Order-α radiative corrections for semileptonic decays of polarized baryons

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Model-independent radiative corrections are calculated for the $\Sigma^- \to ne\bar{\nu}$, $\Lambda \to pe\bar{\nu}$, and $n \to pe\bar{\nu}$ decays with polarized initial baryons. The method of polarization asymmetry calculation is outlined, and the most important formulas are presented. Numerical results for the corrections to two- and one-dimensional asymmetry distributions and totally integrated asymmetries are tabulated for the electron, neutrino, hadron, α, and β asymmetries.

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I. INTRODUCTION

Semileptonic decays of spin-$\frac{1}{2}$ octet baryons play a crucial role in our understanding of the interplay between strong and weak interactions and of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix. The observable quantities for each semileptonic decay depend on CKM matrix elements and on a few form factors. The precise determination of these form factors provides important information about the low-energy dynamics of the strong interaction. There are many theoretical predictions for these form factors [1–19]. Most of these studies are quark model calculations, using various models for the low-energy behavior of the quarks and gluons. The Cabibbo model [2], assuming exact SU(3)-flavor symmetry, has been very successful at describing the semileptonic decays so far. The SU(3) symmetry of the octet baryons has particular relevance to recent discussions of the structure functions for polarized deep-inelastic scattering (the proton spin problem) [20]. It is very likely that lattice QCD calculations for baryon semileptonic decay (BSD) form factors will also be carried out in the near future (lattice QCD studies for semileptonic decays of mesons [21] and for electromagnetic properties of baryons [22] have been published recently).

On the other hand, precise measurements have been made in the last decade both for hyperon semileptonic decays (HSD's) [23–26] and for neutron decay [27–36]. The statistical errors of these experiments are rather small. In order to achieve precise values for the form factors and the CKM matrix elements, serious attention has to be paid to the reduction of the various systematic errors of the measurements and of the off-line analyses. There are two types of systematic errors occurring in these measurements. One of them comes from the various shortcomings of the experimental devices (background, kinematic cuts, detection efficiencies, energy and momentum resolution and calibration, etc.). Ingenious Monte Carlo simulation programs have been used (mainly for HSD off-line analyses) to correct for these errors. The other type of systematic errors is of theoretical nature, and takes its origin from the inaccuracies of the theoretically derived relations of the form factors and the CKM matrix elements with the observable quantities. The main theoretical uncertainties are the following: (i) theoretical assumptions for some form factors; (ii) momentum transfer dependence of the form factors; (iii) radiative corrections.

The connection of the first two points with the observables is rather easy to handle (see Sec. II). On the other hand, the precise and reliable calculation of the radiative corrections to various measurable quantities relevant for experimental analyses is the most difficult theoretical problem of BSD's.

Many radiative correction calculations have been published in the past three decades: for unpolarized-neutron decay [37–43], for polarized-neutron decay [44–47], for semileptonic decays of unpolarized [48–54] and of polarized [55,56] hyperons. In the framework of the $SU(3)_c \times SU(2)_L \times U(1)$ standard model these radiative corrections are free from ultraviolet divergences [57–59]. One can decompose the virtual correction in a gauge-invariant manner into three parts: model-independent (MI) part + asymptotic part + model-dependent (MD) part (see Ref. [54], Sec. II). Only the MI part contains infrared divergent terms. The MI correction is defined as the sum of the MI part of the virtual correction and of the real-photonic (bremsstrahlung) correction (this sum is infrared finite). The MI corrections to various observables can be reliably calculated, there are only technical difficulties due to the phase space integration of the bremsstrahlung (BR) photon. The neglect of MI corrections in the experimental analysis might cause essential systematic errors. The asymptotic part [60] gives the leading asymptotic dependence of the virtual correction on the M_W, M_Z vector-boson masses. This correction is a universal coefficient of the zeroth-order amplitude; therefore, it is not relevant for the determination of the ratio of the axial-vector to the vector coupling constants. The reliable calculation of the MD part is hampered by the uncertainties of our present knowledge about the low-energy dynamics of the strong interaction. One can find some crude estimates for the MD corrections in the literature [40,61,62], suggesting that the contribution of the MD part is smaller than the asymptotic part by one order of magnitude. Calculations for the MD corrections by nonperturbative methods is badly needed (perhaps together with the form factor calculations).

This paper is devoted to give numerical results for the order-α MI corrections to various polarization asymmetries of the $\Sigma^- \to ne\bar{\nu}$, $\Lambda \to pe\bar{\nu}$, and $n \to pe\bar{\nu}$ decays. For the MI part of the virtual correction the definition of Sirlin [42] is used. The pointlike hadron approximation is employed in order to calculate the BR part of the corrections. In our previous papers [53,54] we published numerical results for MI radiative corrections to several measurable quantities in semileptonic decays of unpolarized baryons.

The plan of the paper is the following. In Sec. II we describe the general method of the polarization asymmetry calculations in zeroth order, and we define several polarization asymmetries and measurable quantities. In Sec. III we discuss the method of calculation of the radiative corrections to polarization asymmetries. Section IV contains our numerical results. In Appendix A we present the coefficients of the various form factor combinations of the zeroth-order matrix element squared for semileptonic decays of polarized baryons. Finally, Appendix B is devoted to give some detailed formulas useful for the radiative correction calculations.

II. OBSERVABLE QUANTITIES IN ZEROTH ORDER

The conventions of Refs. [63] and [64] are used in this paper. Indices 1, 2, i, and f refer to an antineutrino, electron, initial (decaying) baryon, and final baryon, respectively. $p, \mathbf{p}, E$, and m denote four-momentum, three-momentum, energy and mass, respectively. $G = G_\mu V_{ud}$ for strangeness-conserving decays, $G = G_\mu V_{us}$ for strangeness-changing decays, where G_μ is the muon decay coupling constant, and V_{ud} and V_{us} are CKM matrix elements.

The most general form of the zeroth-order amplitude in $V-A$ theory is

$$\begin{aligned} \mathcal{M}_0 &= \frac{G}{\sqrt{2}}[\bar{u}_f H_\rho u_i][\bar{u}_2 \gamma^\rho (1-\gamma_5) v_1] , \\ H_\rho &= H_\rho^V + H_\rho^A , \\ H_\rho^V &= f_1(q^2)\gamma_\rho + f_2(q^2)\frac{1}{2m_i}[\gamma_\rho,\gamma_\nu]q^\nu + f_3(q^2)\frac{q_\rho}{m_i} , \\ H_\rho^A &= H_\rho^V[f_j \to g_j]\gamma_5 . \end{aligned} \tag{2.1}$$

Here $q = p_i - p_f$ is the four-momentum transfer. The most complete information about the semileptonic decay of a polarized baryon is given (in zeroth order) by the

$$V_0(E_2, E_f, \cos\theta, \phi) = \frac{G^2}{16 m_i \pi^4}(L_0 + S_0) \tag{2.2}$$

four-dimensional distribution. The E_2 and E_f energies determine the decay triangle (with sides $|\mathbf{p}_2|, |\mathbf{p}_f|$, and $|\mathbf{p}_1|$), and its spatial direction is represented by the θ, ϕ angles. L_0 and S_0 (defined in Appendix A) are quadratic functions of the form factors:

$$\begin{aligned} L_0 &= f_1^2(q^2) L_0[f_1^2] + g_1^2(q^2) L_0[g_1^2] \\ &\quad + f_1(q^2) g_1(q^2) L_0[f_1 g_1] \\ &\quad + 18 \text{ similar terms} , \end{aligned} \tag{2.3}$$

and similar decomposition holds for S_0 (real form factors are assumed). The coefficients of the different form factor combinations depend on the scalar products of the p_1, p_2, p_i, p_f four-momenta; the S_0 contains also $(p_1 s)$, $(p_2 s)$, and $(p_f s)$ scalar products, where s is the spin four-momentum of the initial baryon $[s = (0, \mathbf{s}), \mathbf{s}^2 = 1]$. The coefficients of the quadratic combinations of the f_1, f_2, g_1, and g_2 form factors are presented in Appendix A (the contribution of the terms involving the f_3 and g_3 form factors is negligible for the electronic decays).

