Semi-inclusive rapidity distributions and a critical analysis of the concept of partition temperature

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An analytical computation has been performed of the partition temperature of an n-particle system of total invariant mass M with a transverse-momentum cutoff. The result has been used to compare with the previously obtained fitted values at $\sqrt{s}=540$ GeV and it is shown that some further dynamical assumption is needed in order to reproduce aspects of the data beyond $dn/d\eta$. The effect of the center-of-mass motion of the system on the pseudorapidity distributions is also discussed.

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I. INTRODUCTION

Several years ago, Chou, Yang, and Yen (CYY) proposed to describe a high-energy hadron-hadron collision at a given energy as an incoherent superposition of collisions with different partition temperatures [1]. The model is as follows. Consider, for instance, a $\bar{p}p$ collision at a sufficiently high incident energy $\sqrt{s}$ and events with n (nonleading) particles. The total center-of-mass energy of this n-particle system $W=h\sqrt{s}$ is assumed to be stochastically distributed among n particles with some conveniently parametrized transverse-momentum cutoff factor $g(p_T)$. In other words, the exclusive probability distribution for nonleading particles is described by a *microcanonical ensemble*, i.e.,

$$\text{Probability} = \prod_j \frac{d^3\mathbf{p}_j}{E_j} g(p_{Tj})\delta\left[\sum_i E_i - h\sqrt{s}\right], \qquad (1.1)$$

where all the quantities are given in the center-of-mass frame and h represents the fraction of the total energy $\sqrt{s}$, which remains in the central region. We remark that, in CYY, every quantity is referred to one hemisphere only, with a tacit assumption that the events are symmetrical with respect to the center of mass.

In this case, the single-particle distribution turns out to be given by the *canonical ensemble*

$$\text{Probability} = \alpha\frac{d^3\mathbf{p}}{E} g(p_T)\exp\left[-\frac{E}{T_p}\right], \qquad (1.2)$$

where α is a normalization constant and the parameter T_p, the so-called *partition temperature*, is evidently a uniquely defined function of W and n, once $g(p_T)$ is chosen [2].

At this point, the authors of Ref. [1] perform a fit of semi-inclusive pseudorapidity distribution data [3] in 540-GeV $\bar{p}p$ collisions, by using (1.2) where an empirically determined factor $g(p_T)$ is replaced. With appropriate choices of T_p, all the experimental points for $n_{\text{obs}}>10$ are surprisingly well reproduced in the entire η range where the distributions have been measured and they conclude that (1.2), and consequently also (1.1), is in excellent agreement with experiment. Another report has been given [5] where, with an additional assumption that the inelasticity h is a function of the impact parameter b, fairly good results have been obtained also at different energies of $\bar{p}p$ collider.

However, notwithstanding a good fit of the data, we think we need something more before concluding that (1.1) is really in good agreement with experiment, since those authors have not obtained T_p from the knowledge of W and n as we are going to do, but determined it by fitting only the semi-inclusive pseudorapidity distributions and then calculated W by using the parametrization itself. The main purpose of the present paper is thus to obtain the relationship between T_p, on the one hand, and W and n, on the other hand, and then, by using W and n determined experimentally, to compute T_p and compare with the semi-inclusive (pseudo) rapidity distribution data.

In what follows, starting from (1.1) we derive in the next section the single-particle momentum distribution (1.2) and thereby the functional form of $T_p(W,n)$. A comparison with data is carried out in Sec. III, where first we consider the CYY analysis [1] and then an independent work by Takagi and Tsukamoto [6], where the implicitly implied forward-backward symmetry by Ref. [1] has been removed but two uncorrelated leading particles are now assumed. Conclusions are drawn in Sec. IV.

II. DERIVATION OF THE SEMI-INCLUSIVE MOMENTUM DISTRIBUTION

Consider a system of n particles, having a total invariant mass $M=\sqrt{W^2-\mathbf{P}^2}$, where $(W,\mathbf{P})$ is the energy-momentum four-vector of the system. For simplicity, we assume all the particles in the system to be pions of mass m and neglect the statistics. As far as the single-particle distribution is concerned, we believe that this is a reasonable approximation.

Divide the phase space into small boxes with volumes

$\Delta V_1, \Delta V_2, \ldots, \Delta V_N$. We shall start from a discrete phase space and obtain the continuum limit by letting $\Delta V_i \to 0$ and $N \to \infty$. The probability of finding n_l particles in ΔV_l $(l=1,2,\ldots,N)$ is written

$$\mathcal{P}(\{n_l\}) = \frac{n!}{n_1! \cdots n_N!} q_1^{n_1} \cdots q_N^{n_N} \delta_{n,\sum n_l} \delta\left[W - \sum_{l=1}^{N} n_l E_l\right] \times \delta\left[P_L - \sum_{l=1}^{N} n_l p_{Ll}\right] \delta^2\left[\mathbf{P}_T - \sum_{l=1}^{N} n_l \mathbf{p}_{Tl}\right], \tag{2.1}$$

where $\{n_l\}$ stands for any set $\{n_1, \ldots, n_N\}$ and the probability that a produced particle be found in ΔV_k can be written in terms of the probability density $f(y, \mathbf{p}_T)$ as

$$q_k \equiv f(y_k, \mathbf{p}_{Tk}) \Delta V_k \xrightarrow[N\to\infty]{\Delta V_k \to 0} f(y, \mathbf{p}_T) dy\, d\mathbf{p}_T , \tag{2.2}$$

with the normalization

$$\int f(y, \mathbf{p}_T) dy\, d\mathbf{p}_T = 1 . \tag{2.3}$$

Here, y is the rapidity of the particle.

