Systematics of Q^2 ($\bar{Q}^2$) systems with a chromomagnetic interaction

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Within the framework of a simple model including chromomagnetic interactions only, the energies of diquonia Q^2 ($\bar{Q}^2$) with orbital angular momentum $L=0$ are calculated and compared to the threshold energies. The results are given as functions of mass ratios and all possible diquonia built with u, d, s, c, b quarks and spins $S=0,1,2$ are studied. A number of new systems which could be bound under strong interactions are proposed.

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I. INTRODUCTION

Today quantum chromodynamics (QCD) is recognized as the correct theory for strong interactions; it relies on the color interaction between quarks and gluons. The number of possible color configurations increases dramatically with the number of interacting quarks in a given system. However, the experimental situation forces QCD to be implemented with the physical principle that observable hadrons must be color singlets. The immediate consequence [with a $SU(3)_c$ gauge group] is the triality rule: any observable hadron must be composed in such a way that the number of quarks minus the number of antiquarks is a multiple of 3. This principle allows one to reduce greatly the number of color configurations. Indeed, the simplest systems, mesons ($q\bar{q}$) or baryons (q^3), need only one color wave function. These hadrons are bound by gluon exchange. For more complicated objects the binding due to strong interactions can come either from meson exchange or from gluon exchange. The first category contains the wide and rich variety of atomic nuclei and received a lot of attention, although it is based on less microscopic grounds, for more than fifty years. The second category contains particles of type $q^m\bar{q}^n$ bound by gluons, with the restriction $m-n=3B$, B being the baryon number. The total set of such systems bears the generic name of "multiquarks" and has been also qualified as "exotic" in the past. The mere existence of these multiquarks is already very important because it is the clue of the reliability of QCD. They are not forbidden by QCD and thus they must exist either as bound states or as typical resonances in some definite channels. If they are not seen experimentally, QCD has to be questioned or, at least, some mechanism must be invoked to explain their absence.

Among the possible multiquarks, the diquonia $q^2\bar{q}^2$ with $B=0$ are the simplest samples. Since their introduction by Jaffe [1], a lot of work has been devoted to their study. Even for such simple objects an exact treatment based on QCD is not feasible, and one has to rely upon some approximation. Most of the derived models are called "QCD" inspired." Some of them are very crude, using only special constant matrix elements [2–5]. More refined methods have also been used more recently: relativistic strings [6], QCD sum rules [7], bag models [8,9], potential models [10–16], the simplified Bethe-Salpeter formalism [17].

In addition to the problem of the spectrum, the decay of the diquonia is also an interesting question. One can imagine essentially two different decays. The first one proceeds through a rearrangement of the quark wave function and a subsequent decay into two mesons. If there is a great probability that the wave function of the composite particle $q^2\bar{q}^2$ is of the type diquark+antidiquark with a large angular momentum between these two clusters, one expects that the decay into two mesons is highly disfavored [18–20]. On the other hand, there exists the possibility of decaying into a baryon-antibaryon pair [1,5,21–23]. The point is that diquonia with a structure predominantly formed with diquark-antidiquark pairs in a color sextet (the so-called mock diquonia) cannot accommodate a baryon-antibaryon decay. The above arguments explain why a real possibility exists for bound (or at least narrow-width) diquonia.

On the experimental side some candidates have been proposed. The earliest ones are the mesons $a_0(980)$ and $f_0(975)$ whose decays are hardly compatible with a $q\bar{q}$ structure. Jaffe [1] suggested that they may be four-quark states, and this hypothesis was claimed again [12]. Many candidates have been observed in baryon-antibaryon scattering and got the name of "baryonia." Narrow structures were present in $p\bar{p}$ and $\bar{p}d$ at about 1930 MeV and with widths of order 10 MeV [24]. Broader enhancements have also been detected near 2190 and 2400 MeV ($\Gamma\simeq 80$–250 MeV). $\rho^0\rho^0$ resonances have also been observed in $\bar{p}n$ annihilation [25] and in γ-γ reactions [26]: these resonances have been named $X_0(1480)$ and $X_2(1650)$. All these particles are made of ordinary quarks. Some candidates containing strange quarks have also been proposed: the resonance $\zeta(2.2)$ observed in a Mark III experiment [27] has been interpreted as s^2 ($\bar{s}^2$)

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diquonium [28]. But perhaps the most well-known system is the so-called U(3.1) resonance [29], which appeared as a multiplet of three states U^+(seen in the channel $\Lambda\bar{p}\pi^+\pi^+)=3115$ MeV, $\Gamma<80$ MeV; $U^0(\Lambda\bar{p}\pi^+\pi^+\pi^-)=3095$ MeV, $\Gamma<30$ MeV; $U^-(\Lambda\bar{p}\pi^+\pi^-)=3105$ MeV, $\Gamma<30$ MeV. The most plausible explanation would be an $I=\frac{3}{2}sq(\bar{q})^2$ state [17,30]. The missing partner U^{--} was not seen in the previous experiment. An exhaustive catalogue of diquonia is given in Ref. [24]. Up to now, the experimental situation is not clear and no diquonium has been assigned unambiguously.

From the theoretical point of view, practically all studies have been devoted to diquonia composed of identical quarks, or with only two different quark flavors. The conclusions resulting from these works are that (i) no diquonia containing identical quarks are expected to be bound and (ii) possibilities of bound systems exist for very disymmetric cases [$q^2\bar{Q}^2$ with $m_Q/m_q>10$ (see Refs. [9,14])]. However, the number of studied samples is rather restrictive as compared to the huge number of possible systems allowed by the whole set of flavors. The above arguments open the door for new potential candidates. As far as I know the only study of diquonia with three different flavors can be found in Ref. [3]. The authors showed, using a very crude model, that the possibility of stable multiquarks of type ($cs\bar{u}\bar{d}$) or ($cu\bar{s}\,\bar{u}$) is not excluded. In addition to the possibility of diquonia systems being bound by gluon exchanges, there was in the past a great debate on whether some of them could have a molecular structure, and hence be loosely bound. The pioneers in this field were Weinstein and Isgur [12,31], who considered the $f_0(975)$ and the $a_0(980)$ particles as $K\bar{K}$ molecules. This conclusion was contradicted by Morgan and Pennington [32], who analyzed the DM_2 and Mark III experiments in a model-independent manner to prove that their molecular structure is strongly disfavored. Recently Törnqvist [33] proposed that some diquonia could be bound by meson clouds, just like the deuteron; he called these new objects "deusons" and raised arguments to incorporate the $f_1(1420)$, $f_2'(1525)$,$f_0(1587)$, and $f_2(1720)$ resonances within such a scheme. Whatever the structure may be, Freund and Rosner [34] showed that the density of meson and baryon states must be nearly the same for high energy, and this fact imposes the existence of a great number of diquonia states.

In this paper I make an exhaustive study of all diquonia with a total orbital angular momentum $L=0$. The energies of these states are calculated within the framework of a chromomagnetic Hamiltonian and compared to the corresponding energies of the two meson thresholds. Of course, this model is rather schematic, and no claim is made that it gives precise quantitative results. This program is ambitious in the sense that a systematics is performed over all flavor configurations—a work that has never been done—but it must be considered as a first qualitative step towards more refined future calculations. Thus my aim is twofold. First, I want to investigate, among the great number of new proposed diquonia, whether some of them have a good chance of being bound by strong interactions, an indication that is already obtainable from this simple model. Second, my conclusions can be used as a guiding basis for more elaborate calculations, and, as a consequence, the predictive power of a chromomagnetic interaction can be checked on a large scale and in full detail by more sophisticated models.

The paper is organized as follows. In the next section, the chromomagnetic Hamiltonian is described and discussed. In Sec. III, the energies of the mesons and the thresholds are presented, while Sec. IV is devoted to the calculation of diquonia energies. The corresponding numerical results and conclusions are gathered in the last section.

II. THE MODEL HAMILTONIAN

Extensive calculations on a lattice have shown that the interacting potential between two quarks or between a quark and an antiquark, due to gluon exchange, contains a Coulombic short-range part and a linear long-range part. This central interaction is to a large extent flavor independent. Moreover, the relativistic corrections give rise to the so-called hyperfine interaction proportional to $\alpha_s\delta(\mathbf{r}_{ij})(\lambda_i\lambda_j)(\sigma_i\sigma_j)/m_im_j$ (the λ are the Gell-Mann matrices acting on color degrees of freedom and the σ are the Pauli matrices acting on spin degrees of freedom). Such a term is of prime importance for explaining the short-range repulsion in the nucleon-nucleon interaction [35]. Nowadays, every one agrees that the previous potentials are crucial ingredients for any phenomenological treatment of hadron spectroscopy. Spin-orbit and tensor forces exist as well, but their effects seem to be of minor importance. The hyperfine interaction is the only one which is flavor dependent; when studying diquonia of different flavors it has the major role in the spectroscopic description of these systems, while the effects of the central interaction depend on the masses through the size of the wave function (coming itself from the kinetic-energy operator) and have a smooth behavior. In its ground state, a system has a strong tendency to develop a spatial wave function with the highest symmetry. The predominant part describes a configuration where all pairs are in relative S waves. The consequence is that the ground state has a total orbital angular momentum $L=0$, and that the corresponding wave function is practically symmetric.

The essence of the chromomagnetic model is to retain only the hyperfine potential with the further restriction that $\langle\delta(\mathbf{r}_{ij})\rangle$ on the spatial wave functions is constant for every quark pair *and* for all systems. As a result, we are left with a Hamiltonian of the type

$$H=-a\sum_{i<j}\frac{(\lambda_i\lambda_j)(\sigma_i\sigma_j)}{m_im_j}\ , \tag{2.1}$$

in which the spatial degrees of freedom are neglected, their dynamical effects being included in the constant term a. Of course this is a very crude approximation to reality. First, neglecting the kinetic energy and the central potential terms is not always justified; a qualitative argument is to say that their effect is smooth both in

complex systems and in their thresholds so that their influence in the binding is attenuated. Second, there is no reason why $\langle \delta(\mathbf{r}_{ij}) \rangle$ should be a universal constant; in fact bag models indicate that an increase of the confinement volume of the quarks in the diquonium relative to that of the meson leads to a reduction of the corresponding spatial hyperfine matrix element and would suggest that $a(q^2(\bar{q}^2)) < a(q\bar{q})$.