To specify the θ and ϕ angles, let us choose an $\mathbf{n}$ direction vector as some linear combination of the $\mathbf{n}_1, \mathbf{n}_2$, and $\mathbf{t}$ unit vectors, where

$$\mathbf{n}_j = \frac{\mathbf{p}_j}{|\mathbf{p}_j|} \quad (j = 1,2), \quad \mathbf{t} = \frac{\mathbf{n}_2 \times \mathbf{n}_1}{|\mathbf{n}_2 \times \mathbf{n}_1|} . \tag{2.4}$$

Different choices for the $\mathbf{n}$ vector give different polarization quantities:

electron asymmetry: $\mathbf{n} = \mathbf{n}_2$,

neutrino asymmetry: $\mathbf{n} = \mathbf{n}_1$,

hadron asymmetry: $\mathbf{n} = \mathbf{p}_f / |\mathbf{p}_f| \quad (\mathbf{p}_f = -\mathbf{p}_1 - \mathbf{p}_2)$,

α asymmetry: $\mathbf{n} = \boldsymbol{\alpha}$,

β asymmetry: $\mathbf{n} = \boldsymbol{\beta}$,

triple correlation asymmetry: $\mathbf{n} = \mathbf{t}$,

where

$$\boldsymbol{\alpha} = \frac{\mathbf{n}_2 + \mathbf{n}_1}{|\mathbf{n}_2 + \mathbf{n}_1|}, \quad \boldsymbol{\beta} = \frac{\mathbf{n}_2 - \mathbf{n}_1}{|\mathbf{n}_2 - \mathbf{n}_1|} \tag{2.5}$$

(see Ref. [64]).

Define θ by the $\cos\theta = \mathbf{s} \cdot \mathbf{n}$ equation, where $\mathbf{s}$ is the polarization vector of the initial baryon $(0^\circ \le \theta \le 180^\circ)$. Then we define the $\hat{\mathbf{n}}$ vector in the following way:

$\hat{\mathbf{n}} = \mathbf{n}_1$: if $|\mathbf{n} \cdot \mathbf{n}_1| < 1$;

$\hat{\mathbf{n}} = \mathbf{n}_2$: if $|\mathbf{n} \cdot \mathbf{n}_1| = 1, \quad |\mathbf{n} \cdot \mathbf{n}_2| < 1$

(the $|\mathbf{n}_1 \cdot \mathbf{n}_2| = 1$ case can be regarded as the $|\mathbf{n}_1 \cdot \mathbf{n}_2| \to 1$ limit). We consider the $\mathbf{n}$ vector as a polar axis, and define ϕ by the azimuthal angle of the above introduced $\hat{\mathbf{n}}$ vector around $\mathbf{n}$. The zero value of ϕ is irrelevant in our considerations.

Next we integrate Eq. (2.2) over the ϕ angle (with fixed E_2, E_f, and θ). The L_0 term does not depend on s, and S_0 is linear in s: $S_0 = \sum_j C_j (p_j s)$ $(j = 1, 2, f)$. Therefore

$$\frac{1}{2\pi}\int_0^{2\pi} d\phi L_0 = 1 , \tag{2.6}$$

$$\frac{1}{2\pi}\int_0^{2\pi} d\phi S_0 = -\left[\sum_j C_j p_{j\parallel}\right]\cos\theta , \tag{2.7}$$

where $p_{j\parallel}$ $(j = 1, 2, f)$ is the projection of the $\mathbf{p}_j$ three-momentum to the $\mathbf{n}$ direction:

$$p_{j\parallel} = \mathbf{p}_j \cdot \mathbf{n} . \tag{2.8}$$

After the ϕ integration we get the

$$\Omega_0(E_2,E_f,\cos\theta)=\tfrac{1}{2}[W_0(E_2,E_f)+A_0(E_2,E_f)\cos\theta] \quad (2.9)$$

three-dimensional distribution, where

$$W_0(E_2,E_f)=\frac{G^2}{4m_i\pi^3}L_0\ , \quad (2.10)$$

$$A_0(E_2,E_f)=\frac{G^2}{4m_i\pi^3}S_0^{\parallel}\ , \quad (2.11)$$

$$S_0^{\parallel}=S_0[(p_js)\to -p_{j\parallel};\ j=1,2,f]\ . \quad (2.12)$$

The (p_js) scalar products in S_0 are to be replaced by the $-p_{j\parallel}=-\mathbf{p}_j\cdot\mathbf{n}$ factors, in order to get $S_0^{\parallel}$ from S_0. For example, the substitutions in the case of the electron asymmetry are

$$(p_2s)\to -|\mathbf{p}_2|\ ,$$
$$(p_1s)\to -\frac{\mathbf{p}_1\cdot\mathbf{p}_2}{|\mathbf{p}_2|}\ , \quad (2.13)$$
$$(p_fs)\to -\frac{\mathbf{p}_f\cdot\mathbf{p}_2}{|\mathbf{p}_2|}\ .$$

The two-dimensional $\alpha_0(E_2,E_f)$ asymmetry function is defined by

$$\alpha_0(E_2,E_f)=\frac{A_0(E_2,E_f)}{W_0(E_2,E_f)}\ . \quad (2.14)$$

Integrating Eq. (2.9) over E_f yields the

$$\omega_0(E_2,\cos\theta)=\tfrac{1}{2}[w_0(E_2)+a_0(E_2)\cos\theta] \quad (2.15)$$

distribution, where

$$w_0(E_2)=\int_{E_{f_{\min}}(E_2)}^{E_{f_{\max}}(E_2)} dE_f W_0(E_2,E_f)\ ,$$
$$a_0(E_2)=\int_{E_{f_{\min}}(E_2)}^{E_{f_{\max}}(E_2)} dE_f A_0(E_2,E_f) \quad (2.16)$$

(for $E_{f_{\min}}(E_2), E_{f_{\max}}(E_2)$ see Ref. [54], Eq. (3.7) or Ref. [64], Eq. (2.11)).

The one-dimensional $\alpha_0(E_2)$ asymmetry function is determined as

$$\alpha_0(E_2)=\frac{a_0(E_2)}{w_0(E_2)}\ . \quad (2.17)$$

Finally, we integrate Eq. (2.15) over E_2 to get the

$$v_0(\cos\theta)=\tfrac{1}{2}\rho_0(1+\alpha_0\cos\theta) \quad (2.18)$$

distribution, where

$$\rho_0=\int_{m_2}^{E_{2\max}} dE_2 w_0(E_2) \quad (2.19)$$

is the total decay rate, and

$$\alpha_0=\frac{\int_{m_2}^{E_{2\max}} dE_2 a_0(E_2)}{\rho_0}=2\frac{N_+^0-N_-^0}{N_+^0+N_-^0} \quad (2.20)$$

is the totally integrated asymmetry parameter (N_+^0 and N_-^0 are the zeroth-order decay rates for $\cos\theta>0$ and $\cos\theta<0$, respectively).

The $\alpha_0(E_2,E_f)$ asymmetry functions have remarkable properties at the lower and upper boundaries of the Dalitz plot.

(1), (a) At the lower boundary $[E_f=E_{f_{\min}}(E_2)]$,

$$\alpha_0^{(e)}(E_2,E_f)=-\alpha_0^{(\nu)}(E_2,E_f)=\alpha_0^{(\beta)}(E_2,E_f)=\begin{cases}\alpha_0^{(h)}(E_2,E_f) & \text{if } E_2<E_{2h}\ ,\\ -\alpha_0^{(h)}(E_2,E_f) & \text{if } E_2>E_{2h}\ ,\end{cases} \quad (2.21)$$

where

$$E_{2h}=\frac{1}{2}\left[m_i-m_f+\frac{m_2^2}{m_i-m_f}\right]\ . \quad (2.22)$$

(b) At the upper boundary $[E_f=E_{f_{\max}}(E_2)]$,

$$\alpha_0^{(e)}(E_2,E_f)=\alpha_0^{(\nu)}(E_2,E_f)=\alpha_0^{(\alpha)}(E_2,E_f)=-\alpha_0^{(h)}(E_2,E_f) \quad (2.23)$$

(e,ν,h,α, and β denote the electron, neutrino, hadron, α, and β asymmetries, respectively).

