Black holes and massive remnants

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This paper examines the conundrum faced when one attempts to understand the dynamics of black hole formation and evaporation without abandoning unitary evolution. Previous efforts to resolve this puzzle assume that information escapes in corrections to the Hawking process, that an arbitrarily large amount of information is transmitted by a Planckian energy or contained in a Planck-sized remnant, or that the information is lost to another universe. Each of these possibilities has serious difficulties. This paper considers another alternative: remnants that carry large amounts of information and whose size and mass depend on their information content. The existence of such objects is suggested by attempts to incorporate a Planck-scale cutoff into physics. They would greatly alter the late stages of the evaporation process. The main drawback of this scenario is the apparent noncausal behavior behind the horizon.

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I. THE CONUNDRUM

Consider a pure quantum state corresponding to a distribution of matter of mass M collapsing due to its gravitational self-attraction. Such a state can be described by a density matrix $\rho=|\psi\rangle\langle\psi|$ with vanishing entropy, $S=-\mathrm{Tr}\rho\ln\rho$. For sufficiently large M it is believed that the generic state will eventually pass its event horizon and form a black hole. According to Hawking [1], the resulting black hole will then lose mass by evaporation. It is believed that this process can be treated using Hawking's semiclassical calculation up until quantum-gravitational and black-reaction effects become important. One expects this to happen when the mass of the black hole becomes comparable to the Planck mass M_{Pl}.

Once the black hole has evaporated to $M\sim M_{\mathrm{Pl}}$, most of the initial mass is contained in outgoing Hawking radiation. At any instant in the evaporation process this radiation is approximately described by a thermal density matrix with a nonzero entropy. The total entropy in the outgoing state is estimated to be $S\sim M^2/M_{\mathrm{Pl}}^2$. Since entropy is an absence of information, its nonvanishing indicates that information that was contained in the initial quantum state and has fallen into the black hole does not subsequently escape in the Hawking process. The information problem for black holes is the problem of explaining what happens to this missing information.

Since Hawking's initial results, several possible fates have been proposed for information lost to a black hole.[1] The first possibility is as follows.

(A) *The black hole evaporates completely, and the information contained within it is irretrievably lost.*

This option implies that the evolution of the complete system, including the black hole, is fundamentally nonunitary [3]. Although there is no known basic inconsistency in this scenario, and hence it is possibly right, it is clearly disturbing. Furthermore, there are general arguments [4] that such nonunitary evolution implies violation of either locality or energy-momentum conservation. For these reasons, in the rest of the paper it will be assumed that this possibility is not correct and alternatives will be sought.

A second option is originally due to Dyson [5].

(B) *The black hole disappears completely, but one or more separate universes branch off during the process and carry away the information.*

This proposal appears to neatly accommodate the apparent nonunitarity of the Hawking process while preserving fundamental unitarity. It does so by enlarging the Hilbert space to include states on the separate universes. However, it has been difficult to find a concrete model producing such a result. Furthermore, there is an apparent issue of principle: it seems that arguments similar to those used for baby universes [6,7] can be used to replace the effects of the other universes by arbitrariness in coupling constants in an otherwise unitary theory describing evolution in our Universe. If so, one is still left with the need to explain information lost to the black hole in the resulting unitary theory [8,9].

Two other possibilities raise the question of whether corrections to the Hawking calculation reinstate the missing information. The first of these is the following.

(C) *The black hole disappears completely, and the information is transmitted from the infalling matter to the outgoing Hawking radiation by higher-order effects neglected in the original calculation.*

This scenario, which has been advocated for example by 't Hooft [10],[2] would require that the Hawking radiation extracts *all* information from the ingoing matter, e.g., through scattering as they cross near the horizon. In particular, matter that crosses the horizon and falls to-

[1]For another nice discussion of some aspects of the problem see [2].

[2]See also [34].

wards $r=0$ must therefore have essentially zero information content. For this reason this option seems farfetched to many.

A slightly different alternative follows.

