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Tau anomaly and vectorlike families

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The implications of a recently indicated increase in τ lifetime are discussed. It is stressed that the available experimental constraints (from $\delta\rho, \epsilon_3$, and N_ν, etc.) are satisfied most naturally if the indicated τ anomaly is attributed to the mixing of the τ family with a heavy vectorlike family $Q'_{L,R}$ with masses $\sim$200 GeV to 2 TeV, which is a doublet of $SU(2)_R$ and singlet of $SU(2)_L$, rather than with a heavy fourth family with standard chiral couplings. $L \leftrightarrow R$ symmetry would imply that $Q'_{L,R}$ is accompanied by the parity-conjugate family $Q_{L,R}$ which is a doublet of $SU(2)_L$ and singlet of $SU(2)_R$. Two such vectorlike families, together with an increase in τ_τ, are, in fact, crucial predictions of a recently proposed supersymmetric composite model that possesses many attractive features, in particular, explanations of the origin of diverse scales and family replication. In the context of such a model, it is noted that *an increase in τ_τ due to mixing involving vectorlike families will necessarily imply a correlated decrease in neutrino counting N_ν from the* CERN *e^+e^- collider* LEP *from 3*. Such a decrease in N_ν would be absent, however, if the τ anomaly is attributed to a mixing involving a standard fourth family with chiral couplings. Because of the seesaw nature of the mass matrix of the three chiral and two vectorlike families, that arises naturally in the model, departures from universality in the first two families as well as in $\bar{b}b$ and $\tau^+\tau^-$ channels (linked to down flavors) are strongly suppressed, in accord with observations.

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I. INTRODUCTION

Recent measurements of the τ lifetime indicate a discrepancy with the e-μ-τ universality in that it seems to be longer by 3–8% than that expected. While the branching ratios of $\tau \to e\nu\bar{\nu}$ and $\tau \to \mu\nu\bar{\nu}$ are found to be normal yielding $(G_\mu/G_e)^2 = 1.000 \pm 0.019$, the world average of τ-lifetime measurements yields $(G_\tau/G_{e,\mu})^2 = 0.948 \pm 0.022$, reflecting a 2.3$\sigma$ effect in the departure from unity [1]. This does not, of course, permit a definitive conclusion yet. Improved measurements of τ_τ and m_τ will be very helpful in this regard. Nevertheless, the effect is sufficiently intriguing and, if it holds up, it would clearly have some profound implications. We plan to discuss some of these in this work.

Believing in a gauge principle, the most likely explanation of such a discrepancy would seem to be the existence of a certain heavy neutrino (and/or a heavy lepton) which may belong to a heavy family [2] with masses in the range of a few hundred GeV to a few TeV. The mixing of the heavy neutrino (and/or the heavy lepton) with ν_τ (and/or τ) could, in general, cause apparent departures from universality in τ decay, provided some additional conditions are satisfied. While the relevance of a heavy fourth family to the τ anomaly has generally been noted by several authors [3], our emphasis on the likely nature of the heavy family and its origin would be new.

The purpose of this work is twofold. First, we wish to spell out, within a certain broad theoretical framework, what ought to be the nature of this heavy family so that it would be consistent with known physics, including, in particular, neutrino counting at the CERN e^+e^- collider LEP and precision electroweak tests. While our main conclusion would hold within the standard $SU(2)_L \times U(1)_Y \times SU(3)^C$ gauge symmetry, we will proceed, for the sake of elegance, with the assumptions of left-right symmetry [4] and charge quantization, which minimally lead to the gauge structure $\mathcal{G} = SU(2)_L \times SU(2)_R \times SU(4)^C$ [5]. Within this gauge framework, we would argue that the experimental constraints are most naturally satisfied if the heavy family that is relevant to τ nonuniversality is a vectorlike family $Q'_{L,R}$ with masses $\sim$200 GeV to 2 TeV, whose left and right components transform as a doublet of $SU(2)_R$, singlet of $SU(2)_L$, and $\mathbf{4}^*$ of $SU(4)^C$, rather than like a standard fourth family with familiar chiral couplings. $L \leftrightarrow R$ symmetry implies that $Q'_{L,R}$ would be accompanied by the parity-conjugate vectorlike family $Q_{L,R}$ which is a doublet of $SU(2)_L$, singlet of $SU(2)_R$, and $\mathbf{4}^*$ of $SU(4)^C$. This $SU(2)_L$ doublet family $Q_{L,R}$ would turn out to be irrelevant, however, for τ nonuniversality. The members of these two families and their transformation properties are listed below:

$$\begin{aligned} Q_{L,R} &= (U,D,E,N)_{L,R} \sim (2_L, 1_R, 4^*_C) \,, \\ Q'_{L,R} &= (U',D',E',N')_{L,R} \sim (1_L, 2_R, 4^*_C) \,. \end{aligned} \tag{1}$$

The second point of this work is to discuss the question of a natural origin of the vectorlike families. In this regard, we wish to point out that, while the existence of such *complete vectorlike families* does not seem to be a compelling feature of most models, two such vectorlike families $Q_{L,R}$ and $Q'_{L,R}$ together with an increase in τ_τ are, in fact, crucial predictions of a recently proposed composite model based on local supersymmetry which possesses many attractive features [6–9], in particular, explanations of the origin of diverse scales from $M_{\rm Pl}$ to m_ν and family replication. This is why it is called the "scale unifying model." One interesting feature of the model emphasized in this paper is that an increase in the τ lifetime would necessarily imply a predicted decrease in the LEP neutrino counting from $N_\nu=3$. Such a decrease would be absent, however, if the τ decay nonuniversality is due to mixing involving a standard fourth family with chiral couplings. At the end, we will comment briefly on the possible relevance of other frameworks to τ nonuniversality.

