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Calculation of the quark damping rate in hot QCD

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We present a complete calculation of the quark damping rate at zero momentum, to leading order in the QCD coupling constant g at a nonzero temperature T. All terms of leading order in g are included by the resummation of an infinite subset of higher-loop diagrams. At zero momentum the damping rates for the quark and the plasmino (a collective mode with opposite chirality and/or helicity) are equal: for three flavors of massless quarks, the result is $\gamma \simeq 0.151 g^2 T$. We also examine the recent controversy over the gauge dependence of the quark damping rate, and show how it is resolved when dimensional regularization is used as an infrared regulator of mass singularities.

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The "plasmon problem" of high-temperature QCD is to calculate the damping rate for a gluon at rest (the plasmon is a collective mode of the gluon with longitudinal polarization). When first posed [1], it was noted that the standard one-loop calculation is incomplete, and that some set of higher loop diagrams contribute even to leading order in the QCD coupling constant g. Over the next decade, many one-loop calculations of the plasmon damping rate were carried out. The incompleteness of the one-loop calculation was demonstrated by the fact that the result for this physical quantity appeared to depend upon the gauge-fixing condition, even with methods that are supposedly gauge invariant. The paradox was resolved by Braaten and Pisarski [2,3] who developed a general and systematic method for resumming the higher-loop diagrams which contribute to damping rates at leading order. The diagrams that require resummation were also studied by Frenkel and Taylor [4]. A formal proof that resummation produces gauge-invariant results for the damping rates of both quarks and gluons was given in Refs. [3] and [5], and used to compute the plasmon damping rate at leading order [6].

Recently, Baier, Kunstatter, and Schiff [7] have shown that, because of mass-shell singularities, in covariant gauge there are subtleties in the formal proof of gauge invariance given in Refs. [3] and [5]. In calculating the damping rate for a massless quark at zero momentum in covariant gauges, the straightforward application of the resummation method of Ref. [3] gives a contribution to the damping rate that depends on the gauge-fixing parameter [7]. The problem was resolved by Rebhan [8], who showed that a careful treatment of the mass shell singularity requires an infrared cutoff; similar results were found by Nakkagawa, Niégawa, and Pire [9]. These authors showed that the gauge-dependent contribution to the damping rate vanishes when calculated in the presence of the cutoff, in agreement with the formal proof of gauge invariance [3].

In this paper we calculate explicitly the quark damping rate at zero momentum to leading order in g in the Coulomb gauge. As we were writing this paper we received a paper on the same subject by Kobes, Kunstatter, and Mark [10]. Our value in (18) agrees with their final result. We then analyze the mass singularities of covariant gauge using dimensional regularization as an infrared cutoff. The gauge-dependent term is found to vanish, in agreement with the analysis of Rebhan and of Nakkagawa, Niégawa, and Pire.

We follow the conventions of Ref. [3]. We use the imaginary-time formalism, with Euclidean momenta $P^\mu = (p^0, \mathbf{p})$ so that $P^2 = (p^0)^2 + p^2$, $p = |\mathbf{p}|$. Dirac matrices satisfy the Euclidean algebra $\{\gamma^\mu, \gamma^\nu\} = 2\delta^{\mu\nu}$. In imaginary time, the Euclidean energy is $p^0 = 2j\pi T$ for a boson and $p^0 = (2j+1)\pi T$ for a fermion, where j is an integer. Amplitudes in real time are obtained from those in imaginary time by analytically continuing the discrete p^0 to a continuous Minkowski energy, $p^0 = -iE$. After analytic continuation, a momentum is called soft if all of its components, E and p, are of order gT, and hard if any component is of order T. When all of the external momenta for an amplitude are soft, there are loop corrections, termed hard thermal loops, which contribute at the same order in g as the tree amplitude. In the effective perturbation expansion developed in Ref. [3], the effects of hard thermal loops are included by resummation into effective propagators and vertices. These effective amplitudes can be summarized compactly by an effective Lagrangian which is gauge invariant, although nonlocal [11]. The quark damping rate at zero momentum is calculated to leading order by evaluating one-loop diagrams constructed out of the effective propagators and vertices.

