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Finite-temperature corrections to the effective potential of neutrinos in a medium

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We consider a neutrino of any flavor propagating through a thermal background of charged leptons, nucleons, and neutrinos. The neutrino self-energy is calculated in a general gauge up to terms of order g^2/m_W^4, and it is shown that, although the self-energy depends on the gauge parameter, the dispersion relation is independent of it. On the basis of this result it is argued that the neutrino effective potential must be defined from the dispersion relation and not directly from the self-energy. Explicit formulas for the effective potential are given.

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I. INTRODUCTION

It is now well known that the properties of neutrinos that propagate through a medium differ from those in the vacuum; e.g., the vacuum energy-momentum relation for massless neutrinos $\omega=\kappa$, where ω is the energy and κ the magnitude of the momentum vector, is not valid in the medium [1]. From a macroscopic point of view, the modifications of the neutrino dispersion relation can be represented in terms of an index of refraction or an effective potential. At the microscopic level, within the framework of finite-temperature field theory (FTFT), the modifications arise from the temperature- and density-dependent corrections to the neutrino self-energy [2–5].

To leading order in g^2/m_W^2, the correction to the dispersion relation in a vacuum is proportional to the particle-antiparticle asymmetry in the background. If the asymmetry is small or zero, then corrections of order g^2/m_W^4 are important and could be dominant. This may be the case in the early Universe, where the asymmetry probably was a very small quantity.

Therefore it is of interest to calculate the $O(g^2/m_W^4)$ corrections to the neutrino dispersion relation. In the FTFT formalism, the corrections arise from the momentum-dependent terms of the boson propagators in the self-energy diagrams. Since these terms depend on the gauge parameter, there immediately arises the question of the gauge invariance of the results for the physical quantities. In fact, it is not obvious that the dispersion relation, or, equivalently, the effective potential, is independent of the gauge parameter, since the self-energy in general depends on it.

Two calculations of the $O(g^2/m_W^4)$ corrections to the neutrino dispersion relations have appeared in the literature [3,6]. However, they have been carried out in a particular gauge, and therefore it becomes impossible to conclude anything about the gauge dependence of the results.

In this work we present a detailed calculation of the $O(g^2/m_W^4)$ corrections to the neutrino self-energy in a general background of charged leptons, nucleons, and neutrinos. The calculation is carried out in an arbitrary ξ gauge, and we show explicitly that, although the self-energy depends on ξ, the dispersion relation does not. This result leads us to conclude that the effective potential must be defined from the dispersion relation and not from the self-energy itself. Our results are summarized by a set of formulas that give the dispersion relations up to terms of order g^2/m_W^4 for each neutrino type.

In Sec. II we present a brief summary of those features of the FTFT formalism that are required to carry out the calculation. The main contribution of our work is contained in Sec. III. There we present the results of the calculation of the self-energy for any neutrino flavor, verify explicitly that the dispersion relation is independent of the gauge parameter up to terms of order g^2/m_W^4, and give the explicit formulas for the neutrino effective potential. Some details of the calculation are provided in two appendixes.

II. BASIC FORMALISM

The groundwork for carrying out the present calculation has been laid out before. We borrow from Ref. [5] some results, which are briefly reviewed here.

The properties of a neutrino that propagates through a medium are determined from the Dirac equation, which in momentum space is

$$(\not{k}-\Sigma_{\text{eff}})\psi=0 \ . \tag{2.1}$$

Here k_μ is the neutrino momentum and Σ_{eff} is the neutrino self-energy, which embodies the effects of the background. The chiral nature of the neutrino interactions implies that the self-energy of a (left-handed) neutrino is of the form

$$\Sigma_{\rm eff}=R\Sigma L\ , \tag{2.2}$$

where $R,L=\frac{1}{2}(1\pm\gamma_5)$. In the vacuum the most general form of the matrix Σ is

$$\Sigma=a\not k\ ,$$

where a is a function of k^2. It is easy to see that the only nontrivial solutions to Eq. (2.1) have $k^2=0$, which implies that the neutrino is massless.

In the presence of a medium, Σ depends also on the velocity four-vector of the medium u_μ. It then has the general structure

$$\Sigma=a\not k+b\not u\ , \tag{2.3}$$

where a and b are functions of the invariant variables

$$\omega=k\cdot u\ ,\qquad \kappa=(\omega^2-k^2)^{1/2}\ . \tag{2.4}$$

Some general properties of the coefficients a and b follow from the invariance of the Lagrangian and background under discrete space-time symmetries. For example, if the Lagrangian and background are CP symmetric, then a and b satisfy [7]

$$a^*(-\omega^*,\kappa)=a(\omega,\kappa)\ ,\qquad b^*(-\omega^*,\kappa)=-b(\omega,\kappa)\ . \tag{2.5}$$

These relations also hold if the background is CPT symmetric (assuming, as usual, that the Lagrangian is CPT symmetric).

We will work within the standard model of the electroweak interactions and neglect the effects of CP violation, which in practice is a good approximation. Therefore, for a background that consists of an equal number of particles and antiparticles, Eq. (2.5) holds. However, if the medium is neither CP nor CPT symmetric, we will show explicitly that Eq. (2.5) is no longer valid.