In the following I maintain the assumption of a constant a, keeping in mind nevertheless that the resulting binding energies are probably overestimated. However, the ultimate goal is not to provide quantitative results, but to get an overall qualitative impression of what could be the most serious candidates for bound diquonia. Thus, I am more interested in some order in a hierarchy of states than in the exact energy values within this hierarchy. My hope is that a model Hamiltonian such as the one proposed in (2.1) is enough to work out this plan. Even though it seems quite simple or naive, it includes nevertheless most of the necessary ingredients for grasping an idea of the interesting candidates. Indeed, it was extensively used at one time, and the corresponding calculations bore the name of color chemistry [36]. It was applied to diquonia made of ordinary quarks by Mo and Hogaasen [19]. Within this framework systematic studies of $\bar{Q}q^4$ systems [37] and dibaryon systems [38] were carried out recently. In these studies more elaborate treatments were done for some of the most interesting candidates, and the conclusion was that, although the chromodynamic binding is always overestimated, a Hamiltonian such as (2.1) is able to predict in a correct way the general trends of the state hierarchy. Curiously, the same kind of exhaustive work has not been completed for diquonia systems, and the aim of this paper is precisely to fill this gap and to provide a good starting point for further sophisticated calculations. It will be useful in the following to introduce some reference mass m_f and to scale the eigenenergies in units $A_f = a/m_f^2$. The results will depend on the dimensionless parameters $x_i = m_f/m_i$ since the original Hamiltonian (2.1) now becomes

$$H = -A_f \sum x_i x_j (\lambda_i \lambda_j)(\sigma_i \sigma_j) \ . \tag{2.2}$$

The choice of the reference mass depends on which system is under study; in practice it is fixed in a way that minimizes the number of free parameters x_i. It is also helpful to arrange the constituent quarks in order to impose the relations $0 < x_j < 1$, given the fact that quarks $q(u \text{ or } d), s, c, b$ are listed by increasing mass.

III. MESONS AND THRESHOLDS

Since we are interested only in the decay of diquonia into two mesons, the first step is to compute the meson energies to be put in the corresponding thresholds. As already stated we forget about the spatial wave functions and keep only color, flavor, and spin degrees of freedom. Since the mesons are made of a quark and an antiquark, the Pauli principle never applies.

The color function of a quark (an antiquark) belongs to the 3 ($\bar{3}$) $SU(3)_c$ irreducible representation. The coupling $3 \times \bar{3}$ gives rise to a singlet 1 and an octet 8 representation. Only the singlet is allowed for observable mesons; the contribution of the color part $\lambda_i \lambda_j$ is then $-\frac{16}{3}$. Concerning the spin part there are two possibilities: one with $S=0$ (pseudoscalar mesons) whose contribution is -3, and one with $S=1$ (vector mesons) whose contribution is $+1$.

The flavor dependence comes from the $x_i x_j$ contribution in (2.2). In order to unify the notations, I will denote by q the ordinary quark (u or d), which is a member of an isospin $t = \frac{1}{2}$ doublet, and by Q_i other heavy-flavor quarks with isospin $t=0$, ($Q_i = s, c, b$). The index i will be omitted both in Q_i and in x_i if no confusion arises.

Three types of mesons can be imagined.

(i) The ordinary mesons $q\bar{q}$. In addition to the spin value S it is interesting to introduce also the isospin I, which can take here two values, $I=0$ (isoscalar) and $I=1$ (isovector). The original Hamiltonian being invariant under isospin, the isoscalar and isovector mesons lie at the same energy. The corresponding mesons can be classified through the (I,S) values: $\eta(0,0), \pi(1,0), \omega(0,1), \rho(1,1)$. In this case the reference mass is obviously m_u and the resulting energies in units $A = a/m_u^2$ are pure numbers.

(ii) The mixed mesons $q\bar{Q}$ (or $Q\bar{q}$ because of charge conjugation). Here the isospin is always $I = \frac{1}{2}$, and one encounters pseudoscalar K and vector K^* mesons. Here again the reference mass is $m_f = m_u$ and the energies are functions of $x = m_u/m_Q$. Since $m_Q > m_u$ one always has $0 < x < 1$.

(iii) The exotic mesons $Q_1\bar{Q}_2$ (or $Q_2\bar{Q}_1$). The isospin is $I=0$, and the $S=0$, 1 members are denoted by D_{12}, D_{12}^*, respectively. By convention the reference mass is chosen as $m_f = m_1$; since $Q_1\bar{Q}_2$ and $Q_2\bar{Q}_1$ are degenerate it is always possible to impose $m_1 \le m_2$ so that $0 < x_2 = m_1/m_2 \le 1$. The chromomagnetic energies for the mesons are listed in Table I.

The thresholds T are easily computed by combining a meson M_1 with a meson $M_2, T = M_1 - M_2$. The threshold energy E_T is simply the sum of the meson energies, $E_T = E_1 + E_2$. The total Hamiltonian is still invariant under isospin and spin so that the total isospin I and the total spin S are good quantum numbers. They are restricted by the usual coupling rules

$$|I_1 - I_2| \le I \le I_1 + I_2 \ , \quad |S_1 - S_2| \le S \le S_1 + S_2 \ . \tag{3.1}$$

TABLE I. Chromomagnetic energies (in units of $A_f = a/m_f^2$ as explained in the text) for the various possible mesons. I is the isospin and S the spin of the meson. For K mesons $x = m_1/m_Q$ and for D_{12} mesons $x_2 = m_1/m_2$.

	$q\bar{q}$				$q\bar{Q}$		$Q_1(\bar{Q}_2)$	
Meson	η	ω	π	ρ	K	K^*	D_{12}	D_{12}^*
I	0	0	1	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0
S	0	1	0	1	0	1	0	1
E/A_f	-16	$\frac{16}{3}$	-16	$\frac{16}{3}$	$-16x$	$16x/3$	$-16x_2$	$16x_2/3$

IV. THE DIQUONIA

A. The various flavor types

The diquonium is composed of two quarks and two antiquarks. The quarks and/or the antiquarks may be identical so that the Pauli principle becomes effective. In that case it is very useful to adopt a coupling scheme which makes it easy to deal with; this is clearly achieved through a diquark-antiquark basis. I and S, being good quantum numbers, can be chosen to classify the states; however, because of the presence of one or more heavy quarks, several flavor configurations can lead to the same set of (I,S) numbers. Lastly, for a given configuration, one can build several color, isospin, and spin functions so that there may exist several basis states. An exhaustive study of diquonia needs to consider all the corresponding possibilities. The number of basis states is reduced by the Pauli principle whenever it applies. Another source of simplification comes from the charge-conjugation operator; the diquonium Q_1Q_2 $(\bar{Q}_3\bar{Q}_4)$ and its charge-conjugate partner Q_3Q_4 $(\bar{Q}_1\bar{Q}_2)$ have the same energy; it is enough to consider only one. The G parity might have been used as well but it makes sense only for ordinary quarks; since our program is much more ambitious, the necessity of introducing G parity is not fundamental and I will not consider it in the following.

The set of all the diquonia covered in this paper is presented in Table II. Several remarks are in order. I have gathered the various diquonia by their flavor configuration with a decreasing number of ordinary quarks q (or antiquarks $\bar{q}$). The charge-conjugate partners are written explicitly. The cases with two identical quarks (or antiquarks) are treated separately since, because of the Pauli principle, they give rise to a different number of basis states. For each flavor configuration, the possible I and S values are reported, as they are good quantum numbers. The fourth column N_s gives the number of basis states for a given set (flavor, I,S). In practice it gives the dimension of the Hamiltonian matrix to be diagonalized, and hence the number of different eigenvalue states. These states depend on the mass ratios $x_i = m_f/m_i$. For applications, I have considered three types of heavy-flavor quarks Q_i, namely, s,c, or b, for which a good estimate of the corresponding x_i can be obtained. The number of resulting physical diquonia that can be computed for each case is denoted by N_d in the fifth column.

In summary our study deals with $N_s = 120$ eigenvalues corresponding to $N_d = 584$ different physical diquonia. These (N_s,N_d) values can be grouped by I or S value with the following numbering: $I=2$ (4,4); $I=\frac{3}{2}$ (6,18); $I=1$ (30,110); $I=\frac{1}{2}$ (30,198); $I=0$ (50,254); $S=2$ (21,100); $S=1$ (57,284); $S=0$ (42,200).

If we are interested only in the ground state of the sys-

TABLE II. Systematics of all diquonia studied in this paper. They are classified by their flavor type: q is the ordinary quark (u or d) and Q,Q_i any heavy-flavor quark (s,c,b). I and S are the total isospin and spin, and N_s the number of allowed states for this peculiar configuration. N_d is the number of possible physical diquonia when limiting ourselves to only three heavy flavors.

