$N(\pi\pi)_S$ decay: A flux-tube-model picture

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We calculate the $N(\pi\pi)_S$ decay width of nucleon resonances up to 2 GeV within the frame of a semirelativistic flux-tube model, using the constant amplitude flux-tube-breaking decay mechanism. To deal with the absence of a known $q\bar{q}$ intermediate state of sufficiently low mass, we use a pseudoresonating intermediate state with no assumption on its composition other than the flux-tube structure constraints. Its mass, width, and radius are varied. The agreement with data is comparable to the one obtained for other hadronic strong decay channels calculations if the intermediate state has a mass of about 600 and a typical hadronic radius, a result consistent with $\pi\pi$ strong final-state interaction.

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I. INTRODUCTION

The QCD-inspired flux-tube models [1,2] have been shown successful in reproducing the spectra of hadrons [3-5]. The flux-tube-breaking mechanism also provides a good description of the strong decays of mesons [6,7] and baryons [8].

Among the various baryon decay channels, many have been investigated using either the full flux-tube-breaking mechanism or the $^3P_0$ model which is a good approximation [8] of it; the quality of the agreement with experiment has been found similarly good for the $N\pi$, $\Delta\pi$, $N\rho$, and still other channels [9-11]. The $N(\pi\pi)_S$ channel, however, has not been reproduced with success [11]. This should not be considered a setback for the decay mechanism, since the nature of the $N(\pi\pi)_S$ process remains unclear. Indeed, no experimentally identified particle appears as a possible intermediate state of this three-body decay. Since the calculation of all channels is needed for purposes such as total-width calculation, missing resonances identification, mass shifts, . . . , it is worth trying to construct a model of this decay.

In the present work, we shall assume the flux-tube-breaking mechanism to be the only decay source, and test the possible resonant nature of the process. We recall the essential formulation of the flux-tube model in Sec. II. The description and parametrization of the tested resonantlike intermediate state will be found in Sec. III, together with the decay-width calculation for the corresponding channel. Section IV contains the results and a discussion.

II. THE MODEL: GENERAL OUTLINE

The qqq and $q\bar{q}$ systems are described by a QCD-inspired Hamiltonian [1], containing a relativistic kinetic energy term, a three-body linear confinement potential, a scalar Coulomb term, and hyperfine interaction terms arising from one-gluon exchange [12]. The baryon wave functions are obtained from a variational calculation on the scalar part of the Hamiltonian followed by a diagonalization [13,14] in the space of $56(0^+,2^+)$, $56'(0^+)$, $70(0^+,1^-,2^+)$, and $20(1^+)$ SU(6)-spin flavor multiplets. The explicit form of the Hamiltonian and related hadronic wave functions can be found in Refs. [13] and [14]. The Roper resonance and correlated $P11$ states are treated as in Ref. [14], where a radial form bringing the Roper mass close to the experimental data range has been introduced.

Alternatives to this spectrum calculation exist, such as e.g., instanton-induced forces [15]. Complementary contributions, e.g., a spin-orbit term [16], could also be included that would have some impact on the mixing angles arising from the diagonalization and, hence, on the decay widths calculations. As this is not the main scope of the present article, we shall not discuss these features at length but keep their possible effects in mind in later discussions.

The breaking of the flux tube gives birth to a $q\bar{q}$ pair which bears the quantum number of the vacuum $J^{PC}=0^{++}$. The transition amplitude corresponding to a baryon resonance B^* decay into baryon + meson (Fig. 1) reads [10]

$$\langle BM|T|B^*\rangle=\sum_m \langle 11m\,-m|00\rangle \times\langle \phi_B\phi_M|\phi_{B^*}\phi_{\rm vac}^{-m}\rangle\,{\rm Im}(B^*;B,M) \qquad (1)$$

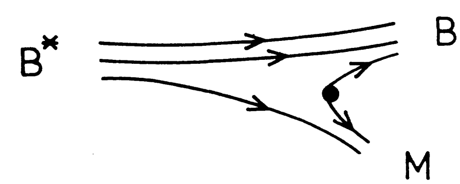

FIG. 1. The elementary decay diagram of an excited baryon B^* into a baryon B and a meson M.