Using Eq. (2.3), Eq. (2.1) can be written as

$$\not V\psi=0\ , \tag{2.6}$$

where

$$V_\mu\equiv(1-a)k_\mu-bu_\mu\ . \tag{2.7}$$

Thus the Dirac equation has nontrivial solutions only for those values of ω and κ such that

$$V^2=0\ . \tag{2.8}$$

This condition is equivalent to requiring that $\det(\not k-\Sigma_{\rm eff})=0$, and it also determines the poles of the propagator, which is given by

$$S_F=\frac{L\not VR}{V^2}\ . \tag{2.9}$$

Equation (2.8) can be written as

$$f(\omega)\bar f(\omega)=0\ , \tag{2.10}$$

where

$$f(\omega)=(1-a)(\omega-\kappa)-b\ ,\qquad \bar f(\omega)=(1-a)(\omega+\kappa)-b\ . \tag{2.11}$$

Therefore its two solutions are obtained by solving

$$f(\omega_\kappa)=0\ ,\qquad \bar f(-\bar\omega_\kappa^*)=0\ , \tag{2.12}$$

where ω_κ and $\bar\omega_\kappa$ are identified with the neutrino and antineutrino energies, respectively.

Notice that, in general $\omega_\kappa\neq\bar\omega_\kappa$; i.e., the neutrino and antineutrino have different dispersion relations. However, whenever Eq. (2.5) holds, f and $\bar f$ satisfy

$$\bar f(\omega)=-f^*(-\omega^*)\ , \tag{2.13}$$

which implies that

$$\bar\omega_\kappa=\omega_\kappa\ . \tag{2.14}$$

As already mentioned, this holds, for example, if the background is CP or CPT symmetric.

For real neutrinos (i.e., the ones that appear in the external lines of a Feynman diagram), the effect of the medium is to modify the dispersion relation, as already discussed, and also the wave function. For most purposes knowledge of the projection operator is sufficient. This can be easily obtained in the rest frame of the medium. If we denote by u_L the neutrino chiral spinor, the projection operator $u_L\bar u_L$ is given by

$$u_L\bar u_L=\frac{\omega_\kappa L\not n}{N_\kappa}\ , \tag{2.15}$$

where

$$n^\mu=\left(1,\frac{\boldsymbol\kappa}{\kappa}\right)\ . \tag{2.16}$$

The normalization factor N_κ is obtained from the function $f(\omega)$ defined in Eq. (2.11) as

$$N_\kappa=\left(\frac{\partial f}{\partial\omega}\right)_{\omega=\omega_\kappa}\ . \tag{2.17}$$

For the antineutrino the corresponding results for the projection operator $v_L\bar v_L$ are obtained from the previous formulas with the substitutions $\omega_\kappa\to\bar\omega_\kappa$, $\boldsymbol\kappa\to-\boldsymbol\kappa$, $f(\omega)\to\bar f(-\omega)$, and $N_\kappa\to\bar N_\kappa$.

One final word concerns the calculation of $\Sigma_{\rm eff}$. The general relation between $\Sigma_{\rm eff}$ and the elements of the 2×2 self-energy matrix of the FTFT formalism has been given previously [8]. In the present work, we restrict ourselves to the real part of $\Sigma_{\rm eff}$. In this case it is sufficient to calculate only the 1-1 element of the self-energy matrix and take its real part. At the one-loop level, the calculation of the 1-1 element of the self-energy proceeds as in the vacuum case, except that the propagator in each internal line of a Feynman diagram must be replaced by the 1-1 element of the thermal propagator matrix. For example, the propagator for an internal fermion line is

$$S_F(p)=(\not p+m)\left[\frac{1}{p^2-m^2}+2\pi i\delta(p^2-m^2)\eta(p\cdot u)\right], \tag{2.18}$$

where m is the fermion mass and

$$\eta(p\cdot u)=\frac{\theta(p\cdot u)}{e^x+1}+\frac{\theta(-p\cdot u)}{e^{-x}+1}. \tag{2.19}$$

Here θ is the step function, and

$$x=\beta(p\cdot u-\mu), \tag{2.20}$$

where $1/\beta$ is the temperature and μ is the chemical potential.

III. CALCULATION

A. Self-energy

Our first task is to calculate in a general ξ gauge the diagrams displayed in Fig. 1 for a neutrino ν_l, where l stands for one of the lepton flavors e, μ, or τ. Note that, since we want to calculate Σ up to terms of order g^2/m_W^4, it is necessary to include the diagram involving the unphysical charged Higgs scalar, as well as the gauge-dependent corrections from Fig. 1(a). None of these contributions have been considered in previous works because either they do not contribute to the leading order [2,4], or they vanish in the unitarity gauge [3,6]. In both cases the issue of the gauge invariance of the dispersion relation cannot be investigated.

