Models of the quark mixing matrix

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The structure of the Cabibbo-Kobayashi-Maskawa (CKM) matrix is analyzed from the standpoint of illustrative composite models. It is shown how electroweak couplings can yield information on substructure. Models are constructed with two and three families of quarks, by taking tensor products of sufficient numbers of spin-$\frac{1}{2}$ representations and imagining the dominant terms in the mass matrix to arise from spin-spin interactions. Assumptions are made about the absence of certain terms. Generic results then obtained include the familiar relation $|V_{us}|=(m_d/m_s)^{1/2}-(m_u/m_c)^{1/2}$, and a less frequently seen relation $|V_{cb}|=\sqrt{2}[(m_s/m_b)-(m_c/m_t)]$. The magnitudes of V_{ub} and V_{td} come out naturally to be of the right order. The phase in the CKM matrix can be put in by hand, but its origin remains obscure.

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I. INTRODUCTION

The pattern of charge-changing weak transitions among quarks undoubtedly is a reflection of deeper physics. In the present article we examine the form this pattern might be expected to take if the underlying physics is that of a composite system. We construct two- or three-level quantum-mechanical models of composite quarks, and analyze the way in which the mass eigenstates are affected by alteration of the identity of one or more of the subconstituents. We thereby obtain an illustration of how the quark mixing matrix, or Cabibbo-Kobayashi-Maskawa (CKM) matrix [1,2], might arise from some underlying substructure.

Our intent is not to present a realistic model of the CKM matrix. Rather, it is to stimulate further efforts devoted to the construction of such a model, by highlighting the gaps in our present knowledge. In particular, a major unsolved problem in such models remains the origin of the *CP*-violating phase in the CKM matrix.

We discuss briefly in Sec. II some reasons for believing quarks are composite objects. We then present, in Sec. III, some results of a model for a two-family system. A more realistic three-family system is described in Sec. IV, while an alternative three-family model is mentioned in Sec. V. Possible origins of the *CP*-violating phase in the present models are treated in Sec. VI, where we also compare results of our three-family model with present data on the phase in the CKM matrix. Our conclusions and suggestions for further study are contained in Sec. VII.

II. MOTIVATION FOR A COMPOSITE MODEL OF QUARKS

A. Electroweak symmetry breaking

The least understood aspect of the present electroweak theory [3] is its symmetry-breaking sector. The breakdown of the full $SU(2)\times U(1)$ gauge group to the $U(1)$ gauge group of electromagnetism is described phenomenologically in terms of one or more doublets of Higgs bosons. The vacuum expectation value of one or more neutral Higgs bosons gives rise to the symmetry breaking, which is manifested directly in terms of masses for the W and Z bosons and indirectly in terms of quark and lepton masses.

The pattern of W and Z masses is specified once we know the gauge couplings and the representations of the Higgs fields which acquire vacuum expectation values. Present electroweak data [4] strongly disfavor any significant contributions from vacuum expectation values of Higgs representations whose weak isospins exceed $\frac{1}{2}$. The effect of the Higgs sector on gauge boson masses can be studied quite productively, as a result of this simplicity.

One approach to the Higgs sector, whose validity we shall implicitly assume here, is to take it to be composite. The sector can even be studied in the limit of zero gauge couplings [5,6], in which case it resembles the physics of low-energy pion-pion scattering, but at an energy scaled up by a factor of about 2650.

Although we now understand pions as states of quarks bound by interactions due to the exchange of gluons, the physics of pion-pion scattering is governed by general constraints of current algebra, unitarity, and crossing symmetry. Whether these suffice to specify the low-energy amplitudes completely is a subject of some current discussion [6,7]. However, there seems little question that one cannot learn about the properties of quarks and gluons from pion-pion scattering alone. In particular, the ρ meson, with isospin $I=1$ and spin $J=1$, is a feature in pion-pion scattering whose properties tell us very little about quarks. The lowest-energy resonance whose couplings and properties are sensitive to assumptions about quarks is the ω, degenerate in mass with the ρ but having $I=0, J=1$.

One can also learn about quark charges and colors from the two-photon decay of the neutral pion, whose amplitude is proportional to a quantity S relating the charges and numbers of the pion's constituents. The measured decay rate implies $S\approx 1$. One can realize this constraint either in terms of a color triplet [8] ($N=3$) of quarks with charges $Q_u=\frac{2}{3}$ and $Q_d=-\frac{1}{3}$, satisfying $S=N(Q_u^2-Q_d^2)=1$, or in terms of a single isospin doublet ($N=1$) of elementary fermions with charges $Q_p=1$, $Q_n=0$, as in the original calculation by Steinberger [9].

The corresponding scenario for the Higgs sector [10] envisions fermionic subunits of Higgs bosons with charges Q_U and $Q_D=Q_U-1$. There may be several such doublets of weak isospin, having various values of $Q_U^{(i)}$. However, a constraint that must be realized in this system leads to an important contrast with the case of quarks and pions. The composite state Π^0 analogous to the neutral pion is the one which mixes with the neutral SU(2) and U(1) bosons to form the longitudinal component of the Z^0. The absence of a Z^0–two-photon coupling in the underlying gauge theory requires one to choose the charges $Q_U^{(i)}$ in such a way that their total contribution to $\Pi^0\to\gamma\gamma$ vanishes. The minimal choice [11,12] is to have a single doublet with $Q_U=\frac{1}{2}$, $Q_D=-\frac{1}{2}$, since then $Q_U^2-Q_D^2=0$.

B. Quark and lepton compositeness

With the minimal choice $Q_U=\frac{1}{2}$, $Q_D=-\frac{1}{2}$ for the assumed constituents of Higgs bosons, a natural extension to the case of quarks and leptons [12] is suggested by the relation between electric charge Q and the third component I_{3L} of weak (left-handed) isospin.

In the standard SU(2)×U(1) electroweak theory, the charge of every quark and lepton is given by $Q=I_{3L}+(Y/2)$, where the single U(1) gauge boson couples to the weak hypercharge Y. At this level of the theory, the hypercharge is just an artificial quantum number invented to make the charge come out right.

The SU(2) of standard electroweak theory is that under which left-handed fermions transform nontrivially. If, however, both left-handed and right-handed SU(2) are good symmetries at some energy, the third component I_{3R} of right-handed isospin also has physical meaning. The weak hypercharge can be expressed [13] as $Y/2=I_{3R}+(B-L)/2$, where B and L are baryon and lepton number. The eigenvalues of $I_{3L}+I_{3R}$ are always $\pm\frac{1}{2}$ for ordinary quarks and leptons. It is then tempting to assume [12] that the fermions U and D in the minimal model mentioned above, with $Q_U=\frac{1}{2}$ and $Q_D=-\frac{1}{2}$, carry the contribution of $I_{3L}+I_{3R}$ to the electric charges of quarks and leptons. Models of this sort have been considered previously [14].