The single-particle momentum distribution is, then written in this discrete version as

$$\langle n_k \rangle \equiv \left\langle \frac{d^3 n}{dy\, d\mathbf{p}_T}\bigg|_{y_k, \mathbf{p}_{Tk}} \Delta V_k \right\rangle_{n,W,\mathbf{P}} = \frac{\sum_{\{n_l\}} n_k \mathcal{P}(\{n_l\})}{\sum_{\{n_l\}} \mathcal{P}(\{n_l\})} = \frac{A}{B} . \tag{2.4}$$

In the continuum limit, A and B are given (see Appendix A) by

$$A = \frac{-n}{(2\pi)^4} f(y, \mathbf{p}_T) dy\, d\mathbf{p}_T \times \int_{\epsilon_0 - i\infty}^{\epsilon_0 + i\infty} ds \int_{\epsilon_1 - i\infty}^{\epsilon_1 + i\infty} dt \int d\mathbf{u}_T [F(s,t,\mathbf{u}_T)]^{n-1} \times \exp[(W - \sqrt{\mathbf{p}_T^2 + m^2}\cosh y)s - (P_L - \sqrt{\mathbf{p}_T^2 + m^2}\sinh y)t - i(\mathbf{P}_T - \mathbf{p}_T)\cdot \mathbf{u}_t] \tag{2.5}$$

and

$$B = -\frac{1}{(2\pi)^4} \int_{\epsilon_0 - i\infty}^{\epsilon_0 + i\infty} ds \int_{\epsilon_1 - i\infty}^{\epsilon_1 + i\infty} dt \int d\mathbf{u}_T [F(s,t,\mathbf{u}_T)]^n \exp[Ws - P_L t - i(\mathbf{P}_T - \mathbf{p}_T)\cdot \mathbf{u}_T] , \tag{2.6}$$

where we have introduced a definition

$$F(s,t,\mathbf{u}_T) \equiv \int dy\, d\mathbf{p}_T f(y, \mathbf{p}_T) \exp[-\sqrt{\mathbf{p}_T^2 + m^2}(s \cosh y - t \sinh y) + i\mathbf{p}_T \cdot \mathbf{u}_T] . \tag{2.7}$$

To avoid unessential complexity, let us forget in this paper the $\mathbf{p}_T$ conservation, since it is well known that $\langle p_T \rangle$ is small compared with $\langle p_L \rangle$, which will be assured by $g(p_T)$ of (1.1) and defined in (2.11) below. Then $F(s,t,\mathbf{u}_T) \to F(s,t)$ in (2.7) and $\mathbf{u}_T$ integrations are suppressed in (2.5) and (2.6). So, (2.4) reduces to

$$\langle n_k \rangle \simeq n f(y, \mathbf{p}_T) dy\, d\mathbf{p}_T \frac{C}{D} , \tag{2.8}$$

with

$$C = \int_{\epsilon_0 - i\infty}^{\epsilon_0 + i\infty} ds \int_{\epsilon_1 - i\infty}^{\epsilon_1 + i\infty} dt [F(s,t)]^{n-1} \exp[(W - \sqrt{\mathbf{p}_T^2 + m^2}\cosh y)s - (P_L - \sqrt{\mathbf{p}_T^2 + m^2}\sinh y)t] \tag{2.9}$$

and

$$D = \int_{\epsilon_0 - i\infty}^{\epsilon_0 + i\infty} ds \int_{\epsilon_1 - i\infty}^{\epsilon_1 + i\infty} dt [F(s,t)]^n \exp(Ws - P_L t) . \tag{2.10}$$

In order to proceed with the computation of the semi-inclusive distribution $\langle n_k \rangle$, we parametrize the probability density $f(y, \mathbf{p}_T)$ [cf. (2.1) and (2.2)] as

$$f(y, \mathbf{p}_T) = \alpha e^{-\beta\sqrt{\mathbf{p}_T^2 + m^2}\cosh y} e^{-\delta\sqrt{\mathbf{p}_T^2 + m^2}} , \tag{2.11}$$

with

$$\int \int f(y, \mathbf{p}_T) dy\, d\mathbf{p}_T = 1 , \tag{2.12}$$

where y is measured with respect to the rest frame of $M (y = y_{\text{c.m.}} - Y, Y =$ rapidity of $M)$.

This ansatz is more general than the one utilized in Ref. [1] insofar as it includes the case of a longitudinal-momentum dependence. That is to say, for $\beta > 0$ it yields an approximate Gaussian in rapidity, which is what one gets for the inclusive particle distribution in Landau's hydrodynamical model [7]. The pure (longitudinal-) phase-space ansatz which appears in (1.1) is recovered in the limit of $\beta \to 0$. We emphasize that due to the constraint of energy conservation the resulting semi-inclusive distribution (2.4) will be *independent* of β, as can readily be seen by substituting (2.11) into (2.1) and (2.4).

Another minor difference between our ansatz and that of Ref. [1] is that here $g(p_T)$ is an exponential not in p_T

but in transverse energy $\sqrt{p_T^2+m^2}$, which is better suited for fitting the data [8]. In Sec. III, we shall choose $\delta(\sim m^{-1})$, by using $\sqrt{s}=540$ GeV $\langle p_T\rangle$ data [9].

