# *CP* violation in the decay $K_L \to \pi^+\pi^-e^+e^-$

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We have calculated the distribution $d\Gamma/d\phi$ for the decay $K_L \to \pi^+\pi^-e^+e^-$, where $\phi$ is the angle between the vectors normal to the $\pi^+\pi^-$ and $e^+e^-$ planes. The result can be written in the form $d\Gamma/d\phi = \Gamma_1\cos^2\phi + \Gamma_2\sin^2\phi + \Gamma_3\sin\phi\cos\phi$, where the last term is *CP* violating and involves the interference of the $M1$ component of $K_L \to \pi^+\pi^-\gamma$ with the bremsstrahlung component as well as a possible direct $E1$ component. Using data on the radiative decays $K_{L,S} \to \pi^+\pi^-\gamma$, we estimate an asymmetry in the $\phi$ distribution of $(3.8\pm1.4)\%$.

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## I. INTRODUCTION

Studies of the photon spectrum in the decay $K_L \to \pi^+\pi^-\gamma$ have revealed the existence of two components in the decay rate [1,2]: (i) a bremsstrahlung component associated with the *CP*-violating decay $K_L \to \pi^+\pi^-$, with a branching ratio [2]

$$\frac{\Gamma^{\rm IB}(K_L\to\pi^+\pi^-\gamma;E_\gamma>20\ \text{MeV})}{\Gamma(K_L\to\pi^+\pi^-)} = (6.90\pm0.21)\times10^{-3}\ , \quad (1)$$

and (ii) a direct emission component, presumably of a *CP*-conserving magnetic dipole nature, with a branching ratio [2]

$$\frac{\Gamma^{\rm DE}(K_L\to\pi^+\pi^-\gamma;E_\gamma>20\ \text{MeV})}{\Gamma(K_L\to\pi^+\pi^-)} = (15.0\pm0.6)\times10^{-3}\ . \quad (2)$$

A similar analysis of the decay $K_S \to \pi^+\pi^-\gamma$ has yielded a branching ratio [2]

$$\frac{\Gamma(K_S\to\pi^+\pi^-\gamma;E_\gamma>20\ \text{MeV})}{\Gamma(K_S\to\pi^+\pi^-)} = (6.69\pm0.20)\times10^{-3}\ , \quad (3)$$

which is slightly lower than, but compatible with, the theoretically expected bremsstrahlung rate of $7.00\times10^{-3}$. A genuine difference in this case could be attributed to a small *CP*-conserving direct emission of $E1$ type, which is coherent with the bremsstrahlung amplitude (see the Appendix).

The simultaneous presence of bremsstrahlung and $M1$ amplitudes in the decay $K_L \to \pi^+\pi^-\gamma$ implies that the final state contains both $CP=+1$ and $-1$ configurations [3–7]. However, no interference between these components is visible as long as the polarization of the photon is not observed [8]. As an alternative to measuring polarization, we consider in this paper the decay $K_L \to \pi^+\pi^-e^+e^-$ resulting from the internal conversion of the photon into an $e^+e^-$ pair. We will demonstrate that the angular correlation of the $e^+e^-$ and $\pi^+\pi^-$ planes contains an explicit *CP*-violating term which is sensitive to the interference between the $M1$ amplitude and bremsstrahlung (and direct $E1$) component. The possibility of *CP* violation in the decay $K_L \to \pi^+\pi^-e^+e^-$ was noted long ago by Dolgov and Ponomarev [9]. A calculation of the decay rate of $K_L \to \pi^+\pi^-e^+e^-$, integrated over angles, was performed by Majumdar and Smith [10]. This latter paper has served as a point of reference for our own calculations.

## II. MATRIX ELEMENT FOR $K_L \to \pi^+\pi^-e^+e^-$

The general matrix element for the radiative decay $K_L \to \pi^+\pi^-\gamma$, with momenta labeled as $K_L(\mathcal{P}) \to \pi^+(p_+) + \pi^-(p_-) + \gamma(k)$, is

$$\mathcal{M}(K_L\to\pi^+\pi^-\gamma) = \mathcal{M}_{\rm brem} + \mathcal{M}^{\rm electric}_{\rm direct} + \mathcal{M}^{\rm magnetic}_{\rm direct}\ . \quad (4)$$

The bremsstrahlung piece $\mathcal{M}_{\rm brem}$ [Fig. 1(a)] is given by

$$\mathcal{M}_{\rm brem} = ef_L\left[\frac{p_{+\mu}}{p_+\cdot k} - \frac{p_{-\mu}}{p_-\cdot k}\right]\epsilon^\mu\ , \quad (5)$$