If the fermions U and D are the sole source of *all* electroweak SU(2) charges in quarks and leptons, the helicity of any quark or lepton must be that of the corresponding U or D which it contains. This requirement can impose strict constraints on models. We shall see below that it is nontrivial to implement.

The above discussion is intended as an illustration of a typical assignment of quantum numbers. In what follows, we shall take the more general point of view that quarks and leptons are composites of fermions, some of which carry weak SU(2), and possibly of other types of subunits such as scalar or vector mesons. We shall investigate the effect on the composite system of changing the charge of a fermionic subunit by absorption or emission of a real or virtual W boson. We now conclude this section with a general discussion of features to be implemented in a composite model of quarks and leptons.

A crucial problem [15] associated with the construction of realistic models is to understand why, if the compositeness mass scale is very high, quarks and leptons are so much lighter than this scale. One usually invokes an unbroken chiral symmetry which prevents left-handed and right-handed fermions from pairing up to form large Dirac mass terms. We assume that the dynamics is such as to solve this problem. (One recent suggestion is given in Ref. [12].) We shall imagine that the families of quarks and leptons are the low-mass states (perhaps massless in some limit) of a quantum-mechanical system whose other states lie at the compositeness scale. We shall assume that this scale is independent of the physics giving rise to any of the quark masses (though we will not be fully comfortable with such an assumption as long as the top quark remains undiscovered).

A similar point of view is familiar when discussing composite models of the ordinary hadrons. We normally tend to view pions as Nambu-Goldstone bosons of a spontaneously broken chiral symmetry, thereby understanding why they have nearly zero mass. However, from the standpoint of the quark model, one could just as well regard pions as spin-singlet quark-antiquark states whose masses are nearly zero by virtue of a strongly attractive hyperfine interaction between the quarks [16] which nearly cancels their dynamical masses (about 300 MeV per quark). The spin-triplet hyperfine partners of the pions, the ρ mesons, have masses which are typical of the strong-interaction scale.

Because of our familiarity with quark models, we shall use a language which is closely related to the nonrelativistic quark model. We recognize that a proper treatment of deeply bound composite systems may require a relativistic treatment [17]. However, we find nonrelativistic discussions a convenient means of counting states and of dealing with dynamically induced masses of the unwanted (i.e., higher-lying) excitations. The treatment is best illustrated by reference to a simple system of three distinct spin-$\frac{1}{2}$ particles, to which we now turn.

III. TWO-LEVEL SYSTEM

A. Three spin-$\frac{1}{2}$ subunits

We wish to consider a quantum-mechanical system consisting of three distinct static spin-$\frac{1}{2}$ subunits with spins $\mathbf{S}_i$ and masses m_i ($i=1,2,3$), in a state in which all relative orbital angular momenta are equal to zero. In the absence of any interactions, the system consists of one spin-$\frac{3}{2}$ and two spin-$\frac{1}{2}$ states, with a total of $2^3=8$ levels which are all degenerate with one another.

Now we imagine the levels of the system to be split from one another by means of interactions with the same form as hyperfine $\mathbf{S}_i\cdot\mathbf{S}_j$ forces:

$$\Delta M=\lambda\sum_{i<j}^{3}\frac{\mathbf{S}_i\cdot\mathbf{S}_j}{m_i m_j}\ . \tag{3.1}$$

We can examine limits of successively more badly broken symmetry.

(1) When all the subunit masses are equal, the mass of the composite system is characterized just by the total spin $\mathbf{S}=\mathbf{S}_1+\mathbf{S}_2+\mathbf{S}_3$, since the hyperfine interaction is proportional to $\mathbf{S}_1\cdot\mathbf{S}_2+\mathbf{S}_1\cdot\mathbf{S}_3+\mathbf{S}_2\cdot\mathbf{S}_3=S(S+1)/2-\frac{9}{8}$. This quantity is equal to $\frac{3}{4}$ for $S=\frac{3}{2}$ and to $-\frac{3}{4}$ for $S=\frac{1}{2}$. If the dynamics were such that the spin-$\frac{1}{2}$ states came out massless, the masses of the states would then be described by the Hamiltonian

$$\begin{aligned} H&=A(\mathbf{S}_1\cdot\mathbf{S}_2+\mathbf{S}_1\cdot\mathbf{S}_3+\mathbf{S}_2\cdot\mathbf{S}_3+\tfrac{3}{4}) \\ &=A[S(S+1)/2-\tfrac{3}{8}] \ . \end{aligned} \tag{3.2}$$

The parameter A would govern the dynamical mass scale associated with compositeness. States of spin $\frac{3}{2}$ would have masses $\frac{3A}{2}$.

(2) When only two subunit masses are equal (say, $m_1=m_2$), the coefficient of one term (say, $\mathbf{S}_1\cdot\mathbf{S}_2$) is different from that of the other two in the Hamiltonian. It then becomes convenient to characterize states in terms of $\mathbf{S}_{12}\equiv\mathbf{S}_1+\mathbf{S}_2$ as well as the total spin S. The two distinct spin-$\frac{1}{2}$ states have $S_{12}=0$ and 1, while the spin-$\frac{3}{2}$ state of course must have $S_{12}=1$. The most general Hamiltonian is of the form

$$H=AS(S+1)/2+BS_{12}(S_{12}+1)/2+C \ . \tag{3.3}$$

If we want both spin-$\frac{1}{2}$ states to be massless, we may take $B=0$, $C=-\frac{3A}{8}$, and this case reduces to the previous one. If we want the $S_{12}=1$ states with $S=\frac{1}{2}$ and $S=\frac{3}{2}$ to be degenerate with one another but the $S_{12}=0$ spin-$\frac{1}{2}$ state to be massless, we may take $A=C=0$. The $S_{12}=1$ states will then have a common mass equal to B.

The charmed nonstrange baryons *cud* composed of a charmed quark c, an up quark u, and a down quark d have a spectrum close to the present example, as illustrated in Fig. 1(a). In the Λ_c, the u and d are in a state of relative spin $S_{ud}=0$, with an attractive hyperfine interaction. In the spin-$\frac{1}{2}$ Σ_c and the spin-$\frac{3}{2}$ Σ_c^*, both of which have $S_{ud}=1$, the u-d hyperfine interaction is repulsive. Hyperfine interactions involving the heavier charmed quark are weaker, and are responsible for the small splitting between the Σ_c and the Σ_c^*.