Let us not compute $F(s,t)$ by replacing in (2.7) $f(y,\mathbf{p}_T)$ parametrized as (2.11). We have

$$F(s,t)=\alpha\int_{-\infty}^{\infty}dy\int d\mathbf{p}_T^2\exp\{-[\delta+(\beta+s)\cosh y - t\sinh y]\sqrt{\mathbf{p}_T^2+m^2}\}\ , \tag{2.13}$$

where the y integration can easily be done and gives

$$F(s,t)=4\pi\alpha\int_m^{\infty}\epsilon e^{-\delta\epsilon}K_0(\epsilon\eta)d\epsilon\ . \tag{2.14}$$

Here, we have made a change of parameters:

$$\left\{\begin{array}{l}\beta+s=\eta\cosh\zeta,\\ t=\eta\sinh\zeta\end{array}\right. \rightarrow \left\{\begin{array}{l}\eta=\sqrt{(\beta+s)^2-t^2}\\ \zeta=\operatorname{arctanh}\dfrac{t}{\beta+s}\end{array}\right. . \tag{2.15}$$

The last integral will be evaluated in Appendix B and reads

$$F(s,t)\simeq 4\pi\alpha\left[\frac{m}{\delta}+\frac{1}{\delta^2}\right]e^{-\delta m}K_0(\langle\epsilon\rangle\eta)\ , \tag{2.16}$$

in the limit of small argument of the Bessel function, with $\langle\epsilon\rangle$ given by (B7).

By introducing this $F(s,t)$ into (2.9) and (2.10) we have, as will be shown in Appendix C,

$$C\simeq -2\pi(n-1)(4\pi\alpha)^{n-1}e^{-(n-1)m\delta}\left[\frac{m}{\delta}+\frac{1}{\delta^2}\right]^{n-1} \times\frac{\exp[-\beta(W-\sqrt{p_T^2+m^2}\cosh y)]}{M'^2}\left[\ln\frac{2}{e^{\gamma}\xi_0'}\right]^{n-2}\left[1-\frac{1}{\ln(2/e^{\gamma}\xi_0')}\right]^{-1/2}\exp\left[\frac{n-1}{\ln(2/e^{\gamma}\xi_0')}\right] \tag{2.17}$$

and

$$D\simeq -2\pi n(4\pi\alpha)^n e^{-nm\delta}\left[\frac{m}{\delta}+\frac{1}{\delta^2}\right]^n\frac{e^{-\beta W}}{M^2}\left[\ln\frac{2}{e^{\gamma}\xi_0}\right]^{n-1}\left[1-\frac{1}{\ln(2/e^{\gamma}\xi_0)}\right]^{-1/2}\exp\left[\frac{n}{\ln(2/e^{\gamma}\xi_0)}\right]\ , \tag{2.18}$$

where

$$M'^2=(W-\sqrt{p_T^2+m^2}\cosh y)^2-(P_L-\sqrt{p_T^2+m^2}\sinh y)^2=M^2\left[1-\frac{2}{M}\sqrt{p_T^2+m^2}\cosh(y-Y)+\frac{p_T^2+m^2}{M^2}\right] \tag{2.19}$$

is the squared mass of the remaining system after the subtraction of the single particle that is being observed. In (2.17) and (2.18), ξ_0 is given by (C9) and ξ_0' is obtained from it by replacing M and n, respectively, by M' and $n-1$.

We are now ready to calculate $\langle n_k\rangle$ by introducing C and D given above into (2.8). First, let us approximate

$$\begin{aligned}\ln\frac{2}{e^{\gamma}\xi_0'}&=\ln\frac{2}{e^{\gamma}\xi_0}\left[1+\left[\ln\frac{2}{e^{\gamma}\xi_0}\right]^{-1}\ln\frac{\xi_0}{\xi_0'}\right]\\&\simeq\ln\frac{2}{e^{\gamma}\xi_0}\left[1+\left[\ln\frac{2}{e^{\gamma}\xi_0}\right]^{-1}\left\{\ln\frac{nM'}{(n-1)M}+\ln\left[1+\left[\ln\frac{2}{e^{\gamma}\xi_0}\right]^{-1}\ln\frac{\xi_0}{\xi_0'}\right]\right\}\right]\\&\simeq\ln\frac{2}{e^{\gamma}\xi_0}\exp\left[\frac{1}{[\ln(2/e^{\gamma}\xi_0)-1](n-1)}\left[1-\frac{n\sqrt{p_T^2+m^2}\cosh(y-Y)}{M}\right]\right]\ ,\end{aligned} \tag{2.20}$$

where we have assumed

$$\sqrt{p_T^2+m^2}\cosh(y-Y)\ll M'<M\ . \tag{2.21}$$

Then,

$$\frac{d^3n}{dy\,d\mathbf{p}_T}\simeq\frac{(n-1)\delta^2e^{m\delta}}{4\pi(m\delta+1)\ln(2/e^{\gamma}\xi_0)}e^{-\delta\sqrt{p_T^2+m^2}} \times\exp\left[-\frac{1}{M}\left[\frac{n}{\ln(2/e^{\gamma}\xi_0)}\left[1-\frac{\ln(2/e^{\gamma}\xi_0)-\frac{1}{2}}{(n-1)[\ln(2/e^{\gamma}\xi_0)-1]^2}\right]-2\right]\sqrt{p_T^2+m^2}\cosh(y-Y)\right]\ . \tag{2.22}$$

This expression is identical to (1.2), provided

$$T_p = M\left[\frac{n}{\ln(2/e^{\gamma}\xi_0)}\left[1-\frac{\ln(2/e^{\gamma}\xi_0)-\frac{1}{2}}{(n-1)[\ln(2/e^{\gamma}\xi_0)-1]^2}\right]-2\right]^{-1}$$
$$\simeq \frac{\langle E\rangle \ln(2/e^{\gamma}\xi_0)}{1-\frac{1}{n}\left[2\ln\frac{2}{e^{\gamma}\xi_0}+\left[\ln\frac{2}{e^{\gamma}\xi_0}-\frac{1}{2}\right]\Big/\left[\ln\frac{2}{e^{\gamma}\xi_0}-1\right]^2\right]} . \quad (2.23)$$

Here, however, E is the particle energy in the rest frame of M. The last term of the denominator of (2.23) is usually a small number, so it may be neglected in the lowest-order approximation. We emphasize that in such a case T_p depends only on $\langle E\rangle$ and not on M and n separately. The latters affect T_p only through the corrections in the denominator.

