PHYSICAL REVIEW D VOLUME 46, NUMBER 4 15 AUGUST 1992

Spontaneous supersymmetry breaking of the Wess-Zumino model at finite temperature

Won-Ho Kye, Sin Kyu Kang, and Jae Kwan Kim

Department of Physics, Korea Advanced Institute of Science and Technology, 373-1, Kusung-dong, Yusung-ku, Taejeon, Korea

(Received 2 March 1992)

We investigate the one-loop effective potential of the supersymmetric model in the high-temperature domain. We observe the fact that supersymmetry can be broken by finite-temperature effects and there is a possibility that broken supersymmetry can be restored in the higher-temperature region.

PACS number(s): 11.30.Pb, 11.15.Bt, 11.30.Qc

I. INTRODUCTION

The study of the vacuum structure by use of the effective-potential method has been investigated by many authors [1] since the possibility of using this method was pointed out by Jona-Lasinio [2]. Jackiw [3] suggested the elegant method, the Feynman path-integral method, which was used to obtain a simple formula for the effective potential. By use of this method, we obtained a deep understanding of vacuum stability and structure in the quantum domain.

Kirzhnits and Linde [4] have suggested that spontaneous symmetry breaking in relativistic field theory will disappear above a critical temperature. By this suggestion the study of the phase transition at finite temperature in quantum field theory has become a matter of interest. It was reported that the broken symmetry of several theories can be restored above the critical temperature. Weinberg [5] suggested that the diagrammatic-functional methods for evaluating effective potentials in quantum field theory might be profitably employed to study temperature effects. Dolan and Jackiw [6] developed the functional method to evaluate directly the temperature-dependent mass and potential shape.

Supersymmetry has fascinated particle physicists since it was first discussed [7]. It allows one to mix bosons and fermions in the same multiplet, which may have relevance for particle-unification schemes. If nature really is described by a supersymmetric theory, the symmetry must be spontaneously broken, because fermions and bosons with degenerate masses do not occur in nature. In terms of the Weyl two-component spinor formalism, the supercharges Q_A and $\bar{Q}_{\dot{A}}$ satisfy the relations $\{Q_A,\bar{Q}_{\dot{A}}\}=2\sigma^{\mu}{}_{A\dot{B}}P_{\mu}$. Multiplying by this relation $(\bar{\sigma}^{\mu})^{\dot{B}A}$, we obtain the energy of the supersymmetric theory:

$$H=\tfrac{1}{4}\{Q_1\bar{Q}_{\dot{1}}+\bar{Q}_{\dot{1}}Q_1+Q_2\bar{Q}_{\dot{2}}+\bar{Q}_{\dot{2}}Q_2\}\ . \tag{1}$$

We define the supersymmetric ground state denoted by $|0\rangle$, such that

$$Q_A|0\rangle=0\ ,\quad A=1,2\ , \tag{2}$$

$$\bar{Q}_{\dot{A}}|0\rangle=0\ ,\quad \dot{A}=\dot{1},\dot{2}\ . \tag{3}$$

Hence the supersymmetric vacuum state corresponds to the zero-energy state $E_{\text{vacuum}}=\langle 0|H|0\rangle=0$. If the vacuum energy is not zero (i.e., $\langle 0|H|0\rangle\neq 0$), then the supersymmetry of the theory breaks down spontaneously. Several authors [8,9] have investigated this issue, which is spontaneous supersymmetry breaking in the finite-temperature region, and they reported the critical temperature, at which supersymmetry breaks down, for some models. On the other hand, it was argued by other authors [10] that supersymmetry is not broken at finite temperature if it is not broken at zero temperature. These considerations are important as a phenomenological and theoretical basis for a unified theory and quantum cosmology [11].

We also investigate this problem and represent some different aspects of the theory. In Sec. II we treat the Wess-Zumino model, explicitly derive the one-loop effective potential, and add a finite-temperature effect, and follow up with discussions.