The complete inverse propagator for a massless quark is $-iP\cdot\gamma - \delta\Sigma(P) - {}^*\Sigma(P)$, where the quark self-energy is separated into the hard thermal loop $\delta\Sigma$ and a remainder ${}^*\Sigma$, termed the effective self-energy. At soft momentum $P \sim gT$, the hard thermal loop in the self-energy is comparable in magnitude to the bare inverse propagator, $\delta\Sigma \sim gT$. The effective self-energy is a perturbative correction down by one power of g, ${}^*\Sigma \sim g^2 T$. The quark damping rate is proportional to the imaginary part of ${}^*\Sigma$.

The hard thermal loop $\delta\Sigma$ in the quark self-energy was computed first by Klimov and Weldon [12]. It is chirally symmetric, $\{\gamma^5, \delta\Sigma(P)\}=0$, and independent of the gauge-fixing condition. As a consequence of the chiral symmetry, the effective quark propagator, defined by ${}^*\Delta_f^{-1} \equiv -iP\cdot\gamma - \delta\Sigma$, can be decomposed into eigenstates of helicity:

$$ {}^*\Delta_f(P) = {}^*\Delta_+(P)\frac{\gamma^0 + i\hat{\mathbf{p}}\cdot\gamma}{2} + {}^*\Delta_-(P)\frac{\gamma^0 - i\hat{\mathbf{p}}\cdot\gamma}{2} , \qquad (1) $$

where $\hat{\mathbf{p}} = \mathbf{p}/p$. The scalar functions ${}^*\Delta_\pm(P)$ are given by

$$ {}^*\Delta_\pm(P)^{-1} = -ip_0 \pm p + \frac{m_f^2}{p}\left[Q_0\left(\frac{ip^0}{p}\right) \mp Q_1\left(\frac{ip^0}{p}\right)\right] . \qquad (2) $$

Here $m_f = \sqrt{C_f/8}\, gT$ is the effective quark mass induced by the thermal medium; in QCD, the Casimir constant for the fundamental representation is $C_f = 4/3$. The Q_n are Legendre functions of the second kind; from the properties of Q_0 and Q_1, the two self-energies are related as ${}^*\Delta_-(p^0,p) = -{}^*\Delta_+(-p^0,p)$. After analytic continuation to Minkowski energies $p^0 = -iE$, the positive-energy poles of ${}^*\Delta_+(-iE,p)$ and ${}^*\Delta_-(-iE,p)$ define the dispersion relations $E=\omega_+(P)$ and $E=\omega_-(p)$ for the quark quasiparticles to leading order in g. The dispersion relation $E=\omega_+(p)$ describes the propagation of a quasiparticle whose chirality is equal to its helicity, and is identified with an ordinary quark. The other dispersion relation, for $E=\omega_-(p)$, represents a quasiparticle with chirality opposite to its helicity. It has been called a plasmino [13] to emphasize that, like the plasmon mode of the gluon, it owes its existence to the collective effects of the plasma. At rest, $p=0$, the helicity is indefinite, and the two modes coincide: $\omega_+(0)=\omega_-(0)=m_f$. Their behavior for small momentum $p \ll m_f$ is $\omega_\pm(p) \sim m_f \pm p/3$. Note that the plasmino dispersion relation is decreasing at small p, so that its minimum occurs at nonzero momentum.