We restrict ourselves to those physical situations in which the temperature is low compared with the W and Z masses. This implies that there are essentially no W and Z bosons present in the medium, and therefore their propagators can be taken to be the same as in the vacuum, namely,

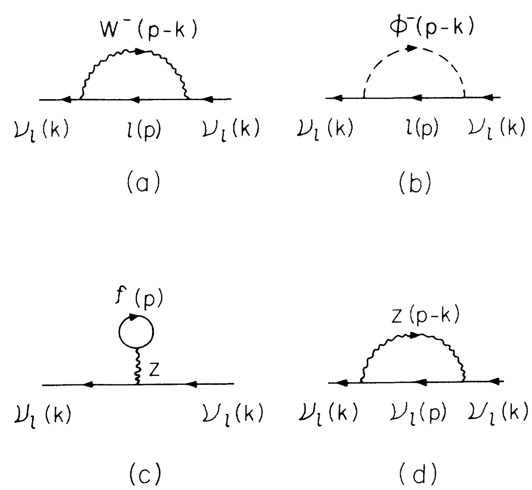

FIG. 1. Self-energy diagrams for a neutrino in a thermal background of charged leptons, nucleons, and neutrinos. In (a) the charged lepton (l) in the loop is of the same flavor as the neutrino. In (c), f stands for any fermion species that is present in the background.

$$\Delta_B^{\mu\nu}(q)=\frac{-1}{q^2-m_B^2}\left\{g^{\mu\nu}-\frac{(1-1/\xi)q^\mu q^\nu}{q^2-m_B^2/\xi}\right\}, \tag{3.1}$$

where $B=W,Z$. The propagator of the corresponding unphysical Higgs boson is given by

$$\Delta_\phi(q)=\frac{1}{q^2-m_B^2/\xi}. \tag{3.2}$$

We first calculate Figs 1(a) and 1(b) and denote their contributions to the self-energy by Σ_W and Σ_ϕ, respectively. Both of these quantities have a part that is independent of the background and is the same as in the vacuum and another one that depends on the background and which arises from the second term of the electron propagator given in Eq. (2.18). In the formulas given below, we will drop the background-independent part. To the order that we are calculating, that part only renormalizes the wave function and does not contribute to the dispersion relation [9].

The background-dependent contributions to Σ_W and Σ_ϕ are given by

$$\Sigma_W^{(T)}=-\frac{g^2}{2}\int\frac{d^4p}{(2\pi)^3}\gamma_\mu\Delta_W^{\mu\nu}(p-k)\not p\gamma_\nu \times\delta(p^2-m_l^2)\eta(p\cdot u), \tag{3.3}$$

$$\Sigma_\phi^{(T)}=-\frac{g^2}{2}\left[\frac{m_l}{m_W}\right]^2\int\frac{d^4p}{(2\pi)^3}\Delta_\phi(p-k)\not p \times\delta(p^2-m_l^2)\eta(p\cdot u). \tag{3.4}$$

It is convenient to decompose the sum $\Sigma_W^{(T)}+\Sigma_\phi^{(T)}$ into two parts and write

$$\Sigma_W^{(T)}+\Sigma_\phi^{(T)}=\Sigma_0^{(T)}+\Sigma_\xi^{(T)}, \tag{3.5}$$

where $\Sigma_\xi^{(T)}$ depends on ξ and $\Sigma_0^{(T)}$ does not. This is accomplished by writing the W propagator in the form

$$\Delta_W^{\mu\nu}(q)=\Delta_0^{\mu\nu}(q)+\Delta_\xi^{\mu\nu}(q), \tag{3.6}$$

where

$$\Delta_0^{\mu\nu}(q)=\frac{1}{q^2-m_W^2}\left[-g^{\mu\nu}+\frac{q^\mu q^\nu}{m_W^2}\right],\qquad \Delta_\xi^{\mu\nu}(q)=\frac{-1}{m_W^2}\frac{q^\mu q^\nu}{q^2-m_W^2/\xi}. \tag{3.7}$$

Using these expressions in Eqs. (3.3) and (3.4), we obtain

$$\Sigma_0^{(T)}=-\frac{g^2}{2}\int\frac{d^4p}{(2\pi)^3}\gamma_\mu\Delta_0^{\mu\nu}(p-k)\not p\gamma_\nu \times\delta(p^2-m_l^2)\eta(p\cdot u), \tag{3.8}$$

$$\Sigma_\xi^{(T)}=-\frac{g^2}{2}\int\frac{d^4p}{(2\pi)^3}\left[\gamma_\mu\Delta_\xi^{\mu\nu}(p-k)\not p\gamma_\nu+\left[\frac{m_l}{m_W}\right]^2\not p\Delta_\phi(p-k)\right] \times\delta(p^2-m_l^2)\eta(p\cdot u). \tag{3.9}$$

In order to evaluate the integrals in Eqs. (3.8) and (3.9), we expand the boson propagators in powers of m_W^{-2}. The computation of $\Sigma_0^{(T)}$ and $\Sigma_\xi^{(T)}$ up to terms of order g^2/m_W^4 is carried out in Appendix A. The coefficient functions a and b are extracted from the defining equation (2.3). These functions can also be decomposed into a gauge-independent and a gauge-dependent part as follows:

$$a = a_0 + a_\xi \, , \qquad b = b_0 + b_\xi \, . \tag{3.10}$$

From the explicit results given in Appendix A, we obtain

$$\begin{aligned} a_0^{(\mathrm{CC})} &= \frac{4\sqrt{2}G_F}{m_W^2}\left\{\frac{2}{3}J_2^{(l)} - \frac{5}{3}m_l^2 J_0^{(l)} + \omega J_1^{(l)}\right\} , \\ b_0^{(\mathrm{CC})} &= 4\sqrt{2}G_F\left\{\left[1+\frac{1}{2m_W^2}(\omega^2-\kappa^2+3m_l^2)\right]J_1^{(l)} + \frac{2\omega}{3m_W^2}(m_l^2 J_0^{(l)} - 4J_2^{(l)})\right\} , \\ a_\xi^{(\mathrm{CC})} &= \frac{4\sqrt{2}G_F}{m_W^2}\xi(m_l^2 J_0^{(l)} - \omega J_1^{(l)}) , \\ b_\xi^{(\mathrm{CC})} &= \frac{4\sqrt{2}G_F}{m_W^2}\xi\frac{(\omega^2-\kappa^2)}{2}J_1^{(l)} , \end{aligned} \tag{3.11}$$

