Photonic parton distributions

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The quark and gluon distributions of the photon are determined in leading and higher order by imposing a vector-meson dominance (VMD) valencelike structure at a low resolution scale adopted from the pion. This leaves only one free parameter, not sufficiently constrained by VMD, to be fixed by experiment. Our predictions are in agreement with presently available data for $F_2^\gamma(x,Q^2)$. Simple parametrizations of the resulting parton distributions are presented in the range $10^{-5} \lesssim x < 1$, $0.3 \lesssim Q^2 \lesssim 10^6$ GeV^2 as obtained from the leading- and higher-order evolution equations.

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The construction of radiatively generated parton distributions of the nucleon [1] and the pion [2] has been shown to reproduce the available deep-inelastic scattering data and to provide, in addition, unique predictions in the still inaccessible small-x region. It is thus natural to proceed and apply this method also to the photon [3–5] where, just as in the previous hadronic case, one needs [4] some input information at some resolution scale $Q^2=Q_0^2$. We shall assume that the resolution scale for a valencelike parton structure is universal [1,2] and accordingly choose $Q_0^2=\mu^2$ where the leading-order (LO) and higher-order (HO) Q^2 evolution start with a valencelike input at [1,2]

$$\mu_{\rm LO}^2=0.25 \text{ GeV}^2 , \quad \mu_{\rm HO}^2=0.3 \text{ GeV}^2 . \tag{1}$$

Utilizing vector-meson dominance (VMD) and the approximate similarity of the vector meson and the pion we take, for the photonic input distributions $f^\gamma=q^\gamma(=\bar q^\gamma), G^\gamma$ [6],

$$f^\gamma(x,\mu^2)=\kappa\frac{4\pi\alpha}{f_\rho^2}f_\pi(x,\mu^2) \tag{2}$$

with $xf_\pi(x,\mu^2)\sim x^a(1-x)^b$ being the *valencelike* (i.e., $a>0$) inputs taken from Ref. [2] and [7] $f_\rho^2/4\pi\simeq 2.2$ with $1\lesssim\kappa\lesssim 2$. The range of κ corresponds to ambiguities related to the inclusion of the ω and φ mesons, in addition to the ρ, in the VMD ansatz and due to their assumed incoherent or coherent addition [7]. The precise value of κ clearly has to be extracted from experiment and, furthermore, can be different in LO and HO calculations. Fits to the data [8] on $F_2^\gamma(x,Q^2)$, shown in Fig. 1, yielded [9]

$$\kappa_{\rm LO}=2 , \quad \kappa_{\rm HO}=1.6 \tag{3}$$

implying that presently available data for F_2^γ can be explained by just one free parameter. The determination of this parameter κ was of course obtained by performing the Q^2 evolution in leading [4,10] and higher [10] orders for different values of κ resulting, via a least-squares fit, in the values stated in Eq. (3). The photonic parton distributions $f^\gamma(x,Q^2)$, consisting of a (inhomogeneous) "pointlike" and a (homogeneous) "hadronic" contribution, $f^\gamma=f_{\rm PL}^\gamma+f_{\rm had}^\gamma$, can now be uniquely calculated in LO and HO using Eqs. (2.12) and (2.13) of Ref. [10], for example.

It should be noted that in HO our input, Eq. (2), refers to the so-called deep-inelastic γ scattering (DIS$_\gamma$) factorization scheme, introduced in Ref. [10], in order to avoid the usual instability problems encountered in HO in the large-x region and to achieve perturbative stability between LO and HO results. This scheme is related to the more commonly used modified minimal subtraction ($\overline{\rm MS}$) scheme by

$$f_{\overline{\rm MS}}^\gamma=f_{{\rm DIS}_\gamma}^\gamma+\delta f^\gamma \tag{4}$$

with

$$\delta q^\gamma=\delta\bar q^\gamma=-3e_q^2\frac{\alpha}{8\pi}B_\gamma , \quad \delta G^\gamma=0 , \tag{5}$$

where $\alpha\simeq 1/137$ and

$$B_\gamma(x)=4\left[(1-2x+2x^2)\ln\frac{1-x}{x}-1+8x(1-x)\right] \tag{6}$$

which represents the troublesome [10], i.e., negative for large values of x, photonic HO contribution. The transformation (4) enables one to use the HO expressions for various other partonic subprocesses in the literature which, so far, have always been given in the $\overline{\rm MS}$ factorization scheme. For F_2^γ the DIS$_\gamma$ scheme implies that the B_γ term is transformed away and, by definition, fully absorbed into the photonic quark distributions leaving us with the result [11]

$$\frac{1}{x}F_2^\gamma(x,Q^2)=\sum_q e_q^2\left\{q^\gamma(x,Q^2)+\bar q^\gamma(x,Q^2) + \frac{\alpha_s(Q^2)}{4\pi}\int_x^1\frac{dy}{y}\left[B_q(x/y)[q^\gamma(y,Q^2)+\bar q^\gamma(y,Q^2)]+\frac{1}{f}B_G(x/y)G^\gamma(y,Q^2)\right]\right\} , \tag{7}$$

