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Heavy-quark symmetry and chiral dynamics

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The flavor and spin symmetry of the heavy quarks and the spontaneously broken approximate $SU(3)_L \times SU(3)_R$ chiral symmetry of the light quarks are exploited to formulate a theory describing the low-energy interactions of the heavy mesons ($Q\bar{q}$ bound states) and heavy baryons (Qq_1q_2 bound states) with the Goldstone bosons π, K, and η. The theory contains only three parameters independent of the number of heavy-quark species involved. They can be determined by the decays $D^* \to D+\pi$, $\Sigma_c \to \Lambda_c + \pi$, and $\Sigma_c^* \to \Sigma_c + \pi$. Theoretically, these coupling constants are related, through partial conservation of axial-vector current, to the axial charges of the heavy mesons and the heavy baryons. They are all calculable in the nonrelativistic quark model by using the spin wave functions of these particles alone. The theory is applied to strong decays and semileptonic weak decays of the heavy mesons and baryons. The implications are also discussed.

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I. INTRODUCTION

The quark contribution to the QCD Lagrangian

$$\mathcal{L}_{\text{quarks}} = \bar{q}(i\not{D} - m_q)q + \bar{Q}(i\not{D} - m_Q)Q$$

separates naturally into two pieces: the first contribution comes from light quarks ($q=u$, d, and s) whereas the second one is due to heavy quarks ($Q=c$, b, and t). Each of the two exhibits a distinct symmetry. The light-quark sector has an approximate $SU(3)_L \times SU(3)_R$ flavor chiral symmetry, because the current quark masses are all very small on the typical hadron energy scale. The symmetry is spontaneously broken to the usual vector SU(3), and the chiral symmetry is reflected in the presence of eight Goldstone bosons: the pions, kaons, and η. Their couplings to hadrons at low energies are determined by PCAC (partial conservation of axial-vector current) and current algebra, or alternatively, by the nonlinear chiral Lagrangians. On the other hand, in the limit of infinite quark masses, the dynamics of a heavy quark in QCD depends only on its velocity and is independent of its mass and spin. As a consequence, a new flavor and spin symmetry appear in the sector of hadrons containing one heavy quark. (In the following we will refer to these as heavy mesons and heavy baryons in distinction from quarkonia which are $\bar{Q}Q$ bound states.) This is known as heavy-quark symmetry and much progress has been made in this area by many authors [1–14]. More precisely, this new symmetry implies that the excitation spectrum of heavy mesons and heavy baryons are independent of the heavy-quark species and heavy-quark spins; so are the transition form factors in weak decays of these heavy hadrons when they are properly defined. All these are a sophisticated generalization of the familiar QED example that a hydrogenlike atom has an excitation spectrum and transition matrix elements independent of the mass and spin of the nucleus.

Since the heavy mesons and heavy baryons contain both heavy and light quarks, one expects both the chiral symmetry of the light quarks and heavy-quark symmetry to have interesting implications for the low-energy dynamics of heavy hadrons interacting with the Goldstone bosons. Experimentally, these circumstances arise in examples of strong decays such as $D^* \to D\pi$ and $\Sigma_c \to \Lambda_c \pi$ involving soft pions and in semileptonic decays of heavy hadrons where there are ample phase space for emission of light mesons. This is the subject of the present work. The preliminary results on mesons were reported earlier by one of us [15].

The chiral properties of a heavy hadron is dictated by its light quark contents. For the heavy mesons, since they contain only a single light quark, each heavy quark Q will give rise to an SU(3) antitriplet $\bar{q}Q$. For the heavy baryons a heavy quark Q will combine the two light quarks to form baryons Qq_1q_2. Here the situation is much more interesting. The two light quarks can form a symmetric **6** or an antisymmetric $\bar{\mathbf{3}}$ in flavor-SU(3) space.

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We will denote these baryons as B_6 and $B_{\bar{3}}$ respectively. For the ground-state baryons in the quark model, the symmetries in the flavor and spin of the "diquarks" are correlated. The SU(3)-symmetric sextet diquarks will have spin 1 whereas the SU(3)-antisymmetric antitriplet diquarks will have spin 0. Thus, for the ground states of the SU(3)-symmetric sextet there are both spin-$\frac{1}{2}$ baryons (B_6) and spin-$\frac{3}{2}$ baryons (B_6^*). For the ground states of the SU(3) antisymmetric antitriplet, there are only spin-$\frac{1}{2}$ baryons ($B_{\bar{3}}$). The correlation between flavor and spin wave functions is outside the realm of QCD, but it is strongly supported by the empirical evidence from the baryon spectroscopy. We will assume this correlation in our subsequent discussions. Once the flavor SU(3) contents of these heavy hadrons are determined, their couplings to the Goldstone bosons can be immediately written down following the standard formalism of nonlinear chiral dynamics. Here we find the nonlinear chiral quark model [16] most useful in our work, although we never have to use it explicitly. Multi-light-meson emissions are of course related by chiral dynamics. Thus only the single-light-meson-emission vertices are independent. Let us call them schematically P^*P^*p, P^*Pp, $B_6^*B_6^*p$, $B_6^*B_6p$, B_6B_6p, $B_6^*B_{\bar{3}}p$, $B_6B_{\bar{3}}p$, and $B_{\bar{3}}B_{\bar{3}}p$. Here P^* and P refer to 1^- and 0^- ground-state vector and pseudoscalar heavy mesons, respectively, and the lower case p denotes a light Goldstone boson. The heavy-quark flavor symmetry tells us how the coupling strengths of these vertices depend on the heavy quark mass. The heavy-quark spin symmetry relates the coupling constants for these vertices. We find in the meson sector that, to leading order in the light meson momentum, there is only one independent coupling constant, and there are two in the baryon sector. To arrive at these conclusions we have made extensive use of the tensor method or the method of interpolating fields [10,12–14]. This is especially true for the case of baryons. Here we find the concepts of diquarks particularly useful.

Since the ground-state baryons have even parity, the diquark in the SU(3) sextet must have spin parity 1^+ which can be represented by an *axial*-vector field ϕ_μ. Similarly, the diquark in the SU(3) antitriplet must have spin parity 0^+ which is represented by a Lorentz scalar field ϕ. The two independent coupling constants in the baryon sector describe the decays $\phi_\mu \to \phi'_\mu + \pi$ and $\phi_\mu \to \phi + \pi$, respectively. As an indication of the power of the heavy-quark spin symmetry we point out the absence of the decay $\phi \to \phi + \pi$ which does not conserve parity. As a result, the $B_{\bar{3}}B_{\bar{3}}p$ coupling vanishes. Finally, the three independent coupling constants can be easily computed in the nonrelativistic quark model. Through PCAC, these coupling constants are related to the matrix elements of the axial vector current between the initial and final heavy hadron states whose values depend only on spin wave functions of these states in the quark model just like the value of g_A for a nucleon. Of course, this is well known for the baryon-pion coupling constants. It is less well known that the meson-pion coupling constant can also be computed in the same fashion. Thus, the heavy-quark symmetry and chiral dynamics together, when supplemented by the nonrelativistic quark model, uniquely determine the low-energy interactions of the heavy mesons and heavy baryons with the light Goldstone bosons.

The paper is organized as follows. In Sec. II we consider the dynamics of heavy mesons interacting with the Goldstone bosons. We discuss the chiral properties of the heavy mesons and derive the nonlinear chiral Lagrangian involving heavy and light mesons. We then investigate the implications of heavy-quark flavor and spin symmetries on the coupling constants. The quark-model calculation of the coupling constants is given. The same discussions for heavy baryons are presented in Sec. III. Here we find the concept of a diquark system extremely useful. In Sec. IV we consider applications of our results to simple examples of strong decays and semileptonic weak decays of the heavy mesons and heavy baryons. Let us mention here two specific cases. In the heavy-quark limit, the weak decay $\Sigma_b \to \Lambda_c + l\nu$ is suppressed, but the decay $\Sigma_b \to \Lambda_c + \pi + l\nu$ should not. So it may become the dominant weak decay mode. In a specific example of the semileptonic decay $B \to D^*\pi l\nu$ we demonstrate how the full power of heavy-quark symmetry and chiral dynamics allows us to write down the complete amplitude involving all possible transition weak form factors in terms of a single universal Isgur-Wise function and a coupling constant which is calculable in the nonrelativistic quark model. In the kinetic region where the leptons carry away most of the energy the amplitude is completely known. In discussing this process, we have worked out two weak transition vertices of the vector and axial vector currents sandwiched between two spin-1 heavy mesons that do not seem to exist in the literature. In Sec. V we make some concluding remarks.

II. DYNAMICS OF HEAVY MESONS

A heavy meson contains a heavy quark Q and a light antiquark $\bar{q}$. The ground states comprise the usual 1^- and 0^- mesons, which will be denoted by P_i^* and P_i, respectively. Their quantum numbers are displayed in the interpolating fields [10,12,14]

$$P_i(v) = \bar{q}\gamma_5 h_v^i \sqrt{M_P} \ , \tag{2.1}$$

$$P_i^*(v,\varepsilon) = \bar{q}\not{\varepsilon} h_v^i \sqrt{M_{P^*}} \ , \tag{2.2}$$

where the heavy-quark field h_v^i destroys a heavy quark of type i and four-velocity v and is related to the conventional field operator Q_i by [8]

$$Q_i(x) = e^{-im_Q v\cdot x} h_v^i \ ; \tag{2.3}$$

the masses M_P and M_{P^*} are those of the pseudoscalar and vector mesons. The factors $\sqrt{M_P}$ and $\sqrt{M_{P^*}}$ are included in (2.1) and (2.2) for our later use. The light quark q stands for a column vector in flavor SU(3):

$$q = \begin{bmatrix} u \\ d \\ s \end{bmatrix} . \tag{2.4}$$

Thus, both $P_i(v)$ and $P_i^*(\nu,\varepsilon)$ are an SU(3) antitriplet.

Before we discuss interactions of heavy mesons with the Goldstone bosons, let us summarize briefly the dynamics of the Goldstone bosons themselves [16]. The nonlinear chiral symmetry is realized by making use of the unitary matrix

$$\Sigma = e^{2iM/\sqrt{2}f_\pi} , \tag{2.5}$$

where M is a 3×3 matrix for the octet of Goldstone bosons

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\sqrt{\tfrac{2}{3}}\eta \end{pmatrix} , \tag{2.6}$$

and

$$f_\pi = 93 \text{ MeV} \tag{2.7}$$

is the pion decay constant. The Lagrangian for the Goldstone bosons is

$$\mathcal{L}_M = \frac{f_\pi^2}{4} \mathrm{tr} \partial_\mu \Sigma^\dagger \partial^\mu \Sigma , \tag{2.8}$$

which contains all $SU(3)_L \times SU(3)_R$-invariant interactions up to two derivatives among the Goldstone bosons. The invariance of the Lagrangian $\mathcal{L}_M$ in (2.8) under $SU(3)_L \times SU(3)_R$ chiral transformations is easily established since the unitary matrix responds to these transformations according to

$$\Sigma \to \Sigma' = L \Sigma R^\dagger , \tag{2.9}$$

where L and R are global transformations in $SU(3)_L$ and $SU(3)_R$ respectively. Since the chiral symmetry $SU(3)_L \times SU(3)_R$ is spontaneously broken down to SU(3), the discussions of the couplings of Goldstone bosons to the ordinary hadrons (including the heavy mesons and baryons) are facilitated by the introduction of a new matrix [16]

$$\xi = \Sigma^{1/2}, \tag{2.10}$$

which transforms under an $SU(3)_L \times SU(3)_R$ as

$$\xi \to \xi' = L \xi U^\dagger = U \xi R^\dagger , \tag{2.11}$$

where U is a unitary matrix depending on L and R and a nonlinear function of the Goldstone fields M. From ξ we can construct a vector field V_μ and an axial vector field A_μ with simple chiral transformation properties:

$$V_\mu = \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) , \tag{2.12a}$$

$$A_\mu = \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) . \tag{2.12b}$$

The vector field is a gauge field under a chiral transformation

$$V_\mu \to V'_\mu = U V_\mu U^\dagger + U \partial_\mu U^\dagger , \tag{2.13}$$

and the axial vector field is an SU(3) octet which transforms as

$$A_\mu \to A'_\mu = U A_\mu U^\dagger . \tag{2.14}$$

The quark triplet (2.4) transforms as [16]

$$q \to q' = Uq . \tag{2.15}$$

We now return to the chiral properties of the heavy mesons. For the moment we will focus on a single species of heavy quark, so the heavy mesons will be simply referred to as P and P^*. The pseudoscalars form an SU(3) antitriplet, so we can endow them with the transformation law

$$P \to P' = P U^\dagger , \tag{2.16}$$

and a similar equation for the vector mesons P^*.