with the Fermi constant identified as $G_F/\sqrt{2} = g^2/8m_W^2$. The superscript (CC) in the coefficients a and b in Eq. (3.11) is used to indicate that these are the contributions arising from the charged-current (W-exchange) diagrams, with the understanding that the contribution from the diagram with the unphysical Higgs boson has been included. The coefficients $J_n^{(f)}$ in Eq. (3.11) are defined by

$$\begin{aligned} J_n^{(f)} &= \int \frac{d^4p}{(2\pi)^3}\delta(p^2-m_f^2)\eta(p\cdot u)(p\cdot u)^n \\ &= \int \frac{d^3\mathcal{P}}{(2\pi)^3 2}\mathcal{E}^{n-1}[f_f(\mathcal{E}) + (-1)^n f_{\bar f}(\mathcal{E})] , \end{aligned} \tag{3.12}$$

for any fermion f in the background. The second line of Eq. (3.12) is obtained by evaluating the integral in the rest frame of the medium, where $p^\mu = (\mathcal{E}, \boldsymbol{\mathcal{P}})$ with $\mathcal{E} = (\mathcal{P}^2 + m_f^2)^{1/2}$. We have also introduced the particle and antiparticle momentum distributions

$$f_{f,\bar f} = \frac{1}{e^{\beta(\mathcal{E}\mp\mu_f)}+1} . \tag{3.13}$$

In terms of the number densities

$$n_{f,\bar f} = g_f \int \frac{d^3\mathcal{P}}{(2\pi)^3} f_{f,\bar f} \tag{3.14}$$

and the thermal average of $\mathcal{E}^n$,

$$\langle \mathcal{E}^n_{f,\bar f}\rangle \equiv \frac{g_f}{n_{f,\bar f}} \int \frac{d^3\mathcal{P}}{(2\pi)^3}\mathcal{E}^n f_{f,\bar f} , \tag{3.15}$$

the functions $J_n^{(f)}$ can be expressed in compact form as

$$J_n^{(f)} = \frac{1}{2g_f}[n_f\langle \mathcal{E}_f^{n-1}\rangle + (-1)^n n_{\bar f}\langle \mathcal{E}_{\bar f}^{n-1}\rangle] . \tag{3.16}$$

The factor g_f is equal to 1 for (left-handed) neutrinos and 2 for charged leptons and nucleons.

Let us now turn the attention to the Z-exchange diagrams. The tadpole diagram is independent of the gauge parameter because the Z propagator is evaluated at zero momentum. The computation of Fig. 1(d) is identical to that of Fig. 1(a). As shown in Appendix B, the total contribution from Figs. 1(c) and 1(d) to the coefficient functions a and b is

$$\begin{aligned} a_0^{(\mathrm{NC})} &= \frac{2\sqrt{2}G_F}{m_Z^2}\left[\frac{2}{3}J_2^{(\nu_l)} + \omega J_1^{(\nu_l)}\right] , \\ b_0^{(\mathrm{NC})} &= 4\sqrt{2}G_F\left[\sum_f X_f J_1^{(f)}\right] + 2\sqrt{2}G_F\left\{\left[1+\frac{1}{2m_Z^2}(\omega^2-\kappa^2)\right]J_1^{(\nu_l)} - \frac{8\omega}{3m_Z^2}J_2^{(\nu_l)})\right\} , \\ a_\xi^{(\mathrm{NC})} &= -\frac{2\sqrt{2}G_F}{m_Z^2}\xi\omega J_1^{(\nu_l)} , \\ b_\xi^{(\mathrm{NC})} &= \frac{\sqrt{2}G_F}{m_Z^2}\xi(\omega^2-\kappa^2)J_1^{(\nu_l)} . \end{aligned} \tag{3.17}$$

In the formula for $b_0^{(\mathrm{NC})}$, the sum extends over all the fermion species that are present in the background, with the factors X_f being the vector-neutral-current couplings of the corresponding fermion. In particular, the sum can include contributions from neutrinos of any flavor. The coefficients a_0 and b_0 are given by

$$a_0 = a_0^{(\mathrm{CC})} + a_0^{(\mathrm{NC})} , \qquad b_0 = b_0^{(\mathrm{CC})} + b_0^{(\mathrm{NC})} , \tag{3.18}$$

and similarly for a_ξ and b_ξ.

If the medium is CP or CPT symmetric, then $n_f = n_{\bar f}$. In that case, $J_1^{(f)}$ vanishes and from Eqs. (3.11) and (3.17) we immediately see that Eq. (2.5) is verified.

B. Dispersion relation

The proof of the gauge independence of the dispersion relation proceeds as follows. The dispersion relation ω_κ is given by the solution to $f(\omega_\kappa)=0$, where f is given in Eq. (2.11). It is convenient to write

$$f = f_0 + f_\xi , \tag{3.19}$$

where

$$f_0 = (1-a_0)(\omega-\kappa) - b_0 , \qquad f_\xi = -a_\xi(\omega-\kappa) - b_\xi . \tag{3.20}$$

The condition for ω_κ to be gauge independent is that both f_0 and f_ξ must vanish at $\omega=\omega_\kappa$. In that case, although the neutrino propagator has a gauge-dependent part, the position of the pole is independent of the gauge. On the other hand, the residue, which is related to the wave-function normalization, in general depends on the gauge.