where B and M are the *emitted* baryon and meson, respectively, and B^* the decaying baryon. The Clebsch-Gordan coefficients account for the $L=1,S=1,J=0$ state of the created $q\bar{q}$ pair. The ϕ's denote the flavor spin structure. The last factor is an overlap integral containing the configuration space hadron wave functions ψ's and the nonlocal $q\bar{q}$ emission operator. In the rest frame of B^*, it reads [8]

$$\mathrm{Im}(B^*,B,M)=-\left[\frac{3}{4\pi}\right]^{1/2}\left[\frac{2}{\pi}\right]^{3/2}\delta(\mathbf{k}_M+\mathbf{k}_B)$$
$$\times\int d^3\rho\, d^3\lambda\, d^3x\,\Psi_R(\boldsymbol{\rho},\boldsymbol{\lambda}+(\tfrac{8}{3})^{1/2}\mathbf{x})\Psi_N(\boldsymbol{\rho},\boldsymbol{\lambda})\exp[(\mathbf{k}_M(\tfrac{2}{3})^{1/2}\boldsymbol{\lambda}+\mathbf{x})]$$
$$\times\boldsymbol{\varepsilon}_m\cdot(\mathbf{k}_M+i\nabla_x)\Psi_m(2\mathbf{x})\gamma(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{r}_c)\ . \qquad (2)$$

$\boldsymbol{\varepsilon}_m$ is the spherical unit vector, $\mathbf{r}_c$ stands for the location of the $q\bar{q}$ pair creation, and γ is a function of the position of the created pair with respect to the flux tube [8]. It has been shown that the naive flux-tube picture, corresponding to $\gamma(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{r}_c)=\gamma_0=$ const, is a very good approximation. This is due to the fact that the falloff of $\gamma(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{r}_c)$ occurs at distances bigger than the hadron sizes to that, in the domain where the rest of the integrand in (2) is not close to zero the breaking amplitude, $\gamma(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{r}_c)$ is essentially a constant.

When the outgoing meson is an identified $q\bar{q}$ system, Ψ_m in (2) is the wave function obtained from the corresponding variational calculation.

III. N $(\pi\pi)_S$ DECAY DESCRIPTION

The final-state quantum numbers of $N\pi\pi$ decay indicate which hadronic resonances may have been an intermediate state in the process. In the case of $N(\pi\pi)_S$, it corresponds to a $I^G=0^+$, $J^{PC}=0^{++}$ bosonic state. The first experimentally observed meson with these quantum numbers is the $f_0(975)$, which has a width of 34 MeV, small at the hadronic scale. The experimental lower limit [17] for $N(1440)$, $N(1680)$, and $N(1710)$ cannot be accommodated by the intermediate f_0 picture as the available phase space volume is zero.

This problem can be solved by admitting that the nature of the process may be a multiquark one and that the narrow resonance picture does not necessarily hold [18], as would be for an identified pure $q\bar{q}$ state.

In the following, we shall assume that all pair creations proceed through $^3P_0$ vertices. We represent the complete process in the diagram of Fig. 2.

The $q\bar{q}$ pair (3,5) must be in a $J^{PC}=0^{++}$ state, as no other total angular momentum contribution enters the box. Accordingly, the spin projection of (3,5) can be $S_z=-1$, 0, or $+1$. This spin projection can be canceled by the one of the created $0^{++}q\bar{q}$ pair that takes place inside the box.

Formula (1) then becomes

$$\langle N;(3,5)|T|B^*\rangle=\sum_m\sum_n\langle 11m-m|00\rangle$$
$$\times\langle 11n-n|00\rangle$$
$$\times\langle\phi_N\phi_{(3,5)}^{-n}|\phi_{B^*}\phi_{\mathrm{vac}}^{-m}\rangle I_{mn}\ , \qquad (3)$$

where the index n refers to the 0^{++} (3,5) pair spin projection, while the index m refers to pair (4,5) as in Eq. (1).