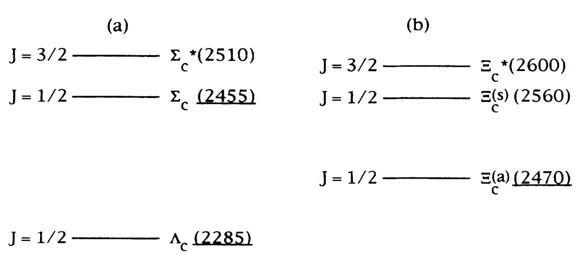

FIG. 1. Pattern of energy levels of charmed baryons. Numbers denote observed masses (for underlined states) or predicted masses, in MeV. (a) States composed of c, u, and d; (b) states composed of c, s, and u. The superscripts on the Ξ_c states denote antisymmetry or symmetry with respect to $s\leftrightarrow u$.

(3) When all three subunit masses are different, the Hamiltonian is no longer diagonal in the combined spin of any two subunits. One may choose a basis of states labeled by total spin S and the spin of any two subunits, say $\mathbf{S}_{12}\equiv\mathbf{S}_1+\mathbf{S}_2$. The state with $S=\frac{3}{2}$ necessarily has $S_{12}=1$. However, the eigenstates of the Hamiltonian with $S=\frac{1}{2}$ are linear combinations of those with $S_{12}=0$ and $S_{12}=1$.

The most general Hamiltonian for this system can be written, adopting the notation of Ref. [18], as

$$H=M_0+\lambda(\sigma_1\cdot\sigma_2+\alpha\sigma_1\cdot\sigma_3+\beta\sigma_2\cdot\sigma_3) \ , \tag{3.4}$$

where $\sigma_i\equiv 2\mathbf{S}_i$. For the spin-$\frac{3}{2}$ state, the expectation value of each $\sigma_i\cdot\sigma_j$ is 1, so

$$M_{3/2}=M_0+\lambda(1+\alpha+\beta) \ . \tag{3.5}$$

The masses of the spin-$\frac{1}{2}$ states may be calculated by diagonalizing a 2×2 matrix, which may be expressed in the basis $S_{12}=0$, 1 as

$$\mathcal{M}_{1/2}=\begin{bmatrix} M_0-3\lambda & \lambda\sqrt{3}(\beta-\alpha) \\ \lambda\sqrt{3}(\beta-\alpha) & M_0+\lambda(1-2\alpha-2\beta) \end{bmatrix} \ . \tag{3.6}$$

It is easy to understand the origin of the terms in (3.6). The upper left-hand entry contains no terms involving α or β, since the expectation value of the operators $\sigma_1\cdot\sigma_3$ and $\sigma_2\cdot\sigma_3$ vanishes in a state of $S_{12}=0$. The expectation value of $\sigma_1\cdot\sigma_2$ in such a state is -3, while it is $+1$ in the state with $S_{12}=1$. When $\alpha=\beta=1$, the matrix should be proportional to the unit matrix, so its lower-right entry is uniquely specified and its off-diagonal entries must be proportional to $\alpha-\beta$. [The matrix element of $\sigma_1\cdot\sigma_2$ between states of different S_{12} clearly vanishes.] A brief calculation, making use of the expansion of $\sigma_i\cdot\sigma_3$ $(i=1,2)$ in terms of raising and lowering operators, then yields the coefficients in the off-diagonal terms.

The eigenvalues of the matrix (3.6) are

$$\begin{aligned} M_{1/2,\pm}=M_0+\lambda[&-(1+\alpha+\beta) \\ &\pm 2(1+\alpha^2+\beta^2-\alpha-\beta-\alpha\beta)^{1/2}] \ . \end{aligned} \tag{3.7}$$

The corresponding eigenvectors are

$$|1/2,-\rangle=\cos\theta|S_{12}=0\rangle-\sin\theta|S_{12}=1\rangle \ , \tag{3.8a}$$

$$|1/2,+\rangle=\sin\theta|S_{12}=0\rangle+\cos\theta|S_{12}=1\rangle \ , \tag{3.8b}$$

where

$$\tan\theta=\frac{\sqrt{3}(\beta-\alpha)}{3+[M_{1/2,+}-M_0]/\lambda} \ . \tag{3.9}$$

The parameters α and β may be expressed in terms of subunit masses, taking account of (3.1), as $\alpha=m_2/m_3$ and $\beta=m_1/m_3$. If any two subunit masses m_i and m_j are equal, the eigenstates correspond to states of definite S_{ij}. For example, when $m_1=m_2$ so that $\alpha=\beta$, the matrix (3.6) is diagonal, with eigenvalues $M_0-3\lambda$ and $M_0+\lambda(1-4\alpha)$ corresponding to the states with $S_{12}=0$ and $S_{12}=1$. When $m_2=m_3$ so that $\alpha=1$, the eigenvalues $M_0-3\lambda\beta$ and $M_0+\lambda(\beta-4)$ correspond to states with

$S_{23}=0$ and $S_{23}=1$. When both α and β are close to 1, the spin-$\frac{1}{2}$ eigenstates are much closer to one another in mass than to the spin-$\frac{3}{2}$ state.

An example of a system to which the above discussion applies is the charmed-strange baryon composed of *csu* (or *csd*) [19]. The mass eigenstates correspond only approximately to states of definite light-quark spin $S_{su}=0,1$. The breaking of SU(3) symmetry induces mixing. The corresponding levels are illustrated in Fig. 1(b).

B. A model for two quark families

Using a model based on three spin-$\frac{1}{2}$ subunits, one can construct only two spin-$\frac{1}{2}$ composites, and hence the present model applies only to the Cabibbo [1] angle. A preliminary account has appeared in Ref. [20].

It has been pointed out [21] that the ansatz

$$M=\begin{bmatrix} 0 & \mu \\ \mu & m \end{bmatrix} . \tag{3.10}$$

for quark mass matrices in a two-family model leads to the reasonably successful relation

$$\theta_{\text{Cabibbo}} \simeq (m_d/m_s)^{1/2}-(m_u/m_c)^{1/2} . \tag{3.11}$$

Equation (3.11) actually agrees with experiment better if the second term is omitted, which is the form in which it was first hypothesized [22]. Here one may use quark masses from Ref. [23], or a more recent set from Ref. [24].