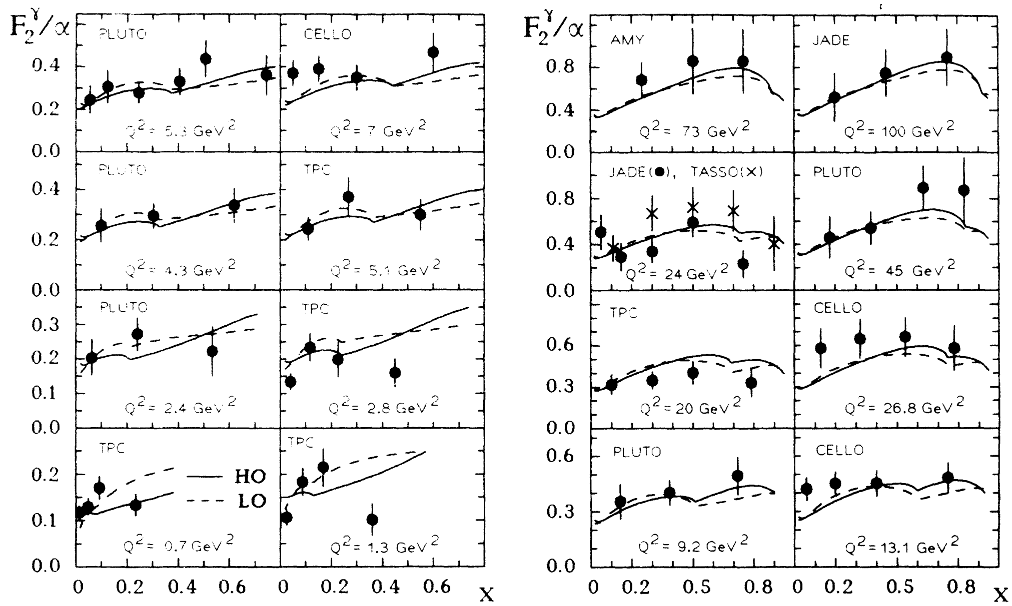

FIG. 1. Comparison of our radiatively generated LO and HO (DIS_γ) predictions, based on the valencelike input in Eq. (2) and the single fit parameter κ in Eq. (3), with the data of Refs. [8,9]. The charm contribution has been added, in the relevant kinematic region $W \geq 2m_c$, according to Eq. (9).

just as for the nucleon [1] structure function F_2^p, with the well-known coefficient functions $B_i(x)$ as given, for example, in Ref. [4] and

$$\frac{\alpha_s(Q^2)}{4\pi} \simeq \frac{1}{\beta_0 \ln Q^2/\Lambda^2} - \frac{\beta_1}{\beta_0^3} \frac{\ln\ln Q^2/\Lambda^2}{(\ln Q^2/\Lambda^2)^2} \tag{8}$$

with $\beta_0 = 11 - 2f/3$ and $\beta_1 = 102 - 38f/3$ and f being the number of active flavors. Expression (7) was used in producing the HO predictions shown in Fig. 1. The LO results are obviously entailed in these expressions by simply dropping all HO terms (B_i, β_1) in Eqs. (7) and (8).

It should be noted that the summation in Eq. (7) extends over all quarks with $m_q^2 \ll W^2 \equiv Q^2(1/x - 1)$. Thus for most experiments of present interest, all the above expressions should correspond to the $f = 3$ active light flavors u, d, and s. The contribution of the heavier quarks $h = c, b, t$ to F_2^γ should, in the threshold region where $W \gtrsim 2m_h$, be calculated according to the lowest-order (Bethe-Heitler) cross section for $\gamma^*(Q^2)\gamma \to h\bar{h}$ given by [3,12]

$$\frac{1}{x} F_{2,h}^\gamma(x,Q^2) = 3e_h^4 \frac{\alpha}{\pi} \left[\beta \left[-1 + 8x(1-x) - x(1-x)\frac{4m_h^2}{Q^2} \right] + \left[x^2 + (1-x)^2 + x(1-3x)\frac{4m_h^2}{Q^2} - x^2 \frac{8m_h^4}{Q^4} \right] \ln \frac{1+\beta}{1-\beta} \right] \tag{9}$$

with $\beta^2 = 1 - 4m_h^2 x/(1-x)Q^2$. Far above the heavy quark threshold region, $W \gg 2m_h$, the heavy quark flavors are treated as the light (massless) u,d,s flavors in the evolution equations [1,13]: The continuity of all parton distributions as well as of $\alpha_s(Q^2)$ across the "threshold" $Q^2 = m_h^2$ yields the boundary condition $h^\gamma(x, m_h^2) = \bar{h}^\gamma(x, m_h^2) = 0$ and a relation between $\Lambda^{(f+1)}$ and $\Lambda^{(f)}$; here $f+1$ denotes the number of the relevant active flavors at $Q^2 > m_h^2$ which should then obviously be used in β_0 and β_1 in Eq. (8) as well as in all flavor factors appearing above. This implies for our choice $m_{c,b,t} = 1.5, 4.5, 100$ GeV, where the precise value for m_t is of minor importance except for $t^\gamma(x,Q^2)$,

$$\Lambda_{\mathrm{LO}}^{(3,4,5,6)} = 232, 200, 153, 82 \text{ MeV}, \qquad \Lambda_{\mathrm{HO}}^{(3,4,5,6)} = 248, 200, 131, 53 \text{ MeV}, \tag{10}$$

where we have fixed $\Lambda_{\mathrm{LO}}^{(4)} \simeq \Lambda_{\mathrm{HO}}^{(4)} = 200$ MeV from recent experimental determinations [14] of $\alpha_s(Q^2)$.