Flavor	I	S	N_s	N_d	Flavor	I	S	N_s	N_d	Flavor	I	S	N_s	N_d
$q^2(\bar{q}^2)$	2	2	1	1	$q^2(\bar{Q}^2)$	1	2	1	3	$qQ_1(\bar{Q}_2^2)$	$\frac{1}{2}$	2	1	9
	2	1	1	1	$Q^2(\bar{q}^2)$	1	1	1	3	$Q_2{}^2(\bar{q}\bar{Q}_1)$	$\frac{1}{2}$	1	3	27
	2	0	2	2		1	0	2	6		$\frac{1}{2}$	0	2	18
	1	2	1	1		0	2	0	0	$qQ_1(\bar{Q}_2\bar{Q}_3)$	$\frac{1}{2}$	2	2	18
	1	1	5	5		0	1	2	6	$2\neq 3$	$\frac{1}{2}$	1	6	54
	1	0	2	2		0	0	0	0	$Q_2Q_3(\bar{q}\bar{Q}_1)$	$\frac{1}{2}$	0	4	36
	0	2	2	2										
	0	1	2	2	$q^2(\bar{Q}_1\bar{Q}_2)$	1	2	1	3	$Q_1{}^2(\bar{Q}_2^2)$	0	2	1	6
	0	0	4	4	$1\neq 2$	1	1	3	9		0	1	1	6
					$Q_1Q_2(\bar{q}^2)$	1	0	2	6		0	0	2	12
$q^2(\bar{q}\bar{Q})$	$\frac{3}{2}$	2	1	3		0	2	1	3					
$qQ(\bar{q}^2)$	$\frac{3}{2}$	1	3	9		0	1	3	9	$Q_1^2(\bar{Q}_2\bar{Q}_3)$	0	2	1	9
	$\frac{3}{2}$	0	2	6		0	0	2	6	$2\neq 3$	0	1	3	27
	$\frac{1}{2}$	2	2	6						$Q_2Q_3(\bar{Q}_1^2)$	0	0	2	18
	$\frac{1}{2}$	1	6	18	$qQ_1(\bar{q}\bar{Q}_2)$	1	2	2	12					
	$\frac{1}{2}$	0	4	12		1	1	6	36	$Q_1Q_2(\bar{Q}_3\bar{Q}_4)$	0	2	2	12
						1	0	4	24		0	1	6	36
						0	2	2	12		0	0	4	24
						0	1	6	36					
						0	0	4	24					

tem $[Q_1Q_2(\bar{Q}_3\bar{Q}_4)]_{I,S}$ our study covers 210 diquonia with the following repartition: $I=2$ (3); $I=\frac{3}{2}$ (9); $I=1$ (39); $I=\frac{1}{2}$ (63); $I=0$ (96); $S=2$ (69); $S=1$ (72); $S=0$ (69).

The top quark t has not been included in our applications because its mass is largely undetermined, but it is implicitly contained in our figures where the mass ratios x_i are plotted as continuous parameters; a good estimate will be provided by taking $x_t=m_f/m_t=0$. The number N_d of corresponding diquonia is greatly increased in that case.

B. Color functions

Each quark (antiquark) belongs to the fundamental 3 ($\bar{3}$) $SU(3)_c$ irreducible representation. Coupling the color degrees of freedom of two quarks to give a diquark (antidiquark) gives rise to a sextet 6 ($\bar{6}$) and a triplet $\bar{3}$ (3). The 6 ($\bar{6}$) representation is symmetric under particle exchange while the $\bar{3}$ (3) representation is antisymmetric. Now we have to couple the diquark to the antidiquark to build a singlet-color function corresponding to an observable multiquark. It is well known that there are two possibilities: the first one leads to the so-called "true" diquonium with the color wave function $|C_1\rangle=[\bar{3}3]_1$; the other one is the "mock" diquonium with color wave function $|C_2\rangle=[6\bar{6}]_1$. In the old time of baryonia phenomenology the true and mock systems were considered as different entities with distinct properties. In fact there exists a color coupling between them and an actual diquonium does not have a pure true or mock structure. Every serious calculation nowadays takes care of this coupling. The calculation of $O_c(i,j)=\lambda_i\lambda_j$ on these states is standard and is not given here.

C. Spin functions

The spins of the quarks couple to S_{12}, the spins of the antiquarks to S_{34}. The total spin S results from the coupling of S_{12} and S_{34}. The only possibilities are $S=0,1,2$ and the spin wave functions of the system are

$$|S_i\rangle=[(12)_{S_{12}}(34)_{S_{34}}]_S \ . \tag{4.1}$$

More explicitly we define the following vectors: spin 0:

$$\begin{aligned} |S_1\rangle&=[(12)_0(34)_0]_0 \ , \\ |S_2\rangle&=[(12)_1(34)_1]_0 \ ; \end{aligned} \tag{4.2a}$$

spin 1:

$$\begin{aligned} |S_3\rangle&=[(12)_0(34)_1]_1 \ , \\ |S_4\rangle&=[(12)_1(34)_0]_1 \ , \\ |S_5\rangle&=[(12)_1(34)_1]_1 \ ; \end{aligned} \tag{4.2b}$$

spin 2:

$$|S_6\rangle=[(12)_1(34)_1]_2 \ . \tag{4.2c}$$

The matrix elements of the spin operator $O_s(i,j)=\sigma_i\sigma_j$ between these basis functions are calculated by usual Racah techniques.

D. Total wave functions

The total Hilbert space is the tensor product of the Hilbert spaces corresponding to color, spin, and flavor degrees of freedom. The basis states are the tensor product of the basis states of each space factor. The color and spin basis states have been studied in the previous subsections. Concerning the flavor degrees of freedom, it is useful to introduce isospin functions in the same way as the spin functions,

$$|T_i\rangle=|[(t_1t_2)_{I_{12}}(t_3t_4)_{I_{34}}]_I \ . \tag{4.3}$$

The isospin t_i of the elementary quarks are $t_u=t_d=\frac{1}{2}$ and $t_s=t_c=t_b=0$. The flavor operator O_f is simply the mass operator in (2.2): $O_f(i,j)=x_ix_j\mathbb{1}$. Once the flavor configuration is fixed, the values of x_i are perfectly determined and one has

$$\langle T_k|O_f(i,j)|T_l\rangle=x_ix_j\delta_{k,l} \ . \tag{4.4}$$

The basis functions for a given system are

$$|\alpha_{klm}\rangle=|C_k\rangle|S_l\rangle|T_m\rangle \ . \tag{4.5}$$

The permutation properties of $|C_k\rangle$, $|S_l\rangle$, and $|T_m\rangle$ are easily calculated, and the same holds for $|\alpha\rangle$; in the case of identical particles, one retains in (4.5) only the basis states which satisfy the Pauli principle. The number N_s of basis states $|\alpha\rangle$ for each configuration is explicitly given in Table II. The physical eigenvectors

$$|\Psi_r\rangle=\sum d_\alpha^r|\alpha\rangle \tag{4.6}$$

are determined through the diagonalization of the Hamiltonian matrix $\langle\alpha'|H|\alpha\rangle$. The total Hamiltonian is given by (2.2), and we explained previously how to calculate the necessary matrix elements. The result is

$$\langle\alpha_{k'l'm'}|H|\alpha_{klm}\rangle=-A_f\sum_{\substack{i<j\\k,l,m,k',l',m'}}x_ix_j\delta_{mm'}\langle C_{k'}|O_c(i,j)|C_k\rangle\langle S_{l'}|O_s(i,j)|S_l\rangle \tag{4.7}$$

The eigenvalues $E_r(x_1,x_2,x_3,x_4)$ are the chromomagnetic energies of the system. They must be compared with the threshold energies $E_T(x_1,x_2,x_3,x_4)$. The threshold energy is defined as the minimum energy for all decay channels compatible with the selection rules (3.1). If the difference $E_r-E_T=-B_r$ (B_r is the binding energy) is negative, the state $|\Psi_r\rangle$ in (4.6) is expected to be bound by strong interactions, at least by the decay into two mesons.

V. RESULTS

A. Choice of the parameters

The free parameters of the problem are the quark masses m_i and the strength constant a. In fact, as al-

ready explained, it is advantageous to choose a reference mass m_f as the mass of the lightest quark of the system. The scaling energy $A_f = a/m_f^2$ is used as energy unit. The matrix elements (4.7) are then calculated analytically in terms of the dimensionless parameters $x_i = m_f/m_i$, where $0 < x_i \leq 1$. The eigenenergies E_r, the threshold energies, E_T, and the binding energies B_r are functions of the x_i parameters.

If the system is composed of identical flavor particles the results are pure numbers. If one deals with two flavors 1 and 2, we have only one parameter $x = m_1/m_2$ and the data can be presented as a function of x. In that case, all the states can be included in the same figure; a number of results correspond to this situation. On the other hand, if the studied diquonium requires three flavors, we have at our disposal two parameters x_1 and x_2 and the data are plotted as biparametric surfaces. Although the study has been done for all the states, it is difficult to display on the same figure the surfaces corresponding to all levels with given quantum numbers. In order to restrict the amount of information, I decided to present the corresponding results only for the ground state. Since I am especially interested by bound candidates, only the ground state is really relevant for the conclusion. Lastly, there is the case of four flavors, and the unique samples $qs(\bar{c}\bar{b})$, $qc(\bar{s}\bar{b})$, and $qb(\bar{s}\,\bar{c})$. One needs three parameters x_i, and this situation is very difficult to represent as a continuous function of the parameters. I choose to make some definite guess for the x_i and give the corresponding data.

My aim is to provide a qualitative feature of what could be the real physical situation. To this end, and in order to delimit the interesting domains where stability can exist, I present the results, whenever I can, as functions of the mass ratios. However, to get a more quantitative feeling, I also made calculations for systems which correspond to the actual quarks. In order to do this, the quark masses are necessary: the set $m_q = m_u = m_d = 330$ MeV, $m_s = 550$ MeV, $m_c = 1650$ MeV, and $m_b = 4715$ MeV seems to me a good estimate. The x values follow from this choice. To determine the coupling constant a in (2.2), or alternatively the energy unit $A = a/m_q^2$, one can, for instance, fit the energy difference $\Delta - N = 8A - (-8A) = 16A = 290$ MeV. This leads to $A = 18$ MeV. The meson system can be used as well by computing the vector-scalar energy difference $\frac{16}{3}A - (-16A) = \frac{64}{3}A$. The problem is that the experimental values are very different for the isovector members $\rho - \pi = 630$ MeV versus the isoscalar members $\omega - \eta = 230$ MeV. A good compromise is to take the centroïd $\frac{1}{4}(3\times 630 + 230) = 530$ MeV, leading to $A = 25$ MeV. Thus, my definite choice is $A = 20$ MeV. We see in passing that $A(q^3) < A(q^2)$ and probably $A(q^4) < A(q^3)$. For simplicity I maintain a constant value for A in both diquonia and mesons appearing in thresholds to that the numerical bindings are probably overestimated.

