Constraints on *CP*-violating nucleon-nucleon interactions in gauge models from atomic electric dipole moment

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In this paper *CP*-violating nucleon-nucleon interactions are studied in several models. The experimental upper bounds on the atomic electric dipole moment (EDM) are used to constrain the *CP*-violating parameters of the underlying weak Lagrangian. We compare the constraints from this consideration with those obtained from the upper bound on the neutron EDM. We find that although the constraints from the former consideration are not yet as sensitive as the latter, in some models the constraints from both considerations are within an order of magnitude.

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I. INTRODUCTION

The origin of *CP* violation is one of the fundamental problems of modern physics. *CP* violation was discovered in 1964 in the neutral kaon system [1], but no other *CP*-violating processes have been found. In these circumstances many models have been proposed to explain the phenomenon. It is very important to find *CP* violation in other systems in order to isolate its source (or sources). The measurement of the electric dipole moment (EDM) of fundamental particles is a very promising avenue. The EDM **D** of a classical charge distribution ρ is given by

$$\mathbf{D}=\int d^3r\,\mathbf{r}\rho\ . \tag{1}$$

In the case of an elementary particle, the only (pseudo)vector that characterizes its state is angular moment (spin) **J**; **D** must be proportional to **J**. The interaction of the electric field **E** with the EDM of a particle is $\propto \mathbf{J}\cdot\mathbf{E}$, which violates both *P* and *T*, as long as the proportionality constant (the electric dipole moment) is nonzero. If *CPT* is a good symmetry, *T* violation implies *CP* violation and vice versa.

Although at present no experiments have measured a nonzero EDM of a fundamental particle, upper bounds on the EDM of the neutron ($d_n < 10^{-25}$ *e* cm) [2] and of the electron ($d_e < 10^{-26}$ *e* cm) [3] have put stringent constraints on *CP*-violating parameters in several models.

Experiments have also been performed to measure the EDM's of atoms, $D(A)$. Upper bounds on the EDM of several atoms have been obtained: $D(^{129}\mathrm{Xe}) = (-0.3\pm1.1)\times10^{-26}$ *e* cm [4], $D(^{199}\mathrm{Hg})=(0.7\pm1.5)\times10^{-26}$ *e* cm [5], $D(^{205}\mathrm{Tl})=(1.6\pm5.0)\times10^{-24}$ *e* cm [3], and $D(\mathrm{Cs})=(-1.8\pm6.7\pm1.8)\times10^{-24}$ *e* cm [6]. It used to be thought that measurement of the atomic EDM would be difficult, and not particularly useful. This was because of a theorem of Schiff [7], which states that the EDM of a nonrelativistic atom vanishes irrespective of whether or not the atomic constituents have an EDM if atoms consist of nonrelativistic particles that interact only electrostatically and if the EDM distribution of each atomic constituent is identical to its charge distribution. This theorem works quite well for the ground-state hydrogen atom, for example. However, in many cases the conditions of the theorem are not met due to the effects of relativistic electrons, spin-orbit interactions, differences between nucleon charge and EDM density distributions, the finite size of the nucleus, and so on. All these effects can in principle give rise to an atomic EDM if *CP*-violating interactions are present. The EDM of atoms due to these effects can be enhanced considerably compared with the EDM of the constituent particles. For example, in Tl, $D(A)$ is enhanced by a factor of about 500–700 compared with the electron EDM [8]. Many *CP*-violating operators can induced EDM's for atoms: the EDM of the nucleon d_N [9], the EDM of the electron d_e [8,10], *T*-odd nucleon-nucleon interactions [11–13], and *T*-odd electron-nucleon interactions [14]. In general the atomic EDM will be a linear combination of the contributions of several *T*-odd interactions to the lowest order, and it can schematically be written as

$$D(A)=R_N d_N+R_e d_e+C_{N\text{-}N}+C_{e\text{-}N}+\cdots\ , \tag{2}$$

where $C_{N\text{-}N}, C_{e\text{-}N}$ are contributions due to *T*-odd nucleon-nucleon and electron-nucleon interactions, respectively. The calculated values of all the quantities R_i and C_i depend on the models of the atom, the nucleus, and the elementary particles.

Obviously if a nonzero $D(A)$ for some atom should be measured, theoretical input would be necessary to pin down its origin. So far only values of $D(A)$ consistent with zero have been measured. It is customary to deduce upper bounds on different contributions by setting other sources of *CP* violation to zero, i.e., to assume that there are no accidental cancellations among different sources. It is difficult to separate these contributions in a model-independent way. However, in a particular model of *CP* violation the strength of each interaction may be determined, and therefore useful information can be obtained. There have been several recent reviews of the EDM of the neutron [15] and the electron [16]. An attempt at a systematic analysis of T-odd nucleon-nucleon interactions has been made in Ref. [17], where some CP-violating dimension-5 and -6 operators are studied. In this paper we will study the T-odd nucleon-nucleon interactions by systematically investigating several models of CP violation and identifying the dominant contributing operators up to dimension 6 in each model. We then compare the constraints on the CP-violating Lagrangian obtained from this study with those obtained from the upper bound on the neutron EDM. The T-odd nucleon-nucleon interactions in which we are interested are

$$\begin{aligned} & i\frac{G_F}{\sqrt{2}}\eta_{nn}\bar{n}\gamma_5 n\bar{n}n, \quad i\frac{G_F}{\sqrt{2}}\eta_{np}\bar{n}\gamma_5 n\bar{p}p , \\ & i\frac{G_F}{\sqrt{2}}\eta_{pn}\bar{p}\gamma_5 p\bar{n}n, \quad i\frac{G_F}{\sqrt{2}}\eta_{pp}\bar{p}\gamma_5 p\bar{p}p , \\ & i\frac{G_F}{\sqrt{2}}\eta'\bar{p}\gamma_5 n\bar{n}p + \text{H.c.} \end{aligned} \tag{3}$$

These operators will induce a P- and T-odd interaction of the nucleus with the atomic electron cloud [12,13] which is proportional to

$$\varphi = 4\pi \mathbf{Q}\cdot\nabla\rho(0) , \tag{4}$$

where $\rho(0)$ is the electron density at the nuclear origin. $\mathbf{Q}$ is called the Schiff moment (SM) and is given by

$$\mathbf{Q} = \sum \tfrac{1}{10} e(\langle r_p^2 \mathbf{r}_p\rangle - \tfrac{5}{3}R_0^2\langle \mathbf{r}_p\rangle) , \tag{5}$$

where $R_0 = r_0 A^{1/3}$ is the nuclear radius (here A is the atomic number), $r_0 = 1.15$ fm, and the summation is over all protons. The interaction φ will generate a nonzero atomic EDM.