The simple form of $F_2^\gamma(x,Q^2)$ presented in Eq. (7) still introduces some insignificant, but nevertheless spurious higher-order terms $O(\alpha_s, \alpha_s^2)$ beyond the order considered. According to the prescriptions in Ref. [10], these can be straightforwardly eliminated from the "pointlike" and "hadronic" contributions to $f^\gamma(x,Q^2) = f_{\mathrm{PL}}^\gamma + f_{\mathrm{had}}^\gamma$ by noting that the pointlike solution is of the general form $f_{\mathrm{PL}}^\gamma = (4\pi/\alpha_s)a(x,Q^2) + b(x,Q^2) + O(\alpha_s)$. In our predictions in Fig. 1 these inconsistent spurious $O(\alpha_s, \alpha_s^2)$ terms are omitted.

Our predicted LO and HO quark and gluon distributions of the photon are presented in Figs. 2–4 for some representative values of Q^2. Because of our valencelike input at a small scale $Q^2 = \mu^2$ in Eq. (2), our predictions for $Q^2 > \mu^2$, in particular in the low-x region, are mainly due to the QCD dynamics and are independent of any *ad hoc* input parametrization for $x \to 0$. This is in contrast

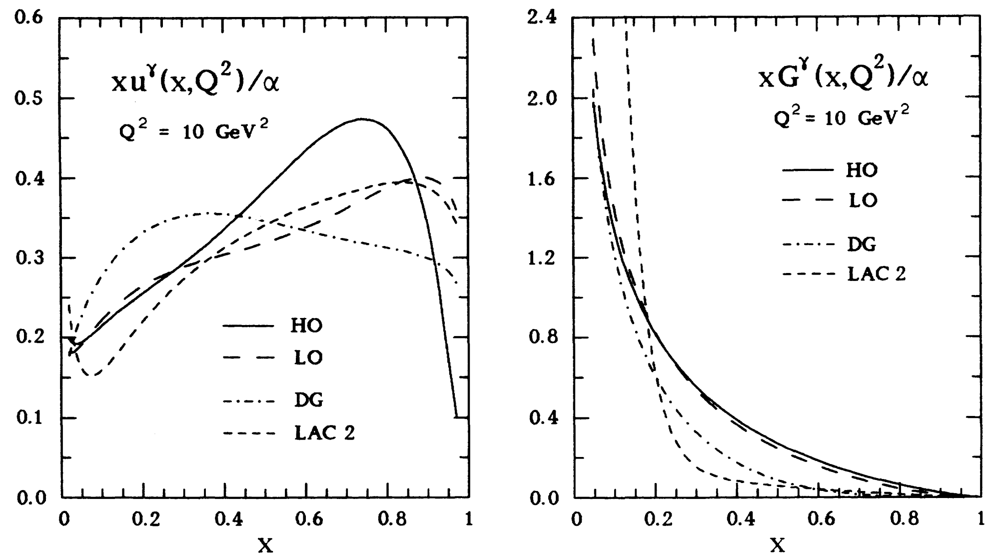

FIG. 2. Comparison of our predicted LO and HO (DIS_γ) distributions $u^\gamma(x,Q^2)$ and $G^\gamma(x,Q^2)$ at $Q^2=10$ GeV^2 with the LO DG (Ref. [16]) and LAC2 (Ref. [17]) parametrizations.

with "conventional" approaches [15–17] of fitting assumed input distributions at $Q^2=Q_0^2=1-5$ GeV^2 which become particularly ambiguous for the experimentally entirely unknown gluon distribution. This is obvious from the vastly different results of the Drees-Grassie (DG) [16] and Levy-Abramowicz-Charchula (LAC) [17] parametrizations in the small-x region in Figs. 2 and 3 where they have no predictive power whatsoever. Note that these LO parametrizations have, for theoretical consistency, always to be compared with our LO predictions, i.e., not with our HO results. The fact that the DG parametrization for u^γ in Fig. 2 differs sizably from other LO results is not surprising since DG [16] had only data from the PLUTO Collaboration at $Q^2=5.9$ GeV^2 available in order to "fix" $u^\gamma(x,Q^2)$ and its less important counterpart d^γ. It is also interesting to note that in the medium- and large-x region our gluon distribution $xG^\gamma(x,Q^2)$ is sizably larger than most of the other LO parametrizations [17] and the (commonly used) DG parametrization [16] as shown in Fig. 2. The detailed small-x behavior of our LO results [denoted by GRV (Glück-Reya-Vogt)] for u^γ and G^γ are shown and compared with other expectations in Fig. 3.