In the configuration space integral I_{mn} we now test the possibility of describing the box as a resonant 0^{++} state, assigning to it a mass M_0, a width Γ_{00}, and a one parameter spatial wave function. To be consistent with other hadron wave functions, we use

$$\Psi_{0^{++}}(r)=Ne^{-\gamma_{1.5}r^{1.5}}\ , \qquad (4)$$

where N is a normalization factor and $\gamma_{1.5}$ the size parameter. Equation (4) exhibits a long distance behavior typical of a linear potential solution. We have also tested a Gaussian wave function and verified that this did not affect much the results presented in Sec. IV.

The value of $\gamma_{1.5}$ can be related to the rms radius of a particle made of 3 and 5. If there exists any $qq\bar{q}\bar{q}$ resonant state, the radius r_{35} should be viewed as a lower bound to the rms radius of that state. Since 3 and 5 do not build up a $q\bar{q}$ meson, Eq. (4) should be taken as an ansatz only.

We insert Eq. (4) into Eq. (2) and use $\gamma(\boldsymbol{\rho},\boldsymbol{\lambda},\mathbf{r}_c)=\gamma_0=$ const, using the value of γ_0 which yields the $\Delta(1232)\to N\pi$ width [8]. The transition amplitude is calculated using a 9-dimensional Monte Carlo integral, performed for various values of parameters $\gamma_{1.5}$ and steps on the allowed mass m of the intermediate state.

For a given mass m the width $B^*\to N(\pi\pi)_S$ in the res-

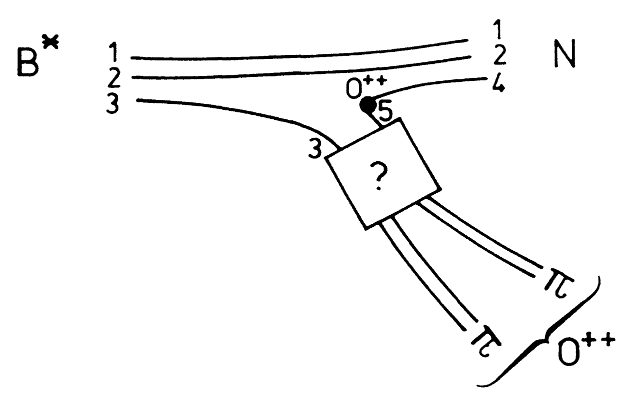

FIG. 2. The $N(\pi\pi)_S$ decay of an excited baryon B^*. The box stands for the unknown structure of the intermediate state.

onance center-of-mass frame then reads

$$\Gamma_{B^*\to N(\pi\pi)_S}(m)=\frac{1}{\pi}\frac{|\langle N;(3,5)|T|B^*\rangle|^2}{2J_{B^*+1}}\frac{kE_{0^{++}}E_N}{M_{B^*}}\ , \tag{5}$$

where k and $E_{0^{++}}$ are the momentum and relativistic kinetic energy of the emitted 0^{++} system, E_N is the nucleon recoil energy, and M_{B^*} the decaying resonance mass.

Since the width of the 0^{++} system is not *a priori* small, the $N(\pi\pi)_S$ width has to be written as the integral over mass of the product of $\Gamma(m)$ by a weight distribution taking the nonzero width into account [19]. Using a relativistic Breit-Wigner distribution, we write

$$\Gamma_{B^*\to N(\pi\pi)_S}(m)=\int_{(2m_\pi)^2}^{(m_R-m_N)^2} dm^2\sigma(m)\Gamma_{B^*\to N(\pi\pi)_S}(m) \tag{6}$$

with

$$\sigma(m)=\frac{1}{\pi}\frac{m\Gamma_\sigma(m)}{(M_0^2-m^2)^2+\Gamma_0^2(m)m^2}\ , \tag{7}$$

where $\Gamma_0(m)$ exhibits a standard form for a 0^{++} meson width, namely,

$$\Gamma_0(m)=\Gamma_{00}\left[\frac{k}{k_0}\right]\left[\frac{M_0}{m}\right]\ , \tag{8}$$

$$k(m)=\sqrt{m^2-4m_\pi^2}\ . \tag{9}$$

In Eq. (8), M_0, and Γ^{00} are the parameters for the intermediate state and $k_0=k(M_0)$.