In the nonrelativistic approximation, terms proportional to η_{ij} induce interactions proportional to $\sigma\cdot\nabla\rho(r)$, where σ is the spin of a particular nucleon and $\rho(r)$ is the core neutron and proton density. The interaction generated by the term proportional to η' is of the form $(\boldsymbol{\sigma}_p\times\boldsymbol{\sigma}_n)\cdot(\mathbf{p}_{fn}+\mathbf{p}_{in}-\mathbf{p}_{fp}-\mathbf{p}_{ip})$, where $\mathbf{p}_{(f,i)(n,p)}$ are the neutron and proton final and initial momenta. In many cases, particularly the cases we will discuss, the latter term does not cause a nonzero $\mathbf{Q}$ [11]. We will neglect it in our later discussions.

Calculations of $\mathbf{Q}$ due to nonzero η_{ij} for some atoms have been carried out in Ref. [13]. The results are given as

	^{129}Xe	^{199}Hg	^{203,205}Tl
Q	$1.75\eta_{np}$	$-1.4\eta_{np}$	$1.2\eta_{pp}-1.4\eta_{pn}$

Here the magnitude of the Schiff moment Q is given in the unit 10^{-8} e fm^3. In these atoms, because $J=\frac{1}{2}$, the magnetic quadrupoles are zero, and the nuclear SM is the only nuclear multipole that leads to the atomic EDM. Also, in these atoms the term proportional to η' does not produce a nonzero $\mathbf{Q}$. There are corrections to the numbers given above due to recoil effects. These corrections can be quite large [13] (about a 30% effect) and also can induce a contribution to Q from η_{nn}. However, the corrections are model dependent, and the order of magnitudes given above will not be changed. In our later discussions, we will neglect these recoil contributions.

Constraints on Q for several atoms have been obtained. Combining the experimental result for $D(^{129}\text{Xe})$ [4] and theoretical calculations [18], $Q(^{129}\text{Xe})$ is estimated to be

$$Q(^{129}\text{Xe}) = (-1\pm4)\times10^{-9}\ e\ \text{fm}^3 . \tag{6}$$

Similarly, $Q(^{199}\text{Hg})$ is determined as

$$Q(^{199}\text{Hg}) = (-1.8\pm3.8)\times10^{-10}\ e\ \text{fm}^3 \tag{7}$$

from experimental data of Ref. [5] and the theoretical calculations of Ref. [12]. Using experimental data on TlF from Ref. [19] and theoretical calculations from Ref. [20], Q(Tl) is estimated in Ref. [19] to be

$$Q(\text{Tl}) = (-1.8\pm3.0)\times10^{-10}\ e\ \text{fm}^3 . \tag{8}$$

At the 90% confidence level, Eqs. (7) and (8) and the relationship between Q and η_{np}, η_{pp} given above imply

$$|\eta_{np}| < 7\times10^{-2}, \quad |\eta_{pp}-1.17\eta_{pn}| < 6\times10^{-2} , \tag{9}$$

respectively.

In the rest of this paper we calculate η_{ij} in several models of CP violation and compare the constraints on the CP-violating parameters of the Lagrangian with those obtained from the upper bound on the neutron EDM. In Secs. II–VII, we estimate η in the minimal standard model, in the model with a CP-violating θ term in QCD, in multi-Higgs-doublet models, in left-right-symmetric models, in supersymmetric extensions of the minimal standard model, and in diquark scalar models. We emphasize the possible new CP-violating sources outside the minimal standard model. In Sec. VIII we make our concluding observations. When we use η_{ij} without explicitly identifying the subscripts i and j it can take with value n or p. An unsubscripted η is used to refer to the η_{ij} generically.

II. η IN THE STANDARD MODEL

In the $\text{SU}(3)_C\times\text{SU}(2)_L\times\text{U}(1)_Y$ model with one Higgs doublet (the minimal standard model), CP violation is due to the nonremovable phase in the quark mixing matrix V_{KM} of the charged current [21]. There must be at least three generations in order to have a nonzero CP-violating phase. The charged-current interaction Lagrangian is

$$L_W = \frac{g}{\sqrt{2}}\bar{U}_L\gamma_\mu V_{\text{KM}} D_L W_\mu^+ + \text{H.c.} , \tag{10}$$

where W is the W-gauge boson and $U=(u,c,t,\ldots)$ and $D=(d,s,b,\ldots)$ are the charge $\frac{2}{3}$ and $-\frac{1}{3}$ quark fields, respectively. In the three generation case, V_{KM} can be parametrized as

$$V_{\text{KM}} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} , \tag{11}$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$. The mixing angles are determined by the analysis of many experimental data.

The allowed range of the CP-violating phase δ is determined from the observed CP violation in the K^0-$\bar{K}^0$ system and is [22]

$$2\times 10^{-4} < s_2 s_3 s_\delta < 10^{-3} \tag{12}$$

when one varies the top-quark mass m_t from 90 to 200 GeV with the maximum being reached for small values of m_t.

The quantity η_{ij} has been studied before by two groups. η_{ij} was found to be of order 10^{-8} in Ref. [11]. In the evaluation of η, Ref. [11] omitted some diagrams which cancel out the leading terms. This was partly corrected in the calculation of Ref. [23]. It was found that the dominant contribution is from baryon-pole diagrams, and the result is smaller by a factor of 25 than in Ref. [11]. In Ref. [23] only η_{pn} was evaluated. In this paper we report a calculation for η_{ij} using a different method. The set of diagrams we will evaluate are shown in Fig. 1. We will use the chiral Lagrangian for the π-K transition

$$L_{\pi\text{-}K} = h e^{i\chi} \mathrm{Tr}(\lambda_+ D^\mu M D_\mu M) + \mathrm{H.c.} \ , \tag{13}$$

where M is the pseudoscalar octet of flavor SU(3),

$$\lambda_+ = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} ,$$

and

$$D_\mu = \partial_\mu M + ie A_\mu [Q, M] \ ,$$

with

$$Q = \begin{bmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} ,$$

is the covariant derivative. This Lagrangian will guarantee the k^2 dependence in the π-K transition which was missed in Ref. [11] and was partly corrected in Ref. [23]. h and the CP-violating phase χ are obtained by relating h and χ to $\langle \pi^+\pi^- | H_W | K \rangle$ using PCAC (partial conservation of axial-vector current):

$$\begin{aligned} h m_K^2 &= i\sqrt{2} f_\pi \langle \pi^+\pi^- | H_W | | K_S \rangle \ , \\ \chi &\approx \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0} \ . \end{aligned} \tag{14}$$

We obtain $h = 1.49\times 10^{-7}$ and $\chi = -3.2\,\mathrm{Im}C_5$, where $\mathrm{Im}C_5$ is approximately $-0.1 s_2 s_3 s_\delta$ [24]. A full evaluation for large m_t can be found in Ref. [25].