The damping rates $\gamma_+(p)$ and $\gamma_-(p)$ for quarks and plasminos are given by the imaginary parts of the poles in the propagator, $\gamma_\pm(p) = -\,\mathrm{Im}\,\omega_\pm(p)$. At rest the hard thermal loop in the quark self-energy is $\delta\Sigma(-i\omega,0) = -(m_f^2/\omega)\gamma^0$, and the equation for the pole in the quark propagator becomes

$$ \left(\omega - \frac{m_f^2}{\omega}\right)\gamma^0 + {}^*\Sigma(-i\omega + 0^+, 0) = 0 . \qquad (3) $$

To lowest order in g, the damping rate for a quark or plasmino at rest is then

$$ \gamma_\pm(0) = \tfrac{1}{8}\,\mathrm{tr}[\gamma^0\, \mathrm{Im}\, {}^*\Sigma(-im_f + 0^+, 0)] . \qquad (4) $$

The Dirac trace is represented by tr, so $\frac{1}{4}\mathrm{tr}\gamma^0$ extracts the coefficient of γ^0 in the effective self-energy. The extra factor of $\frac{1}{2}$ in (4) arises from the solution of the mass-shell condition in (3). Like the real part of the mass, the damping rates for quarks and plasminos are equal at rest, $\gamma_+(0)=\gamma_-(0)$.

We now turn to the calculation of the damping rate for a massless quark. From the analysis of Ref. [3], the only diagrams that contribute at leading order to the discontinuity of ${}^*\Sigma(-im_f + 0^+, 0)$ are one-loop diagrams with soft-loop momenta. From (4.44)–(4.46) of Ref. [3], the expression for the discontinuity is

$$ \mathrm{Disc}\, {}^*\Sigma(P) = -C_f g^2\, \mathrm{Disc\,Tr}\left[\frac{i}{2}\, {}^*\tilde{\Gamma}^{\mu\nu}(P,-P;K,-K)\, {}^*\Delta^{\mu\nu}(K) + {}^*\tilde{\Gamma}^{\mu}(P,K-P;-K)\, {}^*\Delta_f(P-K)\, {}^*\tilde{\Gamma}^{\nu}(P-K,-P;K)\, {}^*\Delta^{\mu\nu}(K)\right] . \qquad (5) $$

The integral over the loop momentum K^μ is $\mathrm{Tr} = T\Sigma_{k^0}\int d^3k/(2\pi)^3$. The discontinuity arises only from the soft region of the integral over k. The first term in (5) involves the effective vertex between a quark pair and two gluons, ${}^*\tilde{\Gamma}^{\mu\nu}$; this vertex is absent at the tree level, and first arises at one-loop order. The second term in (5) is the usual one-loop expression for the quark self-energy, except that the tree amplitudes have been replaced everywhere by effective propagators and vertices. In Ref. [3] we gave a formal proof that (5) is gauge invariant on the effective mass shell defined by ${}^*\Delta_f^{-1}(P)=0$. We return to the question of gauge invariance after calculating the damping rate in Coulomb gauge.

In the Coulomb gauge the only nonzero components of the effective propagator ${}^*\Delta^{\mu\nu}$ for soft gluons are ${}^*\Delta^{00}(K) = {}^*\Delta_l(K)$ and ${}^*\Delta^{ij}(K) = (\delta^{ij} - \hat{k}^i\hat{k}^j)\, {}^*\Delta_t(K)$. The longitudinal and transverse gluon propagators are [14]

$$ {}^*\Delta_l(K)^{-1} = k^2 - 3m_g^2 Q_1\left(\frac{ik^0}{k}\right) , \qquad (6) $$

$$ {}^*\Delta_t(K)^{-1} = K^2 - \frac{3}{5}m_g^2\left[Q_3\left(\frac{ik^0}{k}\right) - Q_1\left(\frac{ik^0}{k}\right) - \frac{5}{3}\right] . \qquad (7) $$

For QCD with N_f flavors of massless quarks in the fundamental representation, the effective gluon mass induced by the thermal medium is $m_g = \sqrt{1+N_f/6}\, gT/\sqrt{3}$. After analytic continuation to real energies $k^0 = -iE$, the poles in E of ${}^*\Delta_t$ and ${}^*\Delta_l$ determine the dispersion relations $\omega_t(k)$ and $\omega_l(k)$ for transverse gluons and plasmons.