II. EFFECTIVE POTENTIAL

Our starting action is the minimally extended Wess-Zumino model. In superspace the action, which apparently has supersymmetry, is given by

$$I=\int d^4x\int d^2\theta\, d^2\bar{\theta}[(\Phi^{\dagger}\Phi-g\Phi-\tfrac{1}{2}m\Phi^2-\tfrac{1}{3}\lambda\Phi^3)\delta^2(\bar{\theta})+\text{H.c.}]\ , \tag{4}$$

where Φ is the chiral superfield and H.c. denotes Hermitian conjugation (we follow the conventions in Ref. [12]). After supercoordinate integration we obtain the action

$$I=\int d^4x\left[\frac{i}{2}\partial_{\mu}\psi\sigma^{\mu}\bar{\psi}-\frac{i}{2}\psi\sigma^{\mu}\partial_{\mu}\bar{\psi}+\frac{1}{2}m^2(\psi^2+\bar{\psi}^2)-A^*\Box A+\lambda(\psi^2A+\bar{\psi}^2A^*)-V(A,A^*)\right]\ , \tag{5}$$

where

$$V(A,A^*)=g^2+gm(A^*+A)+g\lambda(A^{*2}+A^2)+m^2|A|^2+m\lambda|A|^2(A^*+A)+\lambda^2|A|^4\ . \tag{6}$$

The ψ is the two-component Weyl spinor, and A is the complex scalar field; the auxiliary field (which has no kinetic term) has been eliminated by an "on-shell" condition. The superpotential $V(A^*, A)$ has a minimum at

$$A_s = -\frac{m}{2\lambda} \pm \left[\frac{m^2}{4\lambda^2} - \frac{g}{\lambda} \right]^{1/2} .$$

Since $V(A_s^*, A_s)=0$, the system is located at the supersymmetric phase under the classical level.

To obtain the one-loop effective potential, we introduce a spacetime-independent field χ, which corresponds to the zero-momentum state, such that $A(x) \to A(x)+\chi$, $A^*(x) \to A^*(x)+\chi^*$. We obtain the quadratic action in terms of quantum fields:

$$\begin{aligned}\mathcal{L}^{\text{quadr}} = {} & \frac{i}{2}\partial_\mu \psi \sigma^\mu \bar{\psi} - \frac{i}{2}\psi\sigma^\mu \partial_\mu \bar{\psi} + \frac{1}{2}m^2(\psi^2+\bar{\psi}^2) - A^* \Box A \\ & + \lambda(\psi^2\chi + \bar{\psi}^2\chi^*) - g\lambda(A^{*2}+A^2) - m^2|A|^2 \\ & - m\lambda(2|A|^2(\chi+\chi^*) + \chi^* A^2 + \chi A^{*2}) \\ & - \lambda^2(4|\chi|^2|A|^2 + A^2\chi^{*2} + A^{*2}\chi^2) . \end{aligned} \tag{7}$$

Our one-loop partition function $Z[\chi^*,\chi]$ in terms of the zero-momentum field χ,χ^* is

$$Z[\chi^*,\chi] = \frac{1}{N}\int \mathcal{D}\bar{\psi}_M \mathcal{D}\psi_M \exp\left[-\frac{i}{2}\int d^4x [\bar{\psi}_M(i\gamma_W^\mu \partial_\mu - \mathbf{M})\psi_M] \right] \int \mathcal{D}A^* \mathcal{D}A \exp\left[-\frac{i}{2} \left[\int d^4x\, \mathbf{B}^T(\mathbf{O}+\mathbf{N})\mathbf{B} \right] \right] , \tag{8}$$

where

$$\bar{\psi}_M = (\psi^A, \bar{\psi}_{\dot{A}}) , \quad \mathbf{B}^T = (A, A^*) ,$$

$$\gamma_W^\mu = \begin{bmatrix} 0 & \sigma^\mu{}_{A\dot{B}} \\ \bar{\sigma}^{\mu\,\dot{A}B} & 0 \end{bmatrix} , \quad \mathbf{M} = \begin{bmatrix} \eta\delta_A^B & 0 \\ 0 & \eta^*\delta_{\dot{B}}^{\dot{A}} \end{bmatrix} , \tag{9}$$

$$\mathbf{N} = \begin{bmatrix} \xi^* & \rho \\ \rho & \xi \end{bmatrix} , \quad \mathbf{O} = \begin{bmatrix} 0 & \Box \\ \Box & 0 \end{bmatrix} ,$$