$f_L$ being the coupling constant for $K_L \to \pi^+\pi^-$. The remaining pieces [Fig. 1(b)] are direct emission amplitudes for electric- and magnetic-type radiation, which may be parametrized as

$$\mathcal{M}^{\rm electric}_{\rm direct} = G_E[(p_-\cdot k)p_{+\mu} - (p_+\cdot k)p_{-\mu}]\epsilon^\mu\ ,$$
$$\mathcal{M}^{\rm magnetic}_{\rm direct} = G_M\epsilon_{\mu\nu\rho\sigma}\epsilon^\mu k^\nu p_+^\rho p_-^\sigma\ . \quad (6)$$

Here $G_{E,M}$ are form factors depending on $s_\pi = (p_+ + p_-)^2$ and $(p_+ - p_-)\cdot k$. The bremsstrahlung term gives the exact result for the radiative decay as far

 

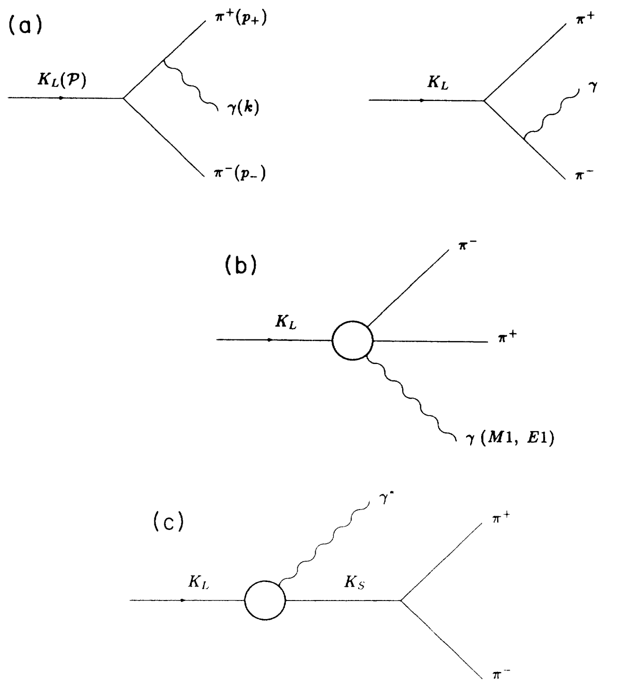


FIG. 1. Diagrams illustrating (a) Bremsstrahlung and (b) direct $M1$ or $E1$ contributions to $K_L \to \pi^+\pi^-\gamma$. Diagram (c) represents an additional $K^0$ charge-radius contribution relevant for $K_L \to \pi^+\pi^- e^+ e^-$.

as terms of order $1/k$ and $k^0$, where $k$ is the photon energy. The direct emission terms, by contrast, vanish in the soft photon limit. Different powers of $(p_+ - p_-)\cdot k$ in the expansion of $G_E$ yield [11] electric multipoles $E1, E2, E3, \ldots$, where the multipole $EJ$ corresponds to a final state with quantum numbers $CP = -(-1)^J$ and $P = +1$. Similarly, different terms in the expansion of $G_M$ yield magnetic multipoles $M1, M2, M3, \ldots$, the multipole $MJ$ corresponding to $CP = (-1)^J$, $P = -1$. (Note that the bremsstrahlung amplitude contains only odd electric multipoles $E1$, $E3$, etc.) In the following we work in the approximation of retaining only dipole terms in the direct emission amplitudes, so that $G_{E,M}$ can be treated as constants. [A dependence on $s_\pi = (p_+ + p_-)^2$, such as that given by the $\rho$ propagator, can be introduced if necessary.]