For the strong-interaction vertices, we use

$$L_S = -\sqrt{2} g_{\pi NN} [\mathrm{Tr}(\bar{B} i\gamma_5 M B) + (2\alpha - 1)\mathrm{Tr}(\bar{B} i\gamma_5 B M)] \ , \tag{15}$$

where $\alpha = 0.64$, $g_{\pi NN}^2/4\pi = 14$, and B is the matrix of baryon-octet fields.

Neglecting small terms, the relevant weak parity-violating $\bar{B}BM$ interaction Lagrangian can be written as

$$\begin{aligned} L_W = \sqrt{2}\{ & f_3 [e^{i\phi_3} \mathrm{Tr}(BM\lambda_+ \bar{B}) + e^{-i\phi_3} \mathrm{Tr}(BM\lambda_- \bar{B})] \\ & + f_4 [e^{i\phi_4} \mathrm{Tr}(\bar{B} M \lambda_+ B) + e^{-i\phi_4} \mathrm{Tr}(\bar{B} M \lambda_- B)]\} \ , \end{aligned} \tag{16}$$

where $\lambda_- = \lambda_+^\dagger$. The parameters f_3 and f_4 are related to the S-wave hyperon decay amplitudes $A(\Sigma^+ \to p\pi^0)$ and $A(\Lambda^0 \to n\pi^-)$. Using the experimental values, one obtains $f_3 = -3.2\times 10^{-7}$, $f_4 = 1.18\times 10^{-7}$. The phases ϕ_3 and ϕ_4 are similarly obtained from the calculated CP-violating amplitudes for hyperon decays [26]; $\phi_3 \approx -0.29\,\mathrm{Im}C_5$, $\phi_4 \approx 0.61\,\mathrm{Im}C_5$.

For the baryon-baryon transition amplitude, we use the SU(3) parametrization

$$L_{BB} = -G_F m_\pi^2 (f f_{i6j} + d d_{i6j}) \ , \tag{17}$$

where f_{i6j} and d_{i6j} are the antisymmetric and symmetric structure constants of the SU(3) group. By fitting P-wave hyperon decay data, f and d are determined to be $f = -0.57$ and $d = 0.65$ GeV. The amplitudes $a_{\Lambda n}$, $a_{\Sigma^0 n}$, and $a_{\Sigma^+ p}$ of Λ^0-n, Σ^0-n, and Σ^+-p transitions are given by $-(3f + d) G_F m_\pi^2/\sqrt{6}$, $-(d - f) G_F m_\pi^2/\sqrt{2}$, and $(d - f) G_F m_\pi^2$, respectively. Their phases $\phi_{\Lambda^0 n}, \phi_{\Sigma^+ p} = \phi_{\Sigma^0 n}$ are calculated to be $\phi_{\Lambda^0 n} \approx \mathrm{Im}C_5$ and $\phi_{\Sigma^+ p} = \phi_{\Sigma^0 n} \approx -0.36\,\mathrm{Im}C_5$, respectively, by using the MIT bag model [27].

The contribution of Fig. 1(a) to the T-odd nucleon-nucleon interaction is

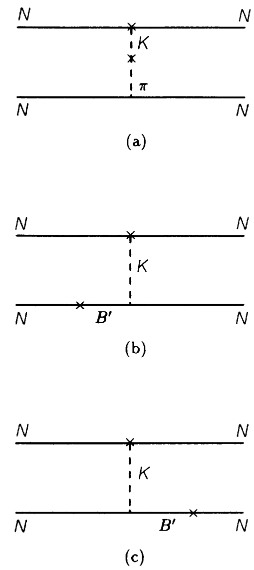

FIG. 1. Diagrams contributing to η in the standard model. B' indicates the intermediate baryon and $\times$ indicates a CP-violating vertex.

$$H=2\sqrt{2}f_{kn}hg_{\pi NN}\sin(\chi-\phi_{kn})\frac{k^2}{(k^2-m_K^2)(k^2-m_\pi^2)}i(\bar{n}\gamma_5 n-\bar{p}\gamma_5 p)\bar{n}n$$
$$+2\sqrt{2}f_3hg_{\pi NN}\sin(\chi-\phi_3)\frac{k^2}{(k^2-m_K^2)(k^2-m_\pi^2)}i(\bar{n}\gamma_5 n-\bar{p}\gamma_5 p)\bar{p}p\ . \tag{18}$$

The baryon-pole contributions [Figs. 1(b) and 1(c)] to the T-odd nucleon-nucleon interaction are

$$H=\frac{4f_{kn}}{k^2-m_K^2}\left[\frac{a_{\Sigma^0 n}g_{\Sigma^0 nK^0}\sin(\phi_{kn}-\phi_{\Sigma^0 n})}{m_N-m_\Sigma}+\frac{a_{\Lambda n}g_{\Lambda nK^0}\sin(\phi_{kn}-\phi_{\Lambda n})}{m_N-m_\Lambda}\right]i\bar{n}\gamma_5 n\bar{n}n$$
$$+\frac{4f_3}{k^2-m_K^2}\left[\frac{a_{\Sigma^0 n}g_{\Sigma^0 nK^0}\sin(\phi_3-\phi_{\Sigma^0 n})}{m_N-m_\Sigma}+\frac{a_{\Lambda n}g_{\Lambda nK^0}\sin(\phi_3-\phi_{\Lambda n})}{m_N-m_\Lambda}\right]i\bar{n}\gamma_5 n\bar{p}p$$
$$+\frac{4f_{kn}}{k^2-m_K^2}\frac{a_{\Sigma^+ p}g_{\Sigma^+ pK^0}\sin(\phi_{kn}-\phi_{\Sigma^+ p})}{m_N-m_\Sigma}i\bar{p}\gamma_5 p\bar{n}n+\frac{4f_3}{k^2-m_K^2}\frac{a_{\Sigma^+ p}g_{\Sigma^+ pK^0}\sin(\phi_3-\phi_{\Sigma^+ p})}{m_N-m_\Sigma}i\bar{p}\gamma_5 p\bar{p}p\ , \tag{19}$$

where $g_{\Sigma^+ pK^0}=-\sqrt{2}g_{\Sigma^0 nK^0}=\sqrt{2}g_{\pi NN}(2\alpha-1)$, $g_{\Lambda nK^0}=-g_{\pi NN}(3-2\alpha)/\sqrt{6}$, and $f_{kn}e^{i\phi_{kn}}=f_3e^{i\phi_3}-f_4e^{i\phi_4}$.