$$\xi = 2(g\lambda + m\lambda\chi + \lambda^2\chi^2) , \tag{10}$$

$$\rho = m^2 + 2m\lambda(\chi+\chi^*) + 4\lambda^2|\chi|^2 , \tag{11}$$

$$\eta = m + 2\lambda\chi , \tag{12}$$

where $\bar{\psi}_M$ is the four-component Majorana spinor and γ_W^μ is the gamma matrix in Weyl basis. Now we obtain the result in momentum space:

$$\begin{aligned} Z[\chi,\chi^*] &= \frac{\mathrm{Det}(\gamma_\mu P^\mu - \mathbf{M})}{\mathrm{Det}(\mathbf{O}+\mathbf{N})} \\ &= \frac{\det[(p^2-|\eta|^2)^2]}{\det[(p^2-\rho-|\xi|)(p^2-\rho+|\xi|)]} , \end{aligned} \tag{13}$$

where we have performed the determinant in terms of discrete space for a second equality. We use the ζ-function method [13] to obtain the continuum-space determinant (note $\det A = e^{-\zeta_A'(0)}$):

$$\begin{aligned} \zeta_{p^2-|\eta|^2}(s) &= \lim_{y\to x}\frac{1}{\Gamma(s)}\int_0^\infty dt\, t^{s-1}\int d^4x \left[\frac{\mu^4}{16\pi^2 t^2}\exp[\mu^2(x-y)^2/(4t) - |\eta|^2 t/(\mu^2)] \right] \\ &= \frac{\mu^4}{16\pi^2}\left[\frac{|\eta|^2}{\mu^2} \right]^{2-s} \frac{\Gamma(s-2)}{\Gamma(s)} \times (\text{volume}) , \end{aligned} \tag{14}$$

where the dimensional parameter μ^2 is introduced to make a dimensionless exponent. We obtain the regulated ζ function such that

$$\zeta'_{p^2-|\eta|^2}(0) = -\frac{1}{32\pi^2}\left\{ |\eta|^4 \left[-\frac{3}{2} + \ln\left[\frac{|\eta|^2}{\mu^2} \right] \right] \right\} , \tag{15}$$

where a prime denotes differentiation with respect to s. The one-loop effective potential for the fermionic loops is

$$V_1^{\text{Fermi}} = 2\zeta'_{p^2-|\eta|}(0) = -\frac{1}{16\pi^2}\left\{ |\eta|^4\left[-\frac{3}{2} + \ln\left[\frac{|\eta|^2}{\mu^2} \right] \right] \right\} . \tag{16}$$

By the same way, we can find the bosonic contribution:

$$\begin{aligned} V_1^{\text{Bose}} &= -\zeta'_{p^2-\rho-|\xi|}(0) - \zeta'_{p^2-\rho+|\xi|}(0) \\ &= \frac{1}{32\pi^2}\left\{ (\rho+|\xi|)^2\left[-\frac{3}{2} + \ln\left[\frac{\rho+|\xi|}{\mu^2} \right] \right] + (\rho-|\xi|)^2\left[-\frac{3}{2} + \ln\left[\frac{\rho-|\xi|}{\mu^2} \right] \right] \right\} . \end{aligned} \tag{17}$$

It is interesting that bosonic and fermionic contributions have the opposite effect to the potential [see Eqs. (16) and (17)]. This difference of sign originates from the different spin character of the two fields. The above ζ-function approach directly gives us the renormalized expression. So all parameters have been already renormalized. In terms of the one-loop level, we find the effective potential

$$V_{\rm eff}^{0}=V_{\rm classical}(\chi,\chi^*)+\hbar(V_1^{\rm Fermi}+V_1^{\rm Bose}) \ . \tag{18}$$

Consider the thermal effect of our system. The thermal contribution can be easily introduced by an imaginary-time formalism [6] such as