Our LO and HO (DIS_γ) predictions [18] for u^γ and G^γ

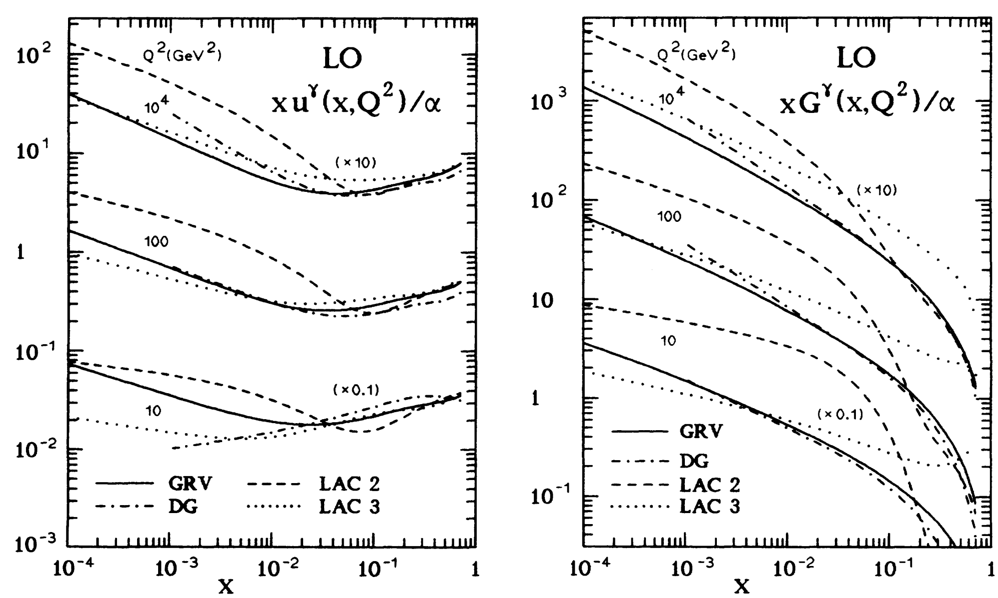

FIG. 3. Comparison of our radiatively generated LO distributions (denoted by GRV) with the LO DG parametrization (Ref. [16]) within its domain of validity ($x \gtrsim 10^{-3}$), and the LO LAC2 and LAC3 parametrizations (Ref. [17]). Note that LAC2 is based on an assumed gluon input $xG^\gamma(x,Q_0^2=4\ \mathrm{GeV}^2)\simeq x^0$ as $x\to 0$. The results for $Q^2=10$ and 10^4 GeV^2 have been multiplied by the corresponding numbers as indicated.

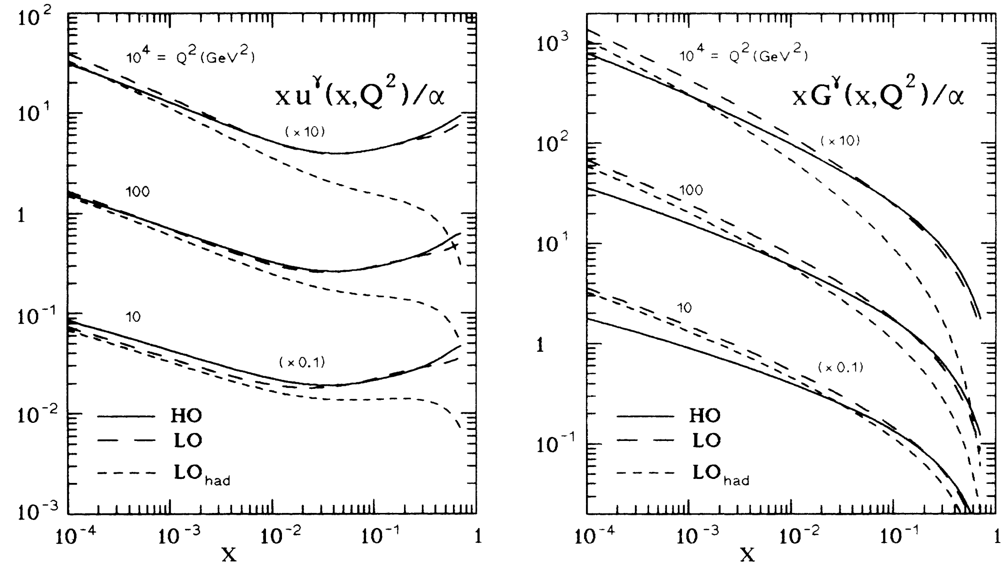

FIG. 4. Detailed small-x behavior of our radiatively generated $u^\gamma=\bar{u}^\gamma$ and G^γ distributions in LO and HO (DIS$_\gamma$) at fixed values of Q^2. The short-dashed curves show the hadronic contribution $f^\gamma_{\rm had}$ to $f^\gamma=f^\gamma_{\rm PL}+f^\gamma_{\rm had}$ in LO. The results for $Q^2=10$ and 10^4 GeV^2 have been multiplied by the corresponding numbers as indicated.