To complete the derivation, we have checked the large-n approximation which, starting from (2.8), (2.11), (2.17), and (2.18), has led to the exponential form (2.22) and verified that, in the entire range of multiplicity and $0<\eta\simeq y<5$ where data exist, the error is less than 5%. Moreover, the asymptotic form of the one-dimensional phase space as argued by Chao [10] is correctly reproduced by our method.

III. COMPARISON WITH DATA

We shall begin by examining the CYY analysis, the main results of which are summarized in Table I (columns 2–4). Compute T_p by replacing CYY's $\langle E\rangle$ into the asymptotic form of (2.23). For doing so, $\langle \epsilon\rangle$ is to be estimated by using (B7) with $m\delta\simeq 0.790$ which gives the CYY value of $\langle p_T(0)\rangle=0.381$ GeV, in the limit of $\delta T_p \gg 1$. Numerically, it corresponds to $\langle\epsilon\rangle\simeq 2.58m \simeq 0.361$ GeV. One sees that the results are in excellent agreement with the input T_p, except at the lowest-n_{ch} interval, where however we do not expect such an agreement. For example, in the range $n_{ch}=41-50$, the computed T_p is $\simeq 6.53$ GeV, which is only 4% smaller than the input T_p. The deviation becomes slightly larger ($\sim 5\%$) and changes sign ($T_p\simeq 7.11$ GeV) if one includes the correction in the denominator of (2.23).

Now, in CYY δ has been fixed constant, T_p determined by fitting (1.2) to experimental $dn/d\eta$ in each n_{ch} interval [3] and finally $\langle E\rangle$ calculated from the curve itself. Then, if one computes $\langle p_T(0)\rangle$ by using $m\delta=$const given above and their T_p values, one sees that it decreases as n_{ch} increases. However, this behavior is just opposite the well-known tendency shown by data [9] in the central-y region ($|y|<2.5$, see Table I). So, unless we regard δ as a new free parameter (in addition to T_p), the model cannot accommodate simultaneously both semi-inclusive $dn/d\eta$ and $\langle p_T(0)\rangle$. Even if this is actually done, still it is not evident *a priori* that the good agreement shown above remains valid when $\langle p_T(0)\rangle$ is fitted to the existent data.

In order to check this new version of the partition temperature, namely, with δ taken as a free parameter, we first estimate δ by using $\langle p_T(0)\rangle$ data [9] with neglect of T_p^{-1} with regard to δ. Then, $\langle\epsilon\rangle$ for each n_{ch} is determined through the use of (B7). Now, as we have changed δ, we need new values of $\langle E\rangle$, in order to compute our T_p with (2.23) and (C9), and also new values of CYY's T_p to compare with. In the zero-mass limit where all the expressions become simpler and the results are, in general, good enough, one obtains

$$\frac{dn}{d\eta}\propto\frac{1}{(1+\cosh\eta/\delta T_p)^2} \quad (3.1)$$

and

$$\langle E\rangle=\frac{2T_p\int_{-\infty}^{\infty}d\eta(1+\delta T_p/\cosh\eta)^{-3}/\cosh\eta}{\int_{-\infty}^{\infty}d\eta(1+\delta T_p/\cosh\eta)^{-2}/\cosh\eta} . \quad (3.2)$$

Equation (3.1) shows that in partition-temperature formalism *the semi-inclusive η distributions depend only on the product (δT_p), in the limit of $m=0$, and not on δ and T_p separately.* This means that we can retain the form of $dn/d\eta$ curves by keeping δT_p constant. We use this condition to redefine CYY's T_p values. With δT_p held constant, (3.2) indicates that $\langle E\rangle$ is proportional to T_p. So,

TABLE I. Comparison of the partition temperature T_p as given by (2.23) and by using independently estimated W with the one obtained by fitting the experimental $d\sigma/d\eta$ (fourth column). The data marked with an asterisk have been taken from Ref. [1].

$n_{ch,obs}$	$n^*_{ch,cal}$	Av. energy per particle in central region $\langle E\rangle^*$ (GeV)	$T_p{}^*$ (GeV)	Experimental [9] $\langle p_T(0)\rangle$ (GeV)	Our results: Total central energy W (GeV)	Our results: T_p (GeV)
≥ 71	99.4	1.64	4.38	0.471	522	12.8
51–70	73.3	2.06	6.25	0.468	423	14.9
41–50	55.0	2.17	6.80	0.456	369	19.0
31–40	44.2	2.57	8.84	0.442	348	24.6
21–30	33.0	3.35	13.8	0.414	252	25.3
11–20	21.2	4.36	23.8	0.380	183	35.0
≤ 10	10.7	6.63	183	0.340	126	101

a comparison with (2.23) tells us that in the lowest-order approximation the agreements obtained before remain valid after the corrections are introduced to fit $\langle p_T(0)\rangle$. Anyhow, a more exact comparison can always be done with the use of (3.2) and (2.23) and one gets, for instance, in the range $n_{\rm ch}=41-50$, $T_p\simeq 8.91$ GeV, whereas the corrected (CYY) input is $T_p\simeq 8.29$ GeV, with $\sim 7.5\%$ deviation.

