physical degrees of freedom.

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