(2) Simple expressions can be obtained for the $\alpha_0(E_2,E_f)$ asymmetries at the above boundaries, if the electron mass is neglected ($m_2=0$ limit).

(a) At the lower boundary,

$$\alpha_0^{(e)}(E_2,E_f)=-1\ . \quad (2.24)$$

(b) At the upper boundary,

$$\alpha_0^{(e)}(E_2,E_f)=-\frac{2f_1g_1}{f_1^2+g_1^2}\ . \quad (2.25)$$

We would like to emphasize that in the $m_2=0$ limit Eqs. (2.24) and (2.25) hold exactly, for arbitrary m_i, m_f baryon masses and f_1,f_2,g_1, and g_2 form factor values (see Appendix A). It has been shown numerically that they are also good approximations for neutron decay (where the electron mass cannot be neglected).

There is an important consequence of Eqs. (2.24) and (2.25) for the E_f dependence of the $\alpha_0(E_2,E_f)$ asymmetries. For $g_1/f_1<0$ the $\alpha_0^{(e)}(E_2,E_f)$ asymmetry function changes sign when E_f goes from $E_{f_{\min}}(E_2)$ to $E_{f_{\max}}(E_2)$. The $-2f_1g_1/(f_1^2-g_1^2)$ ratio is rather close to $+1$ for the $\Lambda\to pe\bar{\nu}$ and $n\to pe\bar{\nu}$ decays. Therefore the $\alpha_0^{(e)}(E_2)$ asymmetry is small for these decays, and its measurement is rather sensitive to the proton-energy dependence of the detection efficiency. On the other hand, the $\alpha_0^{(\nu)}(E_2,E_f)$ and $\alpha_0^{(\nu)}(E_2)$ neutrino asymmetries are everywhere close to 1, and are not sensitive to the g_1/f_1 form factor ratios.

III. RADIATIVE CORRECTIONS

The theoretical framework and our technique of computing the bremsstrahlung (BR) correction for unpolarized quantities was outlined in Ref. [54]. We present below our method to obtain the radiative corrections to polarization quantities. Some results for the virtual and BR integrals may be found in Appendix B.

We start by computing the $\delta\Omega(E_2,E_f,\cos\theta)$ correction to the $\Omega_0(E_2,E_f,\cos\theta)$ distribution [Eq. (2.9)]. $\delta\Omega=\Omega_{\rm virt}+\Omega_{\rm BR}$, where $\Omega_{\rm virt}$ denotes the virtual correction and $\Omega_{\rm BR}$ the BR correction. $\Omega_{\rm virt}$ can be expressed as

$$\Omega_{\rm virt}=\frac{1}{128 m_i \pi^3}\left[\sum_{s_f,s_2} 2\,{\rm Re}(\mathcal{M}_V\mathcal{M}_0^*)\right]_{\parallel} =\tfrac{1}{2}(W_{\rm virt}+A_{\rm virt}\cos\theta) \ . \tag{3.1}$$

Dependences on the E_2 and E_f energies have been omitted. $\mathcal{M}_V$ is the virtual correction to the $\mathcal{M}_0$ amplitude (see Appendix B), $\sum_{s_f,s_2}$ means summation over the final baryon and electron polarizations, and the $\parallel$ mark refers to the $(p_j s)\to -p_{j\parallel}$ $(j=1,2,f)$ substitutions, as explained in Sec. II.

To calculate $\Omega_{\rm BR}$, we start from the

$$V_{\rm BR}(E_2,E_f,\cos\theta,Q,K,\phi_k,\phi) =\frac{1}{2^{12}\pi^7 m_i}\frac{K}{K_0}\sum_{s_f,s_2,\gamma}|\mathcal{M}_{\rm BR}|^2 \tag{3.2}$$

distribution. Here we sum over the polarizations of the final baryon, the electron and the BR photon (see Appendix B). $Q=|\mathbf{Q}|$, $\mathbf{Q}=\mathbf{p}_2+\mathbf{p}_f=-(\mathbf{p}_1+\mathbf{k})$, $K=|\mathbf{k}|$ is the photon energy, ϕ_k is the azimuthal angle of the $\mathbf{k}$ three-momentum of the photon around $\mathbf{Q}$. $K_0=\sqrt{K^2+m_\gamma^2}$ and m_γ is the infrared cutoff.

The θ and ϕ angles are those introduced in Sec. II. In the presence of BR photons, however, we define the $\mathbf{n}_1$ unit vector not with the antineutrino three-momentum, which is not measurable if the BR photons are undetected, but with the measurable $\mathbf{Q}$ vector:

$$\mathbf{n}_1=-\frac{\mathbf{Q}}{Q} \ . \tag{3.3}$$

For $K=0$ the definitions (2.4) and (3.3) are equivalent. The θ and ϕ angles represent the spatial direction of the decay triangle with sides $|\mathbf{p}_2|,|\mathbf{p}_f|$ and Q, and do not depend on the $\mathbf{p}_1$ and $\mathbf{k}$ three momenta.

We integrate first over ϕ (with the other variables kept fixed). The s dependence of $V_{\rm BR}$ is linear:

$$V_{\rm BR}=B+\sum_{j=1,2,f}B_j(p_j s)+B_k(ks) \ ; \tag{3.4}$$

therefore

$$\frac{1}{2\pi}\int_0^{2\pi}d\phi V_{\rm BR}=B-\left[\sum_j B_j p_{j\parallel}+B_k k_{\parallel}\right]\cos\theta \ , \quad p_{j\parallel}=\mathbf{p}_j\cdot\mathbf{n}, \quad k_{\parallel}=\mathbf{k}\cdot\mathbf{n} \ . \tag{3.5}$$

B,B_j $(j=1,2,f)$ and B_k in Eq. (3.4) depend on the scalar products of the p_1,p_2, p_i,p_f, and k four-momenta. The $p_{j\parallel}$ and $k_\parallel$ quantities can also be expressed by these scalar products. The method is the same as in the zeroth-order case: the integration over the ϕ angle is obtained simply by making the

$$(ps)\to -p_\parallel\cos\theta=-\mathbf{p}\cdot\mathbf{n}\cos\theta \quad (p=p_1,p_2,p_f,k) \tag{3.6}$$

replacements.

Integrating over the BR photon phase space we get

$$\Omega_{\rm BR}=\frac{1}{2^{10}\pi^5 m_i}\int_{Q_{\rm min}}^{Q_{\rm max}}dQ\int_{K_{\rm min}}^{K_{\rm max}}dK\frac{K}{K_0}\frac{1}{2\pi}\int_0^{2\pi}d\phi_k\left[\sum_{s_f,s_2,\gamma}|\mathcal{M}_{\rm BR}|^2\right]_{\parallel} =\tfrac{1}{2}(W_{\rm BR}+A_{\rm BR}\cos\theta) \ , \tag{3.7}$$

where $\parallel$ refers to the (3.6) substitutions.