Thus, provided δ is regarded as a free parameter and $\langle p_T(0)\rangle$ is properly taken into account, it seems that the CYY proposal of partition temperature is basically correct and that the semi-inclusive distributions are essentially determined by the pure phase space. However, before establishing such a conclusion, let us examine some other features of multiparticle production. What about $\langle p_T(y)\rangle$ as function of y (or alternatively $\langle p_T(\eta)\rangle$ as function of η)? $\langle p_T(\eta)\rangle$ in the zero-mass approximation has a particularly simple expression and reads

$$\langle p_T(\eta)\rangle \simeq \frac{2}{\delta+\cosh\eta/T_p} \quad (m=0)\ . \tag{3.3}$$

It is clear that as η increases $\langle p_T(\eta)\rangle$ decreases tending to 0 for large η. This happens also for $\langle p_T(y)\rangle$ and is one of the main features of the partition-temperature formalism. Now, what are the corresponding experimental results? Although such semi-inclusive data are not yet available as a function of y at $\sqrt{s}=540$ GeV, the inclusive data [11] at $\sqrt{s}=630$ GeV seems to indicate a near constancy of $\langle p_T(y)\rangle$ in a wide rapidity range ($|y|<5.0\simeq y_{\rm beam}-1.5$), decreasing only near the kinematical boundary. It is also known [12] that at energies of the CERN Intersecting Storage Rings (ISR) the y dependence of $\langle p_T\rangle$ in semi-inclusive data is weak in the entire region $|y|\le y_{\rm beam}-1.5\simeq 2.5$, the maximum lying at the shoulder of $d\sigma/d\eta$. Probably this property remains valid also at $\sqrt{s}=540$ GeV, as already suggested by the UA7 inclusive data [11]. Explicit computation of $\langle p_T(y)\rangle$ in the range $n_{\rm ch}=41-50$, with the parameters δ and T_p fixed above shows that it varies from 0.445 down to 0.182 GeV, when y goes from 0 up to 5, i.e., CYY's $\langle p_T(y)\rangle$ becomes ~ 2.5 times smaller than the expected value at $y=5\simeq y_{\rm beam}-1.5$. This implies of course an important underestimate of $\langle E\rangle$ in CYY.

Let us now calculate T_p as given by (2.23) by making an independent estimate of $\langle E\rangle$ (or W). Assuming $\langle p_T(\eta)\rangle\simeq$const, which is approximately valid if $\langle p_T(y)\rangle=$const, and using its values given in Ref. [9] and $d\sigma/d\eta$ of CYY's Fig. 2, one obtains for W the values which appear in the fifth column of Table I. Now, forward-backward symmetry as in CYY means $W=M$ and $\mathbf{P}=0$ in our notation. So, with an additional assumption of $n=3n_{\rm ch}/2$, T_p is readily obtained and the results are shown in the last column of Table I, where a huge discrepancy with CYY's values is evident, even after the correction due to $\langle p_T(0)\rangle$ as discussed above has been introduced. These results have also been checked by a Monte Carlo method with random-event generation according to (1.1). For $y(\sim\eta)\le 5$, the results so obtained compare with the analytical ones with a few percent deviation in the highest $n_{\rm ch}$ values, giving additional support to our result (2.23). At larger values of y, the analytical curves deviate from the Monte Carlo results, but it is not expected that our approximation remains valid in such a kinematical region.

Thus, going back to the question we raised in the Introduction, one is forced to conclude that *although nice fits have been obtained with* (1.2) by *CYY, the parameter T_p thus determined has nothing to do with the* "true" *partition temperature.* If one assigns more realistic values to $\langle E\rangle$, pseudorapidity distributions within the physical hypothesis of forward-backward symmetry become too low and too broad to reproduce the data.

One may think at this point that the phase-space calculation is nonsense and the concept of partition temperature is completely meaningless in hadronic collisions. However, one may also be a little less categorical and try to see whether it can be used meaningfully, once some precaution is taken. It is clear from the forward-backward asymmetry observed in the multiplicity distribution [4] that event-by-event fluctuation both in multiplicity and energy partition between the two hemispheres is not at all negligible or, in other words, the hypothesis of forward-backward symmetry is valid only globally but too strong if applied to each event as implied by the use of (1.1) as in CYY just to one hemisphere. Evidently, if one takes the center-of-mass motion of the n-particle system into account without changing the event multiplicity and W, particles will appear more concentrated in its proper frame, i.e., with smaller T_p. Another nice feature of this hypothesis is that in doing so we move the maximum of $\langle p_T(y)\rangle$ from the center of mass as in (1.2) to $y=Y$ as in (2.22).

In Ref. [6], such a calculation has been done under a simplified assumption of a constant $m_T=\sqrt{\mathbf{p}_T^2+m^2}$ and with two uncorrelated leading particles with a flat x distribution, which is the standard picture of the hadronic multiparticle production. As can be seen in their comparison (Fig. 4 of that paper), the agreement is still not satisfactory. The difficulties arise especially in the low-multiplicity data ($n_{\rm ch}\le 20$), where the maximum in the large-η values cannot be reproduced, but also arise in higher-multiplicity data where the overall width seems to be systematically wider than the experimental trends.