Notice that our results are invariant under a redefinition of the s-quark phase; as was emphasized in Ref. [28], this is an important consistency check in the construction of models of CP violation. Inserting the numerical values of all quantities, we find the dominant contributions are from baryon poles. The contributions from Fig. 1(a) are much smaller than the baryon-pole contributions and we can neglect them. The results are

$$|\eta_{nn}|=2.2\times10^{-5}|\mathrm{Im}C_5|\ ,$$
$$|\eta_{np}|=1.7\times10^{-5}|\mathrm{Im}C_5|\ . \tag{20}$$

The numerical values for η_{pn} and η_{pp} are of order $10^{-7}\mathrm{Im}C_5$ and thus are much smaller. η_{nn} and η_{np} can be as large as 2×10^{-9}. In our estimates of the finite-range effect for nonzero k^2, we have made the approximation that $k^2\approx-m_\pi^2$ as a mean momentum transfer in the nucleus. If the Kobayashi-Maskawa (KM) mechanism is the only source for CP violation, Xe and Hg will be better places to be looking for atomic electric dipole moments than Tl. Of course, these numbers are still very small compared with the present experimental upper bounds.

III. THE θ TERM IN QCD AND THE η

In this section we study η_{ij} generated by the θ term in QCD. It has long been realized that, due to instanton effects in non-Abelian gauge theory, the total divergence term

$$\tilde{G}_{\alpha\beta}G^{\alpha\beta}=\tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}G_{\alpha\beta}G^{\mu\nu} \tag{21}$$

constructed from the field strength $G^{\mu\nu}$ has nonvanishing physical effects. In the case of QCD, $G_{\mu\nu}$ is the gluon field strength. The full QCD Lagrangian incorporating this term is

$$L_{\mathrm{QCD}}=-\frac{1}{4}G^{\mu\nu}G_{\mu\nu}+\bar{q}(D_\mu\gamma^\mu-m)q-\theta\frac{g_s^2}{32\pi^2}\tilde{G}_{\mu\nu}G^{\mu\nu}\ , \tag{22}$$

where q is the quark field, m is the quark mass, D_μ is the covariant derivative, and θ is a constant parameter introduced as a measure of the strength of the CP-violating term in the Lagrangian.

The last term in L_{QCD} violates P and CP. The physical effects of a nonzero θ have been extensively studied [29,30]. This Lagrangian will generate CP-violating meson-nucleon couplings. In this paper we use the result from the chiral Lagrangian to leading order in $1/N$, where N is the number of colors [30]. One obtains

$$L_{\pi NN}=-\sqrt{2}\bar{N}\tau\cdot\pi(i\gamma_5 g_{\pi NN}+f_{\pi NN})N\ , \tag{23}$$

with

$$f_{\pi NN}=0.027\theta\ . \tag{24}$$

From this effective Lagrangian, we obtain CP-violating nucleon-nucleon interactions by exchange of a neutral pion:

$$H_{\mathrm{eff}}=i\frac{g_{\pi NN}f_{\pi NN}}{m_\pi^2-k^2}\bar{N}\tau_3\gamma_5 N\bar{N}\tau_3 N\ . \tag{25}$$

Here we only need to evaluate contributions due to exchange of π^0 because exchange of charged pions will generate a term proportional to η', which can be neglected in our applications. This interaction has a finite range because of the dependence on the momentum carried by the pion. For an order-of-magnitude estimate we can use our previous approximation that $k^2\approx-m_\pi^2$. We obtain $\eta_{nn}=-\eta_{np}=-\eta_{pn}=\eta_{pp}\approx g_{\pi NN}f_{\pi NN}/\sqrt{2}G_Fm_\pi^2$.

The parameter θ has been constrained to be less than 10^{-9} [29] from the upper bound on the neutron EDM. This implies that $|\eta_{ij}|\approx10^6|\theta|<10^{-3}$. This is about an order of magnitude below the experimental bound.

IV. η IN MULTI-HIGGS-DOUBLET MODELS

In this section we study η in multi-Higgs-doublet models. In multi-Higgs-doublet models, it is possible to have CP violation in the Higgs sector. With more than one Higgs doublet, in general, there will be flavor-changing neutral currents induced by neutral Higgs particles at the tree level. Such dangerous neutral flavor-changing currents can be prevented by imposing certain discrete symmetries. These symmetries will eliminate some terms in the Higgs potential and also eliminate *CP* violation in the Higgs sector in some cases. In order to have neutral flavor current conservation at the tree level and *CP* violation in the Higgs sector at least three Higgs doublets are needed [31]. With three Higgs doublets it is also possible to have spontaneous *CP* violation. It has been shown recently that if *CP* violation is due only to spontaneous symmetry breaking, the three-Higgs-doublet model and many other models are ruled out because they have an unacceptably large θ term ($\gg 10^{-9}$) [32,33]. In the following we will discuss models in which *CP* symmetry is broken explicitly such that the large-θ term can always be canceled out by tuning the relevant parameters.

The interactions of Higgs particles with quarks are given by

$$L_{H^+}=(2\sqrt{2}G_F)^{1/2}(\alpha_i \bar{U}_L V_{\rm KM} M_D D_R + \beta_i \bar{U}_R M_U V_{\rm KM} D_L) H_i^+ \qquad (26)$$

and

$$L_{H^0}=(2\sqrt{2}G_F)^{1/2}(\alpha_{di}\bar{D}M_D D+\beta_{di}\bar{D}iM_D\gamma_5 D + \alpha_{ui}\bar{U}M_U U+\beta_{ui}\bar{U}iM_U\gamma_5 U)H_i^0 \, , \qquad (27)$$

where M_U and M_D are the diagonalized quark mass matrices and H_i^+ and H_i^0 are mass eigenstates of charged Higgs and neutral Higgs particles, respectively. In three Higgs-doublet models, there are two charged and five neutral physical Higgs particles. If in the weak eigenstate of Higgs particles only one of the Higgs doublets couples to up and down quarks, $\alpha_i=\beta_i, \alpha_{di}=\alpha_{ui}$, and $\beta_{di}=-\beta_{ui}$. In this case *CP* violation due to exchange of the charged Higgs particles is solely from the KM matrix. If up and down quarks couple to different Higgs doublets, *CP* violation will occur in both the Higgs sector and the KM sector with exchange of charged Higgs particles. In all cases, exchange of neutral Higgs particles can violate *CP*. In the literature sometimes the *CP* violation is parametrized in terms of [34]

$$\mathrm{Im}Z_i=2\,\mathrm{Im}\alpha_i\beta_i^*, \quad \mathrm{Im}Z_{qq',i}=2\alpha_{qi}\beta_{q'i} \, . \qquad (28)$$

In our later discussions, we will assume that the effect of Higgs-boson exchange is dominated by a single Higgs particle. We will use $\mathrm{Im}Z$ and $\mathrm{Im}Z_{qq'}$ for *CP*-violating couplings of the lightest charged and neutral Higgs bosons, respectively.