The effective vertex ${}^*\tilde{\Gamma}$ is the sum of the tree amplitude $\tilde{\Gamma}$ and a hard thermal loop $\delta\tilde{\Gamma}$, ${}^*\tilde{\Gamma} = \tilde{\Gamma} + \delta\tilde{\Gamma}$. The hard thermal loop is part of the one-loop amplitude, and so we introduce a four-vector Q_μ which is related to the loop momentum: $Q^\mu=(i,\hat{\mathbf{q}})$, with $\hat{\mathbf{q}}$ is a three-vector of unit length, $\hat{\mathbf{q}}^2=1$, so Q^μ is a null vector, $Q^2=0$. For the amplitude between a quark pair and a gluon, at the tree level $\tilde{\Gamma}^\mu=\gamma^\mu$ [16]. The hard thermal loop in this amplitude, $\delta\tilde{\Gamma}^\mu$, is given by

$$\delta\tilde{\Gamma}^\mu(P,K-P;-K)=m_f^2\int\frac{d\Omega}{4\pi}\frac{Q^\mu\gamma\cdot Q}{P\cdot Q(K-P)\cdot Q}\ . \quad (8)$$

The integration over the directions of $\hat{\mathbf{q}}$ is denoted by $d\Omega$. The angular integrals are especially simple at zero three-momentum, $p=0$, and give

$$^*\tilde{\Gamma}^0(P,K-P;-K)=\left[1+\frac{m_q^2}{ip^0k}Q_0'\right]\gamma^0-\frac{m_q^2}{ip^0k}Q_1'i\hat{\mathbf{k}}\cdot\gamma\ , \quad (9)$$

$$(\delta^{ij}-\hat{\mathbf{k}}^i\hat{\mathbf{k}}^j)^*\tilde{\Gamma}^j(P,K-P;-K)=\left[1-\frac{m_q^2}{3ip^0k}(Q_0'-Q_2')\right](\delta^{ij}-\hat{\mathbf{k}}^i\hat{\mathbf{k}}^j)\gamma^j\ , \quad (10)$$

using the shorthand $Q_n'=Q_n(i(p^0-k^0)/k)$.

At the tree level there is no amplitude between a quark pair and two gluons, $\tilde{\Gamma}^{\mu\nu}=0$. This amplitude is induced at the one-loop level through the hard thermal loop, so $^*\tilde{\Gamma}^{\mu\nu}=\delta\tilde{\Gamma}^{\mu\nu}$. The angular integral for the hard thermal loop is [16]

$$^*\tilde{\Gamma}^{\mu\nu}(P,-P;K,-K)=m_f^2\int\frac{\delta\Omega}{4\pi}\left[\frac{Q^\mu Q^\nu\gamma\cdot Q}{(P\cdot Q)^2(P-K)\cdot Q}+(K\to -K)\right]\ . \quad (11)$$

At $p=0$ the required components are

$$^*\tilde{\Gamma}^{00}(P,-P;K,-K)=i\frac{m_f^2}{(ip^0)^2k}(Q_0'\gamma^0-Q_1'i\hat{\mathbf{k}}\cdot\gamma)+(K\to -K)\ , \quad (12)$$

$$(\delta^{ij}-\hat{\mathbf{k}}^i\hat{\mathbf{k}}^j)^*\tilde{\Gamma}^{ij}(P,-P;K,-K)=-\frac{2im_f^2}{3(ip^0)^2k}\left[(Q_0'-Q_2')\gamma^0-\tfrac{3}{5}(Q_1'-Q_3')i\hat{\mathbf{k}}\cdot\gamma\right]+(K\to -K)\ . \quad (13)$$

In the Coulomb gauge the transverse gluon propagator is proportional to $\delta^{ij}-\hat{\mathbf{k}}^i\hat{\mathbf{k}}^j$, so in (10) and (13) we only need the effective vertices $^*\tilde{\Gamma}^i$ and $^*\tilde{\Gamma}^{ij}$ after contraction with this projection operator.