II. PHENOMENOLOGICAL CONSTRAINTS ON HEAVY FAMILIES

(i) Neutrino counting at LEP. The recent LEP result [10] $N_\nu=2.99\pm0.05$ implies that at least the neutrinos $N_{L,R}$ belonging to the $\mathrm{SU}(2)_L$-doublet family ($Q_{L,R}$) and likewise N_L^S belonging to a standard fourth family must be heavier than about $m_Z/2$, because both couple to Z^0:

$$m_N(m_N^S)\geq m_Z/2\simeq 45\ \mathrm{GeV}\ . \tag{2}$$

(ii) τ nonuniversality. $N'_{L,R}$ belonging to the $\mathrm{SU}(2)_R$-doublet family Q' do not, however, couple to Z^0 and, as such, LEP neutrino-counting does not restrict their mass. If, on the other hand, $\nu_\tau-N'$ mixing is to be relevant to τ nonuniversality, N' must still be heavier about 1.8 GeV so that $\tau\to N'e\bar{\nu}$ would be forbidden kinematically. Thus,

$$m_{N'}\geq 1.8\ \mathrm{GeV}\ . \tag{3}$$

(iii) LEP and Collider Detector at Fermilab (CDF) searches. From LEP searches, we may set

$$M_{U,U',D,D',E,E'}(M_{U^s,D^s,E^s})\geq m_Z/2\simeq 45\ \mathrm{GeV}\ . \tag{4}$$

From top-quark searches at CDF which set $m_t\geq 91$ GeV [11], it seems that a similar limit would apply to the exotic heavy quarks as well assuming that they have at least some minimal mixing ($\theta\geq 10^{-4}$, say) with the light ones: say,

$$M_{U,U',D,D'}(M_U^s,D^s)\geq 80\ \mathrm{GeV}\ . \tag{5}$$

The messages from (2)–(5) is that all the exotic fermions including even N' (barring special circumstances) are probably heavier than at least 50–100 GeV.

(iv) ρ parameter. Measurements of the ρ parameter, combining high- and low-q^2 data, yield [12] $\delta\rho=-0.02\pm0.037$. Allowing for a contribution from top quark with $m_t\geq 91$ GeV, this restricts up-down mass splittings in a heavy family (e.g., a standard fourth family) to be less than about 50 GeV (for $m_{U^s}+m_{D^s}\geq 200$ GeV). In other words, up and down members including the leptons should be degenerate to much better than 50%: say,

$$0.6\leq m_{U^s}/m_{D^s}\leq 1.5\ . \tag{6}$$

Before proceeding further, let us pause and discuss the implications of the constraints (2)–(6). It seems to us that neither the heaviness of the neutrino member (i.e., $m_{N^S}\geq 45$ GeV) nor the up-down degeneracy (to better than 50%) fit naturally with a standard fourth family. First, if the neutrinos of the first three families ν_L^e, ν_L^μ, and ν_L^τ become light, utilizing the standard seesaw mechanism, what is special about the fourth family, assuming its gauge couplings are identical to those of the first three, that it is barred from utilizing the same mechanism and, thereby, remain heavy? Second, as regards up-down degeneracy, comparing with $m_c/m_s\approx 10$ and $m_t/m_b\geq 20$, one wonders if there is any good reason why U^s and D^s belonging to a standard fourth family should be so degenerate [see (6)]. (In making this comparison, one is inclined to ignore the electron family which is so light that its mass may be viewed as a correction to tree-level masses.)

(v) The ϵ_3 parameter. Combining once again, high- and low-q^2 data, the ϵ_3 parameter of Ref. [12] [which is related to the S parameter of Ref. [13] by $\epsilon_3=\alpha(m_Z)S/4\sin^2\theta_W^0$] is given by $\epsilon_3^{\rm expt}=(-0.31\pm0.62)\times10^{-2}$. The standard model with three families yields $\epsilon_3^{\rm SM}=+(0.3-0.55)\times10^{-2}$ for the Higgs-boson mass $m_H\simeq 50$ GeV to 1 TeV, which already is near the maximum possible value of $\simeq 0.31\times10^{-2}$ allowed by experiments (within 1σ). A standard fourth family with chiral coupling would increase [12] ϵ_3 by about $=0.16\times10^{-2}$, i.e.,

$$\epsilon_3^{\rm SM}(\text{four families})\simeq(0.46-0.7)\times10^{-2}\ .$$

This seems to be outside of the observed value including error (within 1σ). Thus, a fourth family with chiral couplings seems to be disfavored by ϵ_3 measurements though probably not excluded yet.

We next observe that all three features suggested by observations, i.e., (i) heaviness of neutrinos, (ii) approximate up-down degeneracy and (iii) smallness of positive contribution to ϵ_3, while they do not fit so naturally with a standard fourth family with chiral couplings, are essentially *automatic and compelling features* of vectorlike families Q and Q'. The reasons are as follows.

Being vectorlike, their mass terms $(M\bar{Q}_R 1 Q_L + M'\bar{Q}'_R 1 Q'_L + \mathrm{H.c.})$, whether induced spontaneously or introduced explicitly (see later), conserve the full $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{SU}(4)^C$ symmetry. This ensures that, not only up and down members, but even quark and lepton members of vector families are naturally degenerate barring electroweak and QCD corrections. Thus, the constraint from the ρ parameter is satisfied automatically by the vectorlike families.