A gauge-covariant derivative can then be constructed with the aid of V_μ (V_μ^T is the transpose of the matrix V_μ):

$$D_\mu P^\dagger = (\partial_\mu + V_\mu) P^\dagger , \tag{2.17}$$

$$D_\mu P \equiv P(\overleftarrow{\partial}_\mu + V_\mu^\dagger) = (\partial_\mu + V_\mu^*) P , \tag{2.18}$$

which transforms simply:

$$D_\mu P^\dagger \to (D_\mu P^\dagger)' = U(D_\mu P^\dagger) . \tag{2.19}$$

It is now a simple matter to write down the chiral-invariant Lagrangian involving P and P^* and their couplings to the Goldstone bosons [17]

$$\begin{aligned} \mathcal{L}_{PP^*} = {} & D_\mu P D^\mu P^\dagger - M_P^2 P P^\dagger + f_Q (P A^\mu P_\mu^{*\dagger} + P_\mu^* A^\mu P^\dagger) \\ & - \frac{1}{2} P^{*\mu\nu} P_{\mu\nu}^{*\dagger} + M_{P^*}^2 P^{*\mu} P_\mu^{*\dagger} \\ & + \frac{1}{2} g_Q \varepsilon_{\mu\nu\lambda\kappa} (P^{*\mu\nu} A^\lambda P^{*\kappa\dagger} + P^{*\kappa} A^\lambda P^{*\mu\nu\dagger}) , \end{aligned} \tag{2.20}$$

where

$$P_{\mu\nu}^{*\dagger} = D_\mu P_\nu^{*\dagger} - D_\nu P_\mu^{*\dagger} , \tag{2.21}$$

$$P_{\mu\nu}^* = (\partial_\mu + V_\mu^*) P_\nu^* - (\partial_\nu + V_\nu^*) P_\mu^* . \tag{2.22}$$

This is the most general Lagrangian consistent with chiral invariance that contains one single derivative on the Goldstone boson fields. Namely, these are the leading terms in the expansion of the light meson momenta. The Lagrangian (2.20) contains two coupling constants f_Q and g_Q. As we will see in a moment, the heavy-quark spin symmetry relates g_Q to f_Q, and the heavy-quark flavor symmetry gives the dependence of f_Q on the heavy masses. Thus, there is only one independent coupling constant even if all the heavy-quark species are included. Finally, through PCAC the parameter f_Q is given by a matrix element of the axial-vector current between P and P^* states. In this form, the nonrelativistic quark model has a simple prediction for the value of f_Q.

We will begin with the implication of the heavy-quark spin symmetry for the two coupling constants. Application of PCAC gives the invariant matrix element for the emission of a soft pion:

$$M(A\to B\pi^a(q))_{\rm PCAC}=\frac{1}{f_\pi}\langle\,|q^\mu A^a_\mu|A\,\rangle\ . \qquad (2.23)$$

We now evaluate the right-hand side of (2.23) by the method of interpolating fields which are given by (2.1) and (2.2). Recall that use of these interpolating fields gives a simple derivation of the relations among different weak transition form factors and their dependences on the heavy masses [10,13,14]. We have [18]

$$\begin{aligned}\langle P(v')|q^\mu A^a_\mu|P^*(v,\varepsilon)\rangle&=\langle 0|\bar q_{v'}\gamma_5 h_{v'}(q^\mu A^a_\mu)\bar h_v\not\varepsilon q_v|0\rangle\sqrt{M_PM_{P^*}}\\&=-\langle 0|\gamma_5 h_{v'}\bar h_v\not\varepsilon|0\rangle\langle 0|q_v q^\mu A^a_\mu\bar q_{v'}|0\rangle\sqrt{M_PM_{P^*}}\ ,\end{aligned}\qquad (2.24)$$

where a matrix multiplication with respect to the Dirac indices is implied. We have attached the velocity of the heavy quark to the light quark field with which it is associated. In the soft pion limit, $v\cong v'$ and we have the "propagator" for the heavy quark:

$$\langle 0|h_{v'}\bar h_v|0\rangle=\frac{\not v+1}{2}\ . \qquad (2.25)$$

The matrix

$$\mathcal{M}=\langle 0|q_v q^\mu A^a_\mu\bar q_{v'}|0\rangle \qquad (2.26)$$

is at least first order in q_μ and Lorentz invariance gives

$$\mathcal{M}=u(P^*)^*\frac{\tau_a}{2}u(P)\gamma_5[a\not q+b\not v(v\cdot q)+c\not q\not v] \qquad (2.27)$$

where $u(P^*)$ and $u(P)$ are the isospin wave functions of the heavy mesons, and a, b, and c are constants independent of heavy-quark masses. We notice that

$$\not v\gamma_5\frac{\not v+1}{2}=-\gamma_5\frac{\not v+1}{2}\ . \qquad (2.28)$$

So we have, effectively,

$$\mathcal{M}=u(P^*)^*\frac{\tau_a}{2}u(P)\gamma_5[(a-c)\not q-b(v\cdot q)] \qquad (2.29)$$

and

$$\langle P(v')|q^\mu A^a_\mu|P^*(v,\varepsilon)\rangle=-u(P^*)^*\frac{\tau_a}{2}u(P)\sqrt{M_PM_{P^*}}\,{\rm tr}\left\{\gamma_5\frac{\not v+1}{2}\not\varepsilon\gamma_5[(a-c)\not q-b(v\cdot q)]\right\}\ . \qquad (2.30)$$

The trace is easily evaluated; we obtain

$$M[P^*(v,\varepsilon)\to P(v')+\pi^a(q)]=\frac{1}{f_\pi}\langle P(v')|q^\mu A^a_\mu|P^*(v,\varepsilon)\rangle=\frac{2}{f_\pi}(a-c)\sqrt{M_PM_{P^*}}(\varepsilon\cdot q)u(P^*)^*\frac{\tau_a}{2}u(P)\ . \qquad (2.31)$$

Similarly

$$\begin{aligned}&\langle P^*(v',\varepsilon')|q^\mu A^a_\mu|P^*(v,\varepsilon)\rangle\\&\quad=\langle 0|\bar q_{v'}\not\varepsilon' h_{v'}(q^\mu A^a_\mu)\bar h_v\not\varepsilon q_v|0\rangle M_{P^*}\\&\quad=-{\rm tr}\left[\not\varepsilon'\frac{\not v+1}{2}\not\varepsilon\mathcal{M}\right]M_{P^*}\ .\end{aligned} \qquad (2.32)$$

Here again in the soft pion limit $v\cong v'$, and

$$\not v\not\varepsilon'=-\not\varepsilon'\not v+2\varepsilon'\cdot v=-\not\varepsilon'\not v+O(q)\ . \qquad (2.33)$$

Therefore, the matrix $\mathcal{M}$ in (2.32) effectively takes the same form given by (2.29). So

$$\begin{aligned}&M[P^*(v,\varepsilon)\to P^*(v',\varepsilon')+\pi^a(q)]\\&\quad=-\frac{2}{f_\pi}(a-c)M_{P^*}\\&\qquad\times u(P^*)^*\frac{\tau_a}{2}u(P'^*)i\varepsilon_{\mu\nu\lambda\kappa}q^\mu\varepsilon'^\nu v^\lambda\varepsilon^\kappa\ .\end{aligned} \qquad (2.34)$$

When (2.31) and (2.34) are compared with those obtained from the Lagrangian (2.20) and

$$A_\mu=-\frac{1}{\sqrt{2}f_\pi}\partial_\mu M+\cdots=-\frac{1}{f_\pi}\partial_\mu\left[\frac{1}{2}\tau_a\pi^a\right]+\cdots\ , \qquad (2.35)$$

we find

$$f_Q=2(a-c)\sqrt{M_PM_{P^*}}\ , \qquad (2.36)$$

$$g_Q=(a-c)\ , \qquad (2.37)$$

that is

$$g_Q=\frac{f_Q}{2\sqrt{M_PM_{P^*}}}\ . \qquad (2.38)$$

Equations (2.36) and (2.37) also give the heavy-quark flavor symmetry prediction on the heavy-mass dependences of f_Q and g_Q:

$$f_Q=\sqrt{M_PM_{P^*}}f\ , \qquad (2.39)$$

$$g_Q = g \ , \tag{2.40}$$

where f and g are universal constants independent of heavy-quark masses and species. The relation (2.38) reduces to

$$g = \frac{1}{2} f \ . \tag{2.41}$$

As a by-product we can establish

$$\langle P(v')|q^\mu A^a_\mu|P^*(v,\varepsilon)\rangle = \langle P^*(v',\varepsilon)|q^\mu A^a_\mu|P(v)\rangle \ , \tag{2.42}$$

and hence the constant f_Q is real. Similarly, g_Q is real. The reality of these constants is already incorporated in (2.20).

We now show that the nonrelativistic quark model has a simple prediction for the value of f. Consider the subgroup SU(2) and denote the $\bar{P}^*$ as

$$\bar{P}^* = \begin{bmatrix} \bar{Q}u \\ \bar{Q}d \end{bmatrix} \equiv \begin{bmatrix} \bar{P}^*_{1/2} \\ \bar{P}^*_{-1/2} \end{bmatrix} \ , \tag{2.43}$$

where the subscripts $\pm\frac{1}{2}$ indicate the value of the isospin quantum number I_3. Consider the matrix element

$$\begin{aligned} \langle \bar{P}_{1/2}(p')|A^{1+i2}_\mu|\bar{P}^*_{-1/2}(p,\varepsilon)\rangle &= [\varepsilon_\mu f_Q + (\varepsilon\cdot p')(p+p')_\mu \alpha_+ \\ &\quad + (\varepsilon\cdot p')(p-p')_\mu \alpha_-] \ . \end{aligned} \tag{2.44}$$

In the soft pion limit, $p \approx p'$ and $\varepsilon\cdot p' = O(q)$. Therefore, the first term dominates. Indeed, the coefficient f_Q is identical to the coupling constant that appears in the Lagrangian (2.20). Let us pick $\varepsilon^\mu = (0,0,0,1)$ to be the polarization vector for the helicity zero state, then

$$\langle \bar{P}_{1/2}|A^{1+i2}_3|\bar{P}^*_{-1/2}\rangle = -\sqrt{MM^*} f \ , \tag{2.45}$$

where the states have the Lorentz-invariant normalization [19]

$$\langle \mathbf{p}_1|\mathbf{p}_2\rangle = 2E(2\pi)^3\delta^3(p_1 - p_2) \ . \tag{2.46}$$

It is convenient for our present calculation to use a discrete normalization by enclosing the system in a large volume. Then

$$\langle\langle \mathbf{p}_1|\mathbf{p}_2\rangle\rangle = \delta_{\mathbf{p}_1\mathbf{p}_2} \ , \tag{2.47}$$

and Eq. (2.44) becomes, in the rest frame of $\bar{P}^*$,

$$\langle\langle \bar{P}_{1/2}|\Sigma^3|\bar{P}^*_{-1/2}\rangle\rangle = \tfrac{1}{2} f \ , \tag{2.48}$$

where

$$\Sigma^3 = -\int d^3x \ A^{1+i2}_3 = \int d^3x \ u^\dagger \sigma^3 d \ . \tag{2.49}$$

Now all we need are the spin configurations of these states in the nonrelativistic quark model:

$$|\bar{P}^*_{-1/2}\rangle\rangle = \frac{1}{\sqrt{2}}[|\bar{Q}\uparrow d\downarrow\rangle + |\bar{Q}\downarrow d\uparrow\rangle] \ , \tag{2.50}$$

$$|\bar{P}_{1/2}\rangle\rangle = \frac{1}{\sqrt{2}}[|\bar{Q}\uparrow u\downarrow\rangle - |\bar{Q}\downarrow u\uparrow\rangle] \ , \tag{2.51}$$

where the arrows indicate the spin directions of the quarks. A simple calculation gives

$$\begin{aligned} \langle\langle \bar{P}_{1/2}|\Sigma^3|\bar{P}^*_{-1/2}\rangle\rangle &= \tfrac{1}{2}[\langle u\downarrow|\Sigma^3|d\downarrow\rangle - \langle u\uparrow|\Sigma^3|d\uparrow\rangle] \\ &= -\langle u\uparrow|\Sigma^3|d\uparrow\rangle \\ &= -1 \ , \end{aligned} \tag{2.52}$$

which yields

$$f = -2 \ . \tag{2.53}$$

Implicit in this calculation is the assumption that in the single quark transition $u \to d$ the g_A for the quarks has the value 1. Such an assumption leads to the well-known result

$$g_A^{\text{nucleon}} = \tfrac{5}{3} \quad (g_A^{ud} = 1) \ , \tag{2.54}$$

which disagrees substantially with the experimental value $g_A^{\text{nucleon}} = 1.25$. One may argue that the value for g_A^{ud} is renormalized to give the correct value of g_A^{nucleon} [20]. Then

$$g_A^{ud} = 0.75 \quad (g_A^{\text{nucleon}} = 1.25) \ . \tag{2.55}$$

If this is accepted, we will have

$$f = -2 \quad (g_A^{ud} = 1), \tag{2.56a}$$

$$f = -1.5 \quad (g_A^{ud} = 0.75) \ . \tag{2.56b}$$

These values of f will be used in Sec. IV when we discuss the applications of our results.