Inserting these results into (5), it reduces to

$$\mathrm{Disc}^*\Sigma(p^0,0)=\frac{g^2C_f}{2(ip^0)^2}\gamma^0\mathrm{Disc}\,\mathrm{Tr}\left\{-^*\Delta_l(K)\sum_{\pm}(2ip^0-ik^0\mp k)^{2*}\Delta_\pm(P-K)\right.$$
$$+^*\Delta_t(K)\left[\sum_{\pm}[(p^0-k^0)^2\pm 2ip^0k+k^2+m_f^2]^{2*}\Delta_\pm(P-K)\right.$$
$$\left.\left.-\frac{m_f^2[(p^0-k^0)^2+k^2]}{k^3}Q_0\left[\frac{i(p^0-k^0)}{k}\right]\right]\right\}\ . \quad (14)$$

This expression has been simplified by expressing the Legendre functions Q_n' in the effective vertices (9), (10), (12), and (13) as linear expressions of either $^*\Delta_+^{-1}$ or $^*\Delta_-^{-1}$, (2) with coefficients that are rational functions of the momenta. After canceling factors of $^*\Delta_\pm$ and $^*\Delta_\mp^{-1}$ wherever possible and dropping terms with no discontinuity, the remaining terms give (14).

The integrand in (14) has been reduced to a sum of terms of the form $B(k^0)F(p^0-k^0)$. The discontinuity at $p^0=-im_f$ is computed after evaluating the sum over k^0 for discrete fermonic values of p^0, and then analytically continuing in p^0. We use the identity

$$\mathrm{Disc}T\sum_{k^0}B(k^0)F(p^0-k^0)|_{(p^0=-iE+0^+)}=2\pi i(e^{E/T}+1)\int_{-\infty}^{+\infty}d\omega\, n(\omega)\int_{-\infty}^{+\infty}d\omega'\tilde{n}(\omega')\delta(E-\omega-\omega')\rho_B(\omega)\rho_F(\omega')\ , \quad (15)$$

where $n(\omega)=(e^{\omega/T}-1)^{-1}$ and $\tilde{n}(\omega')=(e^{\omega'/T}+1)^{-1}$ are the thermal distribution functions for bosons and fermions, respectively. The functions $\rho_B(\omega)\equiv\mathrm{Disc}B(-i\omega)/2\pi i$ and $\rho_F(\omega')\equiv\mathrm{Disc}F(-i\omega')/2\pi i$ are the spectral densities for B and F. The spectral densities required in the evaluation of (14) are the quark spectral densities $\rho_\pm(\omega,k)$, the gluon spectral densities $\rho_l(\omega,k)$ and $\rho_t(\omega,k)$ and $\mathrm{Disc}Q_0(\omega/k)/2\pi i=-\theta(k^2-\omega^2)/2$, where $\theta(x)$ is the unit step function. The spectral densities $\rho_\pm(\omega,k)$ for the quark propagators $^*\Delta_\pm(K)$ are related by $\rho_-(\omega,k)=\rho_+(-\omega,k)$, and are given in Ref. [17]. They are positive semidefinite; $\rho_+(\omega,k)$ has support only at the points $\omega=\omega_+(k)$ and $\omega=-\omega_-(k)$ and on the interval $\omega^2<k^2$, while $\rho_-(\omega,k)$ has support at $\omega=\omega_-(k)$, at $\omega=-\omega_+(k)$ and on $\omega^2<k^2$. The spectral densities $\rho_l(\omega,k)$ and $\rho_t(\omega,k)$ for the gluon propagators $^*\Delta_l(K)$ and $^*\Delta_t(K)$ are both odd functions of ω and are given in Ref. [18]. For the transverse spectral function, $\rho_t(\omega,k)/\omega$ is positive semidefinite and has support only at the points $\omega=\pm\omega_t(k)$ and on the interval $\omega^2<k^2$. For the longitudinal spectral function, $-\rho_l(\omega,k)/\omega$ is positive semidefinite and has support at $\omega=\pm\omega_t(k)$ and on $\omega^2<k^2$.