(D) *As the radius of the horizon gradually shrinks, information in the internal state that was previously hidden by the horizon is gradually revealed. When one reaches the Planck radius, essentially all of the information has been released in this fashion.*[3]

Because of the tendency of particles and, hence, information, to focus towards $r=0$, it is questionable whether sufficient information can be shown to escape in this way. Furthermore, one could imagine a process where a continuous flux of incoming coherent radiation precisely balances the outgoing Hawking flux, in which case the horizon does not shrink and an unbounded amount of information is lost.[4]

The final two options involve the details of the end point of evaporation.

(E) *In the late stages of evaporation, during which the energy equivalent of the last few Planck masses is emitted, and where the Hawking calculation breaks down, the new dynamics that replaces it allows all of the missing information to be emitted.*

This last option requires that an unboundedly large amount of information (of order $\exp[M^2/M_{\rm Pl}^2]$, where M is the mass of the initial state, which can be arbitrarily large) be carried by an energy the equivalent of several Planck masses. Locality, causality, and energy conservation put rather stringent constraints on how this may happen.

To see this,[5] note that in order to make up the entropy $S\sim M^2/M_{\rm Pl}^2$, of order $\mathcal{N}=M^2/M_{\rm Pl}^2$, light quanta must be emitted during the last few Planck masses of energy emission. The average wavelength of each such quantum is then of order $\lambda\sim\mathcal{N}l_{Pl}$ where $l_{\rm Pl}$ is the Planck length. This is to be compared with the final size of the black hole, $R_{\rm bh}\sim l_{\rm Pl}$. The very small wave-function overlap then contributes a suppression factor $1/\mathcal{N}^3$ to the emission probability. This means that if the final decay must occur by simultaneous emission of all $\mathcal{N}$ quanta, then the lifetime for the final decay is comparable to the age of the Universe for initial masses a few times the Planck mass. Even if one assumes that the final decay takes place by gradual emission of the $\mathcal{N}$ quanta, the lifetime is of order[6] $\tau\sim\mathcal{N}^4 t_{\rm Pl}$, where $t_{\rm Pl}$ is the Planck time. This lifetime is of order the age of the Universe for even kilogram-sized black holes. Therefore one is forced to conclude that there are long-lived remnants with Planck-sized masses that carry unbounded amounts of information. One can then restate this as a sixth proposal.

(F) *The end product of the Hawking evaporation is a long-lived remnant with mass comparable to M_{Pl} and which carries arbitrarily large amounts of information corresponding to an infinite number of internal states.*

There are also potential problems with this latter possibility. First, this infinite variety of Planck mass particles will appear in loops and in the thermodynamic ensemble. In either case, there appear to be serious difficulties caused by the infinite degeneracy. Furthermore, it is questionable whether it is physically reasonable for an unbounded amount of information to be carried within a Planck volume.[7]

Although one of the alternatives (A)–(F) could conceivably be the key to the black hole information problem, each appears to have sufficient internal difficulties and/or conflicts with basic principles to render it likely unviable. In the following we shall explore yet another alternative that can escape some of these difficulties.

II. CAUTIONARY NOTE

The remainder of this paper rests on speculations about what may be the plausible behavior of gravity at short distances. Clues about this come from string theory, but even within the context of string theory these speculations cannot be confirmed with our present state of knowledge.

Nonetheless, it is felt that it is constructive to apply such speculations to black hole theory and see what the logical outcome may be. The reason for this is twofold.

First, we have seen that it is quite difficult to understand the process of formation and evaporation of a black hole without either abandoning cherished principles such as unitarity and locality or without making various very questionable and potentially unphysical assumptions. The difficulty in arriving at such a plausible picture implies that we must at the present investigate even very speculative scenarios such as (B),(E),(F), or others resting on reasonable assumptions that nonetheless cannot presently be verified. Such investigation should allow us to sharpen our understanding of the constraints on such a scenario. It may also provoke the development of even more precise and reasonable solutions to the black hole information problem. In short, because of the difficulty of imagining reasonable resolutions to the information problem, it is imperative that we leave no stone unturned in the search for even a qualitative picture of black hole evaporation that allows us to retain our principles.