The neutrinos $N_{L,R}$ belonging to $Q_{L,R}$ get a heavy Dirac mass as mentioned above. Since both N_L and N_R belong to $\mathrm{SU}(2)_L$ doublets, however, neither of them can acquire a heavy Majorana mass (unlike the ν_R's belonging to chiral families), as that would need an $\mathrm{SU}(2)_L$-triplet vacuum expectation value (VEV) (such a VEV is assumed to be absent altogether for other reasons, i.e., measurement of ρ). In other words, $N_{L,R}$ naturally retain their $\mathrm{SU}(2)_L\times\mathrm{U}(1)$-invariant Dirac mass. Now, N'_L and

N_R' belonging to Q' also acquire a heavy $SU(2)_L \times U(1)$-invariant Dirac mass as mentioned above. Since these are members of $SU(2)_R$ doublets, however, either one (or both) of these could, in general, acquire a superheavy majorana mass like the ν_R's utilizing a superheavy $SU(2)_R$-triplet VEV. However, if N_L' and N_R' are distinct from the ν_R's because of some quantum numbers, they cannot acquire a superheavy Majorana mass utilizing the same VEV that assign such masses to the ν_R's. This is indeed the case for the model of Ref. [6]. In this case, $N_{L,R}'$ would also naturally retain their $SU(2)_L \times U(1)$-invariant heavy Dirac mass.

Finally, as is well known, the contributions of heavy vectorlike families with $SU(2)_L \times U(1)$ symmetric masses to ϵ_3 are severely damped because they decouple in the large mass limit and precisely vanish in the symmetric limit $M_U = M_D$. Up-down splittings, induced purely through electroweak radiative corrections, lead to negligible contributions $\leq 10^{-5}$ to ϵ_3. Thus, all the phenomenological constraints listed so far are satisfied most naturally if the heavy families in question are vectorlike. In other words, new physics can hide so far most effectively in vectorlike families, in contrast with a heavy chiral family. This is the first main point of this paper.

III. VECTORLIKE FAMILIES AND THE SCALE UNIFYING MODEL

As mentioned before, complete vectorlike families Q and/or Q' with masses of order 200–1000 GeV do not seem to arise in a compelling manner in either grand unified theories or superstring theories with the assumption of elementary quarks and leptons. They can, of course, be introduced by simply postulating, for example, n 16-plets together with m $\overline{16}$-plets of SO(10), or n 27-plets together with m $\overline{27}$-plets of E_6, where $m < n$. In this case, m 16's can be assumed to combine through mass terms, with m $\overline{16}$'s (or m 27's can combine with m $\overline{27}$ as in heterotic superstring theories) to give m families of the Q type and the same number of Q' type. But questions arise: What prevents these vectorlike families from acquiring $SU(2)_L \times U(1)$-invariant masses of order Planck or grand unification scale and why are mixing masses not of the type $\bar{q}_L Q_R$ or $\bar{q}_R Q_L'$, which also preserve $SU(2)_L \times U(1)$, as high as the Planck scale? It seems to us that there is no simple natural answer to these questions within standard grand unification and even superstring theories.

By contrast, two complete vectorlike families Q and Q' as specified in (1), with a seesaw mass matrix for the combined system of three chiral and two vectorlike families, *having just the right mass scales in the various entries,* are the predictions of a recently proposed composite model [6], which possesses many additional desirable features [7–9]. From now on, to be specific, we shall follow the framework of this model. We first outline some of its salient features.

The model assumes that the effective Lagrangian, just below the Planck scale, possesses $N=1$ local supersymmetry and is made out of a set of six positive and six negative massless chiral superfields $\Phi_{\pm}^{a,\sigma} = (\varphi_{L,R}, \psi_{L,R}, F_{L,R})^{a\sigma}$, which represent preonic substructures. These couple to a set of gauge fields (v_μ, λ, D), corresponding to (i) a metacolor gauge symmetry SU(N), that generates the preon binding force, and (ii) the commuting flavor-color gauge symmetry $SU(2)_L \times SU(2)_R \times SU(4)^C$ [5]. The index σ denotes metacolor quantum numbers and runs from 1 to N. The index a denotes flavor-color quantum numbers and runs over six values (x,y,r,y,b, and l). Two of these (x and y) denote the basic flavors (i.e., u and d) and four of them (r,y,b, and l) denote the four basic colors including lepton color in a family. Thus, the model introduces a minimal set of preons which possess the attributes of just one family.

Corresponding to an input value of the metacolor coupling $\bar{\alpha}_M \simeq 0.07-0.05$ at $M_{\mathrm{Pl}}/10$, the asymptotically free metacolor force becomes strong at a scale of $\Lambda_M \sim 10^{11}$ GeV for $N=5-6$. At that point, it makes (a) a set of composites including the known quark-lepton families and (b) a few condensates which, together, break supersymmetry as well as the flavor-color gauge symmetry. As regards composites, although the model introduces the attributes of just one family, it has been shown [7] that supersymmetry, owing to fermion-boson pairing, provides a compelling reason for replication. In fact, with the minimum dimension for the composite operators, the model yields precisely three chiral families ($q_{L,R}^i$), $i=1,2,3$, instead of one. In addition, it also yields [again due to supersymmetry (SUSY)] two complete vectorlike families $Q_{L,R} = (U,D,E,N)_{L,R}$ and $Q_{L,R}' = (U',D',E',N')_{L,R}$ whose composite structure and (thus) binding are on par with those of the chiral families. These couple vectorially to W_L and W_R, respectively, and transform as in (1). Thus, altogether there are five quark-lepton families: three chiral and two vectorlike.