When (15) is applied to (14), the delta function

$\delta(m_f-\omega-\omega')$, together with the behavior of the spectral functions, restricts the support of the remaining integrals to the regions in which ω and ω' are soft. At soft energies the thermal distribution functions can be approximated by $n(\omega)=T/\omega$ and $\tilde{n}(\omega')=\frac{1}{2}$. The final expression for the quark damping rate at zero momentum to leading order in g is

$$\gamma_\pm(0)=a\frac{C_f g^2 T}{16\pi} . \tag{16}$$

The coefficient a is a pure number that depends only on the ratio m_g/m_f; in QCD, this ratio has the value of $\sqrt{8/3}$ for two flavors of massless quarks, and $\sqrt{3}$ with three massless flavors. The normalization of a in (16) is chosen so that a naive one-loop calculation in the Coulomb gauge, using bare propagators and vertices without any resummation, gives $a=1$. The complete value to leading order in g is given after resummation by the integral:

$$a=\int_0^\infty dk\,k^2\int_{-\infty}^{+\infty}d\omega\int_{-\infty}^{+\infty}d\omega'\delta(m_f-\omega-\omega')\times\left[\left[\sum_\pm\frac{2(\omega'+m_f\mp k)^2}{m_f^2}\rho_\pm(\omega',k)\right]\frac{-\rho_l(\omega,k)}{\omega}+\left[\sum_\pm\frac{[\omega'^2-(m_f\pm k)^2]^2}{k^2m_f^2}\rho_\pm(\omega',k)+\frac{k^2-\omega'^2}{k^3}\theta(k^2-\omega'^2)\right]\frac{\rho_t(\omega,k)}{\omega}\right] . \tag{17}$$

Since $\rho_t(\omega,k)/\omega$, $-\rho_l(\omega,k)/\omega$, and $\rho_\pm(\omega',k)$ are all positive semidefinite, (17) is a sum of positive terms. The constant a in (17) is determined numerically to be

$$a\simeq 5.63\ ,\quad N_f=2\ , \tag{18}$$

$$a\simeq 5.71\ ,\quad N_f=3\ . \tag{19}$$

These results are significantly larger than the coefficient $a=1$ found by naive one-loop calculations in the Coulomb gauge.

The damping rate γ for a particle in a thermal medium has a simple physical significance. Because of continual interactions with the medium, such a particle (or more accurately, a quasiparticle) does not have a sharp energy shell, but instead behaves like a resonance with width γ. Only if γ is significantly smaller than its energy should a quasiparticle be regarded as a true physical excitation, because only then will it propagate long enough to have significant physical effects. Equating the damping rate $\gamma_\pm(0)$ in (16) with the thermal quark mass $gT/\sqrt{6}$, we find that quark quasiparticles with zero momentum exist in any meaningful sense only if the coupling constant g is significantly less than 2.7. The calculation of the gluon damping rate in Ref. [6] gives a similar result, implying that gluonic quasiparticles with zero momentum exist only if g is significantly less than 2.5. In making these estimates, we have not worried about the appropriate scales for the running coupling constants appearing in the thermal masses or in the damping rates, nor have we worried about how small the coupling constant g must be in order to use leading-order perturbation theory. These questions can be addressed reliably only after calculations beyond leading order.