B. Qualitative results

In this section, the previous program is carried out for all diquonia in terms of the continuous parameters x_j. I find it convenient to present the results with the order already given in Table II.

1. $q^2(\bar{q}^2)$ *diquonia*

The reference mass is obviously the ordinary quark mass m_q and the energy unit is $A = a/m_q^2$. The results are pure numbers; they are summarized in Table III. Since the Hamiltonian is invariant under isospin symmetry, some data are common to several isospin values. The most bound system corresponds to $I = 0$, $S = 0$ with a binding $B = \frac{4}{3}(\sqrt{241} - 7) = 11.36$. The states with $S = 1$, $I = 0, 1$ are also found below thresholds.

2. $q^2(\bar{q}\bar{Q})$

The reference mass is still m_q and the energy unit $A = a/m_q^2$. The results are plotted in Fig. 1 as a function of the mass ratio $x = m_q/m_Q$. All states with $I = \frac{3}{2}$ are gathered on the right-hand side of the figure. The $S = 2$ level lies at threshold, and there is a possibility of binding for a $S = 1$ state with $x > 0.84$; these values do not correspond to any physical situation so that no bound state exists for isospin $\frac{3}{2}$. On the left-hand side of the figure, I have reported the diquonia with $I = \frac{1}{2}$. A $S = 0$ state is always bound whatever the mass ratio, and the binding is greater for a lighter Q quark. There is also a bound state with $S = 1$, the importance of the binding energy being nearly independent of the heavy flavor.

3. $q^2(\bar{Q}^2)$ *diquonia*

The parameters are the same as previously, and the corresponding results are presented in Fig. 2 for both the

TABLE III. Energies relative to threshold energies for all diquonia formed with ordinary quarks. A negative value of $E - E_T$ is the signature of a bound state.

I	S	Thresholds	$E - E_T$
2	2	ρ-ρ	0
2	1	π-ρ	$\frac{32}{3}$
2	0	π-π	$\frac{4}{3}(25+\sqrt{241})$
			$\frac{4}{3}(25-\sqrt{241})$
1	2	ρ-ρ, ω-ρ	0
1	1	η-ρ, ω-π, π-ρ	$\frac{64}{3}, \frac{64}{3}$
			$\frac{32}{3}$
			$-\frac{8}{3}, -\frac{8}{3}$
1	0	η-π, π-π	$\frac{4}{3}(25+\sqrt{241})$
			$\frac{4}{3}(25-\sqrt{241})$
0	2	ω-ω, ρ-ρ	0,0
0	1	η-ω, π-ρ	$\frac{32}{3}$
			$-\frac{16}{3}$
0	0	η-η, π-π	$\frac{4}{3}(25+\sqrt{241})$
			$\frac{4}{3}(25-\sqrt{241})$
			$\frac{4}{3}(7+\sqrt{241})$
			$\frac{4}{3}(7-\sqrt{241})$

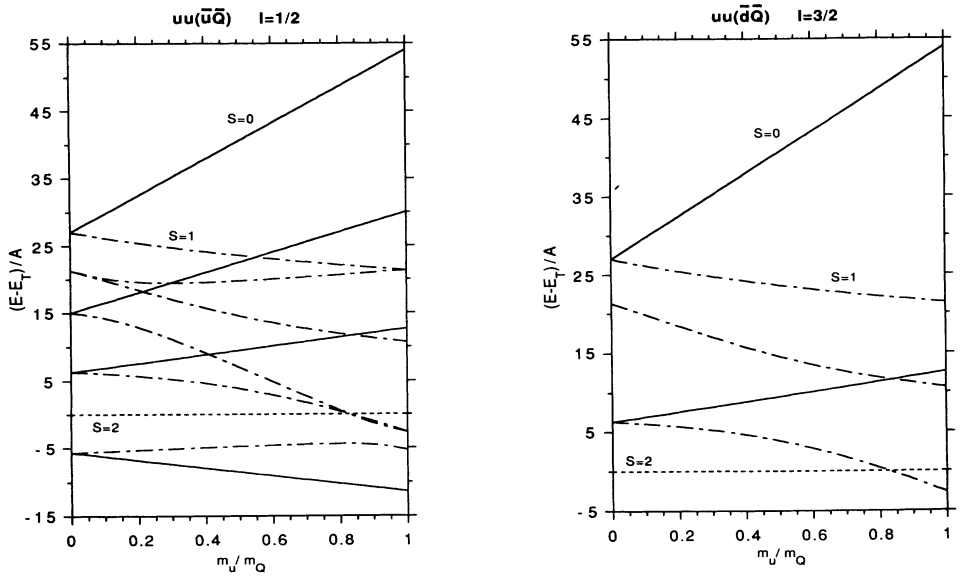

FIG. 1. The chromomagnetic energies for the $q^2(\bar{q}\bar{Q})$ systems are plotted relative to their lowest threshold as a function of the mass ratio m_u/m_Q. All energies are expressed in units of $A=a^2/m_u^2$. The left-hand side corresponds to the lowest isospin value, the right-hand side to the highest one. The states with spin 0 are represented by solid lines, those with spin 1 by dashed-dotted lines, and those with spin 2 by dotted lines. A bound state appears with a negative value.

$I=1$ and $I=0$ levels. No bound states appear in the isospin-1 channel, while one $I=0, S=1$ state is always bound, the binding being greater for a heavier Q quark. This result confirms the conclusions of Refs. [9,14].

4. $q^2(\bar{Q}_1\bar{Q}_2)$ diquonia

The reference mass is the ordinary quark mass $m_f=m_q$ with the scaled energy unit $A=a/m_q^2$. We deal now with a two parameter $(x_1=m_q/m_1, x_2=m_q/m_2)$ surface. The $I=0$ and 1 members are drawn, respectively, in Figs. 3(a) and 3(b). In the $I=1$ sector no bound state exists, except for $S=1$ in a small domain with $x_1, x_2 \simeq 1$. It corresponds to unrealistic values for the heavy-quark masses. In the $I=0$ sector all the multiquarks are bound. In general the bigger is the disymmetry between the ordinary and the heavy quarks, the more important is the stability. We are even in a position for which the first excited $S=1$ state of $q^2(\bar{c}\bar{b})$ is also bound.

5. $qQ_1(\bar{q}\bar{Q}_2)$ diquonia

The parameters are chosen as in the previous study. In this case, the $I=0$ and $I=1$ levels are degenerate and are

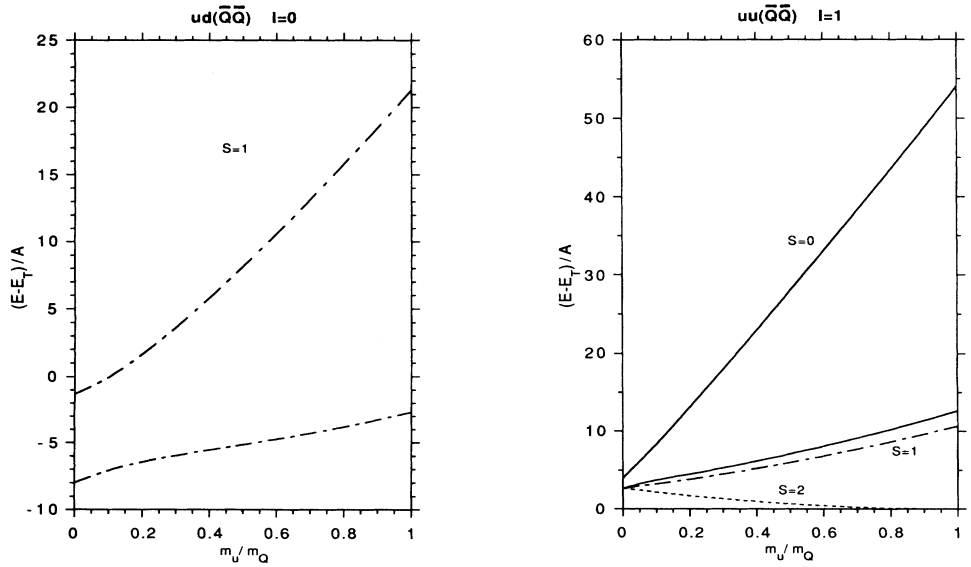

FIG. 2. Same as Fig. 1 for the $q^2(\bar{Q}^2)$ system.

displayed in Fig. 4. All diquonia are stable, although very weakly in the $S=2$ sector. For $S=1$ multiquarks the most stable is found to be $qs(\bar{q}\bar{b})$, while for $S=0$ it is $qs(\bar{q}\,\bar{s})$.

6. $qQ_1(\bar{Q}_2\bar{Q}_2)$ diquonia

The parameters are the same as in the last case. Only $I=\frac{1}{2}$ states exist. The results are summarized in Fig. 5.