Let us denote the total radiative correction by $\delta\Omega$:

$$\delta\Omega(E_2,E_f,\cos\theta)=\Omega_{\rm virt}+\Omega_{\rm BR} =\tfrac{1}{2}[\delta W(E_2,E_f)+\delta A(E_2,E_f)\cos\theta] \ . \tag{3.8}$$

We introduce the $\alpha(E_2,E_f)$ asymmetry function as

$$\alpha(E_2,E_f)=\frac{A(E_2,E_f)}{W(E_2,E_f)}=2\frac{N_+(E_2,E_f)-N_-(E_2,E_f)}{N_+(E_2,E_f)+N_-(E_2,E_f)} \ , \tag{3.9}$$

where

$$A(E_2,E_f)=A_0(E_2,E_f)+\delta A(E_2,E_f) \ , \quad W(E_2,E_f)=W_0(E_2,E_f)+\delta W(E_2,E_f) \ . \tag{3.10}$$

$N_+(E_2,E_f)$ and $N_-(E_2,E_f)$ are the Dalitz distributions for events with $\cos\theta>0$ and $\cos\theta<0$, respectively.

The correction to the zeroth-order asymmetry function is defined as

$$\delta\alpha(E_2,E_f)=\alpha(E_2,E_f)-\alpha_0(E_2,E_f) \ . \tag{3.11}$$

Similar definitions hold for the corrections to $\alpha_0(E_2)$ and α_0:

$$\delta\alpha(E_2)=\alpha(E_2)-\alpha_0(E_2) \ , \tag{3.12}$$

$$\delta\alpha=\alpha-\alpha_0 \ , \tag{3.13}$$

where

$$\alpha(E_2)=\frac{\int dE_f A(E_2,E_f)}{\int dE_f W(E_2,E_f)}=2\frac{N_+(E_2)-N_-(E_2)}{N_+(E_2)+N_-(E_2)} \ , \tag{3.14}$$

$$\alpha=\frac{\int dE_2\int dE_f A(E_2,E_f)}{\int dE_2\int dE_f W(E_2,E_f)}=2\frac{N_+-N_-}{N_++N_-} \ . \tag{3.15}$$

The corrected $\Omega(E_2,E_f,\cos\theta)$, $\omega(E_2,\cos\theta)$, and $v(\cos\theta)$ distributions may be written as

$$\Omega(E_2,E_f,\cos\theta)=\tfrac{1}{2}W(E_2,E_f)[1+\alpha(E_2,E_f)\cos\theta]\ , \quad (3.16)$$

$$\omega(E_2,\cos\theta)=\tfrac{1}{2}w(E_2)[1+\alpha(E_2)\cos\theta]\ , \quad (3.17)$$

$$v(\cos\theta)=\tfrac{1}{2}\rho[1+\alpha\cos\theta]\ , \quad (3.18)$$

where

$$w(E_2)=\int dE_f W(E_2,E_f)\ . \quad (3.19)$$

$$\rho=\int dE_2 w(E_2)\ . \quad (3.20)$$

$W(E_2,E_f), w(E_2)$, and ρ are the corrected Dalitz distribution, electron energy spectrum and total decay rate, respectively (corrected means zeroth order + radiative correction). In our previous paper [54] we published relative corrections to the unpolarized quantities $W_0(E_2,E_f), w_0(E_2)$, and ρ_0. This work is devoted to giving numerical results for the asymmetry corrections $\delta\alpha(E_2,E_f), \delta\alpha(E_2)$, and $\delta\alpha$.

IV. NUMERICAL RESULTS

We present the numerical results of our order-α radiative correction calculations in Tables I–V. The MI part introduced by Sirlin in Ref. [42] has been used for the virtual correction [see Appendix B, Eqs. (B1)–(B12)]. For the BR matrix-element the pointlike hadron approximation has been employed [Appendix B, Eqs. (B13)–(B18)]. The particle masses are those given in Ref. [65]. The following values for the form factor ratios have been used in our computer programs:

$$\Sigma^- \to ne\bar{\nu}\text{: } g_1/f_1=0.34,\ \ f_2/f_1=-0.97\ ;$$
$$\Lambda \to pe\bar{\nu}\text{: } g_1/f_1=-0.72,\ \ f_2/f_1=0.97\ ; \quad (4.1)$$
$$n \to pe\bar{\nu}\text{: } g_1/f_1=-1.261,\ \ f_2/f_1=1.97\ .$$

The q^2 dependence of the form factors and the contribution from the f_3, g_2, and g_3 form factors have been neglected. We have not included the q^2 dependence of the form factors in the zeroth-order calculations, either.

Tables I and II contain radiative corrections to the two-dimensional electron, neutrino and hadron asymmetries for the $\Sigma^- \to ne\bar{\nu}$ and $\Lambda \to pe\bar{\nu}$ decays. The x and y dimensionless quantities are those defined in Ref. [54]:

$$x=\frac{E_2}{E_{2m}},\quad y=\frac{E_f}{m_i}\ , \quad (4.2)$$

where

$$E_{2m}=\frac{m_i^2-m_f^2+m_2^2}{2m_i}\ . \quad (4.3)$$

The $y_{\min}$ numbers are the minimum values of y for zeroth-order decays (without BR photons). The $\delta\alpha(E_2,E_f)$ corrections [see Eq. (3.11)], multiplied by 100, are tabulated in the $y>y_{\min}$ points of the (x,y) plane. In the $y<y_{\min}, x<E_{2h}/E_{2m}$ points [see Eq. (2.22)] the $100\alpha_{\rm BR}(E_2,E_f)$ asymmetries are presented, where

$$\alpha_{\rm BR}(E_2,E_f)=\frac{A_{\rm BR}(E_2,E_f)}{W_{\rm BR}(E_2,E_f)} \quad (4.4)$$

(in this region there are only BR events: the minimum of the BR photon energy is nonzero in each point).

We have studied the behavior of the

$$\alpha_\delta(E_2,E_f)=\frac{\delta A(E_2,E_f)}{\delta W(E_2,E_f)} \quad (4.5)$$

distributions and the $\delta\alpha(E_2,E_f)$ asymmetries near the $E_f=E_{f_{\max}}(E_2)$ and the $E_f=E_{f_{\min}}(E_2), E_2>E_{2h}$ boundaries. We have found that Eqs. (2.21) and (2.23)–(2.25) are valid for the $\alpha_\delta(E_2,E_f)$ asymmetry distributions, too. As a consequence, the $\delta\alpha(E_2,E_f)$ asymmetry corrections approach zero at these boundary curves.

The $\tilde{L}$ and $\tilde{S}$ parts of the virtual corrections [see Appendix B, Eqs. (B9)–(B12)] are negligible for hyperon decays. Therefore

$$W_{\rm virt}\sim ZW_0,\quad A_{\rm virt}\sim ZA_0\quad (\text{for } m_2 \ll m_i-m_f)\ , \quad (4.6)$$

implying that the deviation of the $\delta\alpha(E_2,E_f)$ asymmetries from 0 is mainly the hard BR effect (the soft BR matrix element is also proportional to the zeroth-order matrix element).

In Tables III and IV the one-dimensional asymmetry distributions $[100\alpha_0(E_2)$ and $100\delta\alpha(E_2)]$ and the totally integrated asymmetries ($100\alpha_0$ and $100\delta\alpha$, last column) are tabulated. Table V contains the $100\alpha_0(E_2)$ and $100\delta\alpha(E_2)$ results for the neutron decay. The asymmetry corrections here are remarkably small.

The order-α MI correction to the triple correlation asymmetry have been found to be exactly zero, if real form factors are assumed. This is in agreement with the results of Refs. [66,67].

In order to check our calculations we have made the following tests.

(1) All our REDUCE outputs for the matrix elements squared have been checked by numerical computations (see Appendix B).

(2) All our analytical integral formulas (even the infrared divergent ones) have been checked by numerical integrations. Two different integration methods have been used, and the results have been compared.

(3) We have calculated the order-α radiative correction to the electron energy spectrum and the $\delta\alpha^{(e)}(E_2)$ electron asymmetry correction for the muon decay, with $m_2=0.01$ MeV electron mass. For the virtual correction the formulas of Ref. [68] have been used. Very good agreement has been found with the analytic formulas of Kinoshita and Sirlin [38].

(4) Our corrections to the electron energy spectrum and the electron asymmetry of the neutron decay are in excellent agreement with the analytic formulas of Sirlin [42] and Shann [44].