The condition

$$f_0(\omega_\kappa)=0 \tag{3.21}$$

yields a quadratic equation for ω_κ that can be solved explicitly. Keeping only those terms that are consistent with the order we are working, the dispersion relation reduces to

$$\omega_\kappa=\kappa+b_0(\kappa)+O\left[\frac{g^4}{m_W^2},\frac{g^2}{m_W^6}\right], \tag{3.22}$$

where $b_0(\kappa)$ is the value of b_0 at $\omega=\kappa$. It remains to show that $f_\xi(\omega_\kappa)$ is zero, at least to the order of our calculation. According to the last equation, $\omega_\kappa-\kappa$ is a quantity of order g^2/m_W^2. Therefore, from Eqs. (3.11) and (3.17), we see that

$$b_\xi(\omega_\kappa)=O\left[\frac{g^4}{m_W^6}\right], \tag{3.23}$$

and

$$(\omega_\kappa-\kappa)a_\xi(\omega_\kappa)=O\left[\frac{g^4}{m_W^6}\right], \tag{3.24}$$

which implies that $f_\xi(\omega_\kappa)$ also is of order g^4/m_W^6. A similar result holds for the antineutrino, for which

$$\overline{\omega}_\kappa=\kappa-b_0(-\kappa)+O\left[\frac{g^4}{m_W^2},\frac{g^2}{m_W^6}\right]. \tag{3.25}$$

Thus we conclude that the dispersion relation does not depend on the gauge, although the self-energy does.

C. Effective potential

The previous result indicates that the self-energy itself cannot be directly associated with an observable physical quantity. In Ref. [6], in a way that is unclear at least to us, the authors infer the effective energy V_l directly from the self-energy by identifying it with the coefficient b_0, thermally averaged over the neutrino distribution function.

We insist that the neutrino effective potential must be defined from the dispersion relation. This is the physically relevant quantity, which gives precisely the energy-momentum relation of the particle in the medium. As a matter of convenience, an effective potential is introduced by subtracting the kinetic energy of the particle in a vacuum [10], i.e.,

$$\omega_\kappa=\kappa+V_l\ , \tag{3.26}$$

for a massless particle. According to the previous definition, it is clear from Eq. (3.22) that V_l coincides with b_0 only in the lowest order. In general, V_l will receive contributions from a_0. In higher orders these contributions have to be included to render a gauge-independent result.

Substituting the expressions for $b_0^{(\mathrm{CC})}$ and $b_0^{(\mathrm{NC})}$ given in Eqs. (3.11) and (3.17) into the dispersion relations for the neutrino and antineutrino [Eqs. (3.22) and (3.25)], we obtain the following result for the effective potential:

$$V_l=4\sqrt{2}G_F\left\{\pm\left[\left[1+\frac{3m_l^2}{2m_W^2}\right]J_1^{(l)}+\frac{1}{2}J_1^{(\nu_l)}+\left[\sum_f X_f J_1^{(f)}\right]\right]+\frac{2\kappa}{3m_W^2}(m_l^2J_0^{(l)}-4J_2^{(l)})-\frac{4\kappa}{3m_Z^2}J_2^{(\nu_l)}\right\}. \tag{3.27}$$

In this formula the upper (lower) sign refers to the neutrino (antineutrino). Alternatively, an index of refraction can be introduced,

$$n\equiv\frac{\kappa}{\omega_\kappa}\ ,$$

which is related to the effective potential by

$$n=1-\frac{V_l}{\omega_\kappa}\simeq 1-\frac{V_l}{\kappa}\ .$$

D. Wave-function normalization factor

The normalization factor of the wave function is determined from Eq. (2.17). Explicitly,

$$N_\kappa=\left[1-a-\left[\frac{\partial a}{\partial\omega}\right](\omega-\kappa)-\left[\frac{\partial b}{\partial\omega}\right]\right]\Bigg|_{\omega=\omega_\kappa}\ .$$

Since a is of order m_W^{-4} and $\omega-\kappa$ is of order m_W^{-2}, the term with the derivative of a is of order m_W^{-6} and therefore can be discarded. A straightforward calculation of the remaining terms yields

$$N_\kappa=1-\left[\frac{4\sqrt{2}G_F}{m_W^2}\right]\left\{2\kappa J_1^{(l)}-2J_2^{(l)}+m_l^2(\xi-1)J_0^{(l)}+\frac{m_W^2}{m_Z^2}(\kappa J_1^{(\nu_l)}-J_2^{(\nu_l)})\right\}. \tag{3.28}$$

The normalization factor for antineutrinos, $\overline{N}_\kappa$, is obtained from the last equation with the substitution $\kappa\to-\kappa$. For a CP-symmetric medium, $J_1^{(f)}=0$ and therefore $N_\kappa=\overline{N}_\kappa$, as it should be. N_κ receives corrections only of order m_W^{-4}, in agreement with the result of Ref. (5), where it was shown that, to the order m_W^{-2}, $N_\kappa=\overline{N}_\kappa=1$. Note that N_κ is gauge dependent, as we would expect for the normalization of the wave function. However, this does not pose any problem since it is not a directly observable quantity.