The model assumes the formation of a SUSY-preserving condensate $\langle \Delta_R \rangle \sim \langle (1_L, 3_R, \overline{10}^C) \rangle$ of scale $\Lambda_M \sim 10^{11}$ GeV which breaks $SU(2)_L \times SU(2)_R \times SU(4)^C$ to $SU(2)_L \times U(1) \times SU(3)^C$ and also gives superheavy Majorana masses to the chiral ν_R's. Because of a mismatch of quantum numbers, however, $\langle \Delta_R \rangle$ cannot give a Majorana mass either to N_L' or N_R' [14]. The model also assumes the formation of the metagauging condensate $\langle \lambda \cdot \lambda \rangle$ and the fermion condensates $\langle \bar{\psi}^a \psi^a \rangle$. Each of these breaks supersymmetry and the nonanomalous R symmetry $U(1)_X$. Furthermore, $\langle \bar{\psi}^a \psi^a \rangle$ also break $SU(2)_L \times U(1)_Y$ for $a=x,y$. Owing to the constraints of the index theorem, however, these SUSY-breaking condensates need the collaboration of gravity to form and must therefore be damped by $(\Lambda_M/M_{\mathrm{Pl}})$, i.e., $\langle \lambda \cdot \lambda \rangle = a_\lambda \Lambda_M^3 (\Lambda_M/M_{\mathrm{Pl}})$ and $\langle \bar{\psi}^a \psi^a \rangle = a_{\psi_a} \Lambda_M^3 (\Lambda_M / M_{\mathrm{Pl}})$ [15]. *A priori,* assuming that $\langle \lambda \cdot \lambda \rangle$ and $\langle \bar{\psi} \psi \rangle$ form, we expect $a_\lambda \sim 1$ and a_ψ's to be smaller than a_λ by factors of 3–10 (say) because ψ's are in the fundamental and λ in the adjoint representation of SU(N).

Fermion masses and mixings arise as follows. First, it turns out [6–9] that the masses of the composite chiral $q_{L,R}^i$ and the vectorlike families $Q_{L,R}$ and $Q_{L,R}'$ vanish as long as $U(1)_X$ is preserved. Now, the $\langle \lambda \cdot \lambda \rangle$ condensate, that preserves $SU(2)_L \times U(1)_Y$, breaks $U(1)_X$ by just the right amount to induce flavor-color symmetric Dirac masses for the vectorlike families:

$$M^{(0)}(U,D,E,N)=M^{(0)}(U',D',E',N')$$
$$\equiv\kappa_\lambda=O(a_\lambda)\Lambda_M(\Lambda_M/M_{\rm Pl})\simeq 1\ {\rm TeV}\ .$$

Because of a mismatch of $U(1)_X$, however, neither $\langle\lambda\cdot\lambda\rangle$ nor $\langle\bar\psi\psi\rangle$ can induce direct mass terms for the chiral families, which thus turn out to vanish barring corrections of order 1 MeV: $m^{(0)}_{\rm dir}(q^i_L\to q^i_R)=0+{\rm const}\times(1$ MeV) [6–9]. The chiral families $q^i_{L,R}$ $(i=1,2,3)$ acquire masses (primarily) only through their *mixings* with the two vectorlike families, which are induced by $\langle\bar\psi\psi\rangle$. Thus, suppressing QCD corrections and ignoring $m^{(0)}_{\rm dir}(q^i_L\leftrightarrow q^i_R)$, the Dirac mass matrices of the five-family system for all four sectors, i.e., q_u, q_d, l and ν have the seesaw form [6,9] given by

$$M^{(0)}_{f,c}=\begin{array}{c} \\ \overline{q^i_R} \\ \overline{Q_R} \\ \overline{Q'_R}\end{array}\begin{array}{c}\begin{array}{ccc} q^i_L & Q_L & Q'_L\end{array}\\ \left[\begin{array}{ccc} 0 & X\kappa_f & Y\kappa_c\\ Y'^\dagger\kappa_c & \kappa_\lambda & 0\\ X'^\dagger\kappa_f & 0 & \kappa_\lambda\end{array}\right]\end{array}\ . \tag{7}$$

Here $f=u$ or d corresponding to $a=x$ or y and $c=(r,y,b)$ or l and $\kappa_{f,c}=O(a_{\psi f,c})\Lambda_M(\Lambda_M/M_{\rm Pl})$. The indices $f=u$ and d with $c=(r,y,b)$ represent, respectively, up and down quarks of three colors, while $f=u$ and d with $c=l$ represent, respectively, neutrinos and charged leptons in each family. Following remarks made above, we expect $\kappa_{f,c}\simeq(\frac13-\frac1{10})\kappa_\lambda$. In general, the column matrices X, Y, X', and Y' acting on the family spaces should have entries of order unity. In the absence of electroweak corrections ($\sim$5–10%), $L\leftrightarrow R$ symmetry and $SU(4)^C$ symmetry of the metacolor force guarantee that not only $X=X'$ and $Y=Y'$ but that the same X, Y, and κ_λ apply to all four sectors, q_u, q_d, l, and ν. Furthermore, ignoring electroweak corrections, one can always rotate q^i_L and q^i_R so that $Y^T=Y'^T=(0,0,1)$ and, simultaneously, $X^T=X'^T=(0,p,1)$, with redefined κ_f and κ_c. The model thus has just six effective parameters: p, κ_u, κ_d, κ_r, κ_l, and κ_λ [16]. Furthermore, we know their approximate values (within a factor of 10, say).

Since we expect $(\kappa_f,\kappa_c)\le\kappa_\lambda/3$, we obtain the following eigenvalues at low energies (ignoring electroweak corrections and $m^{(0)}_{\rm dir}$, but including QCD effects) [9]:

$$\begin{aligned}
&m^{(0)}(u,d,e,\tilde\nu)=0\ ,\quad m^{(0)}(c,s)\approx(\kappa_{u,d})(\kappa_r/\kappa_\lambda)(p^2/2)\eta\ ,\\
&m^{(0)}(\tilde\nu_\mu,\mu)\approx(\kappa_{u,d})(\kappa_l/\kappa_\lambda)(p^2/2)\ ,\\
&m^{(0)}(t,b)\approx(\kappa_{u,d})(\kappa_r/\kappa_\lambda)(2\eta)\ ,\\
&m^{(0)}(\tilde\nu_\tau,\tau)\approx(\kappa_{u,d})(\kappa_l/\kappa_\lambda)2\ ,\\
&m(U,D,U',D')\approx\kappa_\lambda\eta,\quad m(E,E')\approx\tilde m(N,N')\approx\kappa_\lambda\ .
\end{aligned}\tag{8}$$