If one arbitrarily chooses $M_0=3\lambda$ in the Hamiltonian (3.4) and the mass matrix (3.6), one obtains the desired zero. The Hamiltonian (3.4) becomes

$$H=\lambda(3+\sigma_1\cdot\sigma_2+\alpha\sigma_1\cdot\sigma_3+\beta\sigma_2\cdot\sigma_3) . \tag{3.12}$$

The mass matrix for spin-$\frac{1}{2}$ states takes the form

$$\mathcal{M}_{1/2}=\lambda\begin{bmatrix} 0 & \sqrt{3}(\beta-\alpha) \\ \sqrt{3}(\beta-\alpha) & 2(2-\alpha-\beta) \end{bmatrix} , \tag{3.13}$$

while the spin-$\frac{3}{2}$ state's mass is

$$M_{3/2}=\lambda(4+\alpha+\beta) . \tag{3.14}$$

An equivalent expression for (3.12) which will be useful in our discussion of a three-family model is

$$H=M_0(J)+a\mathbf{S}_1\cdot\mathbf{S}_3+b\mathbf{S}_2\cdot\mathbf{S}_3 , \tag{3.15}$$

where

$$\begin{aligned} M_0(J) &\equiv \lambda(3+\sigma_1\cdot\sigma_2+\sigma_1\cdot\sigma_3+\sigma_2\cdot\sigma_3) \\ &=\lambda[3+2\{J(J+1)-\tfrac{9}{4}\}] , \end{aligned} \tag{3.16}$$

while

$$a\equiv 4\lambda(\alpha-1), \quad b\equiv 4\lambda(\beta-1) . \tag{3.17}$$

The term $M_0(J)$ vanishes when $J=\frac{1}{2}$ and is equal to 6λ when $J=\frac{3}{2}$. By taking $\lambda\to\infty$ and $(\alpha,\beta)\to 1$ with (a,b) fixed, we may describe masses of spin-$\frac{1}{2}$ states while setting the masses of the spin-$\frac{3}{2}$ states to be arbitrarily high. The mass matrix for spin-$\frac{1}{2}$ states is now

$$\mathcal{M}_{1/2}=\begin{bmatrix} 0 & \sqrt{3}(b-a)/4 \\ \sqrt{3}(b-a)/4 & -(a+b)/2 \end{bmatrix} . \tag{3.18}$$

We may further assume that $|a-b| \ll |a+b|$, anticipating that the two eigenvalues of the quark mass matrices will be very different. The eigenvalues of the matrix (3.10) are approximately $-\mu^2/m$ and m when $\mu\ll m$. In the present case, the eigenvalues of (3.18) are approximately $\frac{3}{8}(a-b)^2/(a+b)$ and $-(a+b)/2$. These are to be identified with the eigenstates $|\frac{1}{2},-\rangle$ and $|\frac{1}{2},+\rangle$ of Eqs. (3.8). (One can always change the sign of a quark mass by multiplying the appropriate quark field by γ_5.) When $\mu\ll m$, the mixing angle θ in (3.8) arising from a matrix of the form (3.10) is

$$\theta=\arctan(\mu/m)\simeq\sqrt{|M_{1/2,-}/M_{1/2,+}|} . \tag{3.19}$$

The Cabibbo angle in (3.11) is then the difference between the mixing angles in the up- and down-quark sectors.

The need to take $M_0=3\lambda$ in Eq. (3.4) to obtain agreement with experiment is using the observed pattern of quark mixing angle to tell us something about quark substructure. Both the terms a and b in Eq. (3.18) must change in the course of a charge-changing weak transition. These features can be used to constrain hypotheses about which subunit or subunits carry the weak SU(2) quantum number.

We have taken an illustrative set of quark masses in order to see if there are some regularities in the parameters of (3.15) that would aid in model building. We choose a representative set of masses [24] all evaluated at a scale of 1 GeV:

$$\begin{aligned} &m_u=5.2\ \text{MeV}, \quad m_d=9.2\ \text{MeV} , \\ &m_s=194\ \text{MeV}, \quad m_c=1.41\ \text{GeV} . \end{aligned} \tag{3.20}$$

One can then solve for the parameters for the (u,c) and (d,s) systems. The results, taking positive values for a and b, are

$$(u,c)\ \text{system:}\quad a=1.51\ \text{GeV}, \quad b=1.31\ \text{GeV} ; \tag{3.21a}$$

$$(d,s)\ \text{system:}\quad a'=0.243\ \text{GeV}, \quad b'=0.145\ \text{GeV} . \tag{3.21b}$$

Here and elsewhere we shall denote quantities associated with the down-quark sector with primes.

No obvious pattern is suggested by Eqs. (3.21). It appears that one cannot identify any one subconstituent as carrying the weak SU(2) quantum number. For example, if we were to take only the third subunit to change its charge and mass in a charge-changing weak transition, we would lose the correlation between the chirality of the composite fermion and that of the subunit, since such a correlation only exists for the $S_{12}=0$ basis state. (In the $S_{12}=1$ basis state, there is anticorrelation between the two chiralities.) Nonetheless, we expect results such as (3.21) to be useful if and when one identifies a viable composite model. With this possibility in mind, we now turn to a three-level system.

IV. THREE-LEVEL SYSTEM

A. Sources of three spin-$\frac{1}{2}$ levels

One can imagine a system of N distinct spin-$\frac{1}{2}$ subunits coupled together to give quarks and leptons, in such a way that the various families are associated with different ways of forming a total spin of $\frac{1}{2}$. We have mentioned the case [14,25] $N=3$ in the previous section. It leads to two spin-$\frac{1}{2}$ composite states. The case $N=5$, examined in Refs. [18] and [26], leads to five spin-$\frac{1}{2}$ composite states. Although this is more than the number of light neutrinos (three) measured in Z decays, one could imagine that the pattern of masses changed after the first three families in such a way as to make the fourth and fifth neutrino species sufficiently massive not to contradict these data. Instead, we seek the simplest model which gives just three spin-$\frac{1}{2}$ states, in order to have a system without extraneous levels wherein one can discuss a 3×3 CKM matrix.

One way to reduce the number of states in a model with many spin-$\frac{1}{2}$ subconstituents is to imagine that two or more of them are identical, and thereby subject to the exclusion principle. For example, if we were to take two of the spin-$\frac{1}{2}$ subunits to be identical except for a hypercolor label with respect to which they were in an antisymmetric state, their total spin would have to be 1. Such is the case for the two u quarks in an S-wave uud state. The quarks are coupled antisymmetrically with respect to color, they are identical in flavor, and hence they have to be symmetric in spin, with $S_{uu}=1$. When coupled to the d quark they can either form a proton with spin $\frac{1}{2}$ or a Δ^+ with spin $\frac{3}{2}$.

It so happens that one can form precisely three spin-$\frac{1}{2}$ levels with the product of three spin-$\frac{1}{2}$ subunits and a spin-1 subunit. The tensor product in question is

$$\tfrac{1}{2}\otimes\tfrac{1}{2}\otimes\tfrac{1}{2}\otimes 1=3(\tfrac{1}{2})\oplus 3(\tfrac{3}{2})\oplus\tfrac{5}{2}\ . \tag{4.1}$$

We shall adopt such a model for quark and lepton families, regarding them as the spin-$\frac{1}{2}$ members of the set (4.1). We do not inquire here into the origin of the subunits. The spin-1 subunit may either arise as a result of the composition of two spin-$\frac{1}{2}$ subunits in the manner mentioned above, or it may be due to some internal orbital angular momentum. In what follows we shall label the spins of the three spin-$\frac{1}{2}$ subunits by $\mathbf{S}_i$ $(i=1,2,3)$ and that of the spin-1 subunit by $\boldsymbol{L}$.