IV. DISCUSSION

The importance of the $O(m_W^{-4})$ corrections depends on the particular situation. For neutrinos propagating through normal matter (i.e., electrons and nucleons), the effective potential can be approximated by

$$V_e \simeq \pm\sqrt{2}G_F[(1+X_e)n_e+X_n n_n+X_p n_p] ,$$
$$V_{\mu,\tau} \simeq \pm\sqrt{2}G_F[X_e n_e+X_n n_n+X_p n_p] , \tag{4.1}$$

where the upper (lower) sign refers to neutrinos (antineutrinos). The quantity that is relevant to the problem of neutrino oscillations in matter [11] is the difference $V_e - V_{\mu,\tau}$, which depends only on the electron density [1]. However, for nontypical matter such as in the early Universe or in the core of a supernova, other contributions could be relevant.

It is believed that the particle-antiparticle asymmetry in the early Universe was a very small quantity, of the order of the present-day ratio of baryons to photons:

$$\frac{n_B-\bar{n}_B}{n_\gamma} \approx 10^{-10} .$$

Under such circumstances the $O(m_W^{-4})$ corrections may be important because they do not depend on the difference between the particle and antiparticle densities. As an example, let us consider the situation in which $T \ll m_\mu$. Then no μ or τ leptons will be present in the background, which is composed of electrons, nucleons, neutrinos of all flavors, and the corresponding antiparticles. In such a case, substituting into Eq. (3.27) the formula for $J_n^{(f)}$ given in Eq. (3.16), the following explicit formulas for effective potentials are obtained:

$$\begin{aligned} V_e &= \pm\sqrt{2}G_F(n_e-n_{\bar{e}}+n_{\nu_e}-n_{\bar{\nu}_e}+Q_Z) \\ &+\frac{2\sqrt{2}G_F\kappa m_e^2}{3m_W^2}\left[n_e\left\langle\frac{1}{E_e}\right\rangle+n_{\bar{e}}\left\langle\frac{1}{E_{\bar{e}}}\right\rangle\right] \\ &-\frac{8\sqrt{2}G_F\kappa}{3m_W^2}[n_e\langle E_e\rangle+n_{\bar{e}}\langle E_{\bar{e}}\rangle] \\ &-\frac{8\sqrt{2}G_F\kappa}{3m_Z^2}[n_{\nu_e}\langle E_{\nu_e}\rangle+n_{\bar{\nu}_e}\langle E_{\bar{\nu}_e}\rangle] , \\ V_{\mu,\tau} &= \pm\sqrt{2}G_F(n_{\nu_{\mu,\tau}}-n_{\bar{\nu}_{\mu,\tau}}+Q_Z) \\ &-\frac{8\sqrt{2}G_F\kappa}{3m_Z^2}[n_{\nu_{\mu,\tau}}\langle E_{\nu_{\mu,\tau}}\rangle+n_{\bar{\nu}_{\mu,\tau}}\langle E_{\bar{\nu}_{\mu,\tau}}\rangle] , \end{aligned} \tag{4.2}$$

where

$$Q_Z = \sum_{f=e,n,p} X_f(n_f-n_{\bar{f}})+\sum_{f=\nu_e,\nu_\mu,\nu_\tau}(n_f-n_{\bar{f}}) \tag{4.3}$$

is the average Z charge of the medium. In writing V_e the term proportional to $(m_e/m_W)^2$ in the coefficient of $J_1^{(e)}$ has been neglected. Note that, in general, Eq. (4.2) contains terms proportional to $m_e^2/\langle E_e\rangle$, which are negligible as long as the temperature $T \gg m_e$. If those corrections are neglected, our results coincide with those of Refs. [3] and [6]. On the other hand, there are physical situations, as in the nucleosynthesis era, where the temperature is of the same order as the electron mass and terms proportional to $m_e^2/\langle E_e\rangle$ could be important.

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APPENDIX A: CALCULATION OF $\Sigma_{W,\phi}^{(T)}$

Here we give the details of the calculation of the functions Σ_0 and Σ_ξ defined in Eqs. (3.8) and (3.9).

Substituting the propagators given in Eq. (3.7) into Eqs. (3.8) and (3.9), we obtain

$$\Sigma_0^{(T)} = -\frac{g^2}{2m_W^2}\{(2m_W^2+m_l^2-k^2)\gamma_\mu\mathcal{P}^\mu(1) +2[k\cdot\mathcal{P}(1)-m_l^2\mathcal{Q}(1)]\not{k}\} , \tag{A1}$$

$$\Sigma_\xi^{(T)} = -\frac{g^2}{2m_W^2}\{k^2\gamma_\mu\mathcal{P}^\mu(\xi)+2[m_l^2\mathcal{Q}(\xi)-k\cdot\mathcal{P}(\xi)]\not{k}\} , \tag{A2}$$

where we have defined the functions

$$\mathcal{Q}(\lambda) = \int\frac{d^4p}{(2\pi)^3}\frac{\delta(p^2-m_l^2)\eta(p\cdot u)}{(p-k)^2-m_W^2/\lambda} , \tag{A3}$$

$$\mathcal{P}_\mu(\lambda) = \int\frac{d^4p}{(2\pi)^3}\frac{\delta(p^2-m_l^2)\eta(p\cdot u)}{(p-k)^2-m_W^2/\lambda}p_\mu . \tag{A4}$$

The same functions appear in Eqs. (A1) and (A2), but evaluated at $\lambda=1$ and ξ, respectively.