(5) Different computer programs have been used for the charged baryon decays and for the neutral baryon decays. It is easy to show that in the $m_i-m_f\to 0$ limit the

corrections do not depend on whether the initial baryon is charged or neutral, provided that the Coulomb correction is omitted. This has been checked in the case of the neutron decay.

Finally, we compare our results with those of earlier publications. As mentioned above, our calculation of the electron asymmetry for neutron decay agrees with the analytic result of Ref. [44]. The MI radiative correction to the neutrino asymmetry of the neutron and hyperon semileptonic decays was found exactly zero in Refs. [47] and [56]. Our result (see Tables III–V) is small, but different from zero. The reason of the disagreement is the following: the authors of Refs. [47] and [56] defined the $\mathbf{n}_1$ vector as the antineutrino three-momentum, while our definition (suitable for experimental analyses) is Eq. (3.3). These two definitions are not equivalent if hard BR photons are present. The situation here is similar to the case of the electron-neutrino correlation [53,54]. The $\delta\alpha$ corrections to the $\alpha_0^{(e)}$ electron asymmetries of the $\Sigma^- \to n e \bar{\nu}$ and $\Lambda \to p e \bar{\nu}$ decays published in Ref. [56] are

TABLE I. $\delta\alpha(E_2,E_f)$ corrections and $\alpha_{BR}(E_2,E_f)$ for the (a) electron asymmetry, (b) neutrino asymmetry, and (c) hadron asymmetry of the $\Sigma^- \to n e \bar{\nu}$ decay.

$100\delta\alpha(E_2,E_f)$ and $100\alpha_{BR}(E_2,E_f)$

(a)

y \										
0.8067	-0.2	-0.1	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0	-0.1
0.8044	16.2	-0.2	-0.1	-0.1	-0.0	-0.0	-0.0	-0.1	-0.1	-0.3
0.8020	**-45.0**	-0.2	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2	
0.7997	**-45.5**	0.8	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2	
0.7974	**-46.8**	**-56.2**	-0.0	-0.1	-0.1	-0.1	-0.2	-0.2	-0.2	
0.7951	**-48.8**	**-56.2**	0.6	-0.1	-0.1	-0.2	-0.2	-0.3	-0.2	
0.7928	**-51.8**	**-59.1**	4.4	0.1	-0.1	-0.2	-0.2	-0.3		
0.7904	**-56.1**	**-63.7**	**-69.0**	0.6	-0.1	-0.2	-0.3	-0.3		
0.7881	**-62.2**	**-70.5**	**-75.2**	**-82.0**	0.1	-0.2	-0.3			
0.7858	**-71.1**	**-80.7**	**-85.5**	**-88.8**	0.7	-0.1	-0.1			
x	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
y_{min}	0.8043	0.7978	0.7925	0.7884	0.7857	0.7847	0.7854	0.7884	0.7939	0.8023

(b)

y										
0.8067	-0.8	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2	-0.4
0.8044	-50.6	-0.6	-0.3	-0.2	-0.2	-0.2	-0.3	-0.3	-0.4	0.7
0.8020	**-59.7**	-1.8	-0.6	-0.4	-0.4	-0.4	-0.4	-0.4	-0.3	
0.7997	**-61.3**	-6.2	-1.1	-0.7	-0.6	-0.5	-0.5	-0.4	0.3	
0.7974	**-62.3**	**-40.5**	-2.1	-1.0	-0.7	-0.6	-0.5	-0.1	0.9	
0.7951	**-62.8**	**-48.9**	-4.6	-1.4	-0.9	-0.6	-0.3	0.3	0.8	
0.7928	**-62.2**	**-49.0**	-16.0	-2.2	-1.0	-0.5	0.1	0.8		
0.7904	**-59.5**	**-45.3**	**-20.2**	-3.9	-1.1	-0.2	0.4	0.8		
0.7881	**-52.0**	**-34.2**	**-9.4**	32.6	-1.2	0.1	0.7			
0.7858	**-28.8**	**2.8**	**39.2**	**60.3**	-2.1	0.2	0.3			
x	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
y_{min}	0.8043	0.7978	0.7925	0.7884	0.7857	0.7847	0.7854	0.7884	0.7939	0.8023

(c)

y										
0.8067	0.6	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
0.8044	50.7	0.3	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.2
0.8020	**60.7**	1.2	0.2	0.1	0.1	0.0	0.0	0.1	0.1	
0.7997	**62.4**	5.4	0.6	0.2	0.1	0.1	0.1	0.1	0.2	
0.7974	**63.7**	**47.3**	1.4	0.4	0.2	0.1	0.1	0.1	0.2	
0.7951	**64.5**	**56.7**	4.0	0.8	0.3	0.2	0.1	0.1	0.1	
0.7928	**64.5**	**58.7**	18.4	1.7	0.6	0.3	0.2	0.1		
0.7904	**62.8**	**58.1**	**45.1**	4.4	1.0	0.4	0.2	0.1		
0.7881	**57.1**	**53.6**	**45.7**	11.1	2.4	0.7	0.2			
0.7858	**39.8**	**37.8**	**33.6**	**25.1**	8.2	1.4	0.1			
x	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
y_{min}	0.8043	0.7978	0.7925	0.7884	0.7857	0.7847	0.7854	0.7884	0.7939	0.8023

in satisfactory agreement with our results. The theoretical framework of the radiative correction calculations used in Ref. [55] is substantially different from ours; therefore, the comparison of our results with those of Ref. [55] is meaningless.

APPENDIX A

Let us introduce

$$T(\Gamma)=\frac{1}{32}\mathrm{Tr}[H_\mu(\not{p}_i+m_i)\Gamma\bar{H}_\nu(\not{p}_f+m_f)]L^{\mu\nu}\ , \tag{A1}$$

where

$$L^{\mu\nu}=\mathrm{Tr}[\gamma^\mu\not{p}_2\gamma^\nu\not{p}_1(1-\gamma_5)],\quad \bar{H}_\nu=\gamma_0 H_\nu^\dagger\gamma_0\ .$$

Then

$$L_0=\frac{1}{32G^2}\sum_{s_i,s_f,s_2}|\mathcal{M}_0|^2=T(1)\ , \tag{A2}$$

$$S_0=T(\gamma_5\not{s})$$

$$\left[L_0+S_0=\frac{1}{16G^2}\sum_{s_f,s_2}|\mathcal{M}_0|^2\right]\ . \tag{A3}$$

The coefficients of the different form factor combinations occurring in L_0 and S_0 are ($q=p_i-p_f$)

TABLE II. $\delta\alpha(E_2,E_f)$ corrections and $\alpha_{\mathrm{BR}}(E_2,E_f)$ for the (a) electron asymmetry, (b) neutrino asymmetry, and (c) hadron asymmetry of the $\Lambda\to pe\bar{\nu}$ decay.