Second, the exploration of possible resolutions to the black hole information problem can act as a guide to the behavior of the microphysics of gravity. If the only imaginable solutions to the problem require particular assumptions about the properties of that microphysics,

[3]This possibility was pointed out to me by K. Kuchař.

[4]I learned this argument from Y. Aharonov.

[5]For related arguments see [3,11].

[6]Preskill has obtained a different bound by thermodynamic arguments [12].

[7]Note, however, that one explanation for how this could appear to happen was proposed in [13–15] within the context of charged dilatonic black holes. For these it was argued that inside what seems, to an outside observer, to be a small volume surrounding the black hole there is, in fact, an infinite-volume tube capable of storing infinite information. This point will be discussed further in Sec. III.

then this is impetus to believe that the assumed properties should arise in a realistic theory of gravity. One should then attempt to see how candidates for microphysics could consistently produce the specified behavior. For example, one might seek to extract the required dynamics from a fundamental theory such as string theory.

III. INFORMATION BOUNDS AND BLACK HOLE CORES

We begin by returning to one of the objections to option (F): it seems physically improbable that an unbounded amount of information could be contained in a volume of Planck size. One should then ask what would be a reasonable bound for information or entropy content.

There is precedent for limits on information content. Within the context of classical mechanics, there is no bound to the number of configurations that may correspond to a given volume of phase space. Quantum mechanics changes this through the uncertainty principle. It does so by introducing a natural unit $\hbar$ for the phase space volume element. The statement that in d dimensions there can be only one quantum state per unit $\hbar^d$ volume restricts the allowed information content.

One can likewise ask if, within the context of gravity, there might be any similar constraints on information content. On dimensional grounds the natural presumption is that there is only one state per Planck volume.

There are physical reasons to suspect the existence of such a bound. In particular, there is the general fact that at distance shorter than the Planck length, spacetime as we know it should cease to exist. One therefore believes that it is meaningless to consider excitations whose wavelength is shorter than the Planck length. Whatever serves as the physical Planck-scale cutoff for gravity should eliminate these excitations.

More specific arguments can be made within the context of a specific theory of quantum gravity such as string theory. In string theory the cutoff at the Planck length is provided by the string scale. String theory presents various pieces of indirect evidence that there are no states at shorter distances.

The first piece of evidence comes from studies of the free string gas. When the entropy density reaches the Planckian density (or equivalently, the energy density reaches the Hagedorn density), attempts to force more energy into the system lead to delocalization of the string ensemble. Within the context of strings in a compactified space, this is evidenced by the production of winding modes [16–18] which wrap around the compact dimensions. In noncompact space, similar attempts again lead to delocalization since the energy is forced into a long string [19,20,16–18]. One can ascribe these phenomena to string theory's attempt to avoid super-Planckian energy and entropy densities.

A second piece of evidence arises in studies of high-energy string scattering [21,22]. It has been shown that one cannot explore distances shorter than the Planck distance using strings as probes. When one increases the incident string energy in an attempt to do so, instead the string probe delocalizes upon interaction with the target. This had led to the suggestion of a new "string uncertainty principle [23,24]," in addition to the traditional uncertainty principle of quantum mechanics. As in quantum mechanics this indicates the absence of degrees of freedom in the theory; here these are the degrees of freedom on scales below the string scale, or Planck length. This implies that information cannot be stored in states below that scale.

Other evidence for the absence of short-distance degrees of freedom arises from duality symmetry. In its simplest form [25,26] this states that a string moving on a circle of radius r is equivalent to a string moving on a circle of radius $1/r$. This has been generalized to much broader contexts, and many believe it to be part of a fundamental symmetry that removes short-distance degrees of freedom in string theory.