$$\begin{aligned}
V_1^{\beta\rm Fermi}&=-4\frac{1}{2\pi^2\beta^4}\int_0^\infty dx\, x^2\ln(1+\exp\{-(x^2+\beta^2|\eta|^2)^{1/2}\})\\
&=-\frac{7\pi^2}{180\beta^4}+\frac{|\eta|^2}{12\beta^2}+\frac{|\eta|^2}{16\pi^2}\ln(|\eta|^2\beta^2)+O(\beta^2)\ ,\\
V_1^{\beta\rm Bose}&=2\frac{1}{2\pi^2\beta^4}\int_0^\infty dx\, x^2\ln(1-\exp\{-[x^2+\beta^2(\rho+|\xi|)]^{1/2}\})+2\frac{1}{2\pi^2\beta^4}\int_0^\infty dx\, x^2\ln(1-\exp\{-[x^2+\beta^2(\rho-|\xi|)]^{1/2}\})\\
&=-\frac{2\pi^2}{45\beta^4}+\frac{\rho}{3\beta^2}-\frac{1}{6\pi\beta}[(\rho-|\xi|)^{3/2}+(\rho+|\xi|)^{3/2}]+O(\beta)\ ,
\end{aligned}\tag{19}$$

where the above expressions include antiperiodic and periodic boundary conditions for the fermion and boson fields, respectively. The total thermal contributions are

$$V_1^{\beta}=V_1^{\beta\rm Fermi}+V_1^{\beta\rm Bose} \tag{20}$$

$$=\left[-\frac{\pi^2}{12\beta^4}+\frac{5m^2}{12\beta^2}+\frac{5}{6\beta^2}[m\lambda(\chi+\chi^*)+2\lambda^2|\chi|^2]+O(1/\beta)\right]\ , \tag{21}$$

where we use a high-temperature perturbation to obtain the above relations. Now we can write the one-loop effective potential at the high-temperature region:

$$V_{\rm eff}^{\beta}=V_{\rm classical}(\chi^*,\chi)+\hbar(V_1^{\rm Fermi}+V_1^{\rm Bose})+\hbar(V_1^{\beta})\ . \tag{22}$$

III. RESULTS AND DISCUSSIONS

We have represented the vacuum structure of the system, which is the minimally extended supersymmetric Wess-Zumino model, by explicitly deriving the one-loop effective potential at a finite-temperature region. As a result of thermal loop corrections, $V_{\rm classical}$ deformed to $V_{\rm eff}^{\beta}$. It is curious that the fermionic contribution to the zero-temperature effective potential has an opposite sign compared with the bosonic one [see Eq. (16)]. At zero temperature there is a competition between fermionic and bosonic contributions. If the contributions of fermions more rapidly grow up than the bosonic ones, then the system is unstable. Hence the parameters of the system should be chosen properly to include the theory on the consistent region.

We can see the free-energy density of an ideal ultrarelativistic free gas in Eq. (21). It can be redefined such that

$$g^2\to g_\beta^2\ , \tag{23}$$

where

$$g_\beta^2=g^2+\hbar\left[-\frac{\pi^2}{12\beta^4}\right]\ . \tag{24}$$

By fitting the other parameters ($\mu\to\mu_\beta$, $m\to m_\beta$, $\lambda\to\lambda_\beta$), we obtain

$$\begin{aligned}
V_{\rm eff}=V_{\rm classical}(\chi_\beta^*,\chi_\beta;\mu_\beta,g_\beta,m_\beta,\lambda_\beta)&+\hbar[V_1^{\rm Fermi}(\chi_\beta^*,\chi_\beta;\mu_\beta,g_\beta,m_\beta,\lambda_\beta)+V_1^{\rm Bose}(\chi_\beta^*,\chi_\beta;\mu_\beta,g_\beta,m_\beta,\lambda_\beta)]\\
&+\hbar\left[\frac{5m_\beta^2}{12\beta^2}+\frac{5}{6\beta^2}[m_\beta\lambda_\beta(\chi_\beta+\chi_\beta^*)+2\lambda_\beta|\chi_\beta|^2]+O(1/\beta)\right]\ .
\end{aligned}\tag{25}$$

These reparametrizations correspond to the changing of the renormalization point, which does not affect the physical analysis. In the high-temperature region, the last term is the dominant one, which is the thermal correction term. As the temperature is raised, the structure in the potential is swept, whatever the vacuum structure at zero temperature may be, and we can expect that the minimum of the effective potential grows up. The vacuum energy is no longer zero, $\langle 0|H|0\rangle \neq 0$, and so a supercharge cannot annihilate the vacuum state $Q_A|0\rangle \neq 0$ and the mass degeneracy has disappeared. This is a general feature of the supersymmetric model which is located in the thermal environment [9]. As the temperature was raised in this temperature region, the vacuum structure wiped out. Then any internal symmetry which was broken at zero temperature can be restored in the finite-temperature region. The supersymmetry restoration can occur under a particular situation (i.e., when there exists a minimum point at which the vacuum energy is zero). Supersymmetry is not the case in this temperature region.