Our point of view is different from that espoused in one early treatment [27] of composite quarks and leptons. In that approach, the family index is explicitly carried by one (h) of three spin-$\frac{1}{2}$ subunits. The approach in Ref. [27] has the virtue that it can preserve the connection between the helicity of the subunit w carrying the weak charge [with $Q(w)=\pm\frac{1}{2}$ as in Sec. II] and the quark or lepton. A third subunit in that model (c) carries the remaining electric charge and the color index, serving to distinguish quarks from leptons.

B. A mass matrix for three spin-$\frac{1}{2}$ levels

We are considering subunits of spin $\frac{1}{2}$ with spins $\mathbf{S}_i$ $(i=1,2,3)$ and a spin-1 subunit with angular momentum $\boldsymbol{L}$. We shall adopt the following basis for description of our states: (1) We imagine $\mathbf{S}_1$ and $\mathbf{S}_2$ to be coupled to a total spin $\mathbf{S}_{12}\equiv\mathbf{S}_1+\mathbf{S}_2$ as in Sec. III. The possible values of S_{12} are 0 and 1. (2) Next, we define $\mathbf{J}_{12}\equiv\mathbf{S}_{12}+\boldsymbol{L}$; (3) finally, we form the total spin $\mathbf{J}=\mathbf{J}_{12}+\mathbf{S}_3$.

For $S_{12}=0$, we must have $J_{12}=1$. When combined with $\mathbf{S}_3$, this leads to a single state with $J=\frac{1}{2}$.

For $S_{12}=1$ we can have $J_{12}=0,1,2$. Only the first two of these can lead to spin-$\frac{1}{2}$ composite states when combined with $\mathbf{S}_3$.

We may then label our three spin-$\frac{1}{2}$ states by $|S_{12},J_{12}\rangle$. The basis states are

$$|0,1\rangle\equiv|u_0\rangle\ , \tag{4.2a}$$

$$|1,1\rangle\equiv|c_0\rangle\ , \tag{4.2b}$$

$$|1,0\rangle\equiv|t_0\rangle\ , \tag{4.2c}$$

where in assigning the weak eigenstate labels we have anticipated a result of Sec. IV C.

We shall take our model Hamiltonian to consist of a constant term and linear combinations of terms proportional to $\mathbf{S}_i\cdot\mathbf{S}_j$ and $\boldsymbol{L}\cdot\mathbf{S}_i$. The matrix elements of these operators between the basis states written in the order (4.2) are

$$\mathbf{S}_1\cdot\mathbf{S}_2=\begin{bmatrix}-\frac{3}{4} & 0 & 0\\ 0 & \frac{1}{4} & 0\\ 0 & 0 & \frac{1}{4}\end{bmatrix}, \tag{4.3a}$$

$$\mathbf{S}_1\cdot\mathbf{S}_3=\begin{bmatrix}0 & 8^{-1/2} & \frac{1}{4}\\ 8^{-1/2} & -\frac{1}{4} & -8^{-1/2}\\ \frac{1}{4} & -8^{-1/2} & 0\end{bmatrix}, \tag{4.3b}$$

$$\mathbf{S}_2\cdot\mathbf{S}_3=\begin{bmatrix}0 & -8^{-1/2} & -\frac{1}{4}\\ -8^{-1/2} & -\frac{1}{4} & -8^{-1/2}\\ -\frac{1}{4} & -8^{-1/2} & 0\end{bmatrix}, \tag{4.3c}$$

$$\boldsymbol{L}\cdot\mathbf{S}_1=\begin{bmatrix}0 & -2^{-1/2} & 0\\ -2^{-1/2} & -\frac{1}{2} & 0\\ 0 & 0 & -1\end{bmatrix}, \tag{4.4a}$$

$$\boldsymbol{L}\cdot\mathbf{S}_2=\begin{bmatrix}0 & 2^{-1/2} & 0\\ 2^{-1/2} & -\frac{1}{2} & 0\\ 0 & 0 & -1\end{bmatrix}, \tag{4.4b}$$

$$\boldsymbol{L}\cdot\mathbf{S}_3=\begin{bmatrix}-1 & 0 & 0\\ 0 & -\frac{1}{2} & 2^{-1/2}\\ 0 & 2^{-1/2} & 0\end{bmatrix}. \tag{4.4c}$$

The sum of all six matrices is $-\frac{7}{4}$ times the unit matrix, as it must be, since

$$\mathbf{S}_1\cdot\mathbf{S}_2+\mathbf{S}_1\cdot\mathbf{S}_3+\mathbf{S}_2\cdot\mathbf{S}_3+\boldsymbol{L}\cdot\mathbf{S}_1+\boldsymbol{L}\cdot\mathbf{S}_2+\boldsymbol{L}\cdot\mathbf{S}_3 =[J(J+1)-3(\tfrac{3}{4})-2]/2 \quad (4.5)$$

which is $-\frac{7}{4}$ for states of $J=\frac{1}{2}$.

We investigated another basis in which the states were labeled by the quantum numbers $(S_{12},S_{123})=(0,\frac{1}{2}),(1,\frac{1}{2}),(1,\frac{3}{2})$. Here $S_{123}\equiv\mathbf{S}_1+\mathbf{S}_2+\mathbf{S}_3$. The matrix elements of $\mathbf{S}_i\cdot\mathbf{S}_j$ and $\boldsymbol{L}\cdot\mathbf{S}_i$ have fewer zeros in this basis, and we were not able to spot any simple regularities leading to a convenient ansatz for mass matrices.

C. Model for three families of quarks

We examine the following simplified form of the mass matrix for quarks, drawing on our experience with the two-family model:

$$M(q)=M_0(J)+a\mathbf{S}_1\cdot\mathbf{S}_3+b\mathbf{S}_2\cdot\mathbf{S}_3+c(1+\boldsymbol{L}\cdot\mathbf{S}_3)\ , \quad (4.6)$$

where $M_0(J)=\text{const}\times[J(J+1)-\frac{3}{4}]$ is chosen so as to make all states with $J>\frac{1}{2}$ arbitrarily heavy, while $M_0(\frac{1}{2})=0$, as in the two-family example. We motivate the inclusion of certain terms and the omission of others in (4.6) by the following arguments.

(1) The terms $\mathbf{S}_1\cdot\mathbf{S}_3$ and $\mathbf{S}_2\cdot\mathbf{S}_3$ mix the first and second families.

(2) To preserve the desired zero in the upper left-hand diagonal element, as in (3.18), the term $\mathbf{S}_1\cdot\mathbf{S}_2$ is omitted. We wish to have a mass matrix leading to the successful relation (3.11) for the first two quark families.