A formal proof of the gauge invariance of the quark damping rate to leading order in g was given in Ref. [3]. It involved straightforward algebraic manipulations using the Ward identities satisfied by the effective vertices:

$$K^\mu\,{}^*\tilde{\Gamma}^\mu(P,K-P;-K)=i[{}^*\Delta_f^{-1}(P)-{}^*\Delta_f^{-1}(P-K)]\ , \tag{20}$$

$$K^\mu K^\nu\,{}^*\tilde{\Gamma}^{\mu\nu}(P,-P;K,-K)=i[2\,{}^*\Delta_f^{-1}(P)-{}^*\Delta_f^{-1}(P+K)-{}^*\Delta_f^{-1}(P-K)]\ . \tag{21}$$

Baier, Kunstatter, and Schiff [7] have shown that in covariant gauges there are mass-shell singularities which appear to invalidate this formal proof. The gauge-dependent term in the discontinuity of the effective self-energy is obtained by substituting ${}^*\Delta^{\mu\nu}(K)\rightarrow\xi K^\mu K^\nu/(K^2)^2$ in (5). After using the Ward identities and dropping terms with no discontinuity, the term proportional to the gauge-fixing parameter ξ is

$$\mathrm{Disc}\,{}^*\Sigma(P)|_{\mathrm{BKS}}=-\xi C_f g^2\,{}^*\Delta_f^{-1}(P)\,\mathrm{Disc}\left[\mathrm{Tr}\frac{1}{(K^2)^2}\,{}^*\Delta_f(P-K)\right]{}^*\Delta_f^{-1}(P)\ . \tag{22}$$

For momenta $P=(-iE,0)$, the inverse propagator ${}^*\Delta_f^{-1}(P)$ vanishes near the mass shell like $(E-m_f)$, and so naively this term is not expected to contribute to the damping rate. But the integral in (22) contains mass-shell singularities: straightforward evaluation of the integral generates a factor of $1/(E-m_f)^2$, which cancels against the $(E-m_f)^2$ from the two inverse propagators to give a finite, gauge dependent contribution to the damping rate. The problem is not merely that a physical quantity, such as the damping rate, is gauge dependent. More fundamentally, since the formal proof of (3) hinges only upon the Ward identities, as in (20) and (21), its breakdown would imply that gauge invariance is violated by a thermal distribution.

This problem was resolved by Rebhan [8] and Nakkagawa, Niégawa, and Pire [9]. They observed that if massless fields are present, as from the gauge-variant part of the gluon propagator in covariant gauges, it is necessary to insert an infrared cutoff before evaluating the integral in (22); only then can the mass shell be approached. Doing so, instead of an infrared finite contribution to the damping rate, there is an infinite contribution to the wave-function renormalization constant of the quark. Of course it is standard to find infrared divergences and gauge dependence in wave-function renormalization constants.

The authors of Refs. [8] and [9] used a momentum cutoff as an infrared regulator. To illustrate this problem in a different light we show what happens when dimensional regularization is used as an infrared regulator; this has the virtues of respecting gauge invariance and avoiding the complication of a new momentum scale. At zero momentum, the contribution to the quark damping rate from the Baier-Kunstatter-Schiff (BKS) integral is

$$\gamma_{\rm BKS} = -2\xi C_f g^2 \lim_{E\to m_f} (E-m_f)^2 \,{\rm Im\,Tr}\frac{1}{(K^2)^2}[{}^*\Delta_+(P-K)+{}^*\Delta_-(P-K)] \ . \tag{23}$$

Dimensional regularization is implemented by replacing the integral over three spatial dimensions by one in $3+\epsilon$ dimensions, with ϵ a positive infintesimal parameter. Using $1/(K^2)^2 = -(\partial/\partial k^2)(1/K^2)$, the sum over k^0 and the discontinuity at $p^0=-iE$ can be evaluated by using (15). Replacing the thermal distribution functions by their soft limits, (23) reduces to

$$\gamma_{\rm BKS} = -2\pi\xi C_f g^2 T \lim_{E\to m_f}(E-m_f)^2 \int \frac{d^{3+\epsilon}k}{(2\pi)^{3+\epsilon}} \int d\omega'[\rho_+(\omega',k)+\rho_-(\omega',k)]\frac{\partial}{\partial k^2}\frac{T}{2k^2}[\delta(E-\omega'-k)+\delta(E-\omega'+k)] \ . \tag{24}$$