In a previous work [13], we have used a fragmentation model and obtained a quite good description of the semi-inclusive pseudorapidity distributions in the low- and intermediate-multiplicity region ($n_{\rm ch}<35$). In that model, one or both of the incident particles were excited into high-temperature states, with a subsequent expansion and decay according to a one-dimensional hydrodynamical model [7]. If we apply the phase-space calculation or equivalently the partition temperature concept to such objects, the results are probably not far from the earlier ones. The distinct ingredient here as compared to Refs. [1] and [6] is the account of the so-called "diffractive" component which is usually assumed to be excluded from the "nondiffractive" data. However, except when the mass of such an excited object is very small, it is hard to recognize this kind of event as "diffractive," even though the forward-backward asymmetry still remains. Thus, the combination of a large fluctuation in the forward-backward multiplicity distributions and the semi-inclusive pseudorapidity spectra with large-$|\eta|$ peaks seems to indicate that the incident-particle fragmentation as described above plays an important role in multiparticle production.

IV. CONCLUSIONS

Starting from the transverse-momentum-cut phase space (1.1), we have derived in this paper the single-particle momentum spectra which, as expected, turned out to be an exponential in particle energy. The inverse of the coefficient in the exponent is to be identified with the previously introduced partition temperature, which is now a definite function of M and n.

Care must be exercised in applying this formalism to analyze the data, because it may lead to a completely wrong conclusion especially with respect to the total central energy. We conclude that a simple application of the model to semi-inclusive particle distributions assuming a forward-backward symmetry does not give consistent results if p_T distributions are also considered. The event-by-event fluctuation in forward-backward multiplicity distributions, which in general show a large asymmetry, is one of the fundamental features of multiparticle production and cannot be neglected.

An inclusion of the fluctuation mentioned above through uncorrelated leading particles is not enough to correctly reproduce the semi-inclusive data. We find that a possible way to give a better account of the existing data is the consideration of particle fragmentation process which clearly contributes to large fluctuations and gives the momentum distribution a more asymmetrical form. Whether or not this mechanism is important may be decided experimentally through a study of pseudorapidity distributions with fixed multiplicities $n_{\rm ch}$ and fixed forward-backward multiplicity ratios $R = n_{\rm F}/n_{\rm B}$. An experimental study of semi-inclusive $\langle p_T(y)\rangle$ distributions are also required to test this mechanism.

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APPENDIX A

In this appendix, we shall give a detailed derivation of (2.5) and (2.6), which have appeared in Sec. II.

First, following Giffon, Hama, and Predazzi (GHP) [14,15], let us rewrite (2.1) by replacing the energy-momentum-conservation δ functions by Fourier-Laplace representations and also the multiplicity-fixing Kronecker δ by its Fourier representation:

$$\begin{aligned}\mathcal{P}(\{n_l\}) = \frac{1}{(2\pi)^3}\frac{n!}{(2\pi i)^2}\int_{\epsilon_0-i\infty}^{\epsilon_0+i\infty} ds\,\exp\left[\left[W-\sum_{l=1}^{N} n_l E_l\right]s\right] \\ \times\int_{\epsilon_1-i\infty}^{\epsilon_1+i\infty} dt\,\exp\left[-\left[P_L-\sum_{l=1}^{N} n_l p_{Ll}\right]t\right] \\ \times\int d\mathbf{u}_T\,\exp\left[-i\left[\mathbf{P}_T-\sum_{l=1}^{N} n_l \mathbf{p}_{Tl}\right]\cdot\mathbf{u}_T\right] \\ \times\int_0^{2\pi} dv\,\exp\left[-i\left[n-\sum_{l=1}^{N} n_l\right]v\right]\left[\frac{q_1^{n_1}}{n_1!}\right]\cdots\left[\frac{q_N^{n_N}}{n_N!}\right],\end{aligned} \tag{A1}$$

where $\epsilon_0 > \epsilon_1$ in order to ensure the convergence of the integral (2.7).

Then, the numerator A of (2.4) can easily be handled and written

$$\begin{aligned}A = \frac{-n!}{(2\pi)^5}\int_{\epsilon_0-i\infty}^{\epsilon_0+i\infty} ds\int_{\epsilon_1-i\infty}^{\epsilon_1+i\infty} dt\int d\mathbf{u}_T\int_0^{2\pi} dv\, e^{(W-E_k)s}e^{-(P_L-p_{Lk})t}e^{-i(\mathbf{P}_T-\mathbf{p}_{Tk})\cdot\mathbf{u}_T}e^{-i(n-1)v}q_k \\ \times\prod_{l=1}^{N}\sum_{n_l=0}^{\infty}\frac{\{q_l\exp[-(E_l s-p_{Ll}t)+i\mathbf{p}_{Tl}\cdot\mathbf{u}_T+iv]\}^{n_l}}{n_l!}\,.\end{aligned} \tag{A2}$$

In the limit of $\Delta V_l \to 0$ and $N\to\infty$, this may be rewritten, on account of (2.2) and

$$E_l = \sqrt{\mathbf{p}_{Tl}^2+m^2}\cosh y_l\ ,\quad p_{Ll} = \sqrt{\mathbf{p}_{Tl}^2+m^2}\sinh y_l \tag{A3}$$

as

$$A=\frac{-n!}{(2\pi)^5}f(y,\mathbf{p}_T)dy\,d\mathbf{p}_T\int_{\epsilon_0-i\infty}^{\epsilon_0+i\infty}ds\int_{\epsilon_1-i\infty}^{\epsilon_1+i\infty}dt\int d\mathbf{u}_T\int_0^{2\pi}dv$$
$$\times\exp[(W-\sqrt{\mathbf{p}_T^2+m^2}\cosh y)s-(P_L-\sqrt{\mathbf{p}_T^2+m^2}\sinh y)t-i(\mathbf{P}_T-\mathbf{p}_T)\cdot\mathbf{u}_T-i(n-1)v]$$
$$\times\exp\left[\int dy\,d\mathbf{p}_T\,f(y,\mathbf{p}_T)\exp[-\sqrt{\mathbf{p}_T^2+m^2}(s\cosh y-t\sinh y)+i\mathbf{p}_T\cdot\mathbf{u}_T+iv]\right]. \tag{A4}$$

Recalling that

$$\exp[e^{iv}]=\sum_{l=0}^{\infty}\frac{e^{ilv}}{l!}, \tag{A5}$$

the integration in v may easily be effected, giving (2.5) of Sec. II.