III. DYNAMICS OF HEAVY BARYONS

A heavy baryon contains a heavy quark and two light quarks, which we will often refer to as a diquark. Each light quark is in a triplet of the flavor SU(3). Since

$$3 \times 3 = \bar{3} + 6 \ , \tag{3.1}$$

there are two different SU(3) multiplets of heavy baryons: a symmetric sextet and an antisymmetric antitriplet. As explained in the Introduction, we will make use of the correlation between the symmetries of the flavor wave functions and the spin wave functions of the ground-state baryons in the quark model. Consequently, the diquark in the flavor-symmetric sextet has spin 1, and the diquark in the flavor-antisymmetric antitriplet has spin 0. When the diquark combines with a heavy quark, the sextet contains both spin-$\frac{1}{2}$(B_6) and spin-$\frac{3}{2}$(B_6^*) baryons. However, the antitriplet contains only spin-$\frac{1}{2}$($B_{\bar{3}}$) baryons. We will adopt the following notation used in Ref. [11]. The spin-$\frac{1}{2}$ heavy baryons in the sextet are

$$\Sigma_Q^{+1} = uuQ \ , \tag{3.2a}$$

$$\Sigma_Q^0 = \frac{1}{\sqrt{2}}(ud + du)Q \ , \tag{3.2b}$$

$$\Sigma_Q^{-1} = ddQ \ , \tag{3.2c}$$

$$\Xi_Q'^{+1/2}=\frac{1}{\sqrt{2}}(us+su)Q\ , \tag{3.2d}$$

$$\Xi_Q'^{-1/2}=\frac{1}{\sqrt{2}}(ds+sd)Q\ , \tag{3.2e}$$

$$\Omega_Q=ssQ\ , \tag{3.2f}$$

where the superscript denotes the value of the isospin quantum number I_3. An asterisk on the symbol will denote a corresponding spin-$\frac{3}{2}$ baryon. For example, the symbol Ω_3^* denotes a spin-$\frac{3}{2}$ baryon of ssQ. The spin-$\frac{1}{2}$ heavy baryons in the antitriplet are

$$\Lambda_Q=\frac{1}{\sqrt{2}}(ud-du)Q\ , \tag{3.3a}$$

$$\Xi_Q^{+1/2}=\frac{1}{\sqrt{2}}(us-su)Q\ , \tag{3.3b}$$

$$\Xi_Q^{-1/2}=\frac{1}{\sqrt{2}}(ds-sd)Q\ . \tag{3.3c}$$

The decomposition

$$\begin{aligned} q_{1i}q_{2j}&=\tfrac{1}{2}(q_{1i}q_{2j}+q_{1j}q_{2i})+\tfrac{1}{2}(q_{1i}q_{2j}-q_{1j}q_{2i})\ ,\\ &=(B_6)_{ij}+\frac{1}{\sqrt{2}}(B_{\bar{3}})_{ij}\ , \end{aligned} \tag{3.4}$$

allows us to assemble the sextet and the antitriplet in a systematic and an antisymmetric 3×3 matrix, respectively,

$$B_6=\begin{bmatrix} \Sigma_Q^{+1} & \frac{1}{\sqrt{2}}\Sigma_Q^0 & \frac{1}{\sqrt{2}}\Xi_Q'^{+1/2} \\ \frac{1}{\sqrt{2}}\Sigma_Q^0 & \Sigma_Q^{-1} & \frac{1}{\sqrt{2}}\Xi_Q'^{-1/2} \\ \frac{1}{\sqrt{2}}\Xi_Q'^{+1/2} & \frac{1}{\sqrt{2}}\Xi_Q'^{-1/2} & \Omega_Q \end{bmatrix}\ , \tag{3.5}$$

$$B_{\bar{3}}=\begin{bmatrix} 0 & \Lambda_Q & \Xi_Q^{+1/2} \\ -\Lambda_Q & 0 & \Xi_Q^{-1/2} \\ -\Xi_Q^{+1/2} & -\Xi_Q^{-1/2} & 0 \end{bmatrix}\ , \tag{3.6}$$

and a matrix for B_6^* similar to B_6.

Now that we have determined the flavor SU(3) contents of the heavy baryons, their chiral transformation laws can be established:

$$B_6\to B_6'=UB_6U^T\ , \tag{3.7}$$

$$B_6^*\to B_6^{*\prime}=UB_6^*U^T\ , \tag{3.8}$$

$$B_{\bar{3}}\to B_{\bar{3}}'=UB_{\bar{3}}U^T\ . \tag{3.9}$$

The transformation laws for the antiparticles can be obtained from above by Hermitian conjugation. The covariant derivatives under chiral transformations for B_6 and $B_{\bar{3}}$ are

$$D_\mu B_6\equiv\partial_\mu B_6+V_\mu B_6+B_6V_\mu^T\ , \tag{3.10}$$

$$D_\mu B_{\bar{3}}\equiv\partial_\mu B_{\bar{3}}+V_\mu B_{\bar{3}}+B_{\bar{3}}V_\mu^T\ . \tag{3.11}$$

A similar equation holds for $D_\mu B_6^*$. At first sight two chiral invariants $\mathrm{tr}(\bar{B}_6A_\mu B_6)$ and $\mathrm{tr}(\bar{B}_6B_6A_\mu^T)$ can be constructed out of $\bar{B}_6$, B_6, and A_μ. However, these two are not independent due to the definite symmetry of the matrices B_6 and $\bar{B}_6$. The chiral-invariant Lagrangian is then

$$\begin{aligned} \mathcal{L}_B=&\tfrac{1}{2}\mathrm{tr}[\bar{B}_{\bar{3}}(i\not{D}-M_{\bar{3}})B_{\bar{3}}]+\mathrm{tr}[\bar{B}_6(i\not{D}-M_6)B_6]\\ &+\mathrm{tr}\{\bar{B}_6^{*\mu}[-g_{\mu\nu}(i\not{D}-M_6^*)+i(\gamma_\mu D_\nu+\gamma_\nu D_\mu)-\gamma_\mu(i\not{D}+M_6^*)\gamma_\nu]B_6^{*\nu}\}\\ &+g_1\mathrm{tr}(\bar{B}_6\gamma_\mu\gamma_5A^\mu B_6)+g_2\mathrm{tr}(\bar{B}_6\gamma_\mu\gamma_5A^\mu B_{\bar{3}})+\mathrm{H.c.}\\ &+g_3\mathrm{tr}(\bar{B}_{6\mu}^*A^\mu B_6)+\mathrm{H.c.}+g_4\mathrm{tr}(\bar{B}_6^{*\mu}A_\mu B_{\bar{3}})+\mathrm{H.c.}+g_5\mathrm{tr}(\bar{B}_6^{*\nu}\gamma_\mu\gamma_5A^\mu B_{6\nu}^*)+g_6\mathrm{tr}(\bar{B}_{\bar{3}}\gamma_\mu\gamma_5A^\mu B_{\bar{3}})\ , \end{aligned} \tag{3.12}$$

where $B_{6\mu}^*$ is a Rarita-Schwinger vector-spinor field for a spin-$\frac{3}{2}$ particle [21] and A_μ is the axial field introduced in (2.12). For a similar reason to that given in the preceding section, we do not need to write the Lagrangian $\mathcal{L}_B$ in terms of velocity-dependent fields [17].

We now show that the heavy-quark spin symmetry reduces the six coupling constants to two independent ones. For this purpose, it is convenient to introduce the interpolating fields [12,14] for the heavy baryons in terms of the diquark fields of the light quarks. Since the ground-state baryons have even parity, the diquarks must have also even parity. Therefore, the diquark in the $B_{\bar{3}}$ multiplet has spin parity 0^+; hence, it is represented by a Lorentz scalar field ϕ. Similarly the diquark in the B_6 multiplet has spin parity 1^+; hence it is represented by an *axial*-vector field ϕ_μ. The interpolating fields are

$$B_{\bar{3}}(v,s)=\bar{u}(v,s)\phi_v h_v\ , \tag{3.13}$$

$$B_6(v,s,\kappa)=\bar{B}_\mu(v,s,\kappa)\phi_v^\mu h_v,\quad \kappa=1,2\ , \tag{3.14}$$

where $\kappa=1$ for spin-$\frac{1}{2}$ baryons and $\kappa=2$ for spin-$\frac{3}{2}$ baryons. The wave function $\bar{B}_\mu$ is given by Georgi [12]:

$$\bar{B}_\mu(v,s,\kappa=1)=\frac{1}{\sqrt{3}}\bar{u}(v,s)\gamma_5(\gamma_\mu+v_\mu)\ , \tag{3.15}$$

$$\bar{B}_\mu(v,s,\kappa=2)=\bar{u}_\mu(v,s)\ , \tag{3.16}$$

with $u_\mu(v,s)$ and $u(v,s)$ being the Rarita-Schwinger vector spinor and usual Dirac spinor respectively. The corresponding interpolating fields for the antibaryons are

$$\bar{B}_{\bar{3}}(v,s)=\bar{h}_v\phi_v^\dagger u(v,s)\ , \tag{3.17}$$

$$\bar{B}_6(v,s,\kappa)=\bar{h}_v\phi_v^{\mu\dagger}B_\mu(v,s\kappa),\quad \kappa=1,2\ , \tag{3.18}$$

where

$$B_\mu(v,s,\kappa=1)=-\frac{1}{\sqrt{3}}(\gamma_\mu+v_\mu)\gamma_5 u(v,s)\ , \tag{3.19}$$

$$B_\mu(v,s,\kappa=2)=u_\mu(v,s)\ , \tag{3.20}$$

the minus sign in (3.19) is a result of the anticommutation relation $\{\gamma_5,\gamma_0\}=0$. The relative sign is important when we compute the relations between the different coupling constants. The function $B_\mu(v,s,\kappa)$ satisfies

$$\not{v}B_\mu(v,s)=B_\mu(v,s)\ , \tag{3.21}$$

$$v^\mu B_\mu(v,s)=0\ . \tag{3.22}$$

With the normalization

$$\bar{u}_\mu(v,s)u^\mu(v,s')=-\delta_{ss'}\ , \tag{3.23}$$

$$\bar{u}(v,s)u(v,s')=\delta_{ss'}\ , \tag{3.24}$$

the coefficients in Eqs. (3.15) and (3.16) are chosen to have the proper relative weight for forming a spin-$\frac{3}{2}$ and a spin-$\frac{1}{2}$ field out of a spin-1 field ϕ_μ and a spin-$\frac{1}{2}$ field Q. The diquark fields ϕ and ϕ_μ are matrices in flavor SU(3) space. No factors such as $\sqrt{M_B}$ or $\sqrt{M_{B^*}}$ appear in the interpolating fields as a result of the normalization for baryon states [19]:

$$\langle B(v,s)|B(v',s')\rangle=\frac{E}{M_B}(2\pi)^3\delta^3(p-p')\delta_{ss'}\ . \tag{3.25}$$

Using the interpolating field (3.13) we find the matrix element for the divergence of the SU(2) axial vector current:

$$\begin{aligned}\langle B_{\bar{3}}(v',s')|q^\mu A_\mu^a|B_{\bar{3}}(v,s)\rangle&=\langle 0|\bar{u}(v',s')\phi_{v'}h_{v'}(q^\mu A_\mu^a)\bar{h}_v\phi_v^\dagger u(v,s)|0\rangle\\&=\langle 0|\bar{u}(v',s')h_{v'}\bar{h}_v u(v,s)|0\rangle\langle 0|\phi_{v'}(q^\mu A_\mu^a)\phi_v^\dagger|0\rangle\ .\end{aligned} \tag{3.26}$$