The tildes on neutrinos denote that they are Dirac masses. Combined with superheavy Majorana masses of $(\nu^i_R\text{'s})_{i=1,2,3}$ they yield light ν^i_L's for the neutrinos of the chiral families. The momentum-dependent QCD renormalization factor η is found to be $\eta(\mu)\simeq2.9$, 3.3, 4.1, and 5.2 for $\mu=1$ TeV, 100 GeV, 5 GeV, and 1 GeV, respectively [17]. We see that the electron family is guaranteed to remain massless [barring contributions from $m^{(0)}_{\rm dir}\sim(1$ MeV$)\eta_{\rm QCD}$]—a fact which is not far from the truth. We also see that the μ-τ mass ratios (evaluating $\eta_{\rm QCD}$ at a fixed momentum for all quarks) are given by

$$(m^{(0)}_c/m^{(0)}_t)\approx(m^{(0)}_s/m^{(0)}_b)$$
$$\approx(m^{(0)}_\mu/m^{(0)}_\tau)\simeq p^2/4\ . \tag{9}$$

Thus, for $p\approx\frac13-\frac14$, which is not too small and natural, we obtain a rather large μ-τ hierarchy of about $\frac1{40}-\frac1{64}$. In this way, the model provides a natural reason for the interfamily hierarchy: $\overline{m_e}\ll\overline{m_\mu}\ll\overline{m_\tau}$.

A reasonable fit [9] to all the known quark-lepton masses is obtained (to within factors of 2–3) by choosing [18] $p\approx0.31$, $\kappa_u\approx80$ GeV, $(\kappa_l/\kappa_\lambda)\approx\frac13$, $(\kappa_r/\kappa_l)\approx0.6$, $(\kappa_d/\kappa_u)\approx\frac1{30}$, and $\kappa_\lambda\approx(3-5)\kappa_u\approx(200-400)$ GeV. These yield (including QCD corrections) $m^{(0)}_u=m^{(0)}_d=m^{(0)}_e=0$, $m^{(0)}_t\approx110$ GeV, $m^{(0)}_b\approx4.7$ GeV, $m^{(0)}_c\approx3.9$ GeV, $m^{(0)}_s\approx130$ MeV, $m^{(0)}_\tau\approx1.7$ GeV, and $m^{(0)}_\mu\approx40$ MeV, while $m(U,D,U',D')\approx(1.5-3)$ TeV and

$$m(E,E')\approx\tilde m(N,N')\approx(200-400)\ {\rm GeV}\ .$$

These results possess at least the desired gross pattern. Inclusion of electroweak corrections ($\sim$5–10%) to X, Y, X', and Y' and also $m^{(0)}_{\rm dir}$ can substantially remove the discrepancies in m_μ and m_c and yield a desired pattern for the Cabibbo-Kobayashi-Maskawa (CKM) elements in the light family sector (see Ref. [9]). In this sense, a seesaw mass matrix of the form (7), with its economy in effective parameters, seems to be a promising and realistic approach to the problem of fermion masses and mixings. Let us now discuss how τ anomaly may be addressed in this framework.

IV. τ ANOMALY

For considerations of τ anomaly, only the mixings of the τ family with the heavy vectorlike families Q and Q' are of primary importance. These are induced by the dominant entries proportional to one in the matrices $X\kappa_f=(0,0,1)\kappa_f$ and $Y\kappa_c=(0,p,1)\kappa_c$. The corresponding entries with or without electroweak corrections ($\sim$5–10%), that mix the e and the μ families with the vectorlike families, are, of course, much smaller. Thus, for considerations of the τ anomaly, the presence of the first two families (e and μ) may be ignored to a good approximation. This is what we do in the following.

Allowing for small mixing angles (compared to unity), the current eigenstates, i.e., the canonical fields, involving the left-handed neutrinos ($\nu^{(0)}_{\tau L}$, $N^{(0)}_L$, and $N'^{(0)}_L$) and the charged leptons $\tau^{(0)}_L$, $E^{(0)}_L$, and $E'^{(0)}_L$ may be expressed in terms of the corresponding mass eigenstates as [19]

$$\begin{aligned}
&\nu^{(0)}_{\tau L}=a_\nu\nu_{\tau L}+\epsilon_\nu N_L+\eta_\nu N'_L\ ,\\
&N^{(0)}_L=-\epsilon_\nu\nu_{\tau L}+b_\nu N_L+\beta_\nu N'_L\ ,\\
&N'^{(0)}_L=-\eta_\nu\nu_{\tau L}-\beta_\nu N_L+c_\nu N'_L\ ;
\end{aligned}\tag{10}$$

$$\begin{aligned} \tau_L^{(0)} &= a_l \tau_L + \epsilon_l E_L + \eta_l E'_L \, , \\ E_L^{(0)} &= -\epsilon_l \tau_L + b_l E_L + \beta_l E'_L \, , \\ E_L'^{(0)} &= -\eta_l \tau_L - \beta_l E_L + c_l E'_L \, . \end{aligned} \tag{11}$$

The neutrinos and charged leptons with right chirality may also be expanded in a similar manner. They will not be relevant, however, for charged-current τ decay because all the right-handed neutrinos $(\nu_{\tau R}, N_R, N'_R)$ are heavier than τ.