The denominator B of (2.4) is computed in a similar way, resulting in (2.6).

APPENDIX B

In this appendix, we shall evaluate the integral in (2.14), namely,

$$F(s,t)=4\pi\alpha\int_m^{\infty}\epsilon e^{-\delta\epsilon}K_0(\eta\epsilon)d\epsilon, \tag{B1}$$

where

$$\eta=\sqrt{(\beta+s)^2-t^2}. \tag{B2}$$

As will become clear later when computing C and D in (2.9) and (2.10), the precise behavior of $F(s,t)$ is required when $|\eta|$ is small. Since $K_0(\eta\epsilon)$ is logarithmic in this limit, a convenient approximation would be to evaluate it at $\epsilon=m$ and put it out in the integration sign. So

$$F(s,t)\simeq 4\pi\alpha\left[\frac{m}{\delta}+\frac{1}{\delta^2}\right]e^{-\delta m}K_0(m\eta). \tag{B3}$$

A somewhat better estimate is obtained by taking $K_0(\eta\epsilon)$ at a point $\epsilon=\langle\epsilon\rangle\simeq m$ such that

$$\int_m^{\infty}\epsilon e^{-\delta\epsilon}[K_0(\eta\epsilon)-K_0(\eta\langle\epsilon\rangle)]d\epsilon=0, \tag{B4}$$

in which case we have exactly

$$F(s,t)=4\pi\alpha\left[\frac{m}{\delta}+\frac{1}{\delta^2}\right]e^{-\delta m}K_0(\langle\epsilon\rangle\eta). \tag{B5}$$

By recalling that $K_0(\eta\epsilon)\simeq-\ln(e^{\gamma}\eta\epsilon/2)$ in the small argument limit, we can easily estimate $\langle\epsilon\rangle$ from Eq. (B4) by partial integration.

$$\begin{aligned}\frac{m\delta+1}{\delta^2}e^{-\delta m}\ln\langle\epsilon\rangle &\simeq\int_m^{\infty}\epsilon e^{-\delta\epsilon}\ln\epsilon\,d\epsilon\\ &=\frac{m\delta+1}{\delta^2}e^{-\delta m}\ln m\\ &\quad+\frac{e^{-\delta m}-\mathrm{Ei}(-m\delta)}{\delta^2}.\end{aligned} \tag{B6}$$

So, we have finally

$$\langle\epsilon\rangle\simeq m\exp\left[\frac{1-e^{m\delta}\mathrm{Ei}(-m\delta)}{1+m\delta}\right]. \tag{B7}$$

APPENDIX C

Let us compute D defined by (2.10), with $F(s,t)$ obtained in the preceding appendix. The evaluation of C is entirely similar, so we shall omit it here.

Define

$$\begin{aligned}W&=M\cosh Y,\quad M>0,\\ P_L&=M\sinh Y.\end{aligned} \tag{C1}$$

Then, D may be rewritten

$$\begin{aligned}D\simeq(4\pi\alpha)^n&\left[\frac{m}{\delta}+\frac{1}{\delta^2}\right]^n e^{-nm\delta}\\ &\times\int_{a-i\infty}^{a+i\infty}[K_0(\langle\epsilon\rangle\eta)]^n\eta\,d\eta\\ &\quad\times\int_{C_\pm}\exp[-M\beta\cosh Y\\ &\qquad+M\eta\cosh(\zeta-Y)]d\zeta,\end{aligned} \tag{C2}$$

where a is a small positive constant and integration paths $C_\pm$ are as shown in Fig. 1. The integral in ζ has been computed previously (see Appendix C of Ref. [12]) and reads

$$\begin{aligned}\int_{C_\pm}&\exp[-M\beta\cosh Y+M\eta\cosh(\zeta-Y)]d\zeta\\ &=2\pi ie^{-M\beta\cosh Y}\left[I_0(M\eta)+\frac{1}{i\pi}K_0(M\eta)\right].\end{aligned} \tag{C3}$$

The last term in (C3) may be dropped because, when integrated over η, it does not give any contribution. Thus,

$$\begin{aligned}D\simeq 2\pi i(4\pi\alpha)^n&\left[\frac{m}{\delta}+\frac{1}{\delta^2}\right]^n e^{-M\beta\cosh Y-nm\delta}\\ &\times\int_{a-i\infty}^{a+i\infty}[K_0(\langle\epsilon\rangle\eta)]^n I_0(M\eta)\eta\,d\eta.\end{aligned} \tag{C4}$$