y \ $100\delta\alpha(E_2,E_f)$ and $100\alpha_{\mathrm{BR}}(E_2,E_f)$										
					(a)					
0.8530	2.8	0.4	0.1	0.1	0.0	−0.0	−0.1	−0.1	−0.3	−0.9
0.8518	22.2	1.3	0.4	0.1	−0.0	−0.2	−0.3	−0.5	−0.8	−1.0
0.8505	**15.8**	2.6	0.7	0.2	−0.1	−0.3	−0.5	−0.7	−1.1	
0.8493	**2.3**	4.8	1.0	0.2	−0.1	−0.4	−0.6	−0.9	−1.0	
0.8480	**−10.9**	9.7	1.5	0.3	−0.1	−0.5	−0.7	−0.9	−0.7	
0.8467	**−23.5**	**−33.2**	2.3	0.5	−0.1	−0.5	−0.7	−0.8	−0.2	
0.8455	**−35.5**	**−43.8**	3.4	0.7	−0.1	−0.5	−0.7	−0.6		
0.8442	**−46.9**	**−55.5**	**−64.7**	1.0	0.0	−0.4	−0.6	−0.3		
0.8429	**−57.8**	**−67.2**	**−74.1**	1.4	0.1	−0.3	−0.4			
0.8417	**−68.2**	**−78.5**	**−85.4**	**−90.7**	0.3	−0.1	−0.0			
x	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
$y_{\min}$	0.8516	0.8480	0.8450	0.8428	0.8415	0.8410	0.8417	0.8434	0.8464	0.8509
					(b)					
0.8530	−0.2	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.2	−0.5
0.8518	−2.0	−0.3	−0.2	−0.2	−0.2	−0.2	−0.3	−0.4	−0.5	−0.0
0.8505	**84.3**	−0.6	−0.4	−0.3	−0.3	−0.4	−0.4	−0.5	−0.4	
0.8493	**78.4**	−1.1	−0.6	−0.5	−0.4	−0.5	−0.5	−0.5	−0.1	
0.8480	**71.8**	−1.9	−0.8	−0.6	−0.5	−0.5	−0.5	−0.3	−0.0	
0.8467	**64.6**	**77.8**	−1.0	−0.7	−0.6	−0.5	−0.3	−0.1	0.0	
0.8455	**57.0**	**70.6**	−1.3	−0.8	−0.6	−0.4	−0.1	−0.0		
0.8442	**48.6**	**63.4**	**81.3**	−0.7	−0.4	−0.2	−0.0	0.0		
0.8429	**39.5**	**56.9**	**75.9**	−0.6	−0.2	−0.1	−0.0			
0.8417	**29.9**	**57.0**	**83.6**	**95.1**	−0.1	−0.0	0.0			
x	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
$y_{\min}$	0.8516	0.8480	0.8450	0.8428	0.8415	0.8410	0.8417	0.8434	0.8464	0.8509
					(c)					
0.8530	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.9
0.8518	1.9	0.4	0.3	0.3	0.3	0.4	0.5	0.6	1.0	1.1
8.8505	**−84.2**	0.8	0.6	0.5	0.6	0.6	0.8	1.0	1.3	
0.8493	**−77.6**	1.4	0.9	0.8	0.8	0.9	1.1	1.3	1.4	
0.8480	**−70.5**	2.4	1.3	1.1	1.1	1.2	1.3	1.5	1.1	
0.8467	**−62.8**	**−72.1**	1.7	1.4	1.4	1.5	1.6	1.5	0.3	
0.8455	**−54.4**	**−62.2**	2.3	1.8	1.7	1.7	1.7	1.3		
0.8442	**−45.2**	**−51.7**	**−64.6**	2.2	2.0	2.0	1.7	0.8		
0.8429	**−34.5**	**−39.5**	**−47.8**	2.8	2.5	2.2	1.5			
0.8417	**−20.2**	**−23.1**	**−27.1**	**−35.2**	3.2	2.3	0.1			
x	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
$y_{\min}$	0.8516	0.8480	0.8450	0.8428	0.8415	0.8410	0.8417	0.8434	0.8464	0.8509

TABLE III. $\delta\alpha(E_2)$ and $\delta\alpha$ corrections to the $\alpha_0(E_2)$ asymmetry distributions and the α_0 total asymmetries of the $\Sigma^-\to ne\bar{\nu}$ decay. Notation e: electron asymmetry; ν: neutrino asymmetry; h: hadron asymmetry; α: α asymmetry; β: β asymmetry; 0:$100\alpha_0(E_2)$ and $100\alpha_0$ (last column); δ:$100\delta\alpha(E_2)$ and $100\delta\alpha$ (last column).

x		0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
e	0:	−55.1	−55.9	−57.4	−59.1	−60.8	−62.6	−64.4	−66.4	−68.4	−70.5	−63.2
	δ:	1.8	0.3	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.0	−0.0	0.1
ν	0:	−46.9	−46.2	−44.6	−42.7	−40.4	−37.9	−35.0	−31.7	−28.0	−23.8	−36.5
	δ:	−4.1	−1.7	−0.7	−0.5	−0.4	−0.4	−0.4	−0.4	−0.4	−0.5	−0.7
h	0:	48.7	49.8	52.6	56.0	60.4	66.2	73.0	75.4	75.1	73.9	66.7
	δ:	4.0	1.7	0.7	0.5	0.4	0.3	0.1	−0.0	−0.0	−0.0	−0.0
α	0:	−67.0	−67.4	−68.1	−68.7	−69.3	−69.7	−69.9	−70.0	−69.9	−69.6	−69.5
	δ:	0.6	−0.0	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1
β	0:	−4.2	−5.2	−7.4	−9.7	−12.2	−14.9	−17.9	−21.1	−24.5	−28.2	−16.1
	δ:	4.9	1.7	0.6	0.3	0.3	0.2	0.2	0.2	0.2	0.3	0.5

$$L_0[f_1^2]=(p_ip_2)(p_1p_f)+(p_ip_1)(p_2p_f)-m_im_f(p_1p_2)\ ,$$

$$L_0[f_1g_1]=2\{(p_ip_2)(p_1p_f)-(p_ip_1)(p_2p_f)\}\ ,$$

$$L_0[f_1f_2]=\frac{1}{m_i}\{m_i[(p_2q)(p_1p_f)+(p_1q)(p_2p_f)+(p_fq)(p_1p_2)]-m_f[(p_2q)(p_ip_1)+(p_1q)(p_ip_2)+(p_iq)(p_1p_2)]\}\ ,$$

$$L_0[f_2g_1]=\frac{m_i+m_f}{m_i}L_0[f_1g_1]\ ,$$

$$\begin{aligned}L_0[f_2^2]=\frac{1}{m_i^2}\{&(p_iq)(p_2q)(p_1p_f)+(p_iq)(p_1q)(p_2p_f)-(p_2q)(p_1q)(p_ip_f)\\&+(p_2q)(p_fq)(p_ip_1)+(p_1q)(p_fq)(p_ip_2)-m_im_f(p_2q)(p_1q)\\&-q^2(p_ip_2)(p_1p_f)-q^2(p_ip_1)(p_2p_f)+\tfrac{1}{2}q^2(p_ip_f)(p_1p_2)-\tfrac{1}{2}q^2m_im_f(p_1p_2)\}\ ,\end{aligned}$$

$$L_0[f_2g_2]=\frac{2}{m_i^2}\{(p_2p_f)(p_1q)(p_iq)-(p_1p_f)(p_2q)(p_iq)+(p_ip_2)(p_1q)(p_fq)-(p_ip_1)(p_2q)(p_fq)\}\ ,$$

$$S_0[f_1^2]=(p_2s)[m_i(p_1p_f)-m_f(p_ip_1)]-(p_1s)[m_i(p_2p_f)-m_f(p_ip_2)]\ ,$$

$$S_0[f_1g_1]=2m_i[(p_2s)(p_1p_f)+(p_1s)(p_2p_f)]\ ,$$

$$\begin{aligned}S_0[f_1f_2]=\frac{1}{m_i}\{&(p_2s)[(p_iq)(p_1p_f)+(p_fq)(p_ip_1)-2m_im_f(p_1q)]\\&-(p_1s)[(p_iq)(p_2p_f)+(p_fq)(p_ip_2)-2m_im_f(p_2q)]+(p_fs)[(p_ip_2)(p_1p_f)-(p_ip_1)(p_2p_f)]\}\ ,\end{aligned}$$

$$\begin{aligned}S_0[f_2g_1]=\frac{1}{m_i}\{&(p_2s)[2(p_iq)(p_1p_f)-(p_1q)(p_ip_f)-m_im_f(p_1q)]\\&+(p_1s)[2(p_iq)(p_2p_f)-(p_2q)(p_ip_f)-m_im_f(p_2q)]\\&+(p_fs)[(p_2q)(p_ip_1)+(p_1q)(p_ip_2)-(p_iq)(p_1p_2)\\&\quad+2(p_ip_2)(p_1p_f)+2(p_ip_1)(p_2p_f)-(p_ip_f)(p_1p_2)+m_im_f(p_1p_2)]\}\ ,\end{aligned}$$