Let us add for completeness that if the width Γ_{00} is not small compared to M_0, expression (7) loses much of its significance. Let us also notice that if m is close to threshold, Eq. (7) departs from the unitarity condition even for typical hadronic scale width. It is then wise to replace $\sigma(m)$ in Eq. (6) by a unitarized form $\sigma_{\rm un}(m)$:

$$\sigma_{\rm un}(m)=\frac{\sigma(m)}{\int_{(2m_\pi)^2}^{\infty}dm^2\sigma(m)} \tag{10}$$

IV. RESULTS AND DISCUSSION

We use as a comparison instrument a χ^2 defined as

$$\chi^2=\sum_{N^*}\frac{(\Gamma^{1/2\,(\rm model)}_{N^*\to N(\pi\pi)_S}-\Gamma^{1/2(\rm expt)}_{N^*\to N(\pi\pi)_S})^2}{(\Delta\Gamma^{1/2(\rm expt)}_{N^*\to N(\pi\pi)_S})^2}\ . \tag{11}$$

$\Gamma^{\rm expt}$ and $\Delta\Gamma^{\rm expt}$ are the experimental width and error range respectively, taken from the Particle Data Group summary table [17]. A lower and an upper bound are given simultaneously for three resonances only: $N(1440)$, $N(1680)$, and $N(1710)$. For one other, only an order of magnitude is given [$N(1535)$]; for five others [$N(1520)$, $N(1650)$, $N(1675)$, $N(1700)$, and $N(1720)$], an upper limit is given [17]. For $N(1535)$ we estimate the uncertainty at 50% on the amplitude. For the latter resonances, the branching ratio reference value has been taken as one-half of the upper limit.

In the absence of a star rating specific to the $N(\pi\pi)_S$ channel, as Particle Data Group publishes for other known decays, [17] it is difficult to assess the reliability of the various data. In a generally low-precision context, the narrow branching ratio range provided for $N(1680)$, namely, 15–20 %, seems questionable, as the experimental interval appears to be the result of a subtraction among other branching ratios. Though these are well known, a precision typical of the best (directly measured) $N\pi$ widths may be too optimistic. In view of this, we used two sets of data. One (set I) is the direct compilation from Ref. [17]. The second (set II) is identical except for the error bar on the amplitude of $N(1680)$ which has been widened by a factor of 2.

The best fit for set I, $\chi^2/N_{\rm DF}=3.2$, is obtained for intermediate state parameter values $M_0=560$ MeV, $\Gamma_{00}=40$ MeV, and rms radius between 3 and 5: 0.25 fm. This small width is due to a kinematical effect, as the amplitude calculated for $N(1680)$ is small, so that the best fit is obtained when the threshold effects [19] are reduced and, hence, is obtained for a small value of Γ_{00}.

The best fit for set II, $\chi^2/N_{\rm DF}=1.8$, is obtained for a mass of 590 MeV, width 200 MeV, and "radius" = 0.25 fm. Though the lowering of the χ^2 is of course partly due to the $N(1680)$ error interval relaxation, the overall pattern looks more sensible.

A $\chi^2/N_{\rm DF}$ of no more than one unit above the minimum can be obtained for any mass between 420 and 700 MeV, with a correspondingly adjusted "radius" value. A similar interval applies for set I. There is not much dependence on the width upwards. The condition $\chi^2/N_{\rm DF}$ no more than one unit above minimum corresponds to $\Gamma_{00}>30$ MeV (set I: 15 MeV), again with correspondingly adjusted mass and "radius" values.