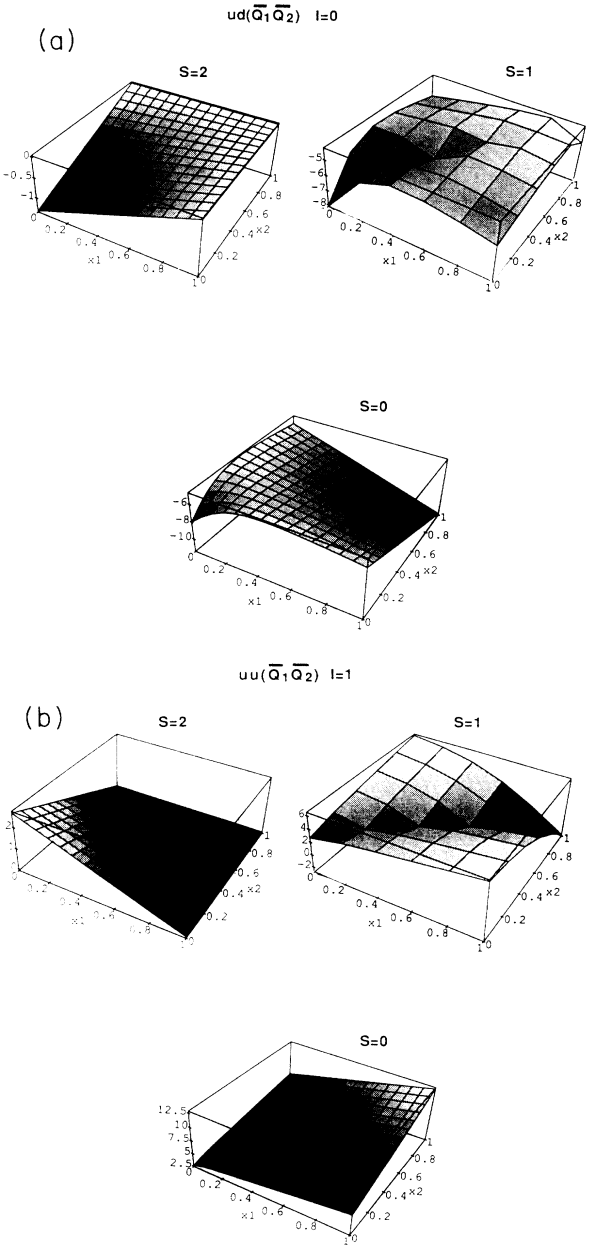

FIG. 3. Chromomagnetic energies for the $q^2(\bar{Q}_1\bar{Q}_2)$ systems relative to their lowest threshold as a function as the mass ratios $x_1=m_u/m_{Q_1}$ (x axis) and $x_2=m_u/m_{Q_2}$ (y axis). The energies represented on the z axis are given in units of $A=a^2/m_u^2$. Only the ground state of the system is displayed for each isospin I and spin S value. Figure 3(a) deals with $I=0$ levels, while Fig. 3(b) is concerned with $I=1$ levels.

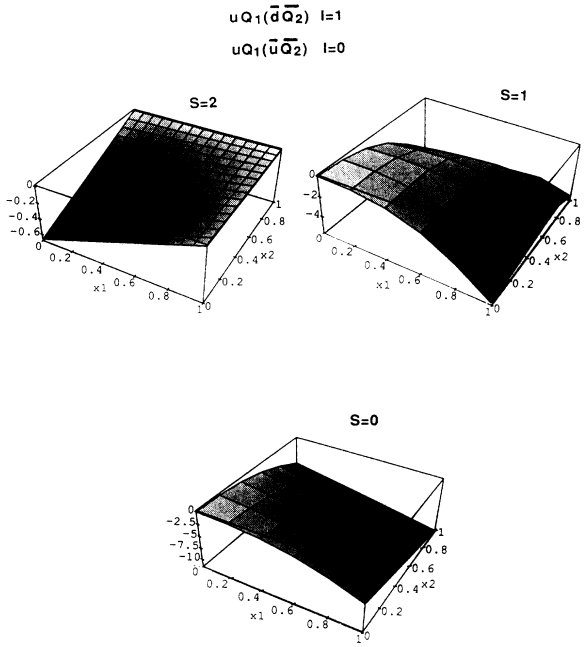

FIG. 4. Same as Fig. 3 for $qQ_1(\bar{q}\bar{Q}_2)$ systems. The levels are degenerate in isospin.

In the $S=2$ sector, the binding takes place only in some restricted domain and is very weak: the only bound diquonia are $qc(\bar{s}\,\bar{s}), qb(\bar{s}\,\bar{s}), qb(\bar{c}\,\bar{c})$. In the $S=1$ sector, the stability is also selective, being important for "light" Q_1 and "heavy" Q_2. No bound state appears in the $S=0$ part.

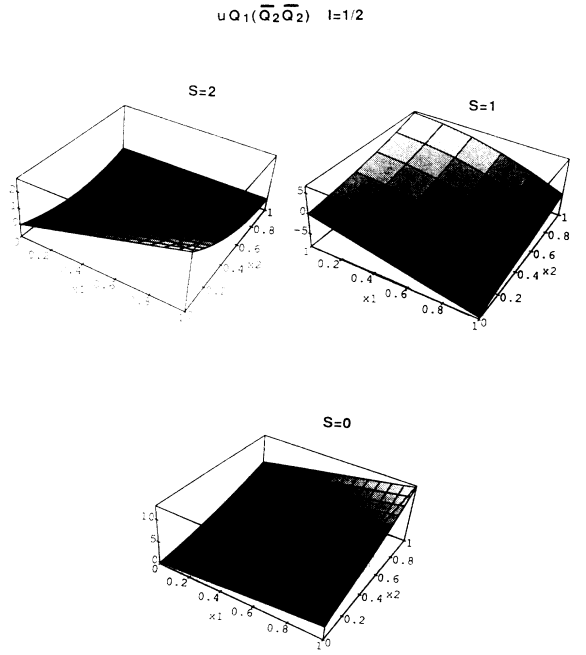

FIG. 5. Same as Fig. 3 for $qQ_1(\bar{Q}_2{}^2)$ systems. Only one isospin value is allowed.

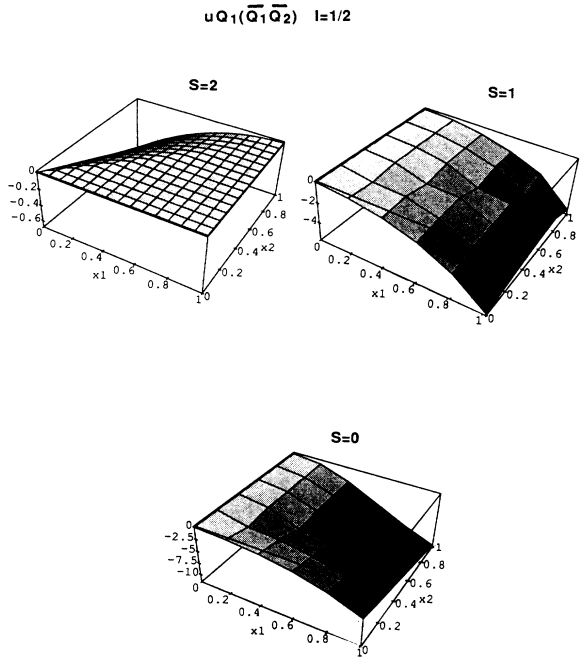

FIG. 6. Same as Fig. 5 for $qQ_1(\bar{Q}_1\bar{Q}_2)$ systems.

7. $qQ_1(\bar{Q}_2\bar{Q}_3)$ *diquonia*

In this case all the particles can support different flavor and the qualitative curves are not easy to draw. We postpone the corresponding study for the next section. Here we present the results when one of the antiquarks has the same flavor as the heavy quark, namely, the results for the $qQ_1(\bar{Q}_1\bar{Q}_2)$ systems. The parameters are the same as previously, and the results are displayed in Fig. 6. The $S=2$ sector shows weak binding. The $S=1$ sector has a more pronounced binding, the most stable diquonium being $qs(\bar{s}\bar{b})$. In the $S=0$ sector the binding seems more important, but only for nonrealistic values of the x parameters. The most stable state corresponds to the $qs(\bar{s}\,\bar{c})$ system.

8. $Q_1^2(\bar{Q}_2^2)$ *diquonia*

Because of charge conjugation properties it is always possible to choose the Q_1 quark as the lightest one; the m_1 mass it then taken as the reference mass and the energies are plotted as functions of $x=m_1/m_2$. The corresponding curves are shown in Fig. 7. All the states are unbounded and consequently rather uninteresting.

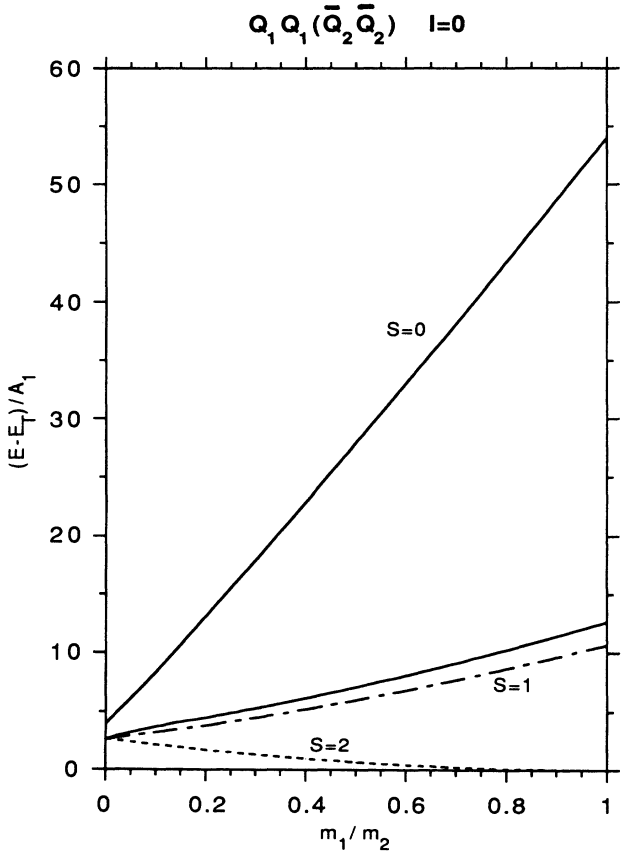

FIG. 7. Same as Fig. 1 for $Q_1^2(\bar{Q}_2)$ systems; the mass ratio parameter is $x=m_1/m_2$ and the energies are given in units of $A_1=a^2/m_1^2$.