The infrared singularities arise from the contributions to the quark spectral functions about their mass shell, $\omega'=\omega_\pm(k)$. Concentrating on the region near the mass shell, $E\sim m_f$, the relevant terms in the spectral functions reduce to $\rho_\pm(\omega',k)\to\delta(\omega'-(m_f\pm k/3))/2$. Using the delta functions to evaluate the integral over ω' and k in (24) and taking the limits $\epsilon\to 0$ and $E\to m_f$ wherever there is no ambiguity, the final result is

$$\gamma_{\rm BKS} = \xi\frac{C_f g^2 T}{4\pi}\lim_{E\to m_f}|E-m_f|^\epsilon \ . \tag{25}$$

The sensitivity to an infrared cutoff is now apparent: the limits of going to the mass shell, $E\to m_f$, and removing the infrared cutoff, $\epsilon\to 0$, do not commute. If we remove the cutoff, $\epsilon\to 0$, before approaching the mass shell, $E=m_f$, we obtain a finite, gauge dependent contribution to the damping rate [7]. Following [8] and [9], however, if we approach the mass shell with an infrared cutoff in place, $\epsilon>0$, the discontinuity of (25) vanishes. This verifies that to leading order the quark damping rate at zero momentum in covariant gauges is independent of the gauge parameter ξ.

These infrared divergences are not seen in the resummed expansion in Coulomb gauge, because then the only gluon modes which contribute to a discontinuity are the physical modes of transverse and plasmon fields. These modes all have thermal masses, and so avoid the mass-shell singularities inherent in the massless modes of covariant gauges. This is why in computing the damping rate in Coulomb gauge we could overlook any infrared regulator.

We conclude by emphasizing that this sensitivity to the presence of an infrared cutoff is generic to *all* theories at nonzero temperature, resummed or not. Without bothering with a cutoff, at zero-temperature field theories coupled to massless fields exhibit logarithmic infrared divergences, from integrals such as $\int dk/k$. At nonzero temperature, the singular behavior of the Bose-Einstein distribution function produces powerlike infrared divergences, from integrals as $\int dk\, n(k)/k \sim T\int dk/k^2$. These infrared divergences require an infrared regulator, with the true test of any regulator being that the proper Ward identities are respected. As we have seen, infrared divergent, gauge-dependent terms are generated for wave-function renormalization; the Ward identities imply similar terms for vertex renormalization. As long as the Ward identities are respected, however, we can be certain that the formal proof of gauge invariance for the damping rate [3,5] goes through.

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[16] To forestall any confusion we should clarify differences in notation between Refs. [3], [15], and here. The hard thermal loop in the amplitude between a quark pair and a gluon with spacetime index μ and color index a is given here by $t^a\delta\tilde{\Gamma}^{\mu}(P,K-P;-K)$, where t^a is the generator for the fundamental representation. This hard thermal loop $\delta\tilde{\Gamma}^{\mu}(P,K-P;-K)$ agrees with (3.27) of Ref. [3]; in Ref. [15] the total amplitude is written in (3.4) as $\delta\tilde{\Gamma}^{\mu}_{a}(P,-K,K-P)$. The angular integral in (8) is given by (3.4) and (4.5) of [15], after using (4.6) to reduce (4.5) to a single term. The complete amplitude between a quark pair and two gluons, with spacetime indices μ and ν, summed over the color indices of the gluons, is given here by $-ig^2C_f\delta\tilde{\Gamma}^{\mu\nu}(P,-P;K,-K)$. The hard thermal loop $\delta\tilde{\Gamma}^{\mu\nu}(P,-P;K,-K)$ differs by an overall sign from that in (3.29) of Ref. [3]. In [15] the complete amplitude is given in (3.5) by $\delta\tilde{\Gamma}^{\mu\nu}_{aa}(P,K,-K,-P)$. The integral in (11) is found from (3.5), (4.5), and (4.6) of (15). Differences in ordering of momenta are due entirely to convention.
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