are compared in Fig. 4 which shows that the photonic HO parton distributions in the DIS$_\gamma$ factorization scheme are perturbatively rather stable, despite the (fitted) difference between the LO and HO input as given in Eq. (3). This stability does not hold for the $\overline{\rm MS}$ scheme as discussed and demonstrated in Ref. [10]. In order to illustrate the importance of the hadronic, nonpointlike contribution to $f^\gamma=f^\gamma_{\rm PL}+f^\gamma_{\rm had}$ we also show the hadronic contribution separately in LO: In contrast with the large-x region ($x \gtrsim 0.1$) where the pointlike contribution dominates, the purely hadronic term becomes sizable, and even dominant, in the very small-x region. This situation is very similar also for our HO results.

Simple parametrizations of our predicted LO and HO photonic parton distributions, valid in the range $10^{-5} \lesssim x < 1$ and $0.3 \lesssim Q^2 \lesssim 10^6$ GeV^2, are given in the Appendix. It should be reemphasized that our HO distributions refer to the DIS$_\gamma$ factorization scheme. These can be easily transformed [10] to the $\overline{\rm MS}$ scheme according to Eqs. (4)–(6) which might be relevant for future HO analyses of resolved photon contributions to leptonic and semihadronic processes where most HO subprocesses have so far been calculated in the $\overline{\rm MS}$ scheme.

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APPENDIX

1. Parametrizations of LO photonic parton distributions

Our predicted LO photonic parton distributions, as obtained from the solutions of the inhomogeneous LO evolution equations [Eqs. (2.12) and (2.13) of Ref. [10], for example] using the valencelike input of Eq. (2), can be simply parametrized in the following way. Define

$$s \equiv \ln \frac{\ln[Q^2/(0.232\ \text{GeV})^2]}{\ln[\mu_{\rm LO}^2/(0.232\ \text{GeV})^2]} \tag{A1}$$

to be evaluated for $\mu^2_{\rm LO}=0.25$ GeV^2. All following parametrizations are then valid for $\mu^2_{\rm LO} \le Q^2 \lesssim 10^6$ GeV^2 (i.e., $0 \le s \lesssim 2.4$) and $10^{-5} \lesssim x < 1$. The u^γ, d^γ, and G^γ distributions can be parametrized as

$$\frac{1}{\alpha} x f(x,Q^2) = [x^a(A+B\sqrt{x}+Cx^b) + s^{\alpha'}\exp(-E+\sqrt{E's^\beta \ln 1/x}\,)](1-x)^D . \tag{A2}$$

For $f=u^\gamma=\bar{u}^\gamma$,

$$\begin{aligned}
&\alpha'=1.717\ ,\quad \beta=0.641\ ,\\
&a=0.500-0.176s\ ,\\
&b=15.00-5.687\sqrt{s}-0.552s^2\ ,\\
&A=0.235+0.046\sqrt{s}\ ,\\
&B=0.082-0.051s+0.168s^2\ ,\\
&C=0.459s\ ,\quad D=0.354-0.061s\ ,\\
&E=4.899+1.678s\ ,\quad E'=2.046+1.389s\ ,
\end{aligned} \tag{A3}$$

for $f=d^\gamma=\bar{d}^\gamma$,

$$\begin{aligned}
&\alpha'=1.549\ ,\quad \beta=0.782\ ,\\
&a=0.496+0.026s\ ,\\
&b=0.685-0.580\sqrt{s}+0.608s^2\ ,\\
&A=0.233+0.302s\ ,\quad B=-0.818s+0.198s^2\ ,\\
&C=0.114+0.154s\ ,\quad D=0.405-0.195s+0.046s^2\ ,\\
&E=4.807+1.226s\ ,\quad E'=2.166+0.664s\ ,
\end{aligned} \tag{A4}$$

and for $f=G^\gamma$,

$$\begin{aligned}
&\alpha'=0.676\ ,\quad \beta=1.089\ ,\\
&a=0.462-0.524\sqrt{s}\ ,\quad b=5.451-0.804s^2\ ,\\
&A=0.535-0.504\sqrt{s}+0.288s^2\ ,\\
&B=0.364-0.520s\ ,\\
&C=-0.323+0.115s^2\ ,\\
&D=0.233+0.790s-0.139s^2\ ,\\
&E=0.893+1.968s\ ,\quad E'=3.432+0.392s\ .
\end{aligned}\tag{A5}$$

The s^γ, c^γ, and b^γ distributions are parametrized as

$$\begin{aligned}
\frac{1}{\alpha}xf'(x,Q^2)=[&(s-s_{f'})x^a(A+B\sqrt{x}+Cx^b)\\
&+(s-s_{f'})^{\alpha'}\exp(-E+\sqrt{E's^\beta\ln 1/x}\,)]\\
&\times(1-x)^D\ .
\end{aligned}\tag{A6}$$