(3) The terms $\boldsymbol{L}\cdot\mathbf{S}_1$ and $\boldsymbol{L}\cdot\mathbf{S}_2$ would give rise to further mixing of the first and second families. The sum $\boldsymbol{L}\cdot\mathbf{S}_1+\boldsymbol{L}\cdot\mathbf{S}_2$ is the matrix diag$(0,-1,-2)$ which could be very useful in fine-tuning our results. For simplicity, however, we omit these terms.

(4) The combination $1+\boldsymbol{L}\cdot\mathbf{S}_3$ mixes only the second and third families.

Clearly the form (4.6) is not unique. Taking account of (4.5), the most general mass matrix will have a total of five parameters for each charge of quark, giving (in principle) more than enough parameters to fit both quark masses and mixing angles. Nevertheless, we wish to see how far one can go with the simplified version (4.6). As we shall see, with this form the mass eigenstates are very close to the basis states (4.2).

The mass operator (4.6) leads to the following mass matrix for spin-$\frac{1}{2}$ quarks:

$$\mathcal{M}_{1/2}=\begin{bmatrix} 0 & \alpha\sqrt{2} & \alpha \\ \alpha\sqrt{2} & \beta & \beta\sqrt{2} \\ \alpha & \beta\sqrt{2} & \gamma \end{bmatrix}, \quad (4.7)$$

where

$$\alpha\equiv(a-b)/4,\quad \beta\equiv(2c-a-b)/4,\quad \gamma=c\ . \quad (4.8)$$

The form (4.7) allows for b- and t-quark masses which are significantly larger than the corresponding ones in the first two quark families. If the hierarchy

$$\alpha\ll\beta\ll\gamma \quad (4.9)$$

is respected, the mass eigenstates will automatically be very close to the basis states (4.2) which we have adopted as weak eigenstates. The corresponding CKM matrix will have diagonal elements close to 1 and small off-diagonal elements.

With separate mass matrices (4.7) for up and down quarks, one has six parameters with which to describe six quark masses. This theory has no CP violation. By respecting the hierarchy (4.9), one can construct approximate eigenvectors. For up quarks, one has

$$u\simeq\begin{bmatrix} 1 \\ -\alpha\sqrt{2}/\beta \\ \alpha/\gamma \end{bmatrix},\quad c\simeq\begin{bmatrix} \alpha\sqrt{2}/\beta \\ 1 \\ -\beta\sqrt{2}/\gamma \end{bmatrix},\quad t\simeq\begin{bmatrix} \alpha/\gamma \\ \beta\sqrt{2}/\gamma \\ 1 \end{bmatrix}, \quad (4.10)$$

where only leading terms are shown. The corresponding eigenvalues are

$$m_u\simeq-2\alpha^2/\beta,\quad m_c\simeq\beta,\quad m_t\simeq\gamma\ . \quad (4.11)$$

The eigenvectors and eigenvalues for down-type quarks may be expressed in terms of primed quantities:

$$d\simeq\begin{bmatrix} 1 \\ -\alpha'\sqrt{2}/\beta' \\ \alpha'/\gamma' \end{bmatrix},\quad s\simeq\begin{bmatrix} \alpha'\sqrt{2}/\beta' \\ 1 \\ -\beta'\sqrt{2}/\gamma' \end{bmatrix},\quad b\simeq\begin{bmatrix} \alpha'/\gamma' \\ \beta'\sqrt{2}/\gamma' \\ 1 \end{bmatrix}, \quad (4.12)$$

and

$$m_d\simeq-2\alpha'^2/\beta',\quad m_s\simeq\beta',\quad m_b\simeq\gamma'\ . \quad (4.13)$$

The CKM matrix elements are obtained as scalar products of the eigenvectors (4.12) and the complex conjugates of the eigenvectors (4.10). Since all quantities are real in the present exercise, we have

$$V_{us}\simeq\left[\frac{-m_d}{m_s}\right]^{1/2}-\left[\frac{-m_u}{m_c}\right]^{1/2}, \quad (4.14a)$$

$$V_{cb}\simeq\sqrt{2}\left[\frac{m_s}{m_b}-\frac{m_c}{m_t}\right], \quad (4.14b)$$

$$V_{ub}\simeq\frac{(-m_dm_s/2)^{1/2}}{m_b}-\frac{m_s}{m_b}\left[\frac{-2m_u}{m_c}\right]^{1/2}+\frac{(-m_um_c/2)^{1/2}}{m_t}, \quad (4.14c)$$

$$V_{td}\simeq\frac{(-m_dm_s/2)^{1/2}}{m_b}-\frac{m_c}{m_t}\left[\frac{-2m_d}{m_s}\right]^{1/2}+\frac{(-m_um_c/2)^{1/2}}{m_t}\ . \quad (4.14d)$$

Additional relations (not shown) are just those one would expect from unitarity of the CKM matrix.

The first of Eqs. (4.14) is just the relation obtained in the two-family model. The second is not a bad approximation to experiment, while the third and fourth are relations of the sort that might be expected to arise in a wide variety of models. The approximation we have used satisfies unitarity explicitly: $V_{ub}+V_{td}=V_{us}V_{cb}$.

Mass matrices of the type (4.7) are among a wider class considered in Refs. [28]. Among the questions we have not yet answered are the significance of the algebraic signs of the various mass eigenvalues, and the origin of the *CP*-violating phase. We shall discuss the question of the phase in Sec. VI.

If one is able to construct a satisfactory mass matrix [along the lines of Eq. (4.7), for example] in which some terms allowed *a priori* are absent, one may be able to learn something about quark substructure. The pattern of parameters a, b, and c for up and down quarks following from a fit of (4.7) to the observed quark masses does not suggest such a pattern at present.

V. AN ALTERNATIVE THREE-LEVEL MODEL

One system with three quantum-mechanical levels consists of a particle traveling in one dimension, subject to a potential consisting of three attractive δ functions of equal strength. In isolation, each δ function would have a single bound state. We can imagine the zero of energy to be chosen in such a way that these three bound states are at nearly zero energy. In the limit in which the distance between each δ function is large compared to the exponential falloff of the wave function, tunneling splits the levels by a small amount.

One can obtain a pattern very similar to the observed one (with one heavy quark and two light ones in each family) by assuming that the δ functions are arranged symmetrically on a ring. The tunneling amplitudes between any two δ functions are then equal, so the mass matrix is of the form

$$\mathcal{M}=\begin{bmatrix}\alpha & \beta & \beta\\ \beta & \alpha & \beta\\ \beta & \beta & \alpha\end{bmatrix} . \tag{5.1}$$

This matrix has two eigenvectors with eigenvalue $\alpha-\beta$ (to be identified with members of the first two families) and one with eigenvalue $\alpha+2\beta$ (to be identified with the third family). By construction, this problem is isomorphic to the one considered in Refs. [29].