It will be difficult to evaluate $\mathcal{P}_\mu(\lambda)$ and $\mathcal{Q}(\lambda)$ in general, but since we are interested in retaining terms up to the order to m_W^{-4}, we expand the denominators in Eqs. (A3) and (A4) as a power series in $\lambda(p-k)^2/m_W^2$, up to second order. Thus, for $\mathcal{Q}$, we obtain

$$\begin{aligned} \mathcal{Q} &= \frac{1}{\mathcal{M}^2}\int\frac{d^4p}{(2\pi)^3}\delta(p^2-m_l^2)\eta(p\cdot u) \times\left[1+\frac{2p\cdot k}{\mathcal{M}^2}+O(\mathcal{M}^{-4})\right] \\ &= \frac{1}{\mathcal{M}^2}\left[J_0^{(l)}+\frac{2}{\mathcal{M}^2}k_\mu I^\mu+O(\mathcal{M}^{-4})\right] , \end{aligned} \tag{A5}$$

where $J_0^{(l)}$ is the function defined in Eq. (3.12) with $n=0$,

$$\mathcal{M}^2 = m_l^2+k^2-\frac{m_W^2}{\lambda} \tag{A6a}$$

and

$$I_\mu = \int\frac{d^4p}{(2\pi)^3}\delta(p^2-m_l^2)\eta(p\cdot u)p_\mu . \tag{A6b}$$

Note that I_μ is manifestly covariant and depends only on the vector u_μ. Therefore it is proportional to u_μ, with the factor of proportionality determined by contracting Eq. (A6) with u_μ. Thus we obtain

$$I_\mu = J_1^{(l)} u_\mu$$

and

$$Q(\lambda) = \frac{1}{\mathcal{M}^2}\left[J_0^{(l)} + \frac{2\omega}{\mathcal{M}^2} J_1^{(l)}\right] .$$

In a similar fashion,

$$\mathcal{P}_\mu(\lambda) = \frac{1}{\mathcal{M}^2}\int \frac{d^4p}{(2\pi)^3}\delta(p^2-m_l^2)\eta(p\cdot u)p_\mu \times\left[1+\frac{2p\cdot k}{\mathcal{M}^2}+O(\mathcal{M}^{-4})\right]$$

$$= \frac{1}{\mathcal{M}^2}\left[I_\mu + \frac{2}{\mathcal{M}^2}k_\nu I^{\mu\nu} + O(\mathcal{M}^{-4})\right] , \tag{A7}$$

where

$$I_{\mu\nu} = \int \frac{d^4p}{(2\pi)^3}\delta(p^2-m_l^2)\eta(p\cdot u)p_\mu p_\nu . \tag{A8}$$

In order to evaluate $I_{\mu\nu}$, note that it must be of the form

$$I_{\mu\nu} = A g_{\mu\nu} + B u_\mu u_\nu .$$

By contracting this expression with $u_\mu u_\nu$ and $g_{\mu\nu}$, we obtain two equations for A and B in terms of the scalars

$$I^\mu_\mu = m_l^2 J_0^{(l)} ,$$

$$u_\mu u_\nu I^{\mu\nu} = J_2^{(l)} .$$

Solving for A and B, these are then determined as

$$\begin{aligned} A &= \tfrac{1}{3}(m_l^2 J_0^{(l)} - J_2^{(l)}) , \\ B &= \tfrac{1}{3}(4J_2^{(l)} - m_l^2 J_0^{(l)}) . \end{aligned} \tag{A9}$$

Substituting the expressions for I_μ and $I_{\mu\nu}$ into Eq. (A7),

$$\mathcal{P}_\mu(\lambda) = \frac{1}{\mathcal{M}^2}\left\{\left[J_1^{(l)} + \frac{2\omega}{3\mathcal{M}^2}(4J_2^{(l)} - m_l^2 J_0^{(l)})\right]u_\mu + \frac{2}{3\mathcal{M}^2}(m_l^2 J_0^{(l)} - J_2^{(l)})k_\mu\right\} . \tag{A10}$$

Now that the explicit expressions for $Q(\lambda)$ and $\mathcal{P}_\mu(\lambda)$ have been derived to order m_W^{-4}, we can substitute these results into Eqs. (A1) and (A2) to find $\Sigma_0^{(T)}$ and $\Sigma_\xi^{(T)}$. Both $\Sigma_0^{(T)}$ and $\Sigma_\xi^{(T)}$ are of the form given in Eq. (2.3). It is then straightforward to read off the coefficient functions a and b as follows:

$$\begin{aligned} a_0^{(\mathrm{CC})} &= \frac{4\sqrt{2}G_F}{m_W^2}\left\{\frac{2}{3}J_2^{(l)} - \frac{5}{3}m_l^2 J_0^{(l)} + \omega J_1^{(l)}\right\} , \\ b_0^{(\mathrm{CC})} &= 4\sqrt{2}G_F\left\{\left[1+\frac{1}{2m_W^2}(\omega^2-\kappa^2+3m_l^2)\right]J_1^{(l)} + \frac{2\omega}{3m_W^2}(m_l^2 J_0^{(l)} - 4J_2^{(l)})\right\} , \\ a_\xi^{(\mathrm{CC})} &= \frac{4\sqrt{2}G_F}{m_W^2}\xi(m_l^2 J_0^{(l)} - \omega J_1^{(l)}) , \\ b_\xi^{(\mathrm{CC})} &= \frac{4G_F}{\sqrt{2}m_W^2}\xi(\omega^2-\kappa^2)J_1^{(l)} . \end{aligned} \tag{A11}$$

These are the results quoted in Eq. (3.11).