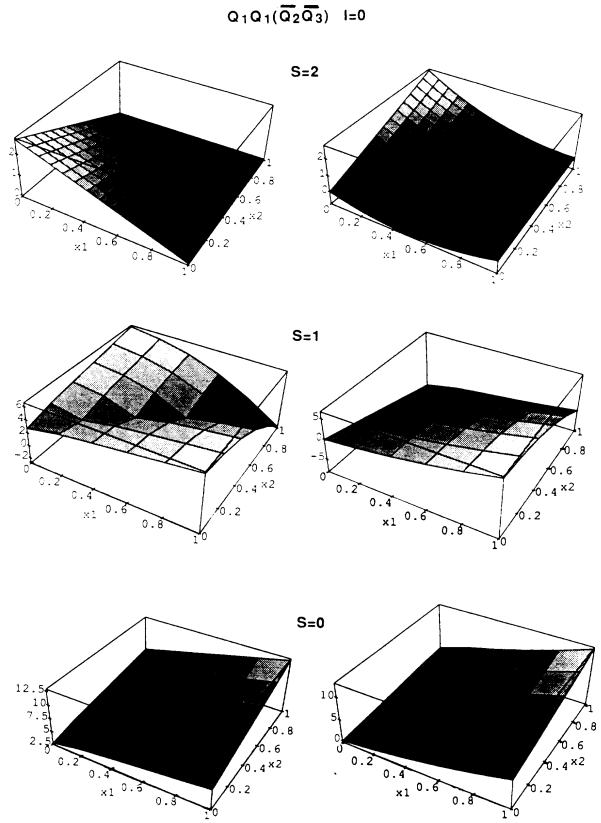

FIG. 8. Same as Fig. 3 for $Q_1^2(\bar{Q}_2\bar{Q}_3)$ systems. On the left-hand side, the lightest particle is Q_1 and the mass ratio parameters are $x_1=m_1/m_2$ and $x_2=m_1/m_3$. On the right-hand side, the lightest particle is Q_2 and the mass ratio parameters are $x_1=m_2/m_1$ and $x_2=m_2/m_3$. In any case the reference mass is the lightest one.

9. $Q_1^2(\bar{Q}_2\bar{Q}_3)$ diquonia

It is always possible to make the convention that $\bar{Q}_2$ is the lightest of the antiquarks (remember that here $\bar{Q}_2 \neq \bar{Q}_3$). In this particular case there are two different possibilities to consider: (a) the quark Q_1 is the lightest particle ($m_1 < m_2$); (b) the antiquark $\bar{Q}_2$ is the lightest particle ($m_2 < m_1$). In either case, the reference mass is chosen as the lightest mass [$m_f = m_1$ in (a), $m_f = m_2$ in (b)] and the energy surfaces are drawn as functions of the ratio between other particles and the reference mass. The results are presented in Fig. 8. The left-hand-side figures correspond to case (a), the right-hand-side ones to case (b). We see that there are few possibilities for binding, and, when it occurs, it is very weak.

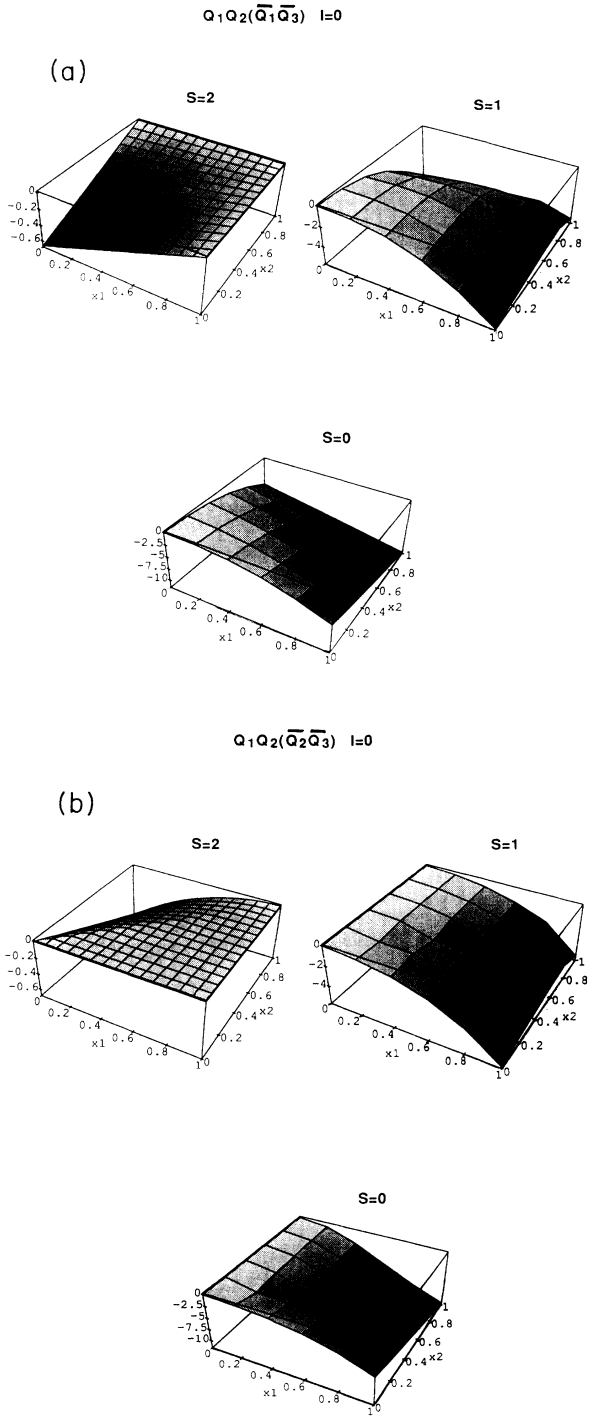

FIG. 9. Same as Fig. 3 for $Q_1Q_2(\bar{Q}_3\bar{Q}_4)$ systems. The reference mass is always m_1 and the mass ratio parameters are $x_1 = m_1/m_2$ and $x_3 = m_1/m_3$. In Fig. 9(a) the lightest antiquark has the same mass as the lightest quark Q_1, while in Fig. 9(b) it has the same mass as the heaviest quark Q_2.

10. $Q_1Q_2(\bar{Q}_3\bar{Q}_4)$ diquonia

We are in a situation where $\bar{Q}_1 \neq \bar{Q}_2$ and $\bar{Q}_3 \neq \bar{Q}_4$ (otherwise we fall into the category of Sec. V B9), but it is impossible that the four particles are different since we have only three types of heavy quarks. It is always possible to take Q_1 as the lightest particle, and my choice is $m_f = m_1$. The first mass ratio parameter is $x_1 = m_1/m_2$. Let $\bar{Q}_3$ be the lightest antiquark; the other parameter is $x_2 = m_1/m_4$. One antiquark must be identical to a quark. We have two configurations.

(a) $Q_3 = Q_1$, and the system is $Q_1Q_2(\bar{Q}_1\bar{Q}_4)$, or by relabeling $Q_1Q_2(\bar{Q}_1\bar{Q}_3)$. This is the case treated in Fig. 9(a).

(b) $Q_3 = Q_2$, and the system is $Q_1Q_2(\bar{Q}_2\bar{Q}_4)$, or by relabeling $Q_1Q_2(\bar{Q}_2\bar{Q}_3)$. This is the case treated in Fig. 9(b).

This situation where Q_4 is identical to one of the quarks is either impossible (because of mass constraint) or can be traced back to one of the previous samples by charge conjugation.

We see from the figure that there exist plenty of possible stable diquonia, particularly in $S=0$ sector. However, remember that the energy unit is $A_1 = a/m_1^2 = (m_q/m_1)^2 A$, which is substantially weaker than the usual A unit.

C. Quantitative results

To get a more quantitative picture in the energy hierarchy of all the multiquarks, I have applied the chromomagnetic formulas with the set of parameters discussed at the beginning of the section, $A = 20$ MeV, $m_q = 330$ MeV, $m_s = 550$ MeV, $m_c = 1650$ MeV, $m_b = 4715$ MeV. The 584 multiquarks have been computed. In fact, most of them (at least every ground state) are contained implicitly in the figures. Among them, 110 correspond to stable diquonia. This is quite a lot, but 62 of them have a binding energy less than 10 MeV. Anyway, there still remain 15 systems bound by more than 100 MeV. I summarized them in Table IV by order of decreasing binding. The thresholds (with the usual nomenclature) are given as indications, although the multiquark states lie below them. It is quite astonishing that the most bound state is composed of identical quarks, since previous studies seem to forbid such a configuration. The answer to this paradox is the special situation of the pion among the mesons. The pion is anomalously bound as compared to other mesons (compare π-ρ to η-ω). The chromomagnetic interaction does not reproduce correctly this feature. More quantitatively, using the above parameters, we find $m_\pi = m_\eta = 340$

TABLE IV. All diquonia bound by more than 100 MeV obtained within the chromomagnetic model and the set of parameters discussed in the text.

$q_1q_2(\bar{q}_3\bar{q}_4)$	I	S	B (MeV)	Thresholds
$ud(\bar{u}\bar{d})$	0	0	227	π-π
$ud(\bar{d}\,\bar{s})$	$\frac{1}{2}$	0	182	π-K
$ud(\bar{b}\,\bar{b})$	0	1	147	B-B^*
$ud(\bar{d}\bar{c})$	$\frac{1}{2}$	0	136	π-$\bar{D}$
$ud(\bar{c}\,\bar{c})$	0	1	129	$\bar{D}$-$\bar{D}^*$
$ud(\bar{d}\,\bar{b})$	$\frac{1}{2}$	0	122	π-B
$us(\bar{d}\,\bar{s})$	1	0	116	π-η_s
$us(\bar{u}\,\bar{s})$	0	0	116	η-η_s
$ud(\bar{c}\bar{b})$	0	1	115	$\bar{D}$-B^*
$ud(\bar{s}\,\bar{c})$	0	0	115	K-$\bar{D}$
$ud(\bar{d}\,\bar{b})$	$\frac{1}{2}$	1	111	π-B^*
$ud(\bar{u}\bar{d})$	0	1	107	π-ρ
$ud(\bar{d}\bar{c})$	$\frac{1}{2}$	1	106	π-$\bar{D}^*$
$ud(\bar{c}\bar{b})$	0	0	106	$\bar{D}$-$\bar{B}$
$ud(\bar{s}\bar{b})$	0	0	105	K-$\bar{B}$

MeV and $m_\omega = m_\rho = 767$ MeV. Thus, the chromomagnetic interaction is quite good for ω and ρ, and still acceptable for η, but it completely fails for the π, which is 200 MeV higher than it should be. The consequence is that the thresholds calculated with the chromomagnetic interaction must be lower by 200 MeV for each π appearing in them. Considering Table IV, this means that we must remove from the list of bound states all the systems decaying by π+anything. Half of the potential candidates disappear after this operation.