TABLE IV. $\delta\alpha(E_2)$ and $\delta\alpha$ corrections to the $\alpha_0(E_2)$ asymmetry distributions and the α_0 total asymmetries of the $\Lambda\to pe\bar{\nu}$ decay (see Table III for notation).

x		0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
e	0:	24.8	22.7	18.2	13.6	8.9	4.1	−0.8	−5.8	−10.8	−15.7	2.2
	δ:	−5.7	−2.2	−0.9	−0.5	−0.4	−0.3	−0.2	−0.2	−0.1	−0.1	−0.0
ν	0:	96.1	96.3	96.8	97.1	97.5	97.8	98.0	98.2	98.4	98.6	97.8
	δ:	−6.6	−2.6	−0.9	−0.5	−0.4	−0.4	−0.3	−0.2	−0.2	−0.2	−0.4
h	0:	−96.5	−96.8	−96.5	−94.1	−88.1	−74.8	−50.5	−28.0	−9.0	7.0	−58.6
	δ:	7.1	3.1	1.5	1.2	1.2	1.3	0.9	0.6	0.3	0.2	−0.1

TABLE V. $\delta\alpha(E_2)$ corrections to the $\alpha_0(E_2)$ asymmetry distributions of the $n\to pe\bar{\nu}$ decay (see Table III for notations).

x		0.4	0.5	0.6	0.7	0.8	0.9
e	0:	−1.66	−6.55	−8.08	−8.90	−9.41	−9.76
	δ:	−0.03	−0.01	−0.01	−0.01	−0.01	−0.01
ν	0:	98.82	98.82	98.81	98.81	98.81	98.81
	δ:	−0.01	−0.02	−0.02	−0.02	−0.03	−0.03
h	0:	−98.36	−83.61	−51.97	−25.53	−9.11	2.30
	δ:	0.01	0.04	0.06	0.04	0.03	0.02

$$S_0[f_2^2]=\frac{1}{m_i^2}\{(p_2s)[m_i(p_1q)(p_fq)-m_f(p_1q)(p_iq)]-(p_1s)[m_i(p_2q)(p_fq)-m_f(p_2q)(p_iq)]$$
$$+(p_fs)[m_i(p_2q)(p_1p_f)-m_i(p_1q)(p_2p_f)+m_f(p_2q)(p_ip_1)-m_f(p_1q)(p_ip_2)]\}\ ,$$
$$S_0[f_2g_2]=\frac{2}{m_i}\{(p_2s)[(p_1q)(p_fq)-q^2(p_1p_f)]+(p_1s)[(p_2q)(p_fq)-q^2(p_2p_f)]$$
$$+(p_fs)[\tfrac{1}{2}q^2(p_1p_2)-(p_1q)(p_2q)-(p_2q)(p_1p_f)-(p_1q)(p_2p_f)]\}\ .$$

The other terms in L_0 and S_0 may be obtained by the

$$f_1\leftrightarrow g_1,\quad f_2\leftrightarrow -g_2,\quad m_f\to -m_f$$

substitution [69] (only the f_1,f_2,g_1, and g_2 form factors are taken into account here).

The above expressions obey the $p_1\leftrightarrow p_2$ interchange theorem of Weinberg [70]. They have been checked by numerical trace computation. We have calculated the coefficients of the various form factor combinations for several totally integrated quantities. Our results agree with the numerical results and approximate analytical formulas of Refs. [64, 71–73].

For $m_2=0$ there exist interesting relations between the form factor coefficients of L_0 and S_0 at the lower and upper boundaries of the Dalitz plot.

(a) At $E_f=E_{f_{\min}}(E_2)$,

$$L_0=-S_0^{e\parallel}\ . \tag{A4}$$

This is true for every form factor coefficient, separately. $S_0^{e\parallel}$ is obtained from S_0 by the (2.13) substitutions.

(b) At $E_f=E_{f_{\max}}(E_2)$,

$$L_0[f_1^2]=L_0[g_1^2]=-\tfrac{1}{2}S_0^{e\parallel}[f_1g_1]\ , \tag{A5}$$

and all the other form factor coefficients are zero.

APPENDIX B

Let us denote

$$Q_0=m_i-E_2-E_f\ .$$

It is useful to introduce the following notation.
If the initial baryon is charged,

$$m=m_i,\quad p=p_i,\quad m'=m_i,\quad \hat{E}_2=E_2\ . \tag{B1}$$

If the initial baryon is neutral,

$$m=m_f,\quad p=p_f,\quad m'=-m_f\ ,\qquad \hat{E}_2=\frac{(E_2+E_f)^2-m_2^2-m_f^2-Q_0^2}{2m_f}\ . \tag{B2}$$

Moreover,

$$\hat{p}_2=\sqrt{\hat{E}_2^2-m_2^2}\ ,\quad \hat{\beta}=\frac{\hat{p}_2}{\hat{E}_2},\quad \hat{N}=\ln\left[\frac{\hat{E}_2+\hat{p}_2}{m_2}\right]\ . \tag{B3}$$

The model-independent part of the virtual correction (with the definition of Ref. [42]) can be written as

$$\mathcal{M}_V^{(\mathrm{MI})}=\tfrac{1}{2}Z\mathcal{M}_0+\tilde{\mathcal{M}}\ , \tag{B4}$$

where

$$Z=\frac{\alpha}{\pi}\left\{\left[\frac{1}{2}-2\frac{\hat{N}}{\hat{\beta}}\right]\ln\left[\frac{m_2}{m_\gamma}\right]+\frac{3}{2}\ln\left[\frac{m}{m_\gamma}\right]-\frac{11}{8}+\frac{\hat{N}}{\hat{\beta}}-\frac{\hat{N}^2}{\hat{\beta}}+\frac{1}{\hat{\beta}}L\left[\frac{2\hat{\beta}}{1+\hat{\beta}}\right]-2\frac{\hat{E}_2}{m'}\left[\ln\left[\frac{\hat{E}_2+\hat{p}_2}{m}\right]-1\right]\right\}+Z_{\mathrm{Cb}} \tag{B5}$$

(here m_γ is the infrared cutoff), and

$$\tilde{\mathcal{M}}=-\frac{G}{\sqrt{2}}\frac{\alpha}{2\pi}\frac{m_2}{m\hat{p}_2}\hat{N}[\bar{u}_fH_\rho u_i][\bar{u}_2\not{p}\gamma^\rho(1-\gamma_5)v_1]\ . \tag{B6}$$

L is the Spence function defined as

$$L(x)=\int_0^x dt\frac{\ln|1-t|}{t}\ . \tag{B7}$$

The Z_{Cb} Coulomb term is zero for charged-baryon decay, and

$$Z_{\mathrm{Cb}}=\frac{\alpha\pi}{\hat{\beta}} \tag{B8}$$

for neutral-baryon decay.

A good approximation is

$$\frac{1}{32G^2}2\sum_{s_f,s_2}\mathrm{Re}(\tilde{\mathcal{M}}\mathcal{M}_0^*)\approx\tfrac{1}{2}(\tilde{L}+\tilde{S}) , \quad \text{(B9)}$$

$$\tilde{L}=-\frac{\alpha}{\pi}\frac{m_2^2}{\hat{p}_2}m\hat{N}(pp_1)(f_1^2+3g_1^2) , \quad \text{(B10)}$$

$$\tilde{S}=\frac{2\alpha}{\pi}\frac{m_2^2}{\hat{p}_2}m^2\hat{N}(p_1s)(g_1^2-f_1g_1) . \quad \text{(B11)}$$

The MI virtual corrections to L_0 and S_0 are

$$ZL_0+\tilde{L} \text{ and } ZS_0+\tilde{S} , \quad \text{(B12)}$$

respectively.