APPENDIX B: Z-EXCHANGE DIAGRAMS

In order to evaluate the contribution of the tadpole diagram [Fig. 1(c)] in a way that is valid for any background fermion f that runs in the loop, we write the Zff coupling in the Lagrangian in the form

$$L_Z = \left[\frac{-g}{2\cos\theta_W}\right]\bar{f}\gamma_\mu(X_f + Y_f\gamma_5)fZ^\mu . \tag{B1}$$

For the electron,

$$X_e = -\tfrac{1}{2} + 2\sin^2\theta_W ,$$

$$Y_e = \tfrac{1}{2} ;$$

for the neutrino,

$$X_\nu = -Y_\nu = \tfrac{1}{2} ;$$

and for nucleons,

$$X_p = \tfrac{1}{2} - 2\sin^2\theta_W ,$$

$$Y_n = -X_n = -Y_p = \tfrac{1}{2} .$$

Each fermion in the loop yields a contribution to the background-dependent part of the self-energy that is given by

$$\Sigma^{(T)}_{\mathrm{tadpole}} = -\left[\frac{g}{2\cos\theta_W}\right]^2 \gamma_\mu \Delta_Z^{\mu\nu}(0) \times \int\frac{d^4p}{(2\pi)^3}\mathrm{Tr}[\gamma_\nu(X_f+Y_f\gamma_5)(\not{p}+m_f)] \times\delta(p^2-m_f^2)\eta(p\cdot u) , \tag{B2}$$

where we have retained only the second term of the fermion propagator given in Eq. (2.18). Since the Z boson is exchanged with zero momentum, there is no contribution to the ξ-dependent terms; i.e.,

$$a_\xi^{(\mathrm{tadpole})} = b_\xi^{(\mathrm{tadpole})} = 0 . \tag{B3}$$

The results for the ξ-independent terms are

$$a_0^{(\text{tadpole})}=0\ ,$$
$$b_0^{(\text{tadpole})}=4\sqrt{2}G_F\sum_f X_f J_1^{(f)}\ , \tag{B4}$$

where we have used the relation $m_W=m_Z\cos\theta_W$ and added the contribution from the different fermion species present in the medium.

The calculation of the diagram Fig. 1(d) follows identical steps as the calculation of $\Sigma_W^{(T)}$, with the replacements

$$m_W \to m_Z\ ,$$
$$m_l \to m_{\nu_l}=0\ , \tag{B5}$$
$$\frac{g}{\sqrt{2}} \to \frac{g}{2\cos\theta_W}\ .$$

The results for the coefficients $a^{(Z)}$ and $b^{(Z)}$ can be borrowed directly from Eq. (A11) with the above substitutions, since the diagram with the unphysical Higgs boson, which is included in that equation, vanished if m_l is set equal to zero. The results are

$$a_0^{(Z)}=\frac{2\sqrt{2}G_F}{m_Z^2}\left[\frac{2}{3}J_2^{(\nu_l)}+\omega J_1^{(\nu_l)}\right]\ ,$$
$$b_0^{(Z)}=2\sqrt{2}G_F\left\{\left[1+\frac{1}{2m_Z^2}(\omega^2-\kappa^2)\right]J_1^{(\nu_l)}-\frac{8\omega}{3m_Z^2}J_2^{(\nu_l)})\right\}\ , \tag{B6}$$
$$a_\xi^{(Z)}=-\frac{2\sqrt{2}G_F}{m_Z^2}\xi\omega J_1^{(\nu_l)}\ ,$$
$$b_\xi^{(Z)}=\frac{\sqrt{2}G_F}{m_Z^2}\xi(\omega^2-\kappa^2)J_1^{(\nu_l)}\ .$$

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[8] See, for example, Ref. [5], and references therein.

[9] In general, if we decompose the coefficient a into a background-dependent $a^{(T)}$ and a background-independent part $a^{(0)}$, respectively, and likewise for b, the Dirac equation can be written in the form

$$(1-a^{(0)})\left\{\left[1-\frac{a^{(T)}}{1-a^{(0)}}\right]\not{k}-\left[\frac{b^{(T)}}{1-a^{(0)}}\right]\not{u}\right\}\psi_L=0$$

and the propagator

$$S_F=L\frac{1}{(1-a^{(0)})\{[1-a^{(T)}/(1-a^{(0)})]\not{k}-[b^{(T)}/(1-a^{(0)})]\not{u}\}}\times R\ .$$

The overall factor of $(1-a^{(0)})$ is absorbed in the vacuum wave-function renormalization constant and does not affect the equation for the dispersion relation. On the other hand, to the order that we are calculating, we can replace, in the rest of the terms,

$$\frac{a^{(T)}}{1-a^{(0)}}\simeq a^{(T)}\ ,$$
$$\frac{b^{(T)}}{1-a^{(0)}}\simeq b^{(T)}\ ,$$

so that the dispersion relations are determined by Eq. (2.12), as already discussed, with the understanding that the symbols a and b stand for the background-dependent part only.

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