For $f'=s^\gamma=\bar{s}^\gamma$,

$$\begin{aligned}
&s_s=0\ ,\quad \alpha'=1.609\ ,\quad \beta=0.962\ ,\\
&a=0.470-0.099s^2\ ,\quad b=3.246\ ,\\
&A=0.121-0.068\sqrt{s}\ ,\quad B=-0.090+0.074s\ ,\\
&C=0.062+0.034s\ ,\quad D=0.226s-0.060s^2\ ,\\
&E=4.288+1.707s\ ,\quad E'=2.122+0.656s\ ,
\end{aligned}\tag{A7}$$

for $f'=c^\gamma=\bar{c}^\gamma$,

$$\begin{aligned}
&s_c=0.888\ ,\quad \alpha'=0.970\ ,\quad \beta=0.545\ ,\\
&a=1.254-0.251s\ ,\quad b=3.932-0.327s^2\ ,\\
&A=0.658+0.202s\ ,\quad B=-0.699\ ,\\
&C=0.965\ ,\quad D=0.141s-0.027s^2\ ,\\
&E=4.911+0.969s\ ,\quad E'=2.796+0.952s\ ,
\end{aligned}\tag{A8}$$

and for $f'=b^\gamma=\bar{b}^\gamma$,

$$\begin{aligned}
&s_b=1.351\ ,\quad \alpha'=1.016\ ,\quad \beta=0.338\ ,\\
&a=1.961-0.370s\ ,\quad b=0.923+0.119s\ ,\\
&A=0.815+0.207s\ ,\quad B=-2.275\ ,\\
&C=1.480\ ,\quad D=-0.223+0.173s\ ,\\
&E=5.426+0.623s\ ,\quad E'=3.819+0.901s\ .
\end{aligned}\tag{A9}$$

2. Parametrizations of HO photonic parton distributions

Our predicted HO photonic parton distributions, as obtained from the solutions of the inhomogeneous HO 2-loop evolution equations in the DIS_γ scheme (Eqs. (2.12) and (2.13) of Ref. [10] with the photonic HO splitting functions $k^{(1)}$ being transformed according to Eq. (3.1) of Ref. [10]) using the valencelike input of Eq. (2), can be simply parametrized in the following way. Define

$$s\equiv\ln\frac{\ln[Q^2/(0.248\ \mathrm{GeV})^2]}{\ln[\mu_{\mathrm{HO}}^2/(0.248\ \mathrm{GeV})^2]}\tag{A10}$$

to be evaluated for $\mu_{\mathrm{HO}}^2=0.3\ \mathrm{GeV}^2$. All following parametrizations are then valid for $\mu_{\mathrm{HO}}^2\leq Q^2\lesssim 10^6\ \mathrm{GeV}^2$ (i.e., $0\leq s\lesssim 2.4$) and $10^{-5}\lesssim x<1$. The functional form of the light photonic parton distributions is as in (A2).

For $f=u^\gamma=\bar{u}^\gamma$,

$$\begin{aligned}
&\alpha'=0.583\ ,\quad \beta=0.688\ ,\\
&a=0.449-0.025s-0.071s^2\ ,\\
&b=5.060-1.116\sqrt{s}\ ,\\
&A=0.103\ ,\quad B=0.319+0.422s\ ,\\
&C=1.508+4.792s-1.963s^2\ ,\\
&D=1.075+0.222\sqrt{s}-0.193s^2\ ,\\
&E=4.147+1.131s\ ,\quad E'=1.661+0.874s\ ,
\end{aligned}\tag{A11}$$

for $f=d^\gamma=\bar{d}^\gamma$,

$$\begin{aligned}
&\alpha'=0.591\ ,\quad \beta=0.698\ ,\\
&a=0.442-0.132s-0.058s^2\ ,\\
&b=5.437-1.916\sqrt{s}\ ,\\
&A=0.099\ ,\quad B=0.311-0.059s\ ,\\
&C=0.800+0.078s-0.100s^2\ ,\\
&D=0.862+0.294\sqrt{s}-0.184s^2\ ,\\
&E=4.202+1.352s\ ,\quad E'=1.841+0.990s\ ,
\end{aligned}\tag{A12}$$

and for $f=G^\gamma$,

$$\begin{aligned}
&\alpha'=1.161\ ,\quad \beta=1.591\ ,\\
&a=0.530-0.742\sqrt{s}+0.025s^2\ ,\quad b=5.662\ ,\\
&A=0.533-0.281\sqrt{s}+0.218s^2\ ,\\
&B=0.025-0.518s+0.156s^2\ ,\\
&C=-0.282+0.209s^2\ ,\\
&D=0.107+1.058s-0.218s^2\ ,\\
&E=2.704s\ ,\quad E'=3.071-0.378s\ .
\end{aligned}\tag{A13}$$