After our long and exhaustive calculations, the most interesting stable diquonia predicted by the chromomagnetic interactions are $ud(\bar{b}\,\bar{b})$, $ud(\bar{c}\,\bar{c})$, $ud(\bar{c}\bar{b})$, $ud(\bar{s}\bar{b})$, $ud(\bar{s}\,\bar{s})$, $ud(\bar{s}\,\bar{c})$ with $I=0, S=1$; $us(\bar{u}\,\bar{s})$, $ud(\bar{c}\bar{b})$, $ud(\bar{s}\bar{b})$, $ud(\bar{s}\,\bar{c})$ with $I=0, S=0$; and $us(\bar{b}\,\bar{b})$ with $I=\frac{1}{2}, S=1$. All these systems are bound by some 100 MeV or so, but, as already explained, the calculated binding energy is probably overestimated. It will be highly instructive to compare these conclusions to more refined computations on the same subject. Will the same configurations persist when changing the potential and the method, and how important will the modifications on the binding energies be? These are open and interesting questions. We have undertaken such a program and this work is in progress.

ACKNOWLEDGMENTS

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[1] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977); **15**, 281 (1977); **17**, 1444 (1978).
[2] H. J. Lipkin, Phys. Lett. **70B**, 113 (1977).
[3] N. Isgur and H. J. Lipkin, Phys. Lett. **99B**, 151 (1981).
[4] C. Rosenzweig, Phys. Rev. Lett. **36**, 697 (1976).
[5] M. Fukugita and T. H. Hansson, Phys. Lett. **84B**, 493 (1979).
[6] L. A. Kondratyuk, B. V. Martemyanov, and M. G. Shchepkin, Yad. Fiz. **46**, 1552 (1987) [Sov. J. Nucl. Phys. **46**, 921 (1987)].
[7] J. I. Latorre and P. Pascual, J. Phys. G **11**, L231 (1985).
[8] L. Heller and J. A. Tjon, Phys. Rev. D **32**, 755 (1985).
[9] J. Carlson, L. Heller, and J. A. Tjon, Phys. Rev. D **37**, 744 (1988).
[10] I. M. Barbour and D. K. Ponting, Nucl. Phys. **B149**, 534 (1979).
[11] J. P. Ader, J. M. Richard, and P. Taxil, Phys. Rev. D **25**, 2370 (1982); J. L. Ballot and J. M. Richard, Phys. Lett. **123B**, 449 (1983).
[12] J. Weinstein and N. Isgur, Phys. Rev. D **27**, 588 (1983).
[13] A. M. Badalyan, B. L. Ioffe, and A. V. Smilga, Nucl. Phys. **B281**, 85 (1987).
[14] S. Zouzou, B. Silvestre-Brac, C. Gignoux, and J. M. Richard, Z. Phys. C **30**, 457 (1986).
[15] C. Semay, Ph.D. thesis, Mons University, 1989.
[16] R. Ceuleneer and C. Semay, J. Phys. G **15**, 1783 (1989).
[17] S. Ono and S. Furui, Z. Phys. C **36**, 651 (1987).
[18] M. B. Gavela *et al.*, Phys. Lett. **79B**, 459 (1978).
[19] Chan Hong Mo and H. Hogaasen, Phys. Lett. **72B**, 121 (1977); Nucl. Phys. **B136**, 401 (1978).
[20] Y. S. Kalashnikova and I. M. Narodetskii, Z. Phys. C **43**, 273 (1989).
[21] J. P. Ader, B. Bonnier, and S. Sood, Nuovo Cimento **68**, 1 (1982); Z. Phys. C **5**, 85 (1980); Phys. Lett. **84B**, 488 (1979).
[22] I. M. Barbour and J. Gilchrist, Z. Phys. C **7**, 225 (1981); **8**, 282(E) (1981).
[23] W. Roberts, B. Silvestre-Brac, and C. Gignoux, Phys. Rev. D **41**, 182 (1990).
[24] L. Montanet, G. C. Rossi, and G. Veneziano, Phys. Rep. **63**, 149 (1980).
[25] K. F. Liu and B. A. Li, Phys. Rev. Lett. **58**, 2288 (1987); D. Bridges *et al.*, *ibid.* **56**, 211 (1986); **56**, 215 (1986).
[26] H. Aihara *et al.*, Phys. Rev. D **37**, 28 (1988).
[27] R. M. Baltrusaitis *et al.*, Phys. Rev. Lett. **56**, 107 (1986).
[28] J. T. Londergan and A. N. Mitra, Indiana University Report No. IUNTC 86/6 (unpublished).
[29] M. Bourquin *et al.*, Phys. Lett. B **172**, 113 (1986).
[30] K. J. Barnes, P. Forgacs, and M. T. Vaughn, Phys. Lett. B **182**, 208 (1986).
[31] J. Weinstein and N. Isgur, Phys. Rev. D **41**, 2236 (1990).
[32] D. Morgan and M. R. Pennington, Phys. Lett. B **258**, 444 (1991).
[33] N. A. Törnqvist, Phys. Rev. Lett. **67**, 556 (1991); Marseille Report No. CPT 91/P.2595 (unpublished).
[34] P. Freund and J. Rosner, Phys. Rev. Lett. **68**, 765 (1992).
[35] M. Oka and K. Yazaki, Prog. Theor. Phys. **66**, 556 (1981); A. Faessler, F. Fernandez, G. Lübeck, and K. Shimizu, Phys. Lett. **112B**, 2011 (1982); Y. Suzuki and K. T. Hecht, Nucl. Phys. **A420**, 525 (1984); B. Silvestre-Brac, J. Carbonell, and C. Gignoux, Phys. Lett. B **179**, 9 (1986).
[36] H. M. Chan *et al.*, Phys. Lett. **76B**, 634 (1978).
[37] J. Leandri and B. Silvestre-Brac, Phys. Rev. D **40**, 2340 (1989).
[38] B. Silvestre-Brac and J. Leandri, Phys. Rev. D **45**, 4221 (1992).

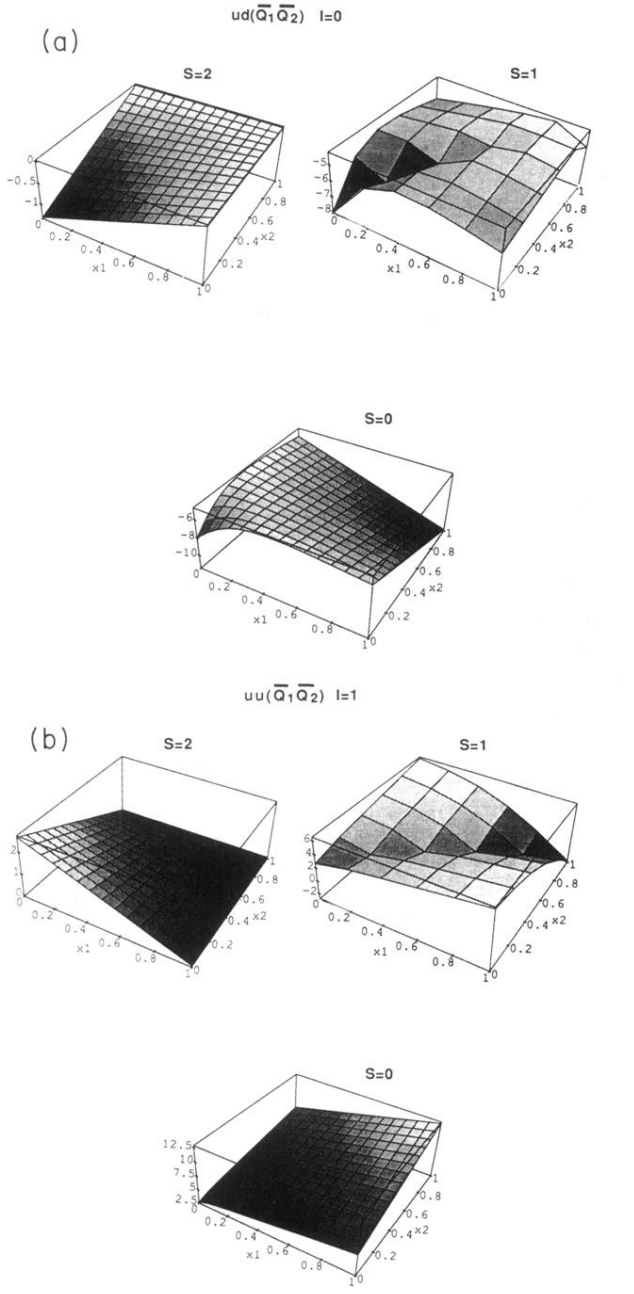

FIG. 3. Chromomagnetic energies for the $q^2(\bar{Q}_1\bar{Q}_2)$ systems relative to their lowest threshold as a function as the mass ratios $x_1 = m_u / m_{Q_1}$ (x axis) and $x_2 = m_u / m_{Q_2}$ (y axis). The energies represented on the z axis are given in units of $A = a^2 / m_u^2$. Only the ground state of the system is displayed for each isospin I and spin S value. Figure 3(a) deals with $I = 0$ levels, while Fig. 3(b) is concerned with $I = 1$ levels.