In the soft pion limit $v'\cong v$, so

$$\begin{aligned}\langle 0|\bar{u}(v',s')h_{v'}\bar{h}_v u(v,s)|0\rangle&=\bar{u}(v',s')\frac{\not{v}+1}{2}u(v,s)\\&=\bar{u}(v',s')u(v,s)\ .\end{aligned} \tag{3.27}$$

Lorentz invariance implies that

$$\langle 0|\phi_{v'}(q^\mu A_\mu^a)\phi_v^\dagger|0\rangle\equiv 0 \tag{3.28}$$

since no pseudoscalar quantity can be formed from v_μ and v'_μ or v_μ and q_μ. By comparing with the one-pion emission matrix element obtained from the Lagrangian (3.12) we find

$$g_6\equiv 0\ . \tag{3.29}$$

This is an interesting and surprising result. It is a statement that in the heavy-quark limit, the pion is emitted from the light quarks and the transition $\phi_v\to\phi_{v'}+\pi$ does not conserve parity. Next, consider the $B_6B_{\bar{3}}$ couplings. We have

$$\begin{aligned}&\langle B_6(v',s',\kappa)|q^\mu A_\mu^a|B_{\bar{3}}(v,s)\rangle\\&\quad=\langle 0|\bar{B}_\nu(v',s',\kappa)h_{v'}\phi_{v'}^\nu(q^\mu A_\mu^a)\phi_v^\dagger\bar{h}_v u(v,s)|0\rangle\\&\quad=\langle 0|\bar{B}_\nu(v',s',\kappa)h_{v'}\bar{h}_v u(v,s)|0\rangle M^\nu(v,q)\ ,\end{aligned} \tag{3.30}$$

where

$$M^\nu(v,q)=\langle 0|\phi_{v'}^\nu(q^\mu A_\mu^a)\phi_v^\dagger|0\rangle \tag{3.31}$$

is a four-vector (both $\phi_{v'}^\nu$ and A_μ^a are axial vectors) and we are only interested in terms linear in q. Therefore,

$$M_\nu(v,q)=\beta_1 q_\nu+\beta_2 v_\nu(v\cdot q)\ . \tag{3.32}$$

In the soft pion and heavy-quark limit,

$$v_\nu=v'_\nu+O\left[\frac{q}{M_B}\right] \tag{3.33}$$

and

$$\bar{B}_\nu(v',s',\kappa)v'^\nu=0\ , \tag{3.34}$$

so finally

$$\langle B_6(v',s',\kappa)|q^\mu A_\mu^a|B_{\bar{3}}(v,s)\rangle=\beta_1\bar{B}_\mu(v',s',\kappa)q^\mu u(v,s)\ , \tag{3.35}$$

which gives

$$\langle B_6^*(v',s')|q^\mu A_\mu^a|B_{\bar{3}}(v,s)\rangle=\beta_1\bar{u}_\mu(v',s')q^\mu u(v,s)\ , \tag{3.36}$$

$$\langle B_6(v',s')|q^\mu A_\mu^a|B_{\bar{3}}(v,s)\rangle=\frac{1}{\sqrt{3}}\beta_1\bar{u}(v',s')\gamma_5(\gamma^\mu+v^\mu)q_\mu u(v,s)\ . \tag{3.37}$$

Now

$$v_\mu=\tfrac{1}{2}(\not{v}\gamma_\mu+\gamma_\mu\not{v})=\tfrac{1}{2}(\not{v}'\gamma_\mu+\gamma_\mu\not{v})+O\left[\frac{q}{M_B}\right] . \quad (3.38)$$

Using the relations

$$\bar{u}(v',s')\not{v}'=\bar{u}(v',s') , \quad (3.39)$$

$$\not{v}u(v,s)=u(v,s) , \quad (3.40)$$

we get

$$\langle B_6(v',s')|q^\mu A^a_\mu|B_{\bar{3}}(v,s)\rangle = -\frac{1}{\sqrt{3}}\beta_1\bar{u}(v',s')\gamma^\mu\gamma_5 q_\mu u(v,s) . \quad (3.41)$$

By comparing Eqs. (3.36) and (3.41) with the one-pion emission matrix elements obtained from the Lagrangian (3.12) one finds the relations

$$g_2=-\frac{1}{\sqrt{3}}\beta_1 , \quad (3.42)$$

$$g_4=\beta_1 . \quad (3.43)$$

In obtaining the results (3.42) and (3.43), we have suppressed the SU(3) flavor quantum numbers and neglected an overall normalization. These are all irrelevant, since we are only interested in the ratio of g_2 to g_4.

We now turn to the B_6B_6 couplings. As before we find

$$\langle B_6(v',s',\kappa')|q^\mu A^a_\mu|B_6(v,s,\kappa)\rangle = \bar{B}_\mu(v',s',\kappa')B_\nu(v,s,\kappa)M^{\mu\nu}(v,q) , \quad (3.44)$$

where

$$M^{\mu\nu}(v,q)=\langle 0|\phi^\mu_v q^\lambda A^a_\lambda \phi^{\nu\dagger}_v|0\rangle , \quad (3.45)$$

which is a second rank pseudotensor. Thus

$$M^{\mu\nu}(v,q)=i\varepsilon^{\mu\nu\lambda\kappa}q_\lambda v_\kappa\delta . \quad (3.46)$$

To simplify (3.44) when (3.46) is substituted, we find the following identity useful:

$$i\varepsilon^{\mu\nu\lambda\kappa}\gamma_\kappa=-\gamma^\mu\gamma^\nu\gamma^\lambda\gamma_5+g^{\mu\nu}\gamma^\lambda\gamma_5 -g^{\mu\lambda}\gamma^\nu\gamma_5+g^{\nu\lambda}\gamma^\mu\gamma_5 . \quad (3.47)$$

It follows from (3.44) and (3.46) that

$$\langle B_6(v',s')|q^\mu A^a_\mu|B_6(v,s)\rangle=\frac{2}{3}\delta\bar{u}(v',s')\gamma^\mu\gamma_5 q_\mu u(v,s) , \quad (3.48)$$

$$\langle B^*_6(v',s')|q^\mu A^a_\mu|B_6(v,s)\rangle=\frac{1}{\sqrt{3}}\delta\bar{u}^{\,\mu}(v',s')q_\mu u(v,s) , \quad (3.49)$$

$$\langle B^*_6(v',s')|q^\mu A^a_\mu|B^*_6(v,s)\rangle = -\delta\bar{u}^{\,\nu}(v',s')\gamma_\mu\gamma_5 q^\mu u_\nu(v,s) . \quad (3.50)$$

Comparing these results with the one-pion emission matrix elements from the Lagrangian (3.12) we obtain

$$g_1=\frac{2}{3}\delta , \quad (3.51)$$

$$g_3=\frac{1}{\sqrt{3}}\delta , \quad (3.52)$$

$$g_5=-\delta . \quad (3.53)$$

Equations (3.29), (3.42), (3.43), and Eqs. (3.51)–(3.53) together give

$$g_3=\frac{1}{2}\sqrt{3}g_1 , \quad (3.54)$$

$$g_5=-\frac{3}{2}g_1 , \quad (3.55)$$

$$g_4=-\sqrt{3}g_2 , \quad (3.56)$$

$$g_6=0 . \quad (3.57)$$

Consequently, only two of the six coupling constants in the baryon sector are independent. Furthermore, these two coupling constants g_1 and g_2 (and all the three other nonzero coupling constants) are independent of the heavy masses.

The quark-model calculation of the coupling constants g_1 and g_2 follow the same procedure as in the meson case. We will do the calculation for g_2 first. The Lagrangian (3.12) leads to the coupling

$$\mathcal{L}_{\Sigma_Q\Lambda_Q\pi}=\frac{g_2}{\sqrt{2}f_\pi}\bar{\Sigma}^{+1}_Q\gamma^\mu\gamma_5\Lambda_Q\partial_\mu\pi^+ . \quad (3.58)$$

We define $g_A^{\Sigma_Q\Lambda_Q}$ by

$$\langle\Sigma^{+1}_Q|A^1_\mu+iA^2_\mu|\Lambda_Q\rangle =\bar{u}(\Sigma^{+1}_Q)\gamma_\mu\gamma_5 g_A^{\Sigma_Q\Lambda_Q}(q^2)u(\Lambda_Q)+\cdots , \quad (3.59)$$

where the unlisted terms vanish at $q=0$. A simple application of PCAC shows that

$$g_2=-g_A^{\Sigma_Q\Lambda_Q}(0) . \quad (3.60)$$

As in the meson case, in the baryon rest frame Eq. (3.59) becomes

$$\langle\langle\Sigma^{+1}_Q\uparrow|\Sigma^3|\Lambda_Q\uparrow\rangle\rangle=g_A^{\Sigma_Q\Lambda_Q}(0) , \quad (3.61)$$

where the notation is the same as introduced in Sec. II. The flavor-spin wave functions needed are

$$|\Lambda_Q\uparrow\rangle\rangle=|Q\uparrow\rangle\frac{1}{\sqrt{2}}(|ud\rangle-|du\rangle)\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle) =\frac{1}{2}|Q\uparrow\rangle(|u\uparrow d\downarrow\rangle-|u\downarrow d\uparrow\rangle-|d\uparrow u\downarrow\rangle+|d\downarrow u\uparrow\rangle) , \quad (3.62)$$

$$|\Sigma_Q^{+1}\uparrow\rangle\rangle=\sqrt{\tfrac{2}{3}}|Q\downarrow\rangle|uu\rangle|\uparrow\uparrow\rangle-\sqrt{\tfrac{1}{3}}|Q\uparrow\rangle|uu\rangle\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)$$
$$=\sqrt{\tfrac{2}{3}}|Q\downarrow\rangle|u\uparrow u\uparrow\rangle-\sqrt{\tfrac{1}{6}}|Q\uparrow\rangle(|u\uparrow u\downarrow\rangle+|u\downarrow u\uparrow\rangle)\ . \tag{3.63}$$

Equations (3.62) and (3.63) then give

$$\langle\langle\Sigma_Q^{+1}\uparrow|\Sigma^3|\Lambda_Q\uparrow\rangle\rangle=-\tfrac{1}{2}\sqrt{\tfrac{1}{6}}(-4)\langle u\uparrow|\Sigma^3|d\uparrow\rangle$$
$$=\sqrt{\tfrac{2}{3}}\ . \tag{3.64}$$

Hence

$$g_2=-\sqrt{\tfrac{2}{3}}\ . \tag{3.65}$$

The calculation of g_1 follows the same steps. The Lagrangian (3.12) contains the coupling

$$\mathcal{L}_{\Xi'_Q\Xi'_Q\pi}=-\frac{g_1}{f_\pi}\bar{\Xi}'_Q\gamma_\mu\gamma_5\frac{1}{2}\tau_a\partial_\mu\pi^a\Xi'_Q\ . \tag{3.66}$$

Let us define a $g_A^{\Xi'_Q}$ according to

$$\langle\Xi_Q'^{+1/2}|A_\mu^1+iA_\mu^2|\Xi_Q'^{-1/2}\rangle$$
$$=\bar{u}(\Xi_Q'^{+1/2})\gamma_\mu\gamma_5u(\Xi_Q'^{-1/2})g_A^{\Xi'_Q}(q^2)+\cdots\ . \tag{3.67}$$

Again, a simple application of PCAC gives the relation

$$g_1=\tfrac{1}{2}g_A^{\Xi'_Q}(0)\ . \tag{3.68}$$

In the baryon rest frame we have

$$g_A^{\Xi'_Q}(0)=\langle\langle\Xi_Q'^{+1/2}\uparrow|\Sigma^3|\Xi_Q'^{-1/2}\uparrow\rangle\rangle\ . \tag{3.69}$$

The spin wave functions of these states are

$$|\Xi_Q'^{+1/2}\uparrow\rangle\rangle=\sqrt{\tfrac{2}{3}}|Q\downarrow\rangle\frac{1}{\sqrt{2}}(|us\rangle+|su\rangle)|\uparrow\uparrow\rangle-\sqrt{\tfrac{1}{3}}|Q\uparrow\rangle\frac{1}{\sqrt{2}}(|us\rangle+|su\rangle)\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)$$
$$=\sqrt{\tfrac{1}{3}}|Q\downarrow\rangle(|u\uparrow s\uparrow\rangle+|s\uparrow u\uparrow\rangle)-\tfrac{1}{2}\sqrt{\tfrac{1}{3}}|Q\uparrow\rangle(|u\uparrow s\downarrow\rangle+|s\uparrow u\downarrow\rangle+|u\downarrow s\uparrow\rangle+|s\downarrow u\uparrow\rangle)\ , \tag{3.70}$$

and a similar equation for $|\Xi_Q'^{-1/2}\uparrow\rangle$ with u replaced by d in the above. One finds

$$g_A^{\Xi'_Q}(0)=\tfrac{2}{3}\ ; \tag{3.71}$$

therefore,

$$g_1=\tfrac{1}{3}\ . \tag{3.72}$$

As in the meson case, the values for g_1 and g_2 obtained so far assume that $g_A^{ud}=1$. Therefore [20]

$$g_A^{ud}=1:\ \ g_1=\tfrac{1}{3},\ \ g_2=-\sqrt{\tfrac{2}{3}}\ , \tag{3.73}$$

$$g_A^{ud}=0.75:\ \ g_1=\tfrac{1}{3}\times0.75,\ \ g_2=-\sqrt{\tfrac{2}{3}}\times0.75\ . \tag{3.74}$$

We will make use of these results in the applications presented in the next section.