Let us therefore take as our basic hypothesis, motivated either by general considerations of quantum gravity or by string theory, that there is an upper bound on the information content within a given volume, and that this bound is determined by the cutoff at the Planck scale. This has an immediate consequence for the black hole problem: the entropy corresponding to that in a black hole formed from an initial mass M requires a volume V_M that grows with M to contain it. One obvious guess is that the allowable entropy density is bounded by the Planck density, which gives the estimate $V_M \sim l_{\rm Pl}^3 (M/M_{\rm Pl})^2$.

This indicates that if one considers the initial black hole of mass M, it should have a finite-sized core in which information is distributed instead of being concentrated at the singularity. One might expect that this has volume of order V_M, and a naive estimate of the radius of the core region is therefore

$$r_M \sim l_{\rm Pl}(M/M_{\rm Pl})^{2/3} . \tag{3.1}$$

In fact, a different argument involving the need to cut off gravity at the Planck scale implies a similar result, albeit with a core radius mass dependence different from the above estimate. In [27], Morgan considers black holes in a model for gravity due to Polchinski [28] in which it is assumed that there is an upper bound on the eigenvalues of the curvature tensor

$$|R| \leq 1/\Lambda^2 , \tag{3.2}$$

where Λ is a cutoff of order $l_{\rm Pl}$. An example of a nontrivial curvature invariant in the Schwarzschild solution is

$$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = 48\frac{M^2}{r^6} . \tag{3.3}$$

Combining this with (3.2) implies that the region inside radius

$$r \sim l_{\rm Pl}(M/M_{\rm Pl})^{1/3} \tag{3.4}$$

should be removed; Morgan argues that it is replaced by a singularity-free core.

Although the qualitative picture of a large core, rather than a singularity, seems robust, one might have hoped for the same mass dependence from both the entropy estimate (3.1) and the curvature estimate (3.4) of the core radius. However, one should note that it is not understood precisely how the cutoff should appear in quantum gravity, and in particular a simple cutoff on the entropy density or on the curvature is probably too naive. Furthermore, there are two different logical possibilities for how a core with size of order V_M can be accommodated within the black hole. The first is that the core extends out to some radius that grows with the mass, for example as given by the entropy estimate above. A second possibility is suggested by the work of Refs. [14,15]: in curved space an arbitrarily large volume can be hidden within a fixed radius. Thus the volume of the core may grow as its radius stays fixed; the core can be thought of as a large internal geometry attached to the outside geometry through a fixed-size neck as in Fig. 1. Gravity with a curvature cutoff presents a picture that includes both of these possibilities. The radius grows with the mass, but the internal solution found in [27] is similar to an expanding de Sitter universe with the cosmological constant given by the cutoff scale.

Now consider the evolution of the core as the black hole evaporates. In accordance with the arguments of the preceding section we assume that the information does not escape in the Hawking radiation or through topology change. If one takes the curvature bound for the core radius, then the core radius shrinks with the mass of the evaporating black hole. However, since the core itself must act as a repository of information one expects it not to shrink. Indeed, in the model of [27] it grows, albeit in a highly cutoff-dependent way. Once the radius of the horizon has shrunk down to the Planck size, the horizon and neck meet. It seems that there are two possible results. One is that the neck can pinch off; the core then becomes a child universe. This possibility returns us to scenario (B) and its attendant difficulties. Another possibility is that the core stays connected to our Universe through the neck. In that case, one views it as a Planck-sized remnant from the outside; the large interior of the core contains the missing information but is not visible without passing through the Planck-sized neck. One is then back at option (F). While one has eluded the prejudice against unbounded information densities, the problems with loops and thermodynamics would still seem to occur,[8] although it is remotely conceivable that they could be cured by suppressed amplitudes for creating such large cores if these amplitudes decrease sufficiently rapidly with the information content. So in either case, we have been returned to the scenarios discussed in the introduction and to their associated difficulties.