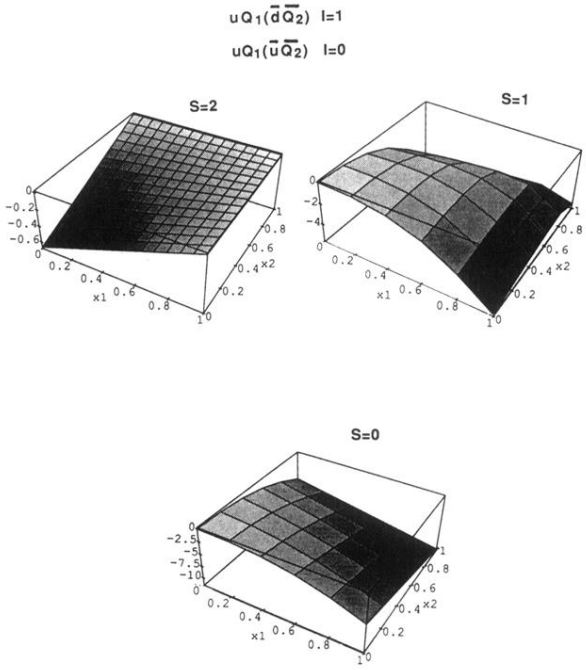

FIG. 4. Same as Fig. 3 for $qQ_1(\bar{q}\bar{Q}_2)$ systems. The levels are degenerate in isospin.

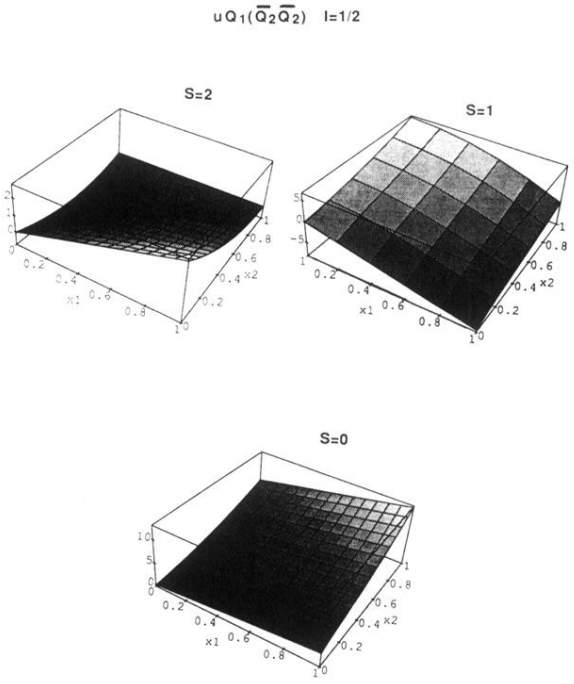

FIG. 5. Same as Fig. 3 for $qQ_1(\overline{Q}_2{}^2)$ systems. Only one isospin value is allowed.

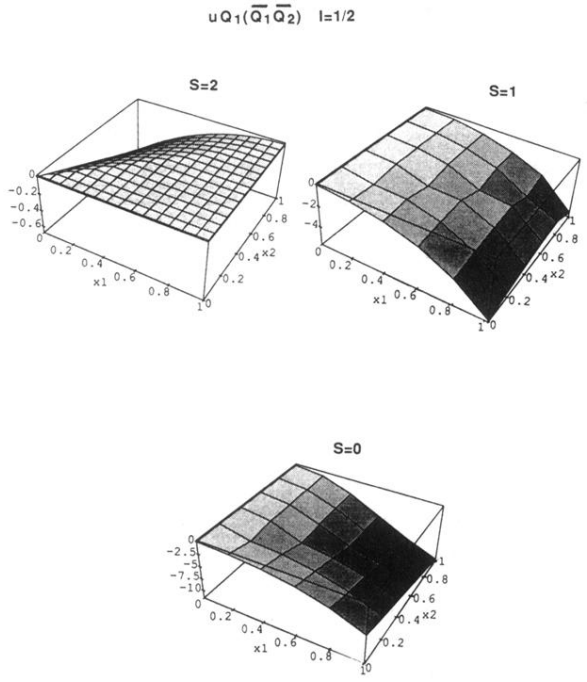

FIG. 6. Same as Fig. 5 for $qQ_1(\overline{Q}_1\overline{Q}_2)$ systems.

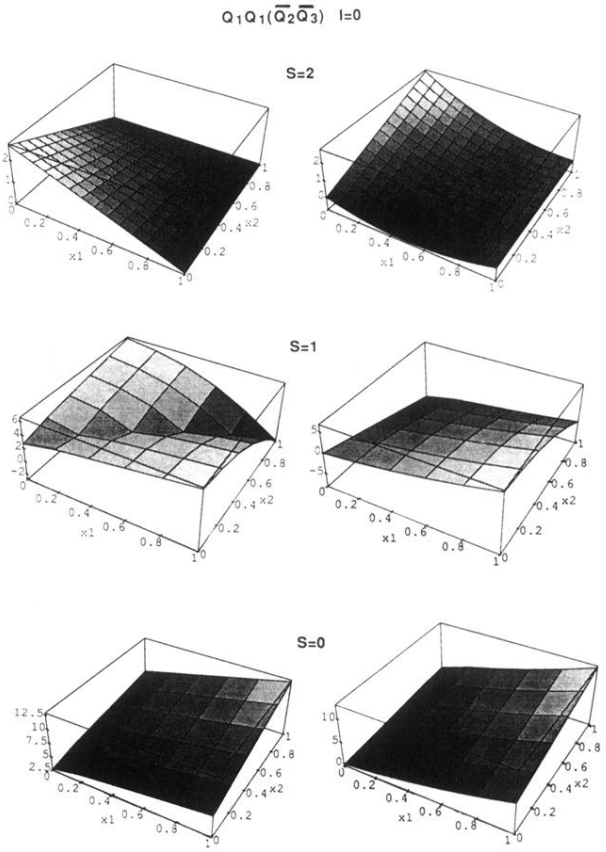

FIG. 8. Same as Fig. 3 for $Q_1{}^2(\overline{Q}_2\overline{Q}_3)$ systems. On the left-hand side, the lightest particle is Q_1 and the mass ratio parameters are $x_1=m_1/m_2$ and $x_2=m_1/m_3$. On the right-hand side, the lightest particle is Q_2 and the mass ratio parameters are $x_1=m_2/m_1$ and $x_2=m_2/m_3$. In any case the reference mass is the lightest one.

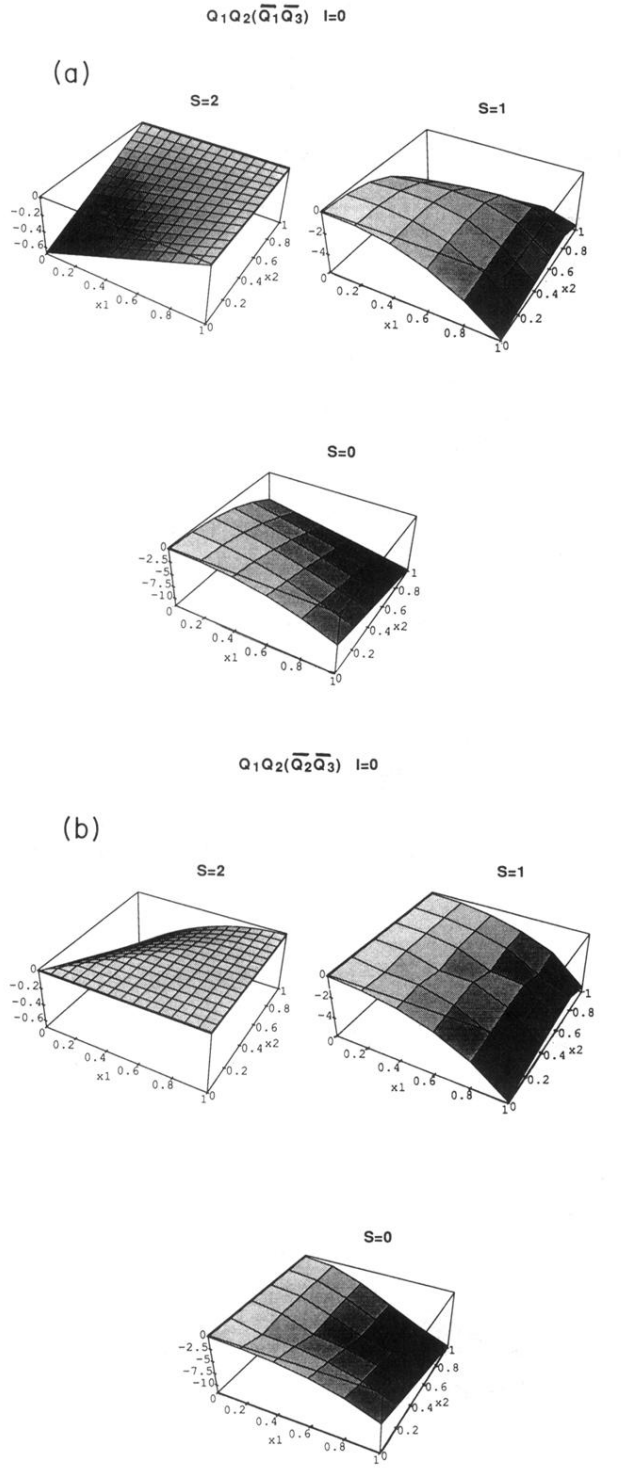

FIG. 9. Same as Fig. 3 for $Q_1Q_2(\bar{Q}_3\bar{Q}_4)$ systems. The reference mass is always m_1 and the mass ratio parameters are $x_1=m_1/m_2$ and $x_3=m_1/m_3$. In Fig. 9(a) the lightest antiquark has the same mass as the lightest quark Q_1, while in Fig. 9(b) it has the same mass as the heaviest quark Q_2.