As we have already seen in Sec. II that η due to the KM mechanism is extremely small, we will study possible large contributions from exchange of Higgs particles. There are several *CP*-violating operators which may have large contributions. Here the dominant ones are

$$Q_{gq}=i\frac{g_s}{2}\bar{q}\sigma_{\mu\nu}\gamma_5\frac{\lambda^a}{2}G^{a\mu\nu}q \, ,$$
$$Q_g=-\frac{g_s^3}{3}f^{abc}\tilde{G}_{\mu\nu}G^b_{\mu\alpha}G^c_{\nu\alpha}, \quad Q_q=i\bar{q}\gamma_5 q\bar{q}'q' \, . \qquad (29)$$

We write the effective Lagrangian as

$$L_{\rm eff}=C_{gq}Q_{gq}+C_qQ_q+C_gQ_g \, . \qquad (30)$$

The coefficients C_i need to be calculated from the model. It turns out that the dominant contributions to C_{gq} [35] and C_g [34,36] are from two-loop diagrams, while C_q is dominated by the exchange of charged and neutral Higgs particles at the tree level. We have

$$C_g=\frac{G_F}{\sqrt{2}(4\pi)^4}\left[\mathrm{Im}Z_{uu}\xi_N h\left[\frac{m_t^2}{m_{H^0}^2}\right] + \mathrm{Im}Z\xi_N' h'\left[\frac{m_t^2}{m_{H^+}^2}\right]\right] , \qquad (31)$$

where

$$\xi_N=\left[\frac{g_s(m_b)}{g_s(m_t)}\right]^{-108/23}\left[\frac{g_s(m_c)}{g_s(m_b)}\right]^{-108/25} \times\left[\frac{g_s(\mu)}{g_s(m_c)}\right]^{-108/27} ,$$

$$\xi_N'=\left[\frac{g_s(m_b)}{g_s(m_t)}\right]^{-28/23}\left[\frac{g_s(m_c)}{g_s(m_b)}\right]^{-108/25} \times\left[\frac{g_s(\mu)}{g_s(m_c)}\right]^{-108/27} ,$$

and m_{H^+} and m_{H^0} indicate the masses of the lightest charged and charged Higgs particles, respectively. The functions $G(z;q)$ and h,h' are given by

$$G(z;u)=[f(z)+g(z)]\mathrm{Im}Z_{uu} \, ,$$
$$G(z;d)=f(z)\mathrm{Im}Z_{ud}+g(z)\mathrm{Im}Z_{du} \, ,$$
$$h(z)=\frac{1}{2}z^2\int_0^1 dx\int_0^1 dy\frac{x^3y^3(1-x)}{[zx(1-xy)+(1-x)(1-y)]^2} \, ,$$
$$h'(z)=\frac{1}{2}\frac{z}{(1-z)^3}\left[-\ln z-\frac{3}{2}+2z-\frac{z^2}{2}\right] ,$$

and

$$f(z)=\frac{1}{2}z\int_0^1 dx\frac{1-2x(1-x)}{x(1-x)-z}\ln\frac{x(1-x)}{z} \, ,$$
$$g(z)=\frac{1}{2}z\int_0^1 dx\frac{1}{x(1-x)-z}\ln\frac{x(1-x)}{z} \, .$$

In multi-Higgs-doublet models C_qQ_q receives contributions from both charged and neutral Higgs particles. For the charged-Higgs-particle contribution, we have

$$C_qQ_q=-i\frac{\sqrt{2}G_F}{12m_{H^+}^2}[\mathrm{Im}Zm_um_d|V_{ud}|^2(\bar{u}\gamma_5u\bar{d}d+\bar{u}u\bar{d}\gamma_5d) + \mathrm{Im}Zm_um_s|V_{us}|^2\bar{u}\gamma_5u\bar{s}s+\cdots] \, , \qquad (32)$$

where a Fierz rearrangement has been made and we have singled out the terms which will give dominant contributions to η when we use the vacuum-saturation and factorization approximation to estimate the matrix element of the four quark operators. For the neutral-Higgs-particle contribution, we have

$$C_q Q_q = i\frac{2\sqrt{2}G_F}{m_{H^0}^2}(\mathrm{Im}Z_{dd} m_d \bar{D} M_D D \bar{d}\gamma_5 d + \mathrm{Im}Z_{du} m_u \bar{D} M_D D \bar{u}\gamma_5 u + \mathrm{Im}Z_{ud} m_d \bar{U} M_U U \bar{d}\gamma_5 d + \mathrm{Im}Z_{uu} m_u \bar{U} M_U U \bar{u}\gamma_5 u)\ . \quad (33)$$

To evaluate η we need to calculate several hadronic matrix elements. We will use the pion-pole-dominance approximation. We first calculate the *CP*-violating $\pi^0 NN$ vertex

$$\langle \pi^0 N | Q_i | N\rangle = B_i\ , \quad (34)$$

and then the π^0 exchange to obtain

$$H_{\text{eff}} = i g_{\pi NN}\frac{B_i C_i}{m_\pi^2 - k^2}\bar{N}\tau_3\gamma_5 N \bar{N}\tau_3 N\ . \quad (35)$$

CP-violating πNN vertices due to Q_{gq} and Q_g have been calculated in Ref. [37]. Using η^0-meson dominance, the parameters B_i are estimated to be

$$B_i \approx -\frac{8\bar{m} a_1 f_i}{4F_\pi^2}\ , \quad (36)$$

where $\bar{m}=(m_u+m_d)/2$, $a_1=(m_\Xi - m_\Sigma)/2(m_s-\bar{m})$, and

$$B_{gq} = -0.22\ \text{GeV},\quad B_g = 0.54\ \text{GeV}^2\ . \quad (37)$$

The contributions to η from Q_g and Q_{gq} are

$$|\eta(Q_g)_{ij}| = 3\times 10^{-4}|0.02\,\mathrm{Im}Z_{uu} h(m_t^2/m_{H^0}^2) + \mathrm{Im}Z h'(m_t^2/m_{H^+}^2)|\ ,$$

$$|\eta(Q_{gq})_{ij}| = \begin{cases} 1.4\times 10^{-5}|G(m_t^2/m_{H^0}^2;u)| & \text{for } q=u\ ,\\ 2.8\times 10^{-5}|G(m_t^2/m_{H^0}^2;d)| & \text{for } q=d\ . \end{cases} \quad (38)$$

We will use the vacuum-saturation and factorization approximation to estimate $\langle \pi^0 N|Q_q|N\rangle$. Within this framework

$$\langle \pi^0 N|Q_u|N\rangle = \langle \pi^0|\bar{u}\gamma_5 u|0\rangle\langle N|\bar{q}q|N\rangle\ , \quad (39)$$

and $\langle \pi^0 N|Q_d|N\rangle$ is obtained by making the obvious substitution $u\to d$.