From the mass matrix (7), it is easy to see that the mixing angles appearing in (10) and (11) [neglecting terms of order $(\kappa_{f,c}/\kappa_\lambda)^3 \leq 10^{-2}$] are given by

$$\begin{aligned} &\epsilon_\nu = \theta_{\nu_{\tau L} N_L} = \kappa_l/\kappa_\lambda \, , \quad \eta_\nu = \theta_{\nu_{\tau L} N'_L} = \kappa_u/\kappa_\lambda \, , \\ &\epsilon_l = \theta_{\tau_L E_L} = \kappa_l/\kappa_\lambda \, , \quad \eta_l = \theta_{\tau_L E'_L} = \kappa_d/\kappa_\lambda \, , \\ &\beta_\nu = \theta_{N_L N'_L} = (\kappa_u \kappa_l/\kappa_\lambda^2) \, , \quad \beta_l = \theta_{E_L E'_L} = (\kappa_d \kappa_l/\kappa_\lambda^2) \, . \end{aligned} \tag{12}$$

Note that $\epsilon_l = \epsilon_\nu$ [20]. Suppressing the first two families and γ_μ, the leptonic current coupled to W_L^+ is given by $(g_2/\sqrt{2})(\bar{\nu}_{\tau L}^{(0)} \tau_L^{(0)} + \bar{N}_L^{(0)} E_L^{(0)} + \bar{N}_R^{(0)} E_R^{(0)})$. Expressing the canonical fields in terms of the mass eigenstates [Eqs. (10) and (11)], the W^+ current containing τ_L is given by

$$J_{W^+} = \frac{g_2}{\sqrt{2}} [\overline{\nu_{\tau L}} \tau_L (a_l a_\nu + \epsilon_l \epsilon_\nu) + \overline{N_L} \tau_L (a_l \epsilon_\nu - \epsilon_l b_\nu) + \overline{N'_L} \tau_L (a_l \eta_\nu - \epsilon_l \beta_\nu) + \text{ other terms}] \, . \tag{13}$$

Since N and N' are heavier than τ, only the first term on the right leading to $\tau_L \to \nu_{\tau L} +$ "W^-" is relevant for τ decay [21]. Thus,

$$\text{Amp}(\tau_L \to \nu_{\tau L} + \text{“}W^-\text{”}) \propto (a_l a_\nu + \epsilon_l \epsilon_\nu) \, . \tag{14}$$

Two features are worth noting: (i) If the Q' family were absent, or equivalently if the mixing between the τ and Q' families were absent, so that $\eta_\nu = \beta_\nu = 0$ and $\eta_l = \beta_l = 0$ [see Eqs. (10) and (11)], then normalization of states would yield $a_\nu^2 + \epsilon_\nu^2 = a_l^2 + \epsilon_l^2 = 1$. Since $\epsilon_l = \epsilon_\nu = \kappa_l/\kappa_\lambda$, it follows that $a_l = a_\nu$. In this case,

$$\frac{\text{Amp}(\tau_L \to \nu_{\tau L} + W^-)}{\text{Amp}(\tau_L \to \nu_{\tau L} + W^-)_{\text{SM}}} = (a_l^2 + \epsilon_l^2) = 1 \, . \tag{15}$$

Thus, the τ decay rate would be universal. The reason for this is simply that *both* (N_L, E_L) and (N_R, E_R) form $SU(2)_L$ doublets and the direct mass term $m_{\text{dir}}^{(0)}$ is negligible. The mixing mass has the invariant form

$$\kappa_l(\bar{\nu}_{\tau L} \bar{\tau}_L) 1 \begin{pmatrix} N_R \\ E_R \end{pmatrix} + \text{H.c.}$$

As a result, in the process of diagonalization of the mass matrices, the angles of rotation in the neutral $(\nu_{\tau L} - N_L)$ system becomes almost equal to that in the charged $(\tau_L - E_L)$ system—i.e., $\theta_{\nu_{\tau L} N_L} = \theta_{\tau_L E_L} = \kappa_l/\kappa_\lambda$ [see Eq. (12)]. Hence, the effective coupling of $W^+ \to \tau_L \nu_{\tau L}$ is unaltered. (ii) If we now add the $SU(2)_R$-doublet family $Q'_{L,R}$, this equality of the rotation angles in the neutral and charged sectors will not hold any longer [compare η_ν and η_l in Eq. (12)] because the relevant mixing masses [following Eq. (7)] now have the form

$$(\bar{\nu}_{\tau L} \bar{\tau}_L) \begin{pmatrix} \kappa_u & 0 \\ 0 & \kappa_d \end{pmatrix} \begin{pmatrix} N'_R \\ E'_R \end{pmatrix} + \text{H.c} \, .$$

In this case,

$$\begin{aligned} a_l a_\nu + \epsilon_l \epsilon_\nu &= \sqrt{1 - (\epsilon_l^2 + \eta_l^2)} \sqrt{1 - (\epsilon_\nu^2 + \eta_\nu^2)} + \epsilon_l \epsilon_\nu \\ &\simeq 1 - \tfrac{1}{2}\eta_l^2 - \tfrac{1}{2}\eta_\nu^2 \end{aligned}$$

(since $\epsilon_l = \epsilon_\nu$). Thus, using (14) and (12), we get

$$\frac{\text{Amp}(\tau_L \to \nu_{\tau L} + W^-)}{\text{Amp}(\tau_L \to \nu_{\tau L} + W^-)_{\text{SM}}} \simeq [1 - \tfrac{1}{2}(\kappa_u/\kappa_\lambda)^2 - \tfrac{1}{2}(\kappa_d/\kappa_\lambda)^2] \, . \tag{16}$$

Since $\kappa_d \ll \kappa_u$, the amplitude is proportional to $\cos\theta_{\nu_{\tau_L} N'_L}$. Thus, the mixing of $\nu_{\tau L}$ with the $SU(2)_R$-doublet leptons $(N', E')_L$, which are, of course, $SU(2)_L$ singlets, is crucial to the origin of τ nonuniversality through vectorlike families.