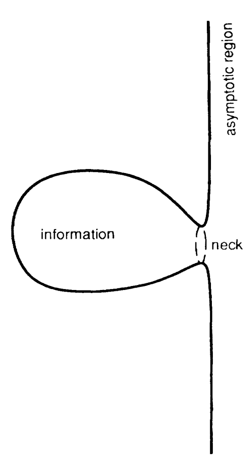

FIG. 1. In curved space, an arbitrarily large amount of information may be contained within a large volume that nonetheless appears to external observers to have Planckian size.

IV. MASSIVE REMNANTS

If one accepts the proposal that information density is bounded but remains concerned about the problems with scenarios (B) and (F), one is left with the possibility that the radius of the core does not shrink to the Planck scale as the black hole evaporates. The persistence of such a finite-sized core indicates both a revision of the Hawking scenario as well as a resolution of the information problem. Assume for example that the core radius r_M is fixed by the initial mass, in accordance with the notion that the core is of finite size to accommodate the bound on information density. In that case the Hawking evaporation should proceed normally only until the point where the horizon size has shrunk to r_M. Then the core begins to protrude and produces a remnant. Subsequently, the standard Hawking calculation is no longer valid.[9]

The radical difference between this scenario and traditional Hawking process therefore offers another resolution of the information problem.

(G) *The Hawking process terminates with the production of a large, massive remnant which carries all of the information missing from the outgoing radiation. Whether the remnant subsequently decays or whether it is long lived, the information stored in it becomes accessible.*

Indeed, since the Hawking calculation is only valid until the core radius is reached, the statement that the lost information is not reemitted is only correct until this time. After this time the core comes into causal contact with the external world. The subsequent evolution of the remnant after the horizon has reached the core radius would seem to depend on the details of the physics introducing the Planckian cutoff. In any case its information content can be investigated. Either the dynamics of the remnant allows the information to subsequently escape

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[8]I thank A. Strominger and J. Preskill for discussions on this issue.

[9]A *superficially* similar modification of the Hawking calculation has recently been noted in the context of a magnetically-charged black hole in the Higgs model [29]. There it was shown that once the hole evaporates to a certain radius, a neighborhood of the horizon makes a transition to the state corresponding to the monopole core of the 't, Hooft–Polyakov monopole. The Hawking calculation for the broken-symmetry vacuum is then no longer valid due to the presence of the core.

(in the extreme case the remnant could immediately explode), or one could directly probe the core to investigate its internal state.

The dependence of the core size (and hence mass) on the information content also offers an escape from the objections to Planck-sized remnants. Such a core is similar to other types of macroscopic bodies whose information content increases with size and mass. For example, since there are only finitely many states below a given energy, the microcanonical ensemble is defined for large remnants, in contrast with those of Planck size. Therefore not only has one escaped the difficulty with storing arbitrarily large amounts of information in a Planck-scale remnant, but the problems of loops and thermodynamics are ameliorated as well.

One possible causal structure for the process of black hole formation in such a picture is shown in Figs. 2 and 3. The surface of the remnant (insofar as it is sharply defined) travels on a spacelike and hence, noncausal, trajectory inside the horizon. However, the trajectory need not be spacelike outside the horizon. Furthermore, this behavior is only encountered when entropy densities become Planckian. One might also have taken solace in the fact that with the curvature estimate for the core radius the surface is encountered only when curvatures become Planckian; it is perhaps not surprising to encounter noncausal phenomena in such regimes. However, if it is assumed that the core does not shrink with the mass, in accordance with the idea that it must stay large to accommodate the information, then one faces the prospect of encountering the core before reaching Planckian curvatures. Therefore, even though it is cloaked by the horizon, the spacelike propagation of the core surface is potentially the most serious drawback of this proposal. One might conjecture that naked causality violation is censored in an attempt to live with this proposal. Alternatively, a possible escape from this objection is the mixing of black hole and white hole states. The resulting causal structure is not easily pictured, and in any case this possibility is equally speculative.[10]

It is an empirical fact that each attempt to resolve the black hole information problem engenders apparently objectionable behavior. Whether this scenario is as offensive as other proposed resolutions of the information problem is unclear. The bottom line on the information problem is that information is either lost, escapes during the Hawking process, remains after the black hole reaches $M \sim M_{\rm Pl}$, or emerges much earlier as a result of new physics. This paper is advocating consideration of the latter possibility.