One could imagine a splitting of the degeneracies by having particles of different masses for up and down quark sectors bound on the ring, and by having the spacings of the δ functions not be uniform around the ring. The physical origin of phases in such a matrix could be associated with complex tunneling amplitudes, such as might arise in systems with absorption or dissipation. We have not yet investigated these possibilities, but they appear promising.

VI. THE *CP*-VIOLATING PHASE

The Hamiltonians we have been investigating all give rise to orthogonal sets of wave functions which may be represented in terms of real parameters. Consequently, the scalar products of these wave functions generate CKM matrix elements which may always be expressed as real quantities.

Without a microscopic theory of *CP* violation, the best we can do in composite models of the present variety is to put in phases by hand. This approach is in the spirit of many other exercises with mass matrices [28,30], but we would hope that such a theory could emerge in the future.

Some guidance in inserting the phase comes from the observation that the relation (3.11) for the Cabibbo angle [i.e., the relation (4.14a) for V_{us}] is best satisfied if one omits the contribution of the u and c quarks. The two terms in these relations have magnitudes of about 0.22 and 0.06, respectively. Equivalently, if the relative phase of these two terms is imaginary, the relation is satisfied almost as well. We shall assume henceforth that $\alpha\rightarrow i\alpha$. Since, from the scalar products of eigenvectors (4.10) and (4.12), we have

$$V_{us}=(\alpha'\sqrt{2}/\beta')-([i\alpha]^*\sqrt{2}/\beta) , \tag{6.1}$$

the two terms then will have a relative imaginary phase, as desired.

Within the spirit of the operator (4.6) it appears we must take symmetric mass matrices, even in the presence of phases. We then assume (α and α' are real)

$$\mathcal{M}_U=\begin{bmatrix}0 & i\alpha\sqrt{2} & i\alpha\\ i\alpha\sqrt{2} & \beta & \beta\sqrt{2}\\ i\alpha & \beta\sqrt{2} & \gamma\end{bmatrix} , \tag{6.2}$$

$$\mathcal{M}_D=\begin{bmatrix}0 & \alpha'\sqrt{2} & \alpha'\\ \alpha'\sqrt{2} & \beta' & \beta'\sqrt{2}\\ \alpha' & \beta'\sqrt{2} & \gamma'\end{bmatrix} , \tag{6.3}$$

for $Q=\frac{2}{3}$ and $Q=-\frac{1}{3}$ quarks, respectively.

Since Eq. (6.2) is not Hermitian, its eigenvectors will not form an orthonormal set. However, we can convert (6.2) to a Hermitian matrix by changing the sign of the right-handed u_0 quarks. This is equivalent to multiplying the first column of (6.2) by -1. The eigenvectors of the corresponding Hermitian matrix are then

$$u\simeq\begin{bmatrix}1\\ i\alpha\sqrt{2}/\beta\\ -i\alpha/\gamma\end{bmatrix},\quad c\simeq\begin{bmatrix}i\alpha\sqrt{2}/\beta\\ 1\\ -\beta\sqrt{2}/\gamma\end{bmatrix},\quad t\simeq\begin{bmatrix}i\alpha/\gamma\\ \beta\sqrt{2}/\gamma\\ 1\end{bmatrix} , \tag{6.4}$$

while the eigenvalues remain (4.11). One can use the approximate expressions (4.11)–(4.13) and (6.4) to construct the CKM matrix elements as scalar products of eigenvectors: $V_{ud}=u^*\cdot d$, and so on. The results are

$$V_{us}\simeq\left[\frac{-m_d}{m_s}\right]^{1/2}-i\left[\frac{-m_u}{m_c}\right]^{1/2} , \tag{6.5a}$$

$$V_{cb}\simeq\sqrt{2}\left[\frac{m_s}{m_b}-\frac{m_c}{m_t}\right] , \tag{6.5b}$$

$$V_{ub}\simeq\frac{(-m_d m_s/2)^{1/2}}{m_b}-i\frac{m_s}{m_b}\left[\frac{-2m_u}{m_c}\right]^{1/2}+i\frac{(-m_u m_c/2)^{1/2}}{m_t} , \tag{6.5c}$$

$$V_{td} \simeq \frac{(-m_d m_s/2)^{1/2}}{m_b} - \frac{m_c}{m_t}\left[\frac{-2m_d}{m_s}\right]^{1/2} - i\frac{(-m_u m_c/2)^{1/2}}{m_t} . \quad (6.5d)$$

We substitute a set of sample quark masses to see the implications of Eqs. (6.5). In addition to the values (3.20), we choose [24]

$$m_b = 6.33 \text{ GeV}, \quad m_t = 200 \text{ GeV} . \quad (6.6)$$

These are values at a scale of 1 GeV. The physical top quark mass will be about $\frac{3}{5}$ the value at 1 GeV [31], for 120 GeV in the above example. In accord with the results $m_u = -2\alpha^2/\beta$, $m_d = -2\alpha'^2/\beta'$, we choose algebraically negative signs for m_u and m_d.

The results before rephasing are

$$V_{us} = 0.218 - 0.061i, \quad V_{cb} = 0.033 ,$$
$$V_{ub} = 0.0047 - 0.0023i, \quad V_{td} = 0.0025 - 0.0003i . \quad (6.7)$$

We perform an approximate rephasing by multiplying the s and b quarks by a phase $\phi = \arctan(0.061/0.218)$ and the c and t quarks by $-\phi$. This gets rid of the phase in V_{us} while keeping the diagonal elements real (which they nearly were to begin with). The results after rephasing are

$$V_{us} = 0.226, \quad V_{cb} = 0.033 ,$$
$$V_{ub} = 0.0052 - 0.0010i, \quad V_{td} = 0.0024 - 0.0010i . \quad (6.8)$$

This solution implies values of the parameters ρ and η as defined in Ref. [32]:

$$\rho = 0.69, \quad \eta = 0.13 . \quad (6.9)$$

Another solution may be obtained by taking an algebraically negative sign for the top quark mass. Here the results, after rephasing, are

$$V_{us} = 0.226, \quad V_{cb} = 0.053 ,$$
$$V_{ub} = 0.0053 - 0.0016i, \quad V_{td} = 0.0067 - 0.0016i ; \quad (6.10)$$

$$\rho = 0.44, \quad \eta = 0.13 . \quad (6.11)$$

The above results are typical consequences of the expressions (6.5). They are not ideal representations of the data, but have some interesting features.