V. NEUTRINO COUNTING AT LEP

The coupling of Z^0 to canonical neutral leptons suppressing first two families and γ_μ is given by

$$J_{Z^0} = \frac{g_2}{2\cos\theta_W} [\bar{\nu}_{\tau L}^{(0)} \nu_{\tau L}^{(0)} + \bar{N}_L^{(0)} N_L^{(0)} + \bar{N}_R^{(0)} N_R^{(0)}] \, .$$

Without the $SU(2)_R$-doublet vector family $Q'_{L,R}$, J_{Z^0}, being Glashow-Iliopoulos-Maiani (GIM) invariant, would retain the same form for arbitrary $(\nu_{\tau L} - N_L)$ mixing. But, if $\nu_{\tau L}$ mixes with N'_L, i.e., with $\theta_{\nu_{\tau L} N'_L} = \eta_\nu = \kappa_u/\kappa_\lambda \neq 0$ [see Eq. (12)], it is clear that the $\bar{\nu}_{\tau L}^{(0)} \nu_{\tau L}^{(0)}$ term will be altered to

$$\text{“}J_{Z^0}\text{”} = \frac{g_2}{2\cos\theta_W} [\cos^2\theta_{\nu_{\tau L} N'_L} (\bar{\nu}_{\tau L} \nu_{\tau L}) + \sin^2\theta_{\nu_{\tau L} N'_L} \bar{N}'_L N'_L + \tfrac{1}{2}\sin 2\theta_{\nu_{\tau L} N'_L} (\bar{\nu}_{\tau L} N'_L + \bar{N}'_L \nu_{\tau L})] \, . \tag{17}$$

If $m_{N'} \geq m_Z$, as is expected in the model, the last two terms would be irrelevant for Z^0 decay, and there would be a net reduction by $\cos^4\theta$ in the rate of $Z^0 \to \bar{\nu}_\tau \nu_\tau$. Since the mixing of the first two families with Q and Q' are too small [22], the rate of $Z^0 \to \bar{\nu}_e \nu_e$ and $Z^0 \to \bar{\nu}_\mu \nu_\mu$ are essentially unaltered, however, by the light-heavy mixing. In this case, the light neutrino number N_ν, which is counted at LEP, is altered to

$$N_\nu = 2 + \cos^4\theta_{\nu_{\tau L} N'_L} = 3 - \Delta N_\nu \, , \tag{18}$$

where $\Delta N_\nu = 1 - \cos^4\theta_{\nu_{\tau L} N'_L}$. In other words, if the τ anomaly has its origin in $\nu_{\tau L} - N'_L$ mixing, N_ν would necessarily be reduced from 3 by a predicted amount: while Γ_τ is reduced by $\cos^2\theta$, N_ν is reduced by

$(1-\cos^4\theta)$. Given $\Gamma_\tau(\text{obs})/\Gamma_\tau(\text{SM})=0.948\pm0.022$ [1], one would then predict $N_\nu=2.9\pm0.045$ (within 1σ), which is at present, consistent with the reported value $N_\nu(\text{obs})=2.99\pm0.05$ [10]. Further improvements in measurements of Γ_τ and N_ν can clearly establish or rule out the *correlation* between those that are predicted above.

While observing the departures from universality in τ decay and in N_ν, due to mixing of the τ family with vectorlike Q and Q' families, it is worth noting that no significant departures from the standard-model predictions are expected in the decays $Z^0\to\tau_L^+\tau_L^-$, τ_R^+,τ_R^-, $\bar{q}_e q_e$, and $\bar{q}_\mu q_\mu$ in such a model because the relevant light-heavy mixing angles are too small [9,23]. In other words, the "large" effects are expected to be only in $Z^0\to\bar{\nu}_\tau\nu_\tau$ and in the kinematically forbidden process $Z^0\to\bar{t}t$, and, of course, in $\tau\to\nu_\tau+W^-$, in short, in processes involving ν_τ and t. This is because (a) both ν_τ and t have up flavor ($\kappa_u \gg \kappa_d$), (b) both belong to the heaviest chiral family, and (c) the mass matrix has a seesaw form.

VI. ALTERNATIVE EXPLANATIONS FOR τ ANOMALY

(i) A standard fourth family. Notwithstanding the problems of (i) naturalness of $m_{N^s}\geq 45$ GeV, (ii) near up-down degeneracy, and (iii) large positive contribution to ϵ_3, one can, of course, account for the τ anomaly by simply assuming the existence of a heavy fourth family with *standard chiral couplings,* in particular, its leptons

$$\begin{bmatrix} N_L^S \\ E_L^S \end{bmatrix},$$

N_R^S, and E_R^s. As in the case of the first three families, the mixing angles $\theta_{\nu_{\tau L}N_L^S}=\theta_\nu^S$ and $\theta_{\tau_L E_L^S}=\theta_\tau^S$ can be very different from each other. That would imply, if N^S is heavy, that the τ decay rate would be reduced by $\cos^2(\theta_\nu^S-\theta_\tau^S)$ compared to its universal value. As regards neutrino counting, since the Z^0 current contains $\nu_{\tau L}$ and N_L^S in the combination $(\bar{\nu}_{\tau L}^{(0)}\nu_{\tau L}^{(0)}+\overline{N}_L^{S(0)}N_L^{S(0)})$, it is invariant in form under rotations in $(\nu_{\tau L}-N_L^S)$ space. Thus, N_ν will remain at 3 if the τ anomaly is due to a standard fourth-family neutrino N_L^S mixing with $\nu_{\tau L}$, as long as $m_N^S\geq m_Z/2$. This is one crucial distinction between a standard fourth family versus a heavy $SU(2)_R$ vectorlike family being responsible for the τ anomaly.