For the BR amplitude ($\mathcal{M}_{\mathrm{BR}}$) the pointlike hadron approximation is employed in our work (the BR photon energy is small compared to the baryon masses). The BR matrix element squared is decomposed into three terms:

$$\sum_{s_f,s_2,\gamma}|\mathcal{M}_{\mathrm{BR}}|^2=-\frac{G^2e^2}{4}\mathrm{Re}\left[\frac{1}{(p_2k)^2}T_e+\frac{1}{(pk)^2}T_h-\frac{2}{(p_2k)(pk)}T_I\right] ; \quad \text{(B13)}$$

$$T_e=H^e_{\mu\nu}L_e^{\mu\nu} ,$$
$$T_h=H^h_{\mu\nu}L_h^{\mu\nu} , \quad \text{(B14)}$$
$$T_I=H^I_{\mu\nu\rho}L_I^{\mu\nu\rho} ;$$

$$L_e^{\mu\nu}=\mathrm{Tr}[(\not p_2+\not k+m_2)\gamma^\rho(\not p_2+m_2)\times\gamma_\rho(\not p_2+\not k+m_2)\gamma^\mu\not p_1\gamma^\nu(1-\gamma_5)] ,$$
$$L_h^{\mu\nu}=\mathrm{Tr}[\not p_2\gamma^\mu\not p_1\gamma^\nu(1-\gamma_5)] , \quad \text{(B15)}$$
$$L_I^{\mu\nu\rho}=\mathrm{Tr}[(\not p_2+\not k+m_2)\gamma^\rho(\not p_2+m_2)\times\gamma^\mu\not p_I\gamma^\nu(1-\gamma_5)] ;$$

$$H^e_{\mu\nu}=\tfrac{1}{2}\mathrm{Tr}[\Gamma^f_{\mu\nu}\Gamma_i] ,$$
$$\Gamma^f_{\mu\nu}=\bar{H}_\nu(\not p_f+m_f)H_\mu , \quad \text{(B16)}$$
$$\Gamma_i=(\not p_i+m_i)(1+\gamma_5\not s) .$$

For charged-baryon decay (such as $\Sigma^-\to ne\bar{\nu}$),

$$H^h_{\mu\nu}=\tfrac{1}{2}\mathrm{Tr}[\Gamma^f_{\mu\nu}(\not p_i-\not k+m_i)\gamma^\rho\Gamma_i\gamma_\rho(\not p_i-\not k+m_i)] ,$$
$$H^I_{\mu\nu\rho}=\tfrac{1}{2}\mathrm{Tr}[\Gamma^f_{\mu\nu}(\not p_i-\not k+m_i)\gamma_\rho\Gamma_i], \quad p=p_i . \quad \text{(B17)}$$

For neutral-baryon decay (such as $\Lambda\to pe\bar{\nu}$),

$$H^h_{\mu\nu}=\tfrac{1}{2}\mathrm{Tr}[\Gamma^i_{\mu\nu}(\not p_f+\not k+m_f)\gamma^\rho(\not p_f+m_f)\times\gamma_\rho(\not p_f+\not k+m_f)] ,$$
$$H^I_{\mu\nu\rho}=\tfrac{1}{2}\mathrm{Tr}[\Gamma^i_{\mu\nu}(\not p_f+m_f)\gamma_\rho(\not p_f+\not k+m_f)] , \quad \text{(B18)}$$
$$\Gamma^i_{\mu\nu}=H_\mu\Gamma_i\bar{H}_\nu, \quad p=p_f .$$

H_μ is taken from (2.1), assuming $f_3=g_3=g_2=0$, and with constant form factors (q^2 dependence neglected). We have also omitted in our calculations the k dependence of the f_2 term and the corresponding direct emission term mentioned in Ref. [54].

The trace calculations and index summations have been carried out by the help of the REDUCE symbolic algebraic software. The output of these computations has the general form [after the (3.6) replacements]

$$T_j=\sum_{n_1,n_2,n_i}C(n_1,n_2,n_i)(p_1k)^{n_1}(p_2k)^{n_2}(p_ik)^{n_i} \quad (j=e,h,I) , \quad \text{(B19)}$$

where the $C(n_1,n_2,n_i)$ coefficients depend on $E_2,E_f,\cos\theta$, the m_i,m_f,m_2 masses and the form factors.

We checked the REDUCE outputs by numerical computation of T_e,T_h, and T_I for a few sets of the p_1,p_2,p_i,k four-momenta. A general FORTRAN subroutine package has been worked out for this purpose. The REDUCE results and the numerical computations coincided with each other by more than 8 digits. We would like to emphasize the importance of this type of numerical control in order to get reliable results for complicated matrix elements occurring in perturbative calculations.

In order to obtain Ω_{BR} [see Eq. (3.7)] we have substituted the $(p_1k)^{n_1}(p_2k)^{n_2}(p_ik)^{n_i}$ factors in (B19) to their phase space integrals:

$$I_{AB}(f;n_1,n_2,n_i)=\int_{Q_{\min}}^{Q_{\max}}dQ\,f(Q)\int_{K_{\min}}^{K_{\max}}dK\frac{K}{K_0}\frac{1}{2\pi}\int_0^{2\pi}d\phi_k\frac{1}{(p_Ak)(p_Bk)}(p_1k)^{n_1}(p_2k)^{n_2}(p_ik)^{n_i} , \quad \text{(B20)}$$

where

$$p_A,p_B:\ p_2,p_i \text{ or } p_f ;$$
$$K_{\min}=\tfrac{1}{2}(Q_0-Q),\quad K_{\max}=\tfrac{1}{2}(Q_0+Q) , \quad \text{(B21)}$$
$$Q_{\min}=||\mathbf{p}_2|-|\mathbf{p}_f||,\quad Q_{\max}=\min(Q_0,|\mathbf{p}_2|+|\mathbf{p}_f|) ,$$
$$Q_0=m_i-E_2-E_f ,$$

f is a general function of Q.

Our general method of evaluating these integrals has been explained in Ref. [54]. We show here another method which is applicable for integrals without infrared divergence ($n_1+n_2+n_i>0$). In order to integrate T_e, we have to calculate the

$$G_{ln}(K)=\int dK\frac{1}{2\pi}\int_0^{2\pi}d\phi_k\frac{K^n}{(p_2k)^l} \quad (l=2,1,0,-1;\ n=0,1,2,3) \quad \text{(B22)}$$

indefinite integrals. (p_2k) can be expressed as

$$\begin{aligned}
(p_2k) &= A - B\cos\phi_k\ ,\\
A &= E_2K - |\mathbf{p}_2|K_{\parallel}\cos\eta\ ,\\
B &= |\mathbf{p}_2|K_{\perp}\sin\eta\ ,\\
\cos\eta &= \frac{\mathbf{p}_2^2-\mathbf{p}_f^2+Q^2}{2|\mathbf{p}_2|Q}\ ,\\
K_{\parallel} &= \frac{Q_0^2-Q^2}{2Q}-\frac{Q_0}{Q}K\ ,\\
K_{\perp} &= \sqrt{K^2-K_{\parallel}^2}\ .
\end{aligned} \tag{B23}$$

Integrating first over ϕ_k,

$$\begin{aligned}
\frac{1}{2\pi}\int_0^{2\pi}\frac{d\phi_k}{(A-B\cos\phi_k)^2} &= \frac{A}{(A^2-B^2)^{3/2}}\ ,\\
\frac{1}{2\pi}\int_0^{2\pi}\frac{d\phi_k}{A-B\cos\phi_k} &= \frac{1}{\sqrt{A^2-B^2}}\ ,
\end{aligned} \tag{B24}$$

where

$$A^2-B^2=aK^2-bK+c\ , \tag{B25}$$

The

$$\int dK\frac{K^n}{(aK^2-bK+c)^{l-1/2}} \tag{B26}$$

integrals may be found in standard integral tables.

We mention that all our analytical integral formulas have been checked by numerical computations.

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