IV. APPLICATIONS

In this section we apply our results obtained so far to the strong decays and the semileptonic decays of the heavy hadrons. Recall that the heavy quark symmetry has greatly reduced the number of parameters in the low-energy interactions among the Goldstone bosons and the heavy hadrons. Among the six coupling constants only three are independent: one in the meson sector and two in the baryon sector. In addition, the nonrelativistic quark model has simple predictions for all three of them. We will first consider the implications of these results in a few examples of the strong decays of these heavy particles. The remainder of the section is devoted to the semileptonic weak decays of the heavy hadrons with one pion emission. Implicit in this application is the assumption that saturation of the intermediate states by the ground-state heavy particles is a reasonable approximation.

The main purpose of the present discussion is to bring out the important features of the formalism. The most striking among these is in the meson sector. Here, the flavor symmetry and the spin symmetry for the heavy quarks and the chiral symmetry of the light quarks completely determine the relative magnitudes and their relative phases among the different strong and weak vertices contributing to different Feynman diagrams of a given decay. The whole amplitude can be expressed in terms of a universal Isgur-Wise function and a single coupling constant all as overall factors. When the lepton pair carries away most of the available energy, the Isgur-Wise function takes the value unity, and the emitted pions are necessarily soft. In this limited corner of phase space, both the heavy-quark symmetry and chiral symmetry, when supplemented by the quark model, combine to determine the dynamics completely.

The situation in the baryon sector is somewhat more complicated. In this case there are two independent coupling constants in the chiral Lagrangian and three independent heavy-quark weak-decay form factors which correspond to one for $\Lambda_{Qi}\to\Lambda_{Qj}+l\nu$, and two for $\Sigma_{Qi}\to\Sigma_{Qj}+l\nu$. Again in the special kinematic region specified above, two of the form factors take the value unity, and a third one will drop out from the decay amplitude. We will restrict our present discussion to this special case. In a future publication we will present, among other things, a more detailed and complete analysis of the examples considered here.

The strong decay of a heavy 1^- meson into a 0^- heavy meson plus a pion is described by the following amplitude derived from the Lagrangian (2.20):

$$M(P^*\to P+\pi^a(q))=u^*(P^*)\frac{\tau_a}{2}u(P)\sqrt{M_P M_{P^*}}\frac{f}{f_\pi}(\varepsilon\cdot q) \tag{4.1}$$

The widths implied by the amplitude (4.1) are

$$\Gamma(P^*_{1/2}\to P_{-1/2}+\pi^+)=\frac{1}{24\pi}\left[\frac{f}{\sqrt{2}f_\pi}\right]^2\left[\frac{M_P}{M_{P^*}}\right]q^3 , \tag{4.2a}$$

$$\Gamma(P^*_{1/2}\to P_{1/2}+\pi^0)=\frac{1}{48\pi}\left[\frac{f}{\sqrt{2}f_\pi}\right]^2\left[\frac{M_P}{M_{P^*}}\right]q^3 . \tag{4.2b}$$

For the decay $D^{*-}\to D^0\pi^-$, with $q=39$ MeV, Eq. (4.2) gives

$$\Gamma(D^{*-}\to D^0\pi^-)=0.045f^2 \text{ MeV} . \tag{4.3}$$

The quark mode predictions for f then yield

$$\Gamma(D^{*-}\to D^0\pi^-)=\begin{cases}0.18 \text{ MeV} & (f=-2) , \quad (4.4a)\\ 0.10 \text{ MeV} & (f=-1.5) . \quad (4.4b)\end{cases}$$

This is consistent with the experimental upper bound [22]

$$\Gamma_{\text{tot}}(D^{*+})<1.1 \text{ MeV} . \tag{4.5}$$

To test the heavy-mass dependence of f_Q (2.39) we must compare the predictions (4.2) for two different heavy-quark species. Unfortunately, the decay $B^*\to B\pi$ is forbidden because the phase space is too small. The other example is $K^*\to K\pi$. However, it is questionable whether the s quark is qualified as a heavy quark. Let us assume it is; then Eq. (4.2) gives

$$\Gamma(K^{*-}\to K^0\pi^-+K^-\pi^0)=\begin{cases}61 \text{ MeV} & (f=-2) , \quad (4.6a)\\ 34 \text{ MeV} & (f=-1.5) , \quad (4.6b)\end{cases}$$

as compared with the experimental value [22]

$$\Gamma(K^{*-}\to K^-\pi^0+K^0\pi^-)=49.8\pm0.8 \text{ MeV} . \tag{4.7}$$

We note that the factor M_P/M_{P^*} in Eq. (4.2) should be unity in the heavy-quark limit. Experimentally, its value is 0.55 for $K^*\to K\pi$ and 0.93 for $D^*\to D\pi$. It is not unreasonable that the heavy-quark symmetry predictions for $K^*\to K\pi$ should differ substantially from the experimental value. Indeed, we find it encouraging that the theoretical prediction even has the right order of magnitude.

Let us compare the results (4.4) with those in some other models [23]. An SU(4) model [24] predicts $\Gamma(D^{*-}\to D^0\pi^-)=17$ keV and $\Gamma(D^{*-}\to D^-\pi^0)=7.4$ keV, and a QCD-sum-rule calculation [24] gives results which are about one-half of those of SU(4). The SU(4) results are easy to understand since the value of the coupling constant in that model is obtained from the rate for $K^*\to K\pi$ and it does not vary with the quark masses. If we remove the mass dependence (2.39), the results (4.4) should be reduced by a factor 8.5 which is how much the mass dependence has changed from the $K^*\to K\pi$ to $D^*\to D\pi$. One calculation is closer to ours in spirit. Based on their experience with quarkonia, Eichten *et al.* [25] proposed a scaling formula

$$\Gamma(P^*\to P+\pi)=A\frac{q^3}{M^*}E_P E_\pi . \tag{4.8}$$

Using the experimental rate for $K^*\to K\pi$ as input, Eichten *et al.* found $\Gamma(D^{*-}\to D^0\pi^-)\sim 53$ keV. For the heavy baryons, the strong decay

$$\Sigma_c^{++}\to\Lambda_c^{+}+\pi \tag{4.9}$$

has been observed experimentally, though its actual rate has not been measured. The chiral Lagrangian (3.12) predicts for the general decay $\Sigma_Q^{+1}\to\Lambda_Q+\pi$

$$\Gamma(\Sigma_Q^{+1}\to\Lambda_Q+\pi^+)=\frac{1}{8\pi}g^2_{\Sigma_Q\Lambda_Q\pi}\frac{(M_{\Sigma_Q}-M_{\Lambda_Q})^2-M_\pi^2}{M^2_{\Sigma_Q}}q , \tag{4.10}$$

where q is the pion momentum in the c.m. system and

$$g_{\Sigma_Q\Lambda_Q\pi}=\frac{M_{\Sigma_Q}+M_{\Lambda_Q}}{\sqrt{2}f_\pi}g_2 \tag{4.11}$$

is the Goldberger-Treiman relation. The quark-model prediction (3.65)

$$g_2=-\sqrt{\tfrac{2}{3}}g_A^{ud} \tag{4.12}$$

gives

$$g_{\Sigma_c\Lambda_c\pi}=\begin{cases}-29.3 & (g_A^{ud}=1) , \quad (4.13a)\\ -22 & (g_A^{ud}=0.75) , \quad (4.13b)\end{cases}$$

which, in turn, leads to

$$\Gamma(\Sigma_c^0\to\Lambda^+\pi^-)=\begin{cases}4.35 \text{ MeV} & (g_A^{ud}=1) , \quad (4.14a)\\ 2.45 \text{ MeV} & (g_A^{ud}=0.75) . \quad (4.14b)\end{cases}$$

The quark-model results are not very different from other predictions. The MIT bag model calculation [26] gives $g_A^{\Sigma_c\Lambda_c}=-\sqrt{\tfrac{2}{3}}(0.65)$ leading to $|g_{\Sigma_c\Lambda_c\pi}|\cong 19$ from the Goldberger-Treiman relation. Another calculation [27] finds $|g_{\Sigma_c\Lambda_c\pi}|=23.5$ by exploiting the null result of Coleman and Glashow [28] for the tadpole model of symmetry breaking. Aside from the observed transition $\Sigma_c \to \Lambda_c + \pi$, the test of the heavy-quark symmetry and chiral symmetry must await the experimental discovery of $\Sigma_b \to \Lambda_b + \pi$ in the future.

We will now study a few examples of the semileptonic decays of the heavy mesons with one-pion emission. We begin by listing all the form factors needed later [1,6]. They are [19]

$$\langle P_j(v')|V_\mu^{ji}|P_i(v)\rangle = \sqrt{M_i M_j}\, C_{ji}\xi(v+v')_\mu \,, \tag{4.15a}$$

$$\langle P_j^*(v',\varepsilon')|V_\mu^{ji}|P_i(v)\rangle = \sqrt{M_i M_j^*}\, C_{ji}\xi i \varepsilon_{\mu\nu\lambda\kappa}\varepsilon'^{\nu} v'^{\lambda} v^{\kappa} \,, \tag{4.15b}$$

$$\langle P_j^*(v',\varepsilon')|A_\mu^{ji}|P_i(v)\rangle = \sqrt{M_i M_j^*}\, C_{ji}\xi[(1+v\cdot v')\varepsilon'_\mu - (\varepsilon'\cdot v)v'_\mu] \,, \tag{4.15c}$$

$$\langle P_j(v')|V_\mu^{ji}|P_i^*(v,\varepsilon)\rangle = \sqrt{M_i M_j^*}\, C_{ji}\xi i \varepsilon_{\mu\nu\lambda\kappa}\varepsilon^{\nu} v'^{\lambda} v^{\kappa} \,, \tag{4.15d}$$

$$\langle P_j(v')|A_\mu^{ji}|P_i^*(v,\varepsilon)\rangle = \sqrt{M_i M_j^*}\, C_{ji}\xi[(1+v\cdot v')\varepsilon_\mu - (\varepsilon\cdot v')v_\mu] \,, \tag{4.15e}$$

$$\langle P_j^*(v',\varepsilon')|V_\mu^{ji}|P_i^*(v,\varepsilon)\rangle = -\sqrt{M_i^* M_j^*}\, C_{ij}\xi[(\varepsilon'\cdot\varepsilon)(v+v')_\mu - (\varepsilon'\cdot v)\varepsilon_\mu - (\varepsilon\cdot v')\varepsilon'_\mu] \,, \tag{4.15f}$$

$$\langle P_j^*(v',\varepsilon')|A_\mu^{ji}|P_i^*(v,\varepsilon)\rangle = -\sqrt{M_i^* M_j^*}\, C_{ji}\xi i \varepsilon_{\mu\nu\lambda\kappa}\varepsilon'^{\nu}\varepsilon^{\lambda}(v+v')^{\kappa} \,, \tag{4.15g}$$

where ξ is the universal Isgur-Wise function depending on the variable $v \cdot v'$ normalized to

$$\xi(v\cdot v'=1)=1 \tag{4.16}$$

and C_{ji} is a renormalization effect which takes into account the large logarithms of M_Q's:

$$C_{ji}(v\cdot v') = \left[\frac{\alpha_s(M_i)}{\alpha_s(M_j)}\right]^{a_I}\left[\frac{\alpha_s(M_j)}{\alpha_s(\mu)}\right]^{a_L(v\cdot v')} \,, \tag{4.17}$$

where, for the $b \to c$ transition [2,5,13],

$$a_I = -\frac{6}{25} \,, \tag{4.18a}$$

$$a_L(w) = \frac{8}{27}[wr(w)-1] \,, \tag{4.18b}$$

$$r(w) = \frac{1}{\sqrt{w^2-1}}\ln(w+\sqrt{w^2-1}) \,. \tag{4.18c}$$

The vector and axial-vector currents are

$$V_\mu^{ji} = \bar{h}_{v'}^{j}\gamma_\mu h_v^i \,, \tag{4.19a}$$

$$A_\mu^{ji} = \bar{h}_{v'}^{j}\gamma_\mu\gamma_5 h_v^i \,, \tag{4.19b}$$

with the heavy-quark fields h_v^i and $h_{v'}^i$ being those introduced in the beginning of Sec. II. The last two equations (4.15f) and (4.15g) in the above list did not seem to exist in the literature but they are easily derived by the method of interpolating fields. In the derivation one encounters the ubiquitous matrix which appears in the derivation of all the results in (4.15) [29]:

$$M \equiv \langle 0|q_v \bar{q}_{v'}|0\rangle = A + B\not{v} + C\not{v}' + D\not{v}\not{v}' \,, \tag{4.20}$$

where Lorentz invariance dictates the structure exhibited on the right-hand side. The universal function ξ is to be identified with [29]

$$\xi = A - B - C + D \,. \tag{4.21}$$

In addition to (4.1), we also need the strong decay matrix elements which follow from the Lagrangian (2.20)

$$M[P_i^*(v,\varepsilon) \to P_i^*(v',\varepsilon') + \pi^a(q)] = -i\frac{g}{f_\pi}M_i^* \varepsilon_{\rho\sigma\lambda\kappa}q^\rho \varepsilon^\sigma \varepsilon'^\lambda (v+v')^\kappa u^*(P_i^*(v)) \times \tfrac{1}{2}\tau_a u(P_i^*(v')) \,. \tag{4.22}$$

The first example we will consider is the semileptonic decay $B \to D + \pi + l\nu$. The amplitude has contributions from the two Feynman diagrams listed in Fig. 1. The vertices can be read off from Eqs. (4.15) and (4.1). Each propagator combines with the $\sqrt{M}$ factors from the adjacent vertices to give

$$\frac{iM_{B^*}}{(P-q)^2 - M_{B^*}^2} = \frac{i}{-2v\cdot q + 2(M_B - M_{B^*})} \,, \tag{4.23a}$$

$$\frac{iM_{D^*}}{(P'+q)^2 - M_{D^*}^2} = \frac{i}{+2v'\cdot q + 2(M_D - M_{D^*}^*)} \,, \tag{4.23b}$$

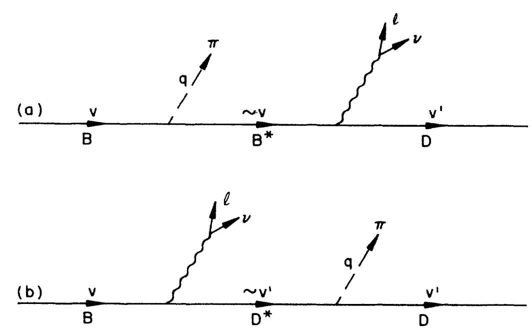

FIG. 1. Feynman diagrams contributing to the decay $B \to D + \pi + l\nu$. The four-velocities of B and D are v and v' respectively. The pion momentum is q.

where v and v' are the four-velocity of the initial B meson and final D meson, respectively, and q is the four-momentum of the emitted pion. We have exercised phenomenological pragmatism in keeping the first-order mass difference between a 1^- and a 0^- meson. We will work out the amplitude due to Fig. 1(a) in some detail. First, in the soft pion limit, the intermediate state B^* can be treated as on shell except for its propagator. So we have

$$\langle D(v')\pi^a(q)|V_\mu^{cb}-A_\mu^{cb}|B(v)\rangle=M_{a\mu}+M_{b\mu}\ , \tag{4.24}$$

where $M_{a\mu}$ and $M_{b\mu}$ are the contributions from Fig. 1(a) and Fig. 1(b), respectively, and

$$M_{a\mu}=u(B)^*\tfrac{1}{2}\tau_a u(D)\sqrt{M_BM_D}C_{cb}\xi\frac{f}{f_\pi}\sum_{\text{pol}}[i\varepsilon_{\mu\nu\lambda\kappa}\varepsilon^\nu v'^\lambda v^\kappa-(1+v\cdot v')\varepsilon_\mu+(\varepsilon\cdot v')v_\mu]\frac{i}{-2v\cdot q+2(M_B-M_{B^*})}(\varepsilon\cdot q)\ . \tag{4.25}$$

This can be simplified further by the polarization sum

$$\sum_{\text{pol}}\varepsilon_\lambda(v)\varepsilon_\kappa(v)=-g_{\lambda\kappa}+v_\lambda v_\kappa\ . \tag{4.26}$$

The final result is

$$\begin{aligned}\langle D(v')\pi^a(q)|V_\mu^{cb}-A_\mu^{cb}|B(v)\rangle=&\,iu(B)^*\tfrac{1}{2}\tau_a u(D)\sqrt{M_BM_D}\frac{f}{f_\pi}C_{cb}(v\cdot v')\xi(v\cdot v')\\&\times\left\{\frac{1}{2v\cdot q-2(M_B-M_{B^*})}[i\varepsilon_{\mu\nu\lambda\kappa}q^\nu v'^\lambda v^\kappa+q\cdot(v+v')v_\mu-(1+v\cdot v')q_\mu]\right.\\&\left.-\frac{1}{2v'\cdot q+2(M_D-M_{D^*})}[i\varepsilon_{\mu\nu\lambda\kappa}q^\nu v'^\lambda v^\kappa+q\cdot(v+v')v'_\mu-(1+v\cdot v')q_\mu]\right\}\ .\end{aligned} \tag{4.27}$$

Even though the lepton pairs and the pions in the two diagrams are emitted from different particles, the heavy-quark symmetry allows us to combine them without ambiguity in relative magnitude and relative phase. The second example $B\to D^*+\pi+l\nu$ is even more interesting. All five transition vertices in Eqs. (4.15) appear in the single decay; so do two different strong pion emission vertices $PP^*\pi$ and $P^*P^*\pi$. Again, their relative magnitudes and relative phases are known. In addition, though the physical process $B^*\to B\pi$ is forbidden by phase space, its coupling is needed for both of our examples. The amplitude for $B\to D^*+\pi+l\nu$ has three Feynman diagrams as depicted in Fig. 2. A calculation similar to the one for $B\to D+\pi+l\nu$ gives

$$\langle D^*(v',\varepsilon')\pi^a(q)|V_\mu^{cb}-A_\mu^{cb}|B(v)\rangle=-iu(B)^*\tfrac{1}{2}\tau_a u(D^*)\sqrt{M_BM_{D^*}}\frac{f}{f_\pi}C_{cb}(v\cdot v')\xi(v\cdot v')(B_{a\mu}+B_{b\mu}+B_{c\mu})\ , \tag{4.28}$$

where

$$\begin{aligned}B_{a\mu}=&\frac{1}{-2v\cdot q+2(M_B-M_{B^*})}\{i\varepsilon_{\mu\nu\lambda\kappa}\varepsilon'^\nu(v+v')^\kappa(q^\lambda-q\cdot vv^\lambda)+[(\varepsilon'\cdot v)(v\cdot q)-\varepsilon'\cdot q](v_\mu+v'_\mu)\\&+(\varepsilon'\cdot v)(q_\mu-q\cdot vv_\mu)+[q\cdot v'-(v\cdot v')(v\cdot q)]\varepsilon'_\mu\}\ ,\end{aligned} \tag{4.29a}$$

$$B_{b\mu}=\frac{1}{2v'\cdot q}\{i\varepsilon_{\rho\sigma\lambda\kappa}q^\rho\varepsilon'^\sigma v'^\lambda[(1+v\cdot v')g_\mu^\kappa-v^\kappa v'_\mu]+(\varepsilon'\cdot v)q_\mu+[(q\cdot v')(v\cdot v')-q\cdot v]\varepsilon'_\mu-(q\cdot v')(\varepsilon'\cdot v)v'_\mu\}\ , \tag{4.29b}$$

$$B_{c\mu}=-\frac{1}{2v'\cdot q+2(M_{D^*}-M_D)}(\varepsilon'\cdot q)(v_\mu+v'_\mu)\ , \tag{4.29c}$$

correspond to the contributions from Figs. 2(a), 2(b), and 2(c), respectively. In obtaining (4.29) the relations (2.38)–(2.41) between f and g have been used. In the region $v\sim v'$ we have $\xi\simeq 1$ and the amplitude is completely determined except for the parameter f. When the nonrelativistic quark-model result (2.53) is employed, the theory leaves nothing unspecified for the semileptonic decays of heavy mesons involving soft pion emission in the region $v\sim v'$.

Finally, we consider some examples of semileptonic decays of the heavy baryons involving one-pion emission. Again, we begin by listing the weak transition form factors needed for our examples [11,12,19,30]:

$$\langle\Lambda_j(v',s')|V_\mu^{ji}|\Lambda_i(v,s)\rangle=C_{ji}\zeta\bar{u}_j(v',s')\gamma_\mu u_i(v,s)\ , \tag{4.30a}$$

$$\langle\Lambda_j(v',s')|A_\mu^{ji}|\Lambda_i(v,s)\rangle=C_{ji}\zeta\bar{u}_j(v',s')\gamma_\mu\gamma_5 u_i(v,s)\ , \tag{4.30b}$$

$$\begin{aligned}&\langle\Sigma_j(v',s')|V_\mu^{ji}|\Sigma_i(v,s)\rangle\\&\quad=-\tfrac{1}{3}C_{ji}\bar{u}_j(v',s')\{[(v\cdot v')\gamma_\mu-2(v_\mu+v'_\mu)]\xi_1+[(1-(v\cdot v')^2)\gamma_\mu-2(1-v\cdot v')(v_\mu+v'_\mu)]\xi_2\}u_i(v,s)\ ,\end{aligned} \tag{4.30c}$$

$$\langle \Sigma_j(v',s')|A^{ji}_\mu|\Sigma_i(v,s)\rangle$$
$$=\tfrac{1}{3}C_{ji}\bar{u}_j(v',s')\gamma_5\{[(v\cdot v')\gamma_\mu+2(v_\mu-v'_\mu)]\xi_1+[(1-(v\cdot v')^2)\gamma_\mu-2(1+v\cdot v')(v_\mu-v'_\mu)]\xi_2\}u_i(v,s)\ , \quad (4.30\text{d})$$

$$\langle \Sigma^*_j(v',s')|V^{ji}_\mu|\Sigma_i(v,s)\rangle=\frac{1}{\sqrt{3}}C_{ji}\bar{u}^{\nu}_j(v',s')\{(2g_{\mu\nu}+\gamma_\mu v_\nu)\xi_1+v_\nu[(1-v\cdot v')\gamma_\mu-2v'_\mu]\xi_2\}\gamma_5 u_i(v,s)\ , \quad (4.30\text{e})$$

$$\langle \Sigma^*_j(v',s')|A^{ji}_\mu|\Sigma_i(v,s)\rangle=-\frac{1}{\sqrt{3}}C_{ji}\bar{u}^{\nu}_j(v',s')\{(2g_{\mu\nu}-\gamma_\mu v_\nu)\xi_1+v_\nu[(1+v\cdot v')\gamma_\mu-2v'_\mu]\xi_2\}u_i(v,s)\ , \quad (4.30\text{f})$$

$$\langle \Sigma_j(v',s')|V^{ji}_\mu|\Sigma^*_i(v,s)\rangle=-\frac{1}{\sqrt{3}}C_{ji}\bar{u}_j(v',s')\gamma_5\{(2g_{\mu\nu}+\gamma_\mu v'_\nu)\xi_1+[(1-v\cdot v')\gamma_\mu-2v_\mu]v'_\nu\xi_2\}u^{\nu}_i(v,s)\ , \quad (4.30\text{g})$$