If such remnants in fact exist, then it should in principle be possible to find them through direct observation. One looks for massive high-density objects[11] that nonetheless do not have horizons and therefore may emit or scatter radiation from their surfaces. Furthermore, such objects could clearly have astrophysical consequences, although determining these consequences would

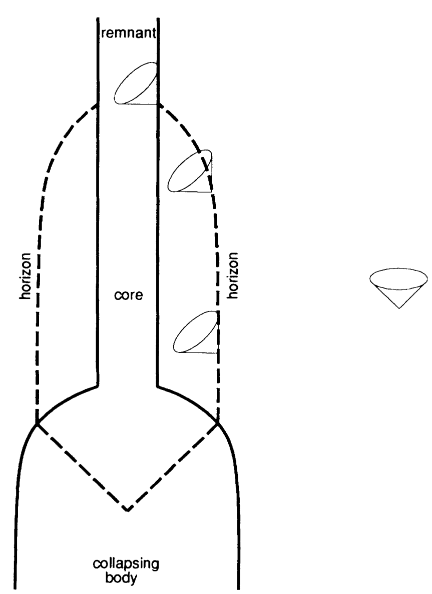

FIG. 2. Shown is a spacetime diagram in which a collapsing body forms a black hole with internal core. The black hole then evaporates until the horizon and core meet. The core then becomes a massive remnant. Some of the light cones are pictured also. Note that this diagram would be equally applicable in the case where the remnant is Planck size; in that case the core and horizon would meet at scales comparable to $l_{\rm Pl}$.

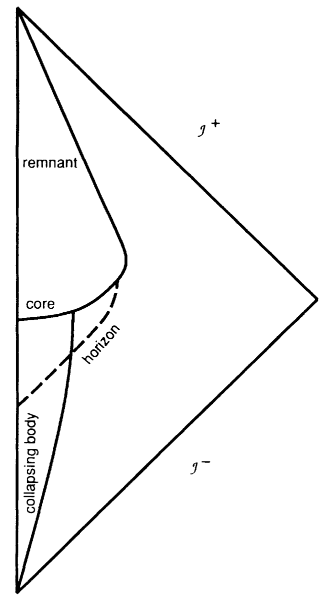

FIG. 3. Pictured is a Penrose diagram corresponding to Fig. 2. Note that this diagram would also be equally applicable to the case of a Planck-sized remnant.

[10]Such mixing has however been considered in a toy model of collapsing shells of matter [30].

[11]For example, $M_{\rm remnant} \sim M_{\rm Pl}(M/M_{\rm Pl})^{2/3}$ and $\rho_{\rm remnant} \sim (M_{\rm Pl}/l_{\rm Pl}^3)(M_{\rm Pl}/M)^{4/3}$ from the naive entropy bound (3.1).

depend both on the initial mass distribution of black holes from which they formed as well as on their dynamics (e.g., stability). The latter in particular is uncertain due to the lack of detailed knowledge about the short-distance physics responsible for their existence. Possible implications for, e.g., dark matter remain to be investigated.

In summary, with the assumption that there is an upper bound on the amount of information that can be contained in a given volume, or with other assumptions about a Planck-scale cutoff in quantum gravity, one is drawn to the conclusion that black holes have finite-sized cores and that these cores could become large, massive remnants after Hawking emission. Such remnants offer an easy escape from the black hole information problem implied by the traditional Hawking evaporation scenario. They also avoid objections raised to other attempts to solve this problem. However, they may encounter problems with causality. They in any case must remain purely conjectural until we have a sufficient understanding of short-distance gravity to definitively predict their existence, or until they are observed.

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