The best-fit values are displayed in Table I (identified particles decay). The radius dependence is interesting as the overlap integral (2) is known to set strong constraints on particle radii [8]. The χ^2 dependence on the rms radius of the $\overline{qq}$ (3,5) distribution is depicted on Fig. 3.

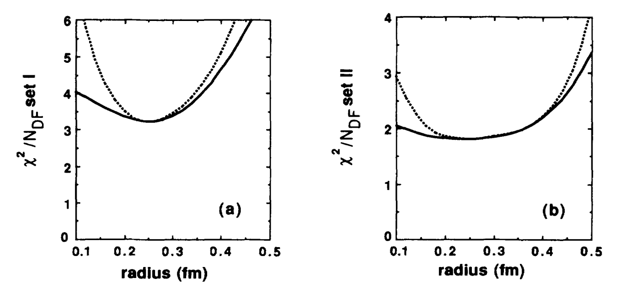

FIG. 3. $\chi^2/N_{\rm DF}$ for set I (a) and set II (b). The full line corresponds to the best χ^2 value for a given radius with corresponding values of M_0 and Γ_{00}. The dotted line stands for the value of χ^2 with M_0 and Γ_{00} fixed at their absolute minimum value. Radius stands for rms radius between quarks 3 and 5 of Fig. 2.

TABLE I. Best fit amplitudes $\Gamma^{1/2}_{N\to N(\pi\pi)_S}$ $(\mathrm{MeV})^{1/2}$ and experiment.

Resonance	Set I	Set II	Experiment
$N(1440)\frac{1}{2}^+$	2.6	3.7	$5.0^{+3.4}_{-2.6}$
$N(1520)\frac{3}{2}^-$	2.1	2.2	<2.6
$N(1535)\frac{1}{2}^-$	2.0	2.6	~ 2.7
$N(1650)\frac{1}{2}^-$	8.5	7.0	<5.5
$N(1675)\frac{5}{2}^-$	0.1	0.1	<3.0
$N(1680)\frac{5}{2}^+$	3.0	2.5	4.5 ± 0.6 or 4.7 ± 1.2[a]
$N(1700)\frac{3}{2}^-$	10.0	8.8	<9.5
$N(1710)\frac{1}{2}^+$	2.3	2.1	$4.7^{+2.0}_{-2.6}$
$N(1720)\frac{3}{2}^+$	0.6–2.9	0.6–2.6	<7.1[b]

[a]Cf. text: first value to be compared with set I, second with set II.
[b]Not used in the χ^2 evaluation due to the interpretation ambiguity in the $N\frac{3}{2}^+$ multiplet; see Table II.

Let us recall that this "radius" is only a lower bound on the radius one would define for a $2q2\bar{q}$ system. Its value is comparable to typical hadronic radii in the flux-tube quark model ($r_\pi = 0.16$ fm, $r_\rho = 0.32$ fm).

The χ^2/N_{DF} compares favorably with that obtained for the well-known $N\pi$ channel, which is close to 3 [8]. However, the present $N(\pi\pi)_S$ data are so scarce that it may be dangerous to draw many conclusions from our results. Table II presents the value of the amplitudes for the unidentified particles decay.

There is not much well-established data concerning the signs of the $N(\pi\pi)_S$ amplitudes. Nevertheless, we wish to present the elements of a comparison. Up to here, the description of the decay given in Sec. III enables us to calculate the magnitude of the amplitudes, as we have made no assumption on the inner quark structure of the 0^{++} intermediate state. To compare the signs with experiment, a quark structure prescription is needed. It was already noted in Ref. [20] that a pure $q\bar{q}$ structure leads to a sign opposite to that of the experimental data. This is confirmed by our calculation of the sign within the $q\bar{q}$ prescription. In Table III, we display the signs of

TABLE II. Best fit amplitudes $\Gamma^{1/2}_{N\to N(\pi\pi)_S}$ $(\mathrm{MeV})^{1/2}$ for unknown decays, (see Refs. [8,13,14] for the detailed multiplet structure).