In going from $K_L \to \pi^+\pi^-\gamma$ to the Dalitz pair process $K_L \to \pi^+\pi^- e^+ e^-$ (with the momenta of $e^+$ and $e^-$ labeled as $k_+$ and $k_-$), we replace $\epsilon^\mu$ in the radiative amplitude by $e/k^2 \cdot \bar{u}(k_-)\gamma^\mu v(k_+)$. Such a replacement, however, is necessarily uncertain by terms of order $k^2$ in the amplitude of the virtual-photon process $K_L \to \pi^+\pi^-\gamma^*$. Such terms include all transitions of the form $K_L \to (\pi^+\pi^-)_{J=0} + \gamma^*$, which are forbidden by angular momentum conservation when $k^2 = 0$, but are possible when $k^2 \neq 0$. A specific example is the contribution of the $K^0$ charge form factor [Fig. 1(c)], which gives rise to a pole in the invariant mass of the $\pi^+\pi^-$ pair at the (unphysical) point $s = m_K^2$. This particular contribution was taken into account by Majumdar and Smith [10] (see also Ref. [13]). For definiteness, we will parametrize our amplitude for $K_L \to \pi^+\pi^- e^+ e^-$ in the same way as in Ref. [10], introducing, in addition to the bremsstrahlung term, phenomenological parameters denoting direct electric dipole, direct magnetic dipole, and a $K^0$ charge-radius contribution.

The matrix element then reads [12]

$$\mathcal{M}(K_L \to \pi^+\pi^- e^+ e^-) = e|f_S| \left[ \frac{g_P}{m_K^2}[k^2 \mathcal{P}_\mu - (\mathcal{P}\cdot k)k_\mu] \frac{1}{k^2 - 2\mathcal{P}\cdot k} + \frac{g_{E1}}{m_K^4}[(\mathcal{P}\cdot k)p_{+\mu} - (p_+\cdot k)\mathcal{P}_\mu] + \frac{g_{M1}}{m_K^4}\epsilon_{\mu\nu\rho\sigma}k^\nu p_+^\rho p_-^\sigma + g_{\rm BR}\left[\frac{p_{+\mu}}{p_+\cdot k} - \frac{p_{-\mu}}{p_-\cdot k}\right]\right] \frac{e}{k^2}\bar{u}(k_-)\gamma^\mu v(k_+) \ . \tag{7}$$

The parameters appearing in the above expression have the following meaning.

(i) $f_S$ is the coupling constant for $K_S \to \pi^+\pi^-$ defined by

$$\Gamma(K_S \to \pi^+\pi^-) = \frac{|f_S|^2}{16\pi m_K}\left[1 - \frac{4m_\pi^2}{m_K^2}\right]^{1/2} . \tag{8}$$

(ii) The parameter $g_{\rm BR}$ is given by

$$g_{\rm BR} = \eta_{+-} \cdot f_S / |f_S| \ . \tag{9}$$

Its phase is $\Phi_{+-} + \delta_0$, where $\Phi_{+-} = \arg(\eta_{+-})$ and $\delta_0$ is the $\pi\pi$ scattering phase in the $I = J = 0$ channel at c.m. energy $\sqrt{s} = m_K$ (we assume here the validity of the $\Delta I = \frac{1}{2}$ rule in $K^0 \to 2\pi$). If the bremsstrahlung term were the only contribution to the decay $K_L \to \pi^+\pi^-\gamma$, the branching ratio would be [14]

$$\frac{\Gamma^{\rm BR}(K_L \to \pi^+\pi^-\gamma; \omega > \omega_{\min})}{\Gamma(K_S \to \pi^+\pi^-)} = |g_{\rm BR}|^2 \frac{\alpha}{\pi}\int_{\omega_{\min}}^{\omega_{\max}} \frac{d\omega}{\omega}\left\{\frac{1+\beta^2}{\beta}\ln\left[\frac{1+\beta}{1-\beta}\right] - 2\right\}\frac{\beta}{\beta_0}\left[1 - \frac{2\omega}{m_K}\right]$$
$$= |g_{\rm BR}|^2 (7.00\times 10^{-3}) \quad \text{for } \omega_{\min} = 20 \text{ MeV} , \tag{10}$$

where $\beta = [1 - 4m_\pi^2/(m_K^2 - 2m_K\omega)]^{1/2}$, $\beta_0 = (1 - 4m_\pi^2/m_K^2)^{1/2}$, and $\omega_{\max} = m_K\beta_0^2/2$.

(iii) The dimensionless parameter $g_{M1}$ measures the strength of the direct $M1$ radiation in $K_L \to \pi^+\pi^-\gamma$. The corresponding decay rate is [14]

$$\frac{\Gamma^{M1}(K_L \to \pi^+\pi^-\gamma; \omega > \omega_{\min})}{\Gamma(K_S \to \pi^+\pi^-)} = \frac{\alpha}{\pi}(g_{M1})^2 \int_{\omega_{\min}}^{\omega_{\max}} \frac{d\omega\, \omega^3}{m_K^4} \frac{\beta^2}{6} \frac{\beta}{\beta_0} \left[1 - \frac{2\omega}{m_K}\right] . \tag{11}$$

Identifying this with the direct emission rate given in Eq. (2), one obtains

$$|g_{M1}| = 0.76 . \tag{12}$$

Since the direct $M1$ transition necessarily produces a $\pi^+\pi^-$ pair in the $I=1$ $p$-wave state, the phase of $g_{M1}$ is equal to $\delta_1(s_\pi)$, the $\pi\pi$ scattering phase in the $\rho$-meson channel at c.m. energy $\sqrt{s_\pi}$.