$$\langle \Sigma_j(v',s')|A^{ji}_\mu|\Sigma^*_i(v,s)\rangle=-\frac{1}{\sqrt{3}}C_{ji}\bar{u}_j(v',s')\{(2g_{\mu\nu}-\gamma_\mu v'_\nu)\xi_1+[(1-v\cdot v')\gamma_\mu-2v_\mu]v'_\nu\xi_2\}u^{\nu}_i(v,s)\ , \quad (4.30\text{h})$$

$$\langle \Sigma^*_j(v',s')|V^{ji}_\mu-A^{ji}_\mu|\Sigma^*_i(v,s)\rangle=C_{ji}\bar{u}_{j\lambda}(v',s')\gamma_\mu(1-\gamma_5)u_{i\kappa}(v,s)[-g^{\lambda\kappa}\xi_1+v^\lambda v'^\kappa\xi_2]\ , \quad (4.30\text{i})$$

where C_{ij} is given in (4.17) and ξ_1 and ξ_2 are the two form factors introduced by Georgi [12] and are related to those of Isgur and Wise [11] by

$$\xi_1(v\cdot v')=\eta(v\cdot v')-\frac{v\cdot v'-1}{2}\iota(v\cdot c')\ , \quad (4.31\text{a})$$

$$\xi_2(v\cdot v')=-\frac{1}{2}\iota(v\cdot v')\ , \quad (4.31\text{b})$$

with the normalization for ξ_1

$$\xi_1(1)=1\ . \quad (4.32)$$

Isgur and Wise [11] have shown that in the heavy-quark limit there are no weak transitions of the type $\Sigma_j\to\Lambda_i+l\nu$ and $\Sigma^*_j\to\Lambda_i+l\nu$. Since it is so simple to establish this fact by the method of interpolating fields, we would like to sketch the proof here. It will demonstrate once again the usefulness of the concepts of diquark fields introduced in Sec. III. Let $\Sigma_j(\kappa,v,s)$ be a spin-$\frac{1}{2}$ baryon if $\kappa=1$ and a spin-$\frac{3}{2}$ baryon if $\kappa=2$. Then for $O^{ji}_\mu=\bar{h}^j_{v'}\Gamma_\mu h^i_v$, we have [18]

$$\langle \Sigma_j(\kappa,v',s')|O^{ji}_\mu|\Lambda_i(v,s)\rangle$$
$$=\langle 0|\bar{B}_\nu(\kappa,v',s')h^j_{v'}\phi^\nu_{v'}\bar{h}^j_{v'}\Gamma_\mu h^i_v\phi^\dagger_v\bar{h}^i_v u(v,s)|0\rangle$$
$$=B_\nu(\kappa,v',s')\Gamma_\mu u(v,s)\langle 0|\phi^\nu_{v'}\phi^\dagger_v|0\rangle\ . \quad (4.33)$$

As emphasized earlier, the diquark fields ϕ_v and $\phi^\nu_{v'}$ are a scalar and an axial vector, respectively. Thus, $\langle 0|\phi^\nu_{v'}\phi^\dagger_v|0\rangle$ is an axial vector. But it is impossible to construct an axial vector out of v_μ and v'_μ. Consequently,

$$\langle 0|\phi^\nu_{v'}\phi^\dagger_v|0\rangle\equiv 0 \quad (4.34)$$

and

$$\langle \Sigma_j(\kappa,v',s')|O^{ji}_\mu|\Lambda_i(v,s)\rangle\equiv 0\ . \quad (4.35)$$

The derivation of all the transition weak form factors in (4.30) follows a procedure similar to the one shown above. For example the form factors ζ, ξ_1, and ξ_2 are related to the "vacuum expectation values" of the interpolating fields:

$$\langle 0|\phi_{v'}\phi^\dagger_v|0\rangle=\zeta(v\cdot v')\ , \quad (4.36\text{a})$$

$$\langle 0|\phi^\mu_{v'}\phi^{\nu\dagger}_v|0\rangle=-g^{\mu\nu}\xi_i(v\cdot v')+v^\mu v'^\nu\xi_2(v\cdot v')$$
$$+v'^\mu v^\nu\xi_3(v\cdot v')+v^\mu v^\nu\xi_4(v\cdot v')$$
$$+v'^\mu v'^\nu\xi_5(v\cdot v')\ , \quad (4.36\text{b})$$

where the form factors ξ_3, ξ_4, and ξ_5 do not appear in the weak transition vertices as a result of

$$v^\mu B_\mu(v,s,\kappa)=0,\quad \kappa=1,2\ . \quad (4.37)$$

The chiral Lagrangian (3.12) implies the following matrix elements for one-pion emission:

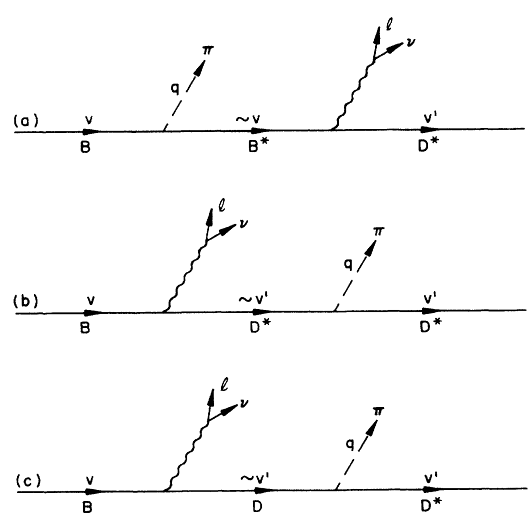

FIG. 2. Feynman diagrams contributing to the decay $B\to D^*+\pi+l\nu$. The four-velocities of B and D^* are v and v' respectively. The pion momentum is q.

$$M(\Sigma_Q^b(v)\to\Sigma_Q^c(v')+\pi^a(q))=\frac{ig_1}{2f_\pi}\varepsilon_{abc}\bar{u}(v')\not{q}\gamma_5 u(v)\ , \tag{4.38a}$$

$$M(\Sigma_Q^{*b}(v)\to\Sigma_Q^c(v')+\pi^a(q))=\frac{ig_3}{2f_\pi}\varepsilon_{abc}\bar{u}(v')q_\nu u^\nu(v)\quad\left[g_3=\frac{\sqrt{3}}{2}g_1\right]\ , \tag{4.38b}$$

$$M(\Sigma_Q^{*b}(v)\to\Sigma_Q^{*c}(v')+\pi^a(q))=\frac{ig_5}{2f_\pi}\varepsilon_{abc}\bar{u}^{\lambda}(v')\not{q}\gamma_5 u_\lambda(v)\quad\left[g_5=-\frac{3}{2}g_1\right]\ , \tag{4.38c}$$

$$M(\Sigma_Q^b(v)\to\Lambda_Q(v')+\pi^a(q))=\frac{g_2}{\sqrt{2}f_\pi}\bar{u}(v')\not{q}\gamma_5 u(v)\delta_{ab}\ , \tag{4.38d}$$

$$M(\Sigma_Q^{*b}(v)\to\Lambda_Q(v')+\pi^a(q))=\frac{g_4}{\sqrt{2}f_\pi}\bar{u}(v')q_\nu u^\nu(v)\delta_{ab}\quad(g_4=-\sqrt{3}g_2)\ . \tag{4.38e}$$

In the above we have given the relations among the coupling constants. We are now in a position to discuss the semileptonic decays of the heavy baryons. To illustrate the various features of the formalism, three examples will be studied: $\Sigma_b\to\Sigma_c+\pi+l\nu$, $\Sigma_b\to\Sigma_c^*+\pi+l\nu$ and $\Sigma_b\to\Lambda_c+\pi+l\nu$. As mentioned earlier, we will restrict ourselves to the special kinematic region $v\sim v'$ where simplification occurs.

The first example $\Sigma_b\to\Sigma_c+\pi+l\nu$ contains four Feynman diagrams as shown in Fig. 3. To evaluate these amplitudes, we have to carry out the spin sums for the intermediate states

$$\sum_{s''}u(v,s'')\bar{u}(v,s')=\frac{\not{v}+1}{2}\ , \tag{4.39}$$

$$\begin{aligned}\sum_{s''}u_\lambda(v,s'')\bar{u}_\kappa(v,s'')&=\frac{\not{v}+1}{2}[-g_{\lambda\kappa}+\tfrac{1}{3}\gamma_\lambda\gamma_\kappa+\tfrac{1}{3}(\gamma_\lambda v_\kappa-\gamma_\kappa v_\lambda)+\tfrac{2}{3}v_\lambda v_\kappa]\\&=[-g_{\lambda\kappa}+\tfrac{1}{3}\gamma_\lambda\gamma_\kappa-\tfrac{1}{3}(\gamma_\lambda v_\kappa-\gamma_\kappa v_\lambda)+\tfrac{2}{3}v_\lambda v_\kappa]\frac{\not{v}+1}{2}\ .\end{aligned} \tag{4.40}$$

With the normalizations (3.23) and (3.24) adopted, the propagator contains a factor $2M$. For example, the propagator which appears in Fig. 3(b) is

$$\frac{2M_{\Sigma_b^*}i}{(P-q)^2-M_{\Sigma_b^*}^2}=\frac{i}{-v\cdot q+M_{\Sigma_b}-M_{\Sigma_b^*}}\ , \tag{4.41}$$

where we have retained a correction term due to the mass difference. Using the weak and strong vertices given in (4.30) and (4.38), we find, for $v\sim v'$,

$$\langle\Sigma_c^f(v',s')\pi^d(q)|V_\mu^{cb}-A_\mu^{cb}|\Sigma_b^e(v,s)\rangle=\varepsilon_{def}\frac{g_1}{2f_\pi}\xi_1 C_{cb}(v'\cdot v)\bar{u}(v',s')[C_{a\mu}+C_{b\mu}+C_{c\mu}+C_{d\mu}]u(v,s)\ , \tag{4.42}$$

where ε_{def} is the totally antisymmetric symbol associated with the isospin of the particles involved, and $C_{a\mu},\ldots,C_{d\mu}$ correspond to the contributions from Figs. 3(a), ..., 3(d), respectively:

$$C_{a\mu}=\frac{1}{3}\frac{1}{-v\cdot q}[\gamma_\mu(1-\gamma_5)+4v_\mu\gamma_5](\not{q}-q\cdot v)\ , \tag{4.43a}$$

$$C_{b\mu}=-\frac{1}{-v\cdot q+M_{\Sigma_b}-M_{\Sigma b^*}}(1-\gamma_5)\times[-q_\mu+\tfrac{1}{3}\gamma_\mu(\not{q}-q\cdot v)+\tfrac{1}{3}v_\mu(\not{q}+2q\cdot v)]\ , \tag{4.43b}$$

$$C_{c\mu}=\frac{1}{3}\frac{1}{v'\cdot q}(\not{q}-q\cdot v')[\gamma_\mu(1-\gamma_5)-4v_\mu\gamma_5]\ , \tag{4.43c}$$

$$C_{d\mu}=-\frac{1}{v'\cdot q+M_{\Sigma_c}-M_{\Sigma_c^*}}\times[-q_\mu+\tfrac{1}{3}(\not{q}-q\cdot v')\gamma_\mu+\tfrac{1}{3}(\not{q}+2q\cdot v')v'_\mu](1+\gamma_5)\ . \tag{4.43d}$$

The approximation $v\approx v'$ is made so only the form factor ξ_1 enters the amplitude. If we set $\xi_1=1$, the value at $v\cdot v'=1$, and $g_1=\frac{1}{3}$, the quark-model result, then the amplitude (4.42) for $\Sigma_b\to\Sigma_c+\pi+l\nu$ is completely known near the region $v\cdot v'\approx 1$.

The second example $\Sigma_b\to\Sigma_c^*+\pi+l\nu$ is somewhat more involved than the first in that it requires more varieties of strong and weak transition vertices. Yet in the same approximation made above the amplitude is

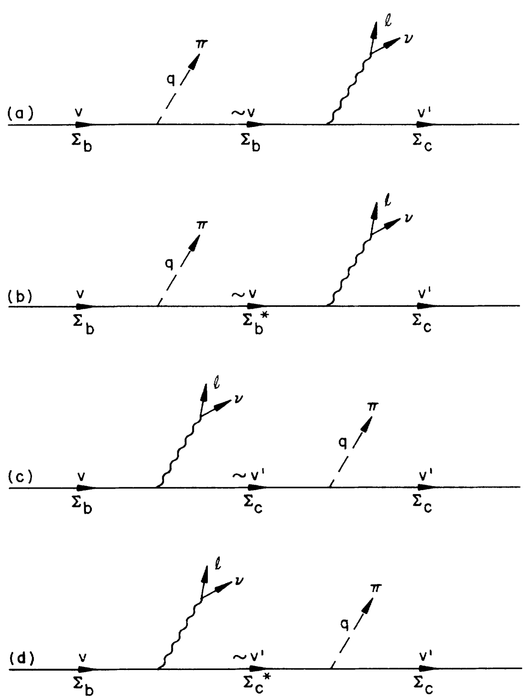

FIG. 3. Feynman diagrams contributing to the decay $\Sigma_b \to \Sigma_c + \pi + l\nu$. The four-velocities of Σ_b and Σ_c are v and v' respectively. The pion momentum is q.