We use

$$2m_u\langle \pi^0|\bar{u}\gamma_5 u|0\rangle = -2m_d\langle \pi^0|\bar{d}\gamma_5 d|0\rangle = -iF_\pi m_\pi^2\ . \quad (40)$$

To evaluate $\langle N|\bar{q}q|N\rangle$, we use the pion-nucleon $\sigma_{\pi N}$ term $\sigma_{\pi N} = \bar{m}\langle N|\bar{u}u+\bar{d}d|N\rangle = 45$ MeV extracted from experiments and calculations of the nucleon mass shift due to SU(3)-breaking quark masses. For heavy quarks, we use

$$\langle N|m_h \bar{h}h|N\rangle = -\left\langle N\left|\frac{\alpha_s}{12\pi}G_{\mu\nu}G^{\mu\nu}\right|N\right\rangle + O\left[\frac{\mu^2}{m_h^2}\right]\ . \quad (41)$$

The second term in the above equation will be neglected. Using $m_u=4.2$, $m_d=7.5$ and $m_s=150$ MeV, we obtain

$$m_u\langle p|\bar{u}u|p\rangle = 18\ \text{MeV},\quad m_d\langle p|\bar{d}d|p\rangle = 26\ \text{MeV}\ ,$$
$$m_u\langle n|\bar{u}u|n\rangle = 14\ \text{MeV},\quad m_d\langle n|\bar{d}d|n\rangle = 32\ \text{MeV}\ , \quad (42)$$
$$m_s\langle N|\bar{s}s|N\rangle = 247\ \text{MeV}\ ,$$
$$\left\langle N\left|-\frac{\alpha_s}{12\pi}G_{\mu\nu}G^{\mu\nu}\right|N\right\rangle = 48\ \text{MeV}\ ,$$

where $N=n,p$. Combining all the information gathered above, we have

$$\langle \pi^0 n|C_q^{H^+}Q_q|n\rangle = \frac{G_F}{\sqrt{2}}\mathrm{Im}Z\frac{m_\pi^2}{m_{H^+}^2}\times 2.2\times 10^{-4}(\text{GeV}^2)\ ,$$
$$\langle \pi^0 p|C_q^{H^+}Q_q|p\rangle = \frac{G_F}{\sqrt{2}}\mathrm{Im}Z\frac{m_\pi^2}{m_{H^+}^2}\times 1.5\times 10^{-4}(\text{GeV}^2)\ , \quad (43)$$
$$\langle \pi^0 N|C_q^{H^0}Q_q|N\rangle = \frac{G_F}{\sqrt{2}}\frac{m_\pi^2}{m_{H^0}^2}\times[0.06(\text{GeV}^2)(\mathrm{Im}Z_{dd}-\mathrm{Im}Z_{du}) + 0.02(\text{GeV}^2)(\mathrm{Im}Z_{uu}-\mathrm{Im}Z_{du})]\ .$$

Here q is summed over u and d quarks. From these we obtain the contributions $\eta^{(+),(0)}$ to η from the charged and neutral Higgs contributions to the operators Q_q:

$$|\eta_{ni}^{(+)}| = 1.5\times 10^{-7}|\mathrm{Im}Z|\frac{(100\ \text{GeV})^2}{m_{H^+}^2}\ ,$$
$$|\eta_{pi}^{(+)}| = 10^{-7}|\mathrm{Im}Z|\frac{(100\ \text{GeV})^2}{m_{H^+}^2}\ , \quad (44)$$
$$|\eta_{ij}^{(0)}| = 4\times 10^{-5}|(\mathrm{Im}Z_{dd}-\mathrm{Im}Z_{du}) + 0.3(\mathrm{Im}Z_{ud}-\mathrm{Im}Z_{uu})|\frac{(100\ \text{GeV})^2}{m_{H^0}^2}\ ,$$

Here i and j indicate n or p. Using a similar method, in Ref. [38] a calculation of the charged-pion coupling to the nucleon has been done. It was found there that its contribution to the neutron EDM is small compared with that from other operators.

η can also be calculated by first evaluating the Higgs-nucleon couplings and then exchanging a Higgs particle between nucleons. The Higgs-nucleon couplings have been calculated by several groups [33,39]. Using the values for $m_q\langle N|\bar{q}q|N\rangle$ from Eq. (42) and the values

$$\begin{aligned} &m_u\langle n|\bar{u}i\gamma_5 u|n\rangle=-419\text{ MeV},\\ &m_d\langle n|\bar{d}i\gamma_5 d|n\rangle=772\text{ MeV},\\ &m_u\langle p|\bar{u}i\gamma_5 u|p\rangle=432\text{ MeV},\\ &m_d\langle p|\bar{d}i\gamma_5 d|p\rangle=-748\text{ MeV},\\ &m_s\langle N|\bar{s}i\gamma_5 s|N\rangle=-165\text{ MeV},\\ &m_h\langle N|\bar{h}i\gamma_5 h|N\rangle=-63\text{ MeV} \end{aligned} \tag{45}$$

determined from the European Muon Collaboration (EMC) data and others [33], we obtain

$$|\eta_{ij}|=4.3\times10^{-5}|\mathrm{Im}Z_{qq}|\frac{100^2\text{ GeV}^2}{m_{H^0}^2}\ . \tag{46}$$

Comparing the contributions discussed before, we see that the charged-Higgs-particle contribution via the operator Q_g may dominate if $\mathrm{Im}Z_{qq}$ and $\mathrm{Im}Z$ have the same order of magnitude.

If terms that softly break the discrete symmetries which prevent neutral flavor-changing current at the tree level are added in the Higgs potential, it is possible to have *CP* violation in the neutral Higgs sector in two Higgs-doublet models. The contribution to η from neutral Higgs particles is similar to the three Higgs-doublet models discussed before. Of course, in this case there is no contribution from the charged Higgs particles.

V. η IN LEFT-RIGHT-SYMMETRIC MODELS

In this section we study η in left-right-symmetric models. Left-right-symmetric models are based on the gauge group $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ [40] with quarks and leptons being assigned as

$$\begin{aligned} &Q_L:(3,2,1,\tfrac{1}{3}),\quad Q_R:(3,1,2,\tfrac{1}{3})\ ,\\ &L_L:(1,2,1,-1),\quad L_R:(1,1,2,-1)\ . \end{aligned} \tag{47}$$