(iv) The dimensionless parameter $g_{E1}$ defines a $CP$-even $E1$ component in the $K_L \to \pi^+\pi^-\gamma$ amplitude, not related to the bremsstrahlung term. Such a term could be induced by a possible direct $E1$ amplitude in $K_1 \to \pi^+\pi^-\gamma$ through the $\epsilon$ impurity of the $K_L$ wave function. From the remarks following Eq. (3), we estimate that $|g_{E1}/g_{M1}| < 0.05$ (see the Appendix); the phase of $g_{E1}$ in this case is related to that of $g_{M1}$ by $\arg(g_{E1}/g_{M1}) = \Phi_\epsilon \approx \Phi_{+-}$. On the other hand, a direct $E1$ amplitude in $K_L \to \pi^+\pi^-\gamma$ could also result from intrinsic $CP$ violation in $K_2 \to \pi^+\pi^-\gamma$, in which case the $E1$ and $M1$ amplitudes are necessarily out of phase [3], i.e., $\arg(g_{E1}/g_{M1}) = \pi/2$. As a consequence, this latter type of $E1$ amplitude does not contribute to the $CP$-violating observable calculated in the next section. For our subsequent discussion, we consider only a possible $\epsilon$-induced $E1$ component, with

$$\left|\frac{g_{E1}}{g_{M1}}\right| < 0.05, \quad \arg\left[\frac{g_{E1}}{g_{M1}}\right] = \Phi_{+-} . \tag{13}$$

(v) Finally, the dimensionless parameter $g_P$ is related to the $K^0$ charge radius by

$$g_P = -\tfrac{1}{3}\langle R^2\rangle m_K^2 \approx 0.15 , \tag{14}$$

where $\langle R^2\rangle \approx -0.07$ fm$^2$. The phase of $g_P$ is $\delta_0(s)$, the $I=0$ $s$-wave phase shift for $\pi\pi$ scattering at energy $\sqrt{s_\pi}$.

## III. DIFFERENTIAL DECAY RATE

From the matrix element given in Eq. (7), we have calculated the differential decay rate of $K_L \to \pi^+\pi^- e^+e^-$ in the following three variables: (i) $x = (p_+ + p_-)^2/m_K^2$ (normalized invariant mass of pions); (ii) $y = (k_+ + k_-)^2/m_K^2$ (normalized invariant mass of electrons); (iii) $\phi =$ angle between normals to the $e^+e^-$ and $\pi^+\pi^-$ planes. The last of these variables is determined in the following way: In the rest frame of the decaying $K_L$, let $(\mathbf{p}_+ + \mathbf{p}_-)$ be parallel to the positive $z$ direction. The unit vectors

$$\mathbf{n}_\pi = (\mathbf{p}_+ \times \mathbf{p}_-)/|\mathbf{p}_+ \times \mathbf{p}_-| \tag{15a}$$

and

$$\mathbf{n}_l = (\mathbf{k}_+ \times \mathbf{k}_-)/|\mathbf{k}_+ \times \mathbf{k}_-| \tag{15b}$$

then lie in the $xy$ plane and have components

$$\mathbf{n}_\pi = (\cos\phi_\pi, \sin\phi_\pi, 0) ,$$

$$\mathbf{n}_l = (\cos\phi_l, \sin\phi_l, 0) ,$$

where $\phi_\pi$ and $\phi_l$ lie between 0 and $2\pi$. The angle $\phi$ is then defined as

$$\phi = \phi_\pi - \phi_l \bmod (2\pi)$$

and ranges from 0 to $2\pi$.