The heavy quark distributions are parametrized as in Eq. (A6) where, for $f'=s^\gamma=\bar{s}^\gamma$,

$$\begin{aligned}
&s_s=0\ ,\quad \alpha'=0.635\ ,\quad \beta=0.456\ ,\\
&a=1.770-0.735\sqrt{s}-0.079s^2\ ,\quad b=3.832\ ,\\
&A=0.084-0.023s\ ,\quad B=0.136\ ,\\
&C=2.119-0.942s+0.063s^2\ ,\\
&D=1.271+0.076s-0.190s^2\ ,\\
&E=4.604+0.737s\ ,\quad E'=1.641+0.976s\ ,
\end{aligned}\tag{A14}$$

for $f'=c^\gamma=\bar{c}^\gamma$,

$$\begin{aligned}
&s_c=0.820\ ,\quad \alpha'=0.926\ ,\quad \beta=0.152\ ,\\
&a=1.142-0.175s\ ,\quad b=3.276\ ,\\
&A=0.504+0.317s\ ,\quad B=-0.433\ ,\\
&C=3.334\ ,\quad D=0.398+0.326s-0.107s^2\ ,\\
&E=5.493+0.408s\ ,\quad E'=2.426+1.277s\ ,
\end{aligned} \tag{A15}$$

and for $f'=b^\gamma=\bar{b}^\gamma$,

$$\begin{aligned}
&s_b=1.297\ ,\quad \alpha'=0.969\ ,\quad \beta=0.266\ ,\\
&a=1.953-0.391s\ ,\quad b=1.657-0.161s\ ,\\
&A=1.076+0.034s\ ,\quad B=-2.015\ ,\\
&C=1.662\ ,\quad D=0.353+0.016s\ ,\\
&E=5.713+0.249s\ ,\quad E'=3.456+0.673s\ .
\end{aligned} \tag{A16}$$

In order to eliminate the spurious higher-order terms $O(\alpha_s,\alpha_s^2)$ in cross sections and structure functions (such as F_2^γ), as explained in Refs. [5,10], one needs, in addition to the full HO parton densities parametrized above, also their leading component $f_0^\gamma(x,Q^2)$. This quantity is defined as $f_0^\gamma=f_{0,\mathrm{PL}}^\gamma+f_{0,\mathrm{had}}^\gamma$ where $f_{0,\mathrm{PL}}^\gamma$ and $f_{0,\mathrm{had}}^\gamma$ refer to the first term in Eqs. (2.18) and (2.13) of Ref. [10], respectively. The light flavor components are parametrized as in Eq. (A2). For $f=u_0^\gamma=\bar{u}_0^\gamma$,

$$\begin{aligned}
&\alpha'=1.447\ ,\quad \beta=0.848\ ,\\
&a=0.527+0.200s-0.107s^2\ ,\\
&b=7.106-0.310\sqrt{s}-0.786s^2\ ,\\
&A=0.197+0.533s\ ,\\
&B=0.062-0.398s+0.109s^2\ ,\\
&C=0.755s-0.112s^2\ ,\quad D=0.318-0.059s\ ,\\
&E=4.225+1.708s\ ,\quad E'=1.752+0.866s\ ,
\end{aligned} \tag{A17}$$

for $f=d_0^\gamma=\bar{d}_0^\gamma$,

$$\begin{aligned}
&\alpha'=1.424\ ,\quad \beta=0.770\ ,\\
&a=0.500+0.067\sqrt{s}-0.055s^2\ ,\\
&b=0.376-0.453\sqrt{s}+0.405s^2\ ,\\
&A=0.156+0.184s\ ,\quad B=-0.528s+0.146s^2\ ,\\
&C=0.121+0.092s\ ,\\
&D=0.379-0.301s+0.081s^2\ ,\\
&E=4.346+1.638s\ ,\quad E'=1.645+1.016s\ ,
\end{aligned} \tag{A18}$$

and for $f=G_0^\gamma$,

$$\begin{aligned}
&\alpha'=0.661\ ,\quad \beta=0.793\ ,\\
&a=0.537-0.600\sqrt{s}\ ,\quad b=6.389-0.953s^2\ ,\\
&A=0.558-0.383\sqrt{s}+0.261s^2\ ,\quad B=-0.305s\ ,\\
&C=-0.222+0.078s^2\ ,\\
&D=0.153+0.978s-0.209s^2\ ,\\
&E=1.429+1.772s\ ,\quad E'=3.331+0.806s\ .
\end{aligned} \tag{A19}$$

The heavy quark components are parametrized as in Eq. (A6) where, for $f'=s_0^\gamma=\bar{s}_0^\gamma$,

$$\begin{aligned}
&s_s=0\ ,\quad \alpha'=1.578\ ,\quad \beta=0.863\ ,\\
&a=0.622+0.332s-0.300s^2\ ,\quad b=2.469\ ,\\
&A=0.211-0.064\sqrt{s}-0.018s^2\ ,\\
&B=-0.215+0.122s\ ,\\
&C=0.153\ ,\quad D=0.253s-0.081s^2\ ,\\
&E=3.990+2.014s\ ,\quad E'=1.720+0.986s\ ,
\end{aligned} \tag{A20}$$