(1) The magnitude of V_{cb} is governed by the ratios of quark masses, rather than their square roots. As a result, one expects smaller typical values for this quantity than for V_{us}. This point was noticed in Ref. [28]. The value inferred from the study of semileptonic B decays to charmed particles [33] is

$$V_{cb} = 0.041 \pm 0.002 \pm 0.004 , \quad (6.12)$$

where the first error is statistical and the second is systematic, mainly associated with theoretical uncertainties in the spectrum shape. The first term in (6.5b) ranges from 0.022 to 0.045 for $m_b = 6.33$ GeV and m_s ranging from 100 to 200 MeV. (As noted in Ref. [23], one knows more about the ratios of the masses of the three light quarks than about their actual values.) Thus, there is the possibility of compatibility between (6.5b) and (6.12) for either relative sign of the two terms, though constructive interference appears favored for the lighter strange quark masses. As in the case of V_{us}, it appears as if one would be better off without the second term.

(2) The orders of magnitude of $|V_{ub}|$ and $|V_{td}|$ are correct. The estimate in Ref. [33], $V_{ub}/V_{cb} = 0.11 \pm 0.03$, is compatible with our range of 0.1 to 0.16 for this ratio. The rather small value of $|V_{td}|$ in the present solution is compatible with information obtained from B-$\bar{B}$ mixing if the B meson decay constant f_B is rather large [34]. Larger values of $|V_{td}|$ do not seem to follow naturally from the mass matrices (6.2) and (6.3). The dominant contributions to the real parts of V_{ub} and V_{td} in Eqs. (6.5c) and (6.5d) come from the first term, and hence the real parts of both quantities are of the same sign. Thus, the present solutions are all ones with $\rho > 0$, entailing small values of $|V_{td}|$. Negative values of ρ appear to require modifications of the mass matrices [28].

(3) The imaginary parts of V_{ub} and V_{td} tend to be too small to accommodate the observed value of the CP-violating parameter ϵ in neutral kaon decays. Since in the limit of a very heavy top quark this parameter is proportional to $\mathrm{Im}(V_{td}^2)m_t^2$ times a slowly varying function of m_t, one expects this conclusion to be true for a wide range of m_t. A related model of mass matrices [35] encounters similar problems. An interesting possibility is that the limit of an extremely heavy top quark in this model might encounter no more serious contradiction with the data than any other value. Such a possibility has been explored for other forms of the mass matrices [36].

(4) The parameters in the mass matrices (6.2) and (6.3) may be related to those in the mass operator (4.6) via Eqs. (4.8) (with corresponding expressions for primed quantities). In the solution in which the signs of the masses of the four heaviest quarks are all positive, a, b, and c are all of order m_t, with small splittings among these quantities responsible for the masses of u and c, and with the CKM phase arising from $a - b = 2i(2m_u m_c)^{1/2}$. Similarly, a', b', and c' are all of order m_b, with small splittings responsible for m_d and m_s. The presence of $\mathbf{S}_3$ in nearly all the terms of the mass operator (4.6) then would suggest that the spin-$\frac{1}{2}$ subunit carrying S_3 changes its identity under a charge-changing weak transition. However, as in the two-family example treated in Sec. III, it seems difficult to preserve the connection between $\mathbf{S}_3$ and the spin of the composite fermion.

Note added in proof. It is possible to insert a relative phase of i between the two terms of Eq. (6.5b), while preserving that in (6.5a), by the substitutions $\alpha \to -i\alpha$ and $\gamma \to i\gamma$ in the mass matrix (4.7) for up quarks. The resulting CKM elements, after rephasing, are

$$V_{us} = 0.226 , \quad V_{cb} = 0.044 , \quad (6.13)$$

$$V_{ub} = 0.0051 - 0.0024i , \quad V_{td} = 0.0049 - 0.0024i ;$$

$$\rho = 0.51 , \quad \eta = 0.24 . \quad (6.14)$$

The values of V_{cb} and η are somewhat closer to the experimental ones than those in (6.8)–(6.11).

VII. SUMMARY AND OUTLOOK

We have explored models for two and three families of quarks based on coupling the spins of several subunits together to form composite systems of spin $\frac{1}{2}$. The higher-spin excitations can be placed at an arbitrarily high (compositeness) scale by means of a spin-spin interaction respecting symmetry under interchange of the subunits, while the spin-$\frac{1}{2}$ levels are split from one another by symmetry-breaking effects. While our discussion was based on a nonrelativistic model of hyperfine interactions, we expect many of the same features to be present in a more realistic treatment. These include correlations between quark masses and mixing angles, and the ability of patterns in these correlations to give us information about the subconstituents.

One open question is the relation of leptons to quarks in models such as the ones we have discussed. In both the two-family and three-family models, we have identified the subunit carrying the spin $\mathbf{S}_3$ as a candidate for the subunit which changes its charge when a W is absorbed or emitted. Thus, we would expect one or more of the other subunits to be responsible for the distinction between a quark and a lepton. In the usual manner of many grand unified theories, we would expect $Q=\frac{2}{3}$ quarks to be related to neutrinos and $Q=-\frac{1}{3}$ quarks to be related to charged leptons. One might expect at the compositeness scale that $m_b=m_\tau$ and $m_t=m_{\nu_\tau}^{(\mathrm{Dirac})}$. The recent analysis of Ref. [24] places the scale at which $m_b=m_\tau$ somewhere above 10^6 GeV, but considerably below a typical grand unification scale of 10^{15} GeV.

Several other recent attempts have been made to connect compositeness with the structure of quark mass matrices. Shrock [37] has explored models in which the origin of fermion masses is largely independent of electroweak symmetry breaking. He has paid particular attention to the fact that the top-quark mass is large, and to new possibilities for neutrino masses. Rau [38] has drawn an analogy between atomic systems and families of quarks and leptons which has much in common in spirit, if not in realization, with the present discussion. Królikowski [39] has constructed a model in which different quark and lepton families consist of different numbers of subunits. Although not based on composite models, the discussion of Dimopoulos, Hall, and Raby [40] is another recent attempt to understand fermion mass matrices, via an ansatz in the context of supersymmetric grand unified theories.

We are still no closer to a fundamental understanding of the origin of the CP-violating phase in the present discussion. We can speculate that it might arise outside the overall framework of compositeness. One possibility becomes more natural once quarks and leptons are being discussed on an equal footing. Additional degrees of freedom are available for forming CP-violating masses in the neutrino sector [41], because of the possibility of forming lepton-number-violating Majorana masses. If CP violation in the lepton sector gets communicated at the compositeness scale to the quark sector, one might expect some regularity of the pattern of the CP-violating phases in the quark mass matrix. Furthermore, since lepton-number and baryon-number violation are linked by the electroweak anomaly [42], an origin of CP violation in the lepton sector might provide a common basis for the two manifestations of CP violation known at present—the baryon asymmetry of the Universe, and the behavior of neutral kaons.

(Note added in proof. Excited higher-spin quarks have been considered in Ref. [43].)

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