Our result for the differential decay rate is

$$\frac{\Gamma(K_L \to \pi^+\pi^- e^+e^-)}{\Gamma(K_S \to \pi^+\pi^-)} = \frac{\alpha^2}{16\pi^2\lambda^{1/2}(1,\mu^2,\mu^2)} \int_{4\nu^2}^{(1-2\mu)^2} dy \frac{\lambda^{1/2}(y,\nu^2,\nu^2)}{y^2} \int_{4\mu^2}^{(1-\sqrt{y})^2} dx\, \lambda^{1/2}(1,x,y) \int_0^{2\pi} \frac{d\phi}{2\pi} F(x,y,\phi) , \tag{16}$$

where we have introduced the notation

$$\mu^2 = \frac{m_\pi^2}{m_K^2}, \quad \nu^2 = \frac{m_e^2}{m_K^2} , \tag{17}$$

and $F(x,y,\phi)$ is given by (neglecting the electron mass)

$$\begin{aligned} F(x,y,\phi) = {} & \frac{1}{3}\left[\frac{|g_P|^2}{(x-1)^2} + \mathrm{Re}\left[\frac{g_{E1}g_P^*}{x-1}\right]\right]\frac{y}{x}\lambda(1,x,y)\lambda^{1/2}(x,\mu^2,\mu^2) \\ & + \frac{1}{18}|g_{M1}|^2\frac{1}{x^2}\lambda(1,x,y)\lambda^{3/2}(x,\mu^2,\mu^2)[\tfrac{1}{2}(1+2\cos^2\phi)] \\ & + \frac{1}{18}|g_{E1}|^2\frac{1}{x^2}\lambda^{1/2}(x,\mu^2,\mu^2)\{[\lambda(1,x,y)+6y]\lambda(x,\mu^2,\mu^2)[\tfrac{1}{2}(1+2\sin^2\phi)] + 2y\lambda(1,x,y)(x-\mu^2)\} \\ & + \frac{8}{3}\mathrm{Re}(g_{E1}g_{\mathrm{BR}}^*)\left[\lambda^{1/2}(x,\mu^2,\mu^2) + \frac{-xy+2\mu^2(1-x+y)}{\lambda^{1/2}(1,x,y)}\ln(L)\right] \end{aligned}$$

$$+\frac{8}{3}|g_{BR}|^2\left[\frac{-16\lambda^{1/2}(x,\mu^2,\mu^2)x\mu^2}{x^2(x+y-1)^2-\lambda(1,x,y)\lambda(x,\mu^2,\mu^2)}+\frac{4(x-2\mu^2)}{(x+y-1)\lambda^{1/2}(1,x,y)}\ln(L)[\tfrac{1}{2}(1+2\sin^2\phi)]\right]$$
$$+\frac{8}{3}\mathrm{Re}(g_{M1}g^*_{BR})\frac{1-x-y}{\lambda^{1/2}(1,x,y)}\left[-2\frac{\mu^2(1-x-y)^2+y\lambda(x,\mu^2,\mu^2)}{(x+y-1)\lambda^{1/2}(1,x,y)}\ln(L)+\lambda^{1/2}(x,\mu^2,\mu^2)\right][\sin\phi\cos\phi]$$
$$+\frac{1}{9}\mathrm{Re}(g_{M1}g^*_{E1})\frac{1}{x^2}\lambda^{3/2}(x,\mu^2,\mu^2)\lambda^{1/2}(1,x,y)(1-x+y)[\sin\phi\cos\phi]+\Delta(x,y)[\tfrac{1}{2}(1-2\sin^2\phi)] \ . \quad (18)$$

Here we have used the abbreviations

$$L=\frac{(x+y-1)x+\lambda^{1/2}(1,x,y)\lambda^{1/2}(x,\mu^2,\mu^2)}{(x+y-1)x-\lambda^{1/2}(1,x,y)\lambda^{1/2}(x,\mu^2,\mu^2)} \ ,$$
$$\lambda(x,y,z)=x^2+y^2+z^2-2(xy+yz+zx) \ . \quad (19)$$

The function $\Delta(x,y)$ is given in the Appendix. Since this term is proportional to $(1-2\sin^2\phi)$, it contributes neither to the $\phi$-integrated decay rate nor to the asymmetry under $\phi\to\pi-\phi$ discussed below. If one wants to study the effect of the $\rho$ propagator on the direct $E1$ and $M1$ contributions, the couplings $g_{E1}$ and $g_{M1}$ have to be multiplied by $D_\rho(x)=\sigma^2/(\sigma^2-x)$, $\sigma^2=m_\rho^2/m_K^2$.