(ii) A heavy mirror family $\tilde{Q}_{L,R}$. Mirror families couple chirally to W and Z, except that their chiralities are reversed compared to the normal fermions. These *a priori* face the same three problems as a standard fourth family mentioned above. In addition, there is the question of why the $SU(2)_L\times U(1)$-preserving mass terms, which could mix ordinary fermions with the mirror fermions, are not as large as 10^{14} or 10^{19} GeV. Ignoring such questions, a mirror family with leptons

$$\begin{bmatrix} \tilde{N} \\ \tilde{E} \end{bmatrix}_R,$$

$\tilde{N}_L$, and $\tilde{E}_L$ can also explain the τ anomaly through $\nu_{\tau L}\tilde{N}_L$ and $\tau_L\tilde{E}_L$ mixings if $m_{\tilde{N}}\geq m_\tau$. As regards neutrino counting, since the $\bar{\tilde{N}}_L^{(0)}\tilde{N}_L^{(0)}$ term does not appear in J_{Z^0}, $(\nu_{\tau L}-\tilde{N}_L)$ mixing would again reduce N_ν by $(1-\cos^4\theta)$, where θ is the mixing angle.

(iii) E_6 superstring fermions. In E_6 models, in addition to the chiral neutrinos ν_L and ν_R, there are three neutral leptons per 27-plet. If they mix with the chiral neutrinos, the τ anomaly may be explained. In the context of $E_8\times E_8'$ heterotic superstring theories, we note that, due to the absence of the 351′-dimensional Higgs representation, the seesaw mechanism is not effective. It is then a challenge to explain the smallness of ordinary neutrino masses. We have examined three different proposals [24] which achieve this goal for their relevance to the τ anomaly: (i) use of nonrenormalizable terms in the superpotential, (ii) discrete symmetries that forbid tree-level Dirac masses of chiral neutrinos, and (iii) R parity-violating interactions via the VEV of the scalar ν_R. In case (ii), there is no mixing of neutrinos with the heavy leptons while in (iii) such mixing is too small (of order 10^{-3}). The τ anomaly with a reduction in N_ν can be accommodated in case (i) if a suitable hierarchy among the superheavy particles (of order 10^{11} GeV) that varies from one family to another is inserted. This, however, is not a natural or compelling feature of the model.

Before concluding, we observe that, if the τ anomaly is established and if that is also accompanied by the expected departure of N_ν from 3 (see discussions above), the most natural explanation of both phenomena, it seems to us, would be the existence of a heavy $SU(2)_R$ vectorlike family $Q'_{L,R}$. In the context of the model of Refs. [6–9], observation of these phenomena would determine the $(\nu_{\tau L}-N_L')$ mixing angle and, thereby, the crucial parameter κ_u/κ_λ [see (12)]. While this is expected to be in the range of $\frac{1}{3}-\frac{1}{10}$, a precise determination of this from any one experiment would help fix other predictions of the model, such as $Z\to t\bar{c}$ and departures from unitarity in the 3×3 part of the CKM matrix (see Ref. [9]).

In short, the τ anomaly, if it persists, may well be the tip of an iceberg hiding much new and richer physics.

Note added. After the submission of the paper, new data seem to have been collected on the τ mass, τ decay lifetime, and branching ratio. We have been informed by several colleagues that these new data amount to a departure from universality in the τ lifetime in the range of 1.3–1.7, rather 2.3 standard deviations [1]. These new data do not, of course, detract from the main thrust of our paper because, as stressed in the last paragraph, precise determinations of τ_τ and/or N_ν would either reveal new physics or limit κ_u/κ_λ, the expected range for which is $\frac{1}{3}-\frac{1}{10}$.

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ence on High Energy Physics, Geneva, Switzerland, 1991, edited by S. Hegarty, K. Potter, and E. Quercigh (World Scientific, Singapore, 1992).

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[14] The MSW resolution of the solar neutrino puzzle prefers the scenario [case (ii) of Ref. [8]] where $N'_{L,R}$ do not acquire superheavy Majorana masses.

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[16] If one allows $\langle \bar{\psi}^C \psi^C \rangle$ to have both 15^C and 1^C components, one would need seven instead of six parameters with p for quarks being different from that for leptons. But the main analysis of Refs. [8,9] will remain unaltered.

[17] K. S. Babu, J. C. Pati, and H. Stremnitzer (unpublished).

[18] Once a scale for κ_u is chosen (based on m_W), the absolute scale of κ_λ does not enter into known quark and lepton masses. One would, however, expect $\kappa_\lambda \approx (3-10)\kappa_u$ on reasonable grounds.

[19] Since N and N' are nearly degenerate with a common mass of κ_λ, small corrections arising from $\kappa_{u,l}$ tend to mix them maximally. This will not be relevant, however, for the discussion of τ nonuniversality. It will be sufficient to work with the eigenstates given in Eqs. (9) and (10).

[20] This, in general, would not hold if $m_{\rm dir}^{(0)} \neq 0$. With $m_{\rm dir}^{(0)} = 0$, $\epsilon_l = \epsilon_\nu + O(\kappa_d/\kappa_\lambda)^2(\kappa_l/\kappa_\lambda) = \epsilon_\nu + \text{const} \times 10^{-4}$.

[21] Mixing of the τ family with the e and μ families would not affect τ decay universality, because these families (unlike Q') contribute to the W^+ current in a *universal manner* before mixing, and ν_e, ν_μ, and ν_τ are all lighter than τ.

[22] To be quantitative, $\theta_{eQ} \approx (\text{few } \%)(\kappa_{f,c}/\kappa_\lambda) \leq 1\%$ and $\theta_{\mu Q} \approx p(k_{f,c}/\kappa_\lambda) \leq 5\%$ for the up flavors, and considerably smaller for down flavors. Such hierarchical mixing is due to the seesaw nature of the mass matrix [Eq. (7)].

[23] For example, $\theta_{\tau_L E'_L} \approx \theta_{\tau_R E'_R} \approx (\kappa_d/\kappa_\lambda) \leq 1\%$. Of course, $\theta_{\tau_L E_L} \approx \theta_{\tau_R E_R} \approx \kappa_l/\kappa_\lambda \approx 10-20\,\%$ (Ref. [9]), but these mixings have no effects due to GIM invariance in the corresponding terms in the Z^0 current.

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