On the other hand, let us consider the extremely-high-temperature domain. In that case the thermal contribution is the most dominant term in the effective potential. We pick up the most dominant power $1/\beta^2$ term such that

$$\lim_{\beta\to 0} V_{\mathrm{eff}} \approx V_1^\beta = \left[\frac{5m_\beta^2}{12\beta^2} + \frac{5}{6\beta^2}[m_\beta\lambda_\beta(\chi_\beta+\chi_\beta^*)+2\lambda^2|\chi_\beta|^2]\right] . \tag{26}$$

In the extremely-high-temperature domain, our system is dictated by the above vacuum structure. We can easily find the minimum value by requiring the condition

$$\left[\frac{\partial V_1^\beta}{\partial(\mathrm{Re}(\chi))}\right]_{\mathrm{Re}(\chi_\beta^{\min})} = 0 , \qquad \left[\frac{\partial V_1^\beta}{\partial(\mathrm{Im}(\chi))}\right]_{\mathrm{Im}(\chi_\beta^{\min})} = 0 . \tag{27}$$

The minimum point is $\mathrm{Re}(\chi_\beta^{\min}) = -m_\beta/2\lambda_\beta$, $\mathrm{Im}(\chi_\beta^{\min})=0$. At this point the value of the potential is $V_1^\beta(\chi_\beta^{min},\chi_\beta^{*\min})=0$ (i.e., $\lim_{\beta\to 0}\langle 0|H|0\rangle=0$). We can see the degenerated thermal masses from Eq. (26) such that

$$\left[\frac{\partial^2 V_2^\beta}{\partial(\mathrm{Re}^2(\chi))}\right]_{\chi=0} = \frac{10\lambda_\beta^2}{3\beta^2} , \qquad \left[\frac{\partial^2 V_1^\beta}{\partial(\mathrm{Im}^2(\chi))}\right]_{\chi=0} = \frac{10\lambda_\beta^2}{3\beta^2} . \tag{28}$$

These aspects (which are the facts that there are the degenerated massive modes and the value of the vacuum energy is zero at the minimum point) imply that supersymmetry can be restored at the extremely-high-temperature domain; this is very curious.

Intuitively, the contribution from the fermionic loop has an opposite sign compared with the bosonic one, and so there is competition between the bosonic and fermionic corrections. At the extremely-high-temperature region, the competition is stabilized as Eq. (26). It is not clear at this stage that these features are the general property of the supersymmetric system or confined within our model. These observations are limited under the one-loop approximation at extremely high temperature. It is noticeable that the stable structure, which gives rise to the restoration of supersymmetry, can be maintained when higher-loop effects are considered.

ACKNOWLEDGMENTS

We would like to thank S. I. Hong for helpful comments. This work was supported in part by KOSEF (Korea Science and Engineering Foundation).

[1] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973); S. Coleman, R. Jackiw, and H. D. Politzer, *ibid.* **10**, 2491 (1974).
[2] G. Jona-Lasinio, Nuovo Cimento **34**, 1790 (1964).
[3] R. Jackiw, Phys. Rev. D **9**, 1686 (1974).
[4] D. A. Kirzhnits and A. D. Linde, Phys. Lett. **42B**, 471 (1972).
[5] S. Weinberg, Phys. Rev. D **9**, 3357 (1974).
[6] L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974).
[7] J. Wess and B. Zumino, Nucl. Phys. **B70**, 39 (1974).
[8] G. Woo, Phys. Rev. D **12**, 975 (1975).
[9] A. Das and M. Kaku, Phys. Rev. D **18**, 4540 (1978); D. Boyanovsky, *ibid.* **29**, 743 (1984).
[10] L. Van Hove, Nucl. Phys. **B207**, 15 (1982).
[11] R. H. Brandenberger, Rev. Mod. Phys. **57**, 1 (1985).
[12] H. J. W. Müller-Kirsten and A. Widemann, *Supersymmetry* (World Scientific, Singapore, 1987).
[13] D. G. C. McKeon and T. N. Sherry, Phys. Rev. D **35**, 3854 (1987).