There are new *CP*-violating interactions in the charged current due to the existence of the right-handed gauge boson W_R of $SU(2)_R$. The interactions of mass eigenstates of charged gauge bosons $W_{1,2}$ with quarks have the form

$$\begin{aligned} L_{W_q}=&\frac{1}{\sqrt{2}}(g_L\bar{U}_L\gamma_\mu V_L D_L\cos\zeta+g_R\bar{U}_R\gamma_\mu V_R D_R\sin\zeta)W_1^\mu\\ &+\frac{1}{\sqrt{2}}(-g_L\bar{U}_L\gamma_\mu V_L D_L\sin\zeta\\ &+g_R\bar{U}_R\gamma_\mu V_R D_R\cos\zeta)W_2^\mu\ , \end{aligned} \tag{48}$$

where all fields are in their mass eigenstates and ζ is the mixing angle between W_L of $SU(2)_L$ and W_R of $SU(2)_R$. V_L is the KM matrix for the left-handed charged current and V_R is an analogous KM matrix involving the right-handed current. If we parametrize V_L in the usual KM way, then for n generations of quarks there are $(n-1)(n-2)/2$ CP-violating phases in V_L. V_R can have a different number of phases depending on the possible models. In the manifest left-right-symmetric models, $V_L=V_R$. In pseudomanifest left-right-symmetric models in which $|(V_L)_{ij}|=|(V_R)_{ij}|$, there are $2n-1$ additional phases in V_R compared with V_L. If there is no relation between V_L and V_R, there are $n(n+1)/2$ phases in V_R. It is no longer necessary to have three generations of quarks in order to have *CP* violation. To see how the new *CP*-violating phases may generate nonzero η with large values, we consider the case, for simplicity, of two generations and $g_L=g_R$. In this case, V_L and V_R can be parametrized as

$$\begin{aligned} &V_L=\begin{pmatrix}\cos\theta_L & -\sin\theta_L\\ -\sin\theta_L & \cos\theta_L\end{pmatrix},\\ &V_R=e^{i\gamma}\begin{pmatrix}e^{-i\delta_2}\cos\theta_R & e^{-i\delta_1}\sin\theta_R\\ -e^{i\delta_1}\sin\theta_R & e^{i\delta_2}\cos\theta_R\end{pmatrix}. \end{aligned} \tag{49}$$

We find that the operator which dominates the contribution to η is due to exchange of W_1 at the tree level. The four-quark effective Lagrangian which contains the *CP*-violating interaction is

$$\begin{aligned} L_{LR}=&4\frac{G_F}{\sqrt{2}}\cos\zeta\sin\zeta(\bar{U}_L\gamma_\mu V_L D_L\bar{D}_R\gamma_\mu V_R^\dagger U_R\\ &+\bar{U}_R\gamma_\mu V_R D_R\bar{D}_L\gamma_\mu V_L^\dagger U_L)\ . \end{aligned} \tag{50}$$

We again use the vacuum-saturation and factorization approximation to evaluate η from the above Lagrangian. After a Fierz rearrangement, we obtain the operator which gives the dominant contribution to η:

$$L=-i\frac{4G_F}{3\sqrt{2}}\cos\zeta\sin\zeta[\mathrm{Im}(V_{Lua}V_{Rud}^*)(\bar{u}\gamma_5 u\bar{d}d-\bar{u}u\bar{d}\gamma_5 d)+\mathrm{Im}(V_{Lus}V_{Rus}^*)\bar{u}\gamma_5 u\bar{s}s]\ . \tag{51}$$

The *CP*-violating πNN vertex is

$$f_{\pi NN}=\langle \pi N|L_{LR}|N\rangle=-i\frac{4}{3}\frac{G_F}{\sqrt{2}}\cos\zeta\sin\zeta\left[\mathrm{Im}(V_{Lud}V^*_{Rud})\langle\pi^0|2m_u\bar{u}\gamma_5 u|0\rangle\left\langle N\left|\frac{\bar{u}u}{2m_u}+\frac{\bar{d}d}{2m_d}\right|N\right\rangle\right.$$
$$\left.+\mathrm{Im}(V_{Lus}V^*_{Rud})\langle\pi^0|\bar{u}\gamma_5 u|0\rangle\langle N|\bar{s}s|N\rangle\right]$$
$$\approx -90\frac{G_F}{\sqrt{2}}m_\pi^2\cos\zeta\sin\zeta\,\mathrm{Im}(V_{Lud}V^*_{Rud})\ . \qquad (52)$$

Inserting the known values into the above equation and assuming $\theta_L=\theta_R$, we obtain

$$|\eta_{ij}|\approx 0.6\times 10^3|\cos\zeta\sin\zeta\sin(\gamma-\delta_2)|\ . \qquad (53)$$

There are also some other contributions to η, for example, the color-dipole-moment contribution. We have evaluated this contribution and found that it is smaller by several orders of magnitude than the operator discussed above.

VI. η IN SUPERSYMMETRY MODELS

In supersymmetric extensions of the standard model there are new sources of *CP* violation. In this section we study the contribution to η due to a *CP*-violating quark-squark-gluon interaction. This interaction can be parametrized as

$$L_{\tilde{g}\tilde{d}d}=i\sqrt{2}g_s\tilde{d}^+\tilde{G}_a\frac{1}{2}\lambda^a\left[\Gamma_L\frac{1-\gamma_5}{2}+\Gamma_R\frac{1+\gamma_5}{2}\right]d\ , \qquad (54)$$

where $\tilde{d}=(\tilde{d}_L,\tilde{d}_R)$ are the squarks, $\tilde{g}$ is the gluino, and the coupling matrices $\Gamma_{L,R}$ are each 6×3 matrices which are related to the squark mass matrix and contain new *CP*-violating phases.

The above Lagrangian will generate a color dipole moment of quarks at the one-loop level. The down-quark color dipole moment is give by [41]

$$f_d=\frac{\alpha_s}{4\pi m_{\tilde{g}}}\mathrm{Im}(\Gamma_L^{id}\Gamma_R^{id*})[3E(z_i)-\tfrac{8}{3}D(z_i)]\ . \qquad (55)$$

Here $D(z)$ and $E(z)$ are given by

$$D(z)=\frac{1}{2(z-1)^2}\left[1+z+\frac{2z}{1-z}\ln z\right]\ ,$$
$$E(z)=\frac{1}{(1-z)^2}(1-z+z\ln z)\ ,$$

where $z=\tilde{m}_i^2/\tilde{m}_{\tilde{g}}^2$, and $\tilde{m}_i$ and $m_{\tilde{g}}$ are the squark and gluon masses, respectively. The contribution from f_d to η is the dominant one in this model. We find that the color dipole moment induces a value of η given by $|\eta_{ij}|=0.9\times 10^7|f_d|\times 1$ GeV.