for $f'=c_0^\gamma=\bar{c}_0^\gamma$,

$$\begin{aligned}
&s_c=0.820\ ,\quad \alpha'=0.929\ ,\quad \beta=0.381\ ,\\
&a=1.228-0.231s\ ,\quad b=3.806-0.337s^2\ ,\\
&A=0.932+0.150s\ ,\quad B=-0.906\ ,\\
&C=1.133\ ,\quad D=0.138s-0.028s^2\ ,\\
&E=5.588+0.628s\ ,\quad E'=2.665+1.054s\ ,
\end{aligned} \tag{A21}$$

and for $f'=b_0^\gamma=\bar{b}_0^\gamma$,

$$\begin{aligned}
&s_b=1.297\ ,\quad \alpha'=0.970\ ,\quad \beta=0.207\ ,\\
&a=1.719-0.292s\ ,\quad b=0.928+0.096s\ ,\\
&A=0.845+0.178s\ ,\quad B=-2.310\ ,\\
&C=1.558\ ,\quad D=-0.191+0.151s\ ,\\
&E=6.089+0.282s\ ,\quad E'=3.379+1.062s\ .
\end{aligned} \tag{A22}$$

For example, the consistent form of the HO expression (7) for F_2^γ is

$$\frac{1}{x}F_2^\gamma(x,Q^2)=\sum_q e_q^2\left\{q^\gamma(x,Q^2)+\bar{q}^\gamma(x,Q^2)+\frac{\alpha_s(Q^2)}{4\pi}\int_x^1\frac{dy}{y}\left[B_q(x/y)[q_0^\gamma(y,Q^2)+\bar{q}_0^\gamma(y,Q^2)]+\frac{1}{f}B_G(x/y)G_0^\gamma(y,Q^2)\right]\right\}\ . \tag{A23}$$

It should be noted that for F_2^γ there is no significant quantitative difference (less than 10%) between Eqs. (7) and (A23).

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[9] We included in our fits only those data points which correspond to $W\geq 2$ GeV. It should be noted that the large-x (low-Q^2) bins, where some disagreement in Fig. 1 might be observed, lie in the resonance region $\langle W\rangle \simeq 1.3-1.5$ GeV and that the experiments did not record the significant events with two charged tracks and no neutrals which are expected to be particularly important in the resonance region $W<2$ GeV. These events can give rise to large positive corrections [J. F. Field, in *Proceedings of the VIIIth International Workshop on Photon-Photon Collisions*, Shoresh, Israel, 1988, edited by U. Karshon (World Scientific, Singapore, 1988), p. 349].

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[11] For completeness it should be mentioned that the photonic parton distributions f^{γ} in the DIS_{γ} scheme in Eq. (7) evolve according to inhomogeneous evolution equations whose photonic HO splitting functions $k^{(1)}$ in the solution (2.12) of Ref. [10] are transformed from the $\overline{\mathrm{MS}}$ to the DIS_{γ} scheme according to Eq. (3.1) of Ref. [10].

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[16] M. Drees and K. Grassie, Z. Phys. C **28**, 451 (1985).

[17] H. Abramowicz, K. Charchula, and A. Levy, Phys. Lett. B **269**, 451 (1991). The LAC1 and LAC2 parametrizations for the quarks $q^{\gamma}(x,Q^2)=\bar{q}^{\gamma}(x,Q^2)$ appear to be rather unphysical. For example, at $Q^2=10$ GeV^2 and $x=0.1$, the LAC1 parametrization implies that $c^{\gamma}(=1.6\alpha)$ is larger than $u^{\gamma}(=1.2\alpha)$ and $d^{\gamma}(=0.95\alpha)$, with $s^{\gamma}(=2.8\alpha)$ being more than twice as large as u^{γ} and d^{γ}. A similar, but less pronounced situation holds for LAC2. The LAC3 quark densities are intuitively more acceptable ($u^{\gamma}=2.3\alpha$, $d^{\gamma}=1.6\alpha$, $s^{\gamma}=2.2\alpha$, $c^{\gamma}=0.7\alpha$ at $Q^2=10$ GeV^2 and $x=0.1$) but the gluon distribution is flat and very large in the medium- and large-x region where G^{γ} is about a factor of 10 larger than all other available expectations for $G^{\gamma}(x,Q^2)$. Nevertheless we shall use the LAC2 and LAC3 parametrizations for comparing with our dynamical predictions.

[18] It is interesting to note that the ratio of the fractional momenta of the photon carried by quarks $\Sigma^{\gamma}\equiv\sum_f(q^{\gamma}+\bar{q}^{\gamma})$ and gluons G^{γ}, i.e., $\int_0^1 x\Sigma^{\gamma}(x,Q^2)dx/\int_0^1 xG^{\gamma}(x,Q^2)dx$ is, in LO, predicted to be 1.38, 1.52, 2.11, 2.63, and 2.9 for $Q^2=\mu_{\mathrm{LO}}^2$, 1, 10, 10^2, and 10^3 GeV^2, respectively. The flavor-independent asymptotic ($Q^2\to\infty$) QCD expectation for this ratio is 99/32 [W. R. Frazer and J. F. Gunion, Phys. Rev. D **20**, 147 (1979)]. Similar results are obtained in HO.