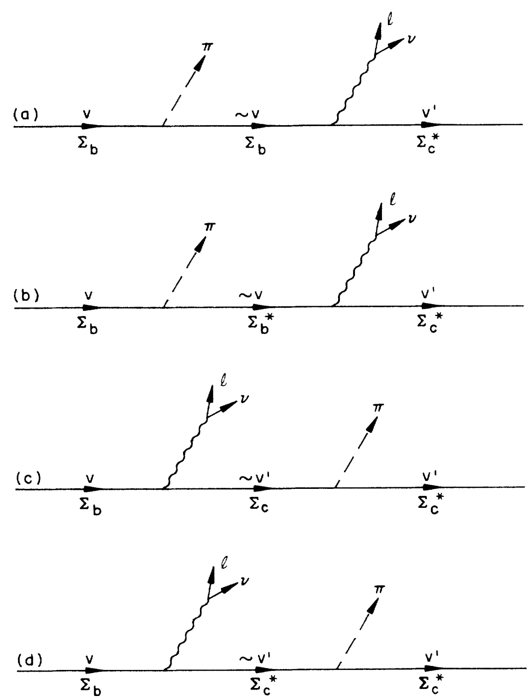

FIG. 4. Feynman diagrams contributing to the decay $\Sigma_b \to \Sigma_c^* + \pi + l\nu$. The four-velocities of Σ_b and Σ_c^* are v and v' respectively. The pion momentum is q.

completely specified. In Fig. 4 we show the Feynman diagrams contributing to the decay $\Sigma_b \to \Sigma_c^* + \pi + l\nu$. The amplitude is, for $v\cdot v' \approx 1$,

$$\langle \Sigma_c^{*f}(v',s')\pi^d(q)|V_\mu^{cb} - A_\mu^{cb}|\Sigma_b^e(v,s)\rangle$$
$$= \varepsilon_{def}\frac{g_1}{2f_\pi}\xi_1 C_{cb}(v\cdot v')\bar{u}_\lambda(v',s)$$
$$\times [D_{a\mu}^\lambda + D_{b\mu}^\lambda + D_{c\mu}^\lambda + D_{d\mu}^\lambda]u(v,s)\ , \qquad (4.44)$$

where $D_{a\mu}^\lambda, \ldots,$ and $D_{d\mu}^\lambda$ are the contributions from each of the Feynman diagrams in Fig. 4, respectively:

$$D_{a\mu}^\lambda = \frac{2}{\sqrt{3}}\frac{1}{-v\cdot q} g_\mu^\lambda(1+\gamma_5)(\not{q} - q\cdot v)\ , \qquad (4.45a)$$

$$D_{b\mu}^\lambda = \frac{\sqrt{3}}{2}\frac{1}{-v\cdot q + M_{\Sigma_b} - M_{\Sigma_b^*}}$$
$$\times [-\gamma_\mu(1-\gamma_5)q^\lambda + \tfrac{2}{3}g_\mu^\lambda(1+\gamma_5)(\not{q} - q\cdot v)]\ , \qquad (4.45b)$$

$$D_{c\mu}^\lambda = \frac{1}{2\sqrt{3}}\frac{1}{v'\cdot q + M_{\Sigma_c^*} - M_{\Sigma_c}} q^\lambda[\gamma_\mu(1-\gamma_5) - 4v_\mu]\ , \qquad (4.45c)$$

$$D_{d\mu}^\lambda = \sqrt{3}\frac{1}{v'\cdot q}[-g_\mu^\lambda(\not{q} - q\cdot v') + \tfrac{2}{3}q^\lambda(\gamma_\mu - v'_\mu)](1+\gamma_5)\ . \qquad (4.45d)$$

We now come to the third and final example of the semileptonic decay of heavy baryons: $\Sigma_b \to \Lambda_c + \pi + l\nu$. This is possibly the dominant semileptonic weak decay of Σ_b since the simpler process $\Sigma_b \to \Lambda_c + l\nu$ is suppressed in the heavy-quark limit. What is interesting about this decay is that in the general case all the three weak form factors ζ, ξ_1, and ξ_2 are required to describe the decay. In the region $v\cdot v' \approx 1$, we have $\zeta = \zeta_1 = 1$ and ξ_2 drops out, and the amplitude is completely determined if the quark-model result is supplemented. The Feynman diagrams contributing to this decay are shown in Fig. 5. The $\Sigma\Sigma\pi$ and $\Sigma\Sigma^*\pi$ vertices could not appear as a result of the suppression of $\Sigma_b \to \Lambda_c$ and $\Sigma_b^* \to \Lambda_c$ transitions. We have

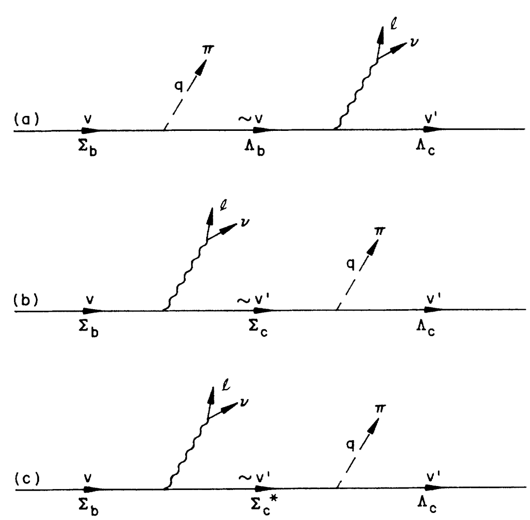

FIG. 5. Feynman diagrams contributing to the decay $\Sigma_b \to \Lambda_c + \pi + l\nu$. The four-velocities of Σ_b and Λ_c are v and v' respectively. The pion momentum is q.

$$\langle \Lambda_c(v',s')\pi^d(q)|V_\mu^{cb}-A_\mu^{cb}|\Sigma_b^e(v,s)\rangle = i\frac{g_2}{\sqrt{2}f_\pi}\delta_{ed}C_{cb}(v'\cdot v)\bar{u}(v',s')[E_{a\mu}+E_{b\mu}+E_{c\mu}]u(v,s) \ . \tag{4.46}$$

Again, $E_{a\mu}$, $E_{b\mu}$, and $E_{c\mu}$ are the contributions from Figs. 5(a), 5(b), and 5(c), respectively. We have, for $v\cdot v'\approx 1$,

$$E_{a\mu}=\zeta\frac{1}{-v\cdot q+M_{\Sigma_b}-M_{\Lambda_b}}\gamma_\mu(1-\gamma_5)(\not{q}-q\cdot v) \ , \tag{4.47a}$$

$$E_{b\mu}=-\frac{1}{3}\xi_1\frac{1}{v'\cdot q+M_{\Lambda_c}-M_{\Sigma_c}}(\not{q}-q\cdot v')[\gamma_\mu(1-\gamma_5)-4v'_\mu\gamma_5] \ , \tag{4.47b}$$

$$E_{c\mu}=-2\xi_1\frac{1}{v'\cdot q+M_{\Lambda_c}-M_{\Sigma_c^*}}[-q_\mu+\tfrac{1}{3}(\not{q}-q\cdot v')\gamma_u+\tfrac{1}{3}(\not{q}+2q\cdot v')v'_\mu](1+\gamma_5) \ . \tag{4.47c}$$

In the above, we have left ζ and ξ_1 unspecified to remind us where they come from.

To fully appreciate the implications of the formalism presented in this work, it is essential to work out the various distributions of the particles in the final states. These will include the energy and angular distributions of the leptons, of the pions, and of the heavy particle, etc. We will present analysis of this type in a future publication.

V. CONCLUSIONS

In this paper we have presented a formalism to describe the chiral dynamics of the heavy mesons and heavy baryons interacting with the Goldstone bosons. Thanks to the heavy-quark symmetry, there are only three parameters independent of how many heavy-quark species there are or will be. Furthermore, these parameters are, through PCAC, related to the axial charges of the heavy mesons and heavy baryons, and therefore they all find simple answers in the quark model just as the g_A of the nucleon did some years ago. It is interesting that the interplay between the symmetries of heavy quarks and light quarks almost defines the theory completely. Of course, the applicability of chiral Lagrangians requires that the light mesons emitted be soft. On the other hand, it is known to the practitioners in the subject that, when treated as a pole model, the chiral Lagrangians work well beyond the soft pion limit.

We have applied the theory to the strong and weak decays of the heavy hadrons. The examples considered illustrate the important features of the formalism. However, these examples involve only a single pion. When two or more pions are emitted, we must take into account the couplings to the vector field V_μ in addition to those to the axial vector field A_μ which are responsible for one pion processes. The full implications of the theory can only be revealed by a detailed analysis of some specific processes in the semileptonic decays accompanied by soft pion emissions. Furthermore, the examples of the semileptonic decays considered so far are all Cabibbo favored processes. It will be interesting to study some of the Cabibbo suppressed semileptonic and nonleptonic decays of the heavy mesons and baryons. Work along these lines is in progress.

In addition, there are many interesting issues we are studying. Let us mention a few here. First of all, we should like to incorporate symmetry-breaking effects into our formalism. These will include the finite-mass effects from the light quarks and the Λ_{QCD}/m_Q corrections from the heavy quarks. Next, it is a simple generalization to incorporate the photon into the theory so radiative decays of these heavy particles can be studied. Also, we feel that the idea of a diquark should be useful in other areas, especially in the nonleptonic decays of heavy baryons. Another topic concerns the dynamics of the excited states of these heavy mesons and baryons. Several of the excited charmed mesons have already been found and undoubtedly many more with other quantum numbers will emerge in the future. It is important to study their strong and weak interactions [31]. How much constraint will the symmetries of heavy and light quarks impose on the theory when the excited states are included? Finally, how do we extend the theory to include the interactions with light vector mesons such as ρ, ω, K^*, and ϕ?

Note added. After completion of this work we have become aware of a recent paper by M. B. Wise, Phys. Rev. D **45**, 2188 (1982), in which he constructs an effective Lagrangian for heavy mesons interacting with the Goldstone bosons. He has considered many aspects in the heavy meson sector which we have discussed in the text.

Notes added in proof

(1) After this paper was submitted for publication, we became aware of the work by G. Burdman and J. Donoghue, Phys. Lett. B **280**, 287 (1992). These authors also considered the implications in the meson sector of combining the heavy quark symmetry and the chiral symmetry.

(2) For the semileptonic decays with one pion emission of heavy baryons considered in Sec. IV, we have found that the pole contributions from the excited heavy baryons vanish at the kinematic point $v=v'$.

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[14] The above is only a partial list of publications on the subject. It is not possible for us to give a comprehensive list. Many more can be found in three excellent review articles on the subject: M. B. Wise, in *Particle Physics—The Factory Era,* Proceedings of the Lake Louise Winter Institute, Lake Louise, Canada, 1991, edited by B. A. Campbell, A. N. Kamal, P. Kitching, and F. C. Khanna (World Scientific, Singapore, 1991); J. D. Bjorken, in *Proceedings of the 18th SLAC Summer Institute on Particle Physics,* Stanford, California, 1990, edited by J. Hawthorne (SLAC, Stanford, 1991); H. Georgi, lectures at TASI, Report No. HUTP-91-A039, 1991 (unpublished).
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[17] For our lowest order Lagrangian presented here, it is not essential to introduce velocity-dependent fields for the heavy particles through relations analogous to Eq. (2.3). The Lagrangians given here allow us to treat phenomenologically some of the nonleading terms such as the mass differences between different spin states. The redefinition of fields becomes important when we include higher order terms in Goldstone-boson momentum q and/or heavy mass corrections $1/M$. For a detailed discussion of the issue, see E. Jenkins and A. V. Manohar, Phys. Lett. B **255**, 558 (1991).
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[30] We have paid special attention to the relative signs among different weak vertices. They are important when different pieces of a decay amplitude are combined. Particularly relevant is the minus sign in Eq. (3.19).
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