VII. η DUE TO DIQUARK SCALARS

In this section we study contributions to η from diquark scalars. Diquark scalars are potential sources of large *CP* violation in neutral flavor-conserving processes. A list of possible diquark scalars which couple to standard-model quarks and some of their phenomenological implications can be found in Ref. [42]. There are two diquark scalars which can induce *CP* violation at the tree level. These are H_9 and H_{10} in the notation of Ref. [42]. They transform under the standard-model group as $(3,1,-\frac{2}{3})$ and $(\bar{6},1,-\frac{2}{3})$, respectively. In the following we consider the contribution to η from H_9. The contribution of H_{10} can be similarly worked out. The couplings of H_9 to up and down quarks are

$$L=(\lambda_9\bar{u}^c_{Ri}d_{Rj}+\lambda'_9\bar{u}^c_{Li}d_{Lj})\epsilon^{ijk}H_{9k}+\mathrm{H.c.}\ , \qquad (56)$$

where i,j,k are color indices, c indicates charge conjugation, and λ_9 and λ'_9 are complex numbers. Exchange of H_9 will generate a four quark interaction

$$L_{\mathrm{int}}=-\frac{1}{m_H^2}(\lambda_9\bar{u}^c_{Ri}d_{Rj}+\lambda'_9\bar{u}^c_{Li}d_{Lj})\times(\lambda_9^*\bar{d}_{Rj'}u^c_{Ri'}+\lambda'_9\bar{d}_{Lj'}u^c_{Li'})\epsilon^{ijk}\epsilon^{i'j'k}\ , \qquad (57)$$

where m_H is the H_9 scalar mass. This effective Lagrangian will induce a *CP*-odd πNN coupling and then will generate a nonzero η. This is the operator which will generate the dominant contribution to η. The calculations are similar to those in Sec. V. After a Fierz rearrangement, we obtain the dominant term which contributes to η:

$$L_{\mathrm{int}}=\frac{i}{m_H^2}\frac{2}{3}\mathrm{Im}(\lambda_9\lambda_9'^*)(\bar{u}u\bar{d}\gamma_5 d+\bar{d}d\bar{u}\gamma_5 u)+\dots\ . \qquad (58)$$

From this we compute

$$|\eta_{iN}|=\frac{1}{\sqrt{2}G_F}g_{\pi NN}\frac{1}{m_H^2}\frac{2}{3}\left|\mathrm{Im}(\lambda_9\lambda_9'^*)\frac{F_\pi}{2m_u m_d}(m_u\langle N|\bar{u}u|N\rangle-m_d\langle N|\bar{d}d|N\rangle)\right|$$
$$=\begin{cases}0.7\times 10^7(\mathrm{GeV}^2)|\mathrm{Im}(\lambda_9\lambda_9'^*)|/m_H^2, & N=p\ ,\\ 1.4\times 10^7(\mathrm{GeV}^2)|\mathrm{Im}(\lambda_9\lambda_9'^*)|/m_H^2, & N=n\ .\end{cases} \qquad (59)$$

VIII. DISCUSSION AND CONCLUSIONS

In the previous sections we have calculated *CP*-odd nucleon-nucleon interactions in several models. From these calculations we can put constraints on *CP*-violating parameters in different models. It is useful to check whether these constraints are really meaningful when other constraints have been taken into account. For this purpose, let us compare these constraints with the ones obtained from the upper bound on the neutron EDM. In the standard model, $d_n = 10^{-31}-10^{-33}$ [28,15] and $\eta_{nn}, \eta_{np} = 2\times 10^{-9}-10^{-10}$ are both very small compared with the experimental upper bounds. No significant information can be extracted from experiments at this stage. One can only hope that future experimental sensitivity will reach the region of theoretical predictions and provide us with useful information. However, should experiments measure d_n or $D(A)$ at the level much larger than the standard-model predictions, we have to go beyond the minimal standard model to explain the results. The studies of other models in the previous sections are a first step in this direction. The upper bound on the strong *CP*-violating θ term of QCD from η is about 10^{-8}, which is weaker by one order of magnitude compared with the one obtained from the neutron EDM constraint.

In multi-Higgs-doublet models, the constraints for the *CP*-violating parameters from $D(A)$ at the present are weaker than the ones from the upper bound of the neutron EDM by two orders of magnitude [34–36,43]. The reason is that the operators which dominate the contributions to η also dominate d_n. For example, the color dipole moment f_d of the down quark contributes to both d_n and η. The contribution to d_n is given by $d_n(f_d) = 4ef_d/9$ [15] and to η by $\eta_{ij} \approx g_{\pi NN} B_{gd} f_d / \sqrt{2} G_F m_\pi^2$. If we require that $d_n(f_d)$ satisfies the upper bound $d_n < 10^{-25}$ e cm, we would have η less than 10^{-4}, which is about two orders of magnitude below the direct experimental constraint on η. Our result for η is about two orders of magnitude smaller than that obtained in Ref. [17]. The main difference is in the evaluation of B_{gq}. If we use the result of Ref. [17], the constraint on f_d from η is comparable to the one from the upper bound on the neutron EDM.

In left-right-symmetric models, the constraint from η on the *CP*-violating parameter $[|\sin\zeta \cos\zeta \sin(\gamma - \delta_2)| < 4\times 10^{-5}]$ is stronger than the existing ocnstraint from the upper bound on the neutron EDM by one order of magnitude [44,15]. However, the same $\pi^0 NN$ *CP*-violating vertex will also generate d_n at the one-loop level by exchanging π^0 and n in the loop [45]. The photon will couple to the neutron through its anomalous magnetic dipole moment, and we have

$$d_n = \frac{e g_{\pi NN} f_{\pi NN}}{8\pi^2} \frac{\kappa_n}{2m_n} F_n(m_\pi^2) , \tag{60}$$

where κ_n is the anomalous magnetic dipole moment of the neutron and $F_n(m_\pi^2)$ is from the loop integral, which can be found in Refs. [15,45]. Requiring d_n to be less than the experimental upper bound, we find that the constraint on $f_{\pi NN}$ is about a factor of 10 better than the one from η. A similar analysis for η in left-right-symmetric models has been carried out previously in Ref. [17]. In our analysis, we have used more recent values for the matrix elements $\langle N|\bar{q}q|N\rangle$.

In the supersymmetry (SUSY) model, the dominant contribution to η is from the color dipole moment of the down quark. The constraint is weaker than that obtained from the neutron EDM, as pointed out above. Using our result of *CP*-odd $\pi^0 NN$ coupling, due to *CP* violation in quark-squark-gluon interaction discussed in Sec. VI, η is predicted to be less than 10^{-4}.

In the diquark scalar model, we obtain $\mathrm{Im}(\lambda_9 \lambda_9'^*)/m_H^2 < 2\times 10^{-9}$ GeV^{-2} from experimental constraints on η. The situation is similar to that for left-right-symmetric models; this constraint is about a factor of 10 less stringent than that obtained from the upper bound on the neutron EDM.

From the above discussion we see that atomic EDM measurement give interesting constraints on *CP*-violating parameters. The importance of these constraints should not be underestimated. In some cases these constraints are within an order of magnitude of those obtained from the upper bound on the neutron EDM. With improved sensitivity in the experiments, information extracted from atomic EDM will play a more important role.

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