Resonance		Set I	Set II
$N(1990)\frac{7}{2}^+$		1.0	0.9
$N(2000)\frac{5}{2}^+$	No. 2	2.0	1.8
	No. 3	5.5	4.9
$N(1720)\frac{3}{2}^+$	No. 1	2.5	2.1
	No. 2	0.6	0.6
	No. 3	2.9	2.6
	No. 4	4.2	3.7
	No. 5	3.7	3.3
$N(2100)\frac{1}{2}^+$	No. 4	1.3	1.1
	No. 5	2.6	2.2

TABLE III. Signs of the $N(\pi\pi)_S$ amplitudes. An asterisk indicates that Manley [20] and Longacre [21] agree on the sign. A double asterisk indicates that the Particle Data Group considers the sign to be experimentally well established [22].

Resonance	$q\bar{q}$	Anti-$q\bar{q}$	Experiment	
$N(1440)\frac{1}{2}^+$	+	−	?	
$N(1520)\frac{3}{2}^-$	−	+	?	
$N(1535)\frac{1}{2}^-$	−	+	+	**
$N(1650)\frac{1}{2}^-$	−	+	+	*
$N(1675)\frac{5}{2}^-$	?	?	?	
$N(1680)\frac{5}{2}^+$	−	+	+	**
$N(1700)\frac{3}{2}^-$	+	−	+	*
$N(1710)\frac{1}{2}^+$	−	+	−	
$N(1720)\frac{3}{2}^+$	+	−	−	*

the $N(\pi\pi)_S$ amplitudes: the first column presents the $q\bar{q}$ prescription sign and the second one the anti-$q\bar{q}$ sign. The $q\bar{q}$ sign is opposite to that of four of the five data sets for which the experimental analysis of Manley *et al.* [20] and Longacre *et al.* positively agree [21], including the two the Particle Data Group considers as well established [22]. Here follow some specific comments on Table III.

$N(1440)\frac{1}{2}^+$: Manley (+) and Longacre (−) contradict each other. Longacre claims that the coupling is well determined.

$N(1520)\frac{3}{2}^-$: Manley does not support any sign and questions the sign obtained by Longacre (−) because the resonant amplitude is rotated more than 70° away from the imaginary axis, but the latter author claims his coupling sign to be well determined.

$N(1675)\frac{5}{2}^-$: It is admitted both experimentally and theoretically that this coupling is extremely small.

$N(1710)\frac{1}{2}^+$: Manley and Longacre agree on the sign but Manley warns that the resonant amplitude is rotated far away from the imaginary axis, which may make the result questionable.

$N(1720)\frac{3}{2}^+$: Manley and Longacre agree on this sign. However, Manley states that no clear evidence for a $P13$ resonance at 1710 MeV has been seen in $N\pi\pi$. Instead the observed inelasticity in the $P13$ wave seems to be mainly associated with the decay of a resonance near 1850 MeV [20].

We predict [8] that the first $P13$ state, mainly a $^2N(56,2^+)$ state has a mass close to the 1710 value but has a very large $N\pi$ width (> 150 MeV) which may make it confused in the partial wave analysis with the second state, mainly $^4N(70,0^+)$ that we predict to have a mass close to 1900 MeV and a small width in $N\pi$. As we have calculated the signs for the first two $P13$ states and found them identical, we may consider the sign predictions unambiguous.

Tentative conclusions would be (1) that we obtain confirmation that the $^3P_0$ vertex predicts the correct sign as far as the vertex is concerned, since the relative signs do agree, and (2) that the 0^{++} intermediate state, if any, is not a pure $q\bar{q}$ state.

More experimental investigations are certainly needed.

In any case [though this conclusion may be enhanced depending on the interpretation of the $N(1680)$ data], the model appears to be an approximation comparable to the ones used in other channels, and is definitely an improvement over the anterior situation [11].

The effective description of this process using an intermediate pseudo-resonating state of typical $L \leq 1$ meson radius, a mass around 600 MeV, and a presumably large width also agrees with what is known of $\pi\pi$ strong final-state interaction [23].

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