Since $M\gg n\langle\epsilon\rangle\simeq nm$, the integrand above is rapidly

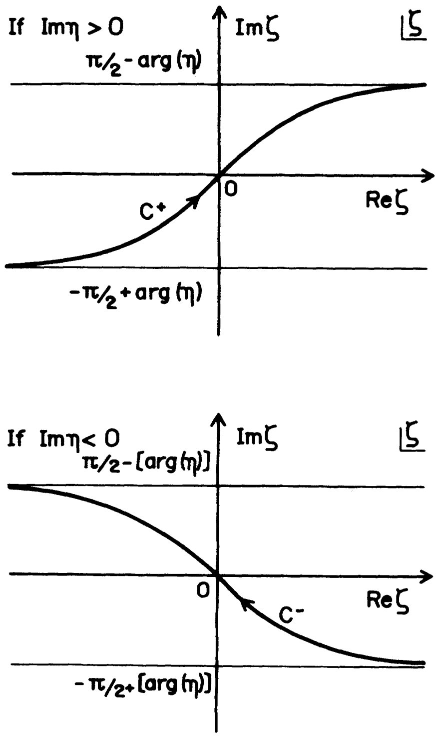

FIG. 1. Integration paths $C_\pm$ which appear in (C2).

dominated by $I_0(M\eta)$ when $|\eta|\to\infty$, so as stated below (B2) only the dominant behavior of K_0 for $m\eta\to 0$ is needed to evaluate this integral. Let us evaluate it (call it I) by the saddle-point method. We have

$$I=\langle\epsilon\rangle^{-2}\int_{a-i\infty}^{a+i\infty}[K_0(\xi)]^n I_0\left[\frac{M}{\langle\epsilon\rangle}\xi\right]\xi\, d\xi$$
$$=\langle\epsilon\rangle^{-2}\int_{a-i\infty}^{a+i\infty}e^{\Phi(\xi)}d\xi\ , \tag{C5}$$

where we have put

$$\Phi(\xi)=\ln\left[[K_0(\xi)]^n I_0\left[\frac{M}{\langle\epsilon\rangle}\xi\right]\xi\right]\ . \tag{C6}$$

The saddle point ξ_0 is on the real axis and, as usual, obtained by equating the first derivative $\Phi'(\xi)$ to 0, namely,

$$\Phi'(\xi_0)=-\frac{nK_1(\xi_0)}{K_0(\xi_0)}+\frac{MI_1(M\xi_0/\langle\epsilon\rangle)}{\langle\epsilon\rangle I_0(M\xi_0/\langle\epsilon\rangle)}+\frac{1}{\xi_0}=0\ . \tag{C7}$$

Now, a direct numerical computation of $e^{\Phi(\xi)}$ shows that, for typical values of $M(\simeq 400\text{ GeV})$ and $n(\simeq 100)$, ξ_0 is in the range $\xi_0=0.01\sim 0.02$. So, recalling (B7), we may neglect the last term in (C7) and replace there

$$K_0(\xi_0)\simeq\ln\frac{2}{e^{\gamma}\xi_0}\ ,\quad K_1(\xi_0)\simeq\frac{1}{\xi_0}$$

and (C8)

$$I_0\left[\frac{M\xi_0}{\langle\epsilon\rangle}\right]\simeq I_1\left[\frac{M\xi_0}{\langle\epsilon\rangle}\right]\simeq\frac{\exp(M\xi_0/\langle\epsilon\rangle)}{(2\pi M\xi_0/\langle\epsilon\rangle)^{1/2}}\ .$$

We have, thus

$$\xi_0\simeq\frac{n\langle\epsilon\rangle}{M\ln(2/e^{\gamma}\xi_0)}\simeq\frac{n\langle\epsilon\rangle}{M\ln((2M/e^{\gamma}n\langle\epsilon\rangle)\ln\{(2M/e^{\gamma}n\langle\epsilon\rangle)\ln[(2M/e^{\gamma}n\langle\epsilon\rangle)\cdots]\})}\ . \tag{C9}$$

In the same approximation, we have

$$\Phi''(\xi_0)\simeq\frac{M^2}{n\langle\epsilon\rangle^2}\left[\ln\frac{2}{e^{\gamma}\xi_0}-1\right]\ , \tag{C10}$$

so, $\Phi''(\xi_0)>0$ for M and n values of our interest as it should be. As for $\Phi(\xi_0)$, it is given as

$$e^{\Phi(\xi_0)}\simeq\frac{\langle\epsilon\rangle}{M}\left[\frac{n}{2\pi\ln(2/e^{\gamma}\xi_0)}\right]^{1/2}\left[\ln\frac{2}{e^{\gamma}\xi_0}\right]^n\exp\left[\frac{n}{\ln(2/e^{\gamma}\xi_0)}\right]\ . \tag{C11}$$

Then,

$$I\simeq\frac{i}{\langle\epsilon\rangle^2}\left[\frac{2\pi}{\Phi''(\xi_0)}\right]^{1/2}e^{\Phi(\xi_0)}$$
$$\simeq\frac{in}{M^2}\left[\ln\frac{2}{e^{\gamma}\xi_0}\right]^{n-1}\left[1-\frac{1}{\ln(2/e^{\gamma}\xi_0)}\right]^{-1/2}\exp\left[\frac{n}{\ln(2/e^{\gamma}\xi_0)}\right]\ . \tag{C12}$$

So, by inserting this I into (C4), we have finally

$$D\simeq -2\pi n(4\pi\alpha)^n e^{-nm\delta}\left[\frac{m}{\delta}+\frac{1}{\delta^2}\right]^n\frac{1}{M^2}\left[\ln\frac{2}{e^{\gamma}\xi_0}\right]^{n-1}\left[1-\frac{1}{\ln(2/e^{\gamma}\xi_0)}\right]^{-1/2}\exp\left[-M\beta\cosh Y+\frac{n}{\ln(2/e^{\gamma}\xi_0)}\right]\ . \tag{C13}$$

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