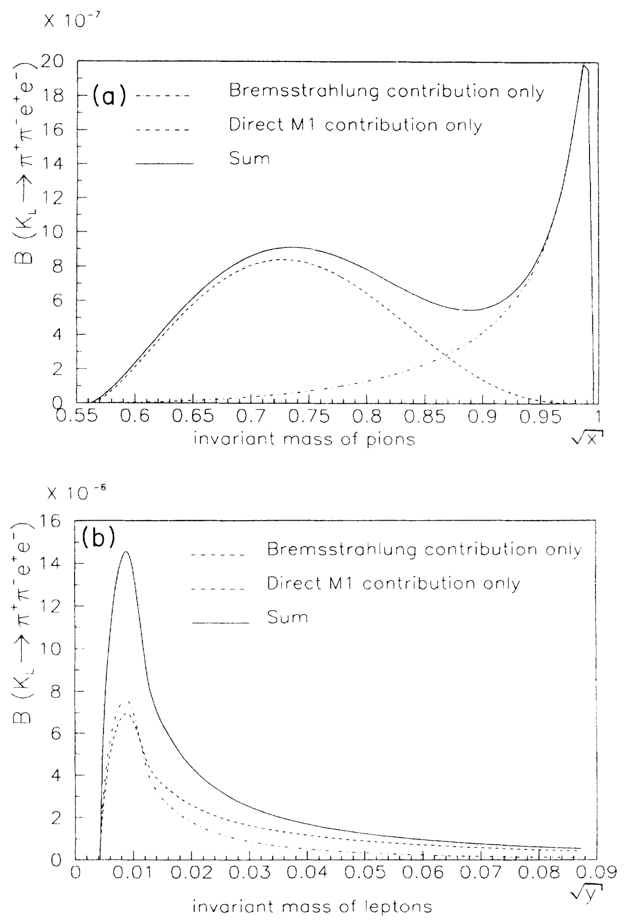


FIG. 2. Differential spectrum $d\Gamma/d\sqrt{x}$ and $d\Gamma/d\sqrt{y}$ for $K_L\to\pi^+\pi^-e^+e^-$, where $\sqrt{x}$ and $\sqrt{y}$ are the invariant masses of $\pi^+\pi^-$ and $e^+e^-$, normalized to $m_K$.

The following general observations may be made on the above formula.

(i) The dependence on the variable $\phi$ is of the general form

$$\frac{d\Gamma}{d\phi}=\Gamma_1\cos^2\phi+\Gamma_2\sin^2\phi+\Gamma_3\sin\phi\cos\phi \ . \quad (20)$$

The last term changes sign under $\phi\to\pi-\phi$ and produces an asymmetry in the distribution of the angle $\phi$ between the normal vectors of the $\pi^+\pi^-$ and $e^+e^-$ planes.

(ii) The fact that the term $\sin\phi\cos\phi$ is $CP$ violating may be judged from the fact that it may be written in the form

$$\sin\phi\cos\phi=(\mathbf{n}_l\times\mathbf{n}_\pi)\cdot\hat{\mathbf{z}}(\mathbf{n}_l\cdot\mathbf{n}_\pi) \ , \quad (21)$$

where $\hat{\mathbf{z}}=(\mathbf{p}_++\mathbf{p}_-)/|\mathbf{p}_++\mathbf{p}_-|$. Noting that, under $C$, $\mathbf{k}_\pm\to\mathbf{k}_\mp$ $\mathbf{p}_\pm\to\mathbf{p}_\mp$ and that, under $P$, $\mathbf{k}_\pm\to-\mathbf{k}_\pm$, $\mathbf{p}_\pm\to-\mathbf{p}_\pm$, we see that the quantity in Eq. (21) changes sign under $CP$.

(iii) The coefficient of the asymmetric term $\sin\phi\cos\phi$ involves the coupling constant combinations $\mathrm{Re}(g_{M1}g^*_{BR})$ and $\mathrm{Re}(g_{M1}g^*_{E1})$, which represent interferences of amplitudes with opposite $CP$ values and are thus manifestly $CP$ violating.

(iv) The pure $M1$ contribution proportional to $|g_{M1}|^2$ has a $\phi$ dependence

$$(1+2\cos^2\phi)=(\sin^2\phi+3\cos^2\phi) \ .$$

This agrees with the result obtained in the paper of Chew [11]; the rate for this contribution was calculated in Refs. [15] and [16].

(v) The differential cross section $d\Gamma/dx\,dy$ obtained after integration over $\phi$ agrees with the result of Majumdar and Smith [10] after certain typographical errors are corrected in their result [17]. The distribution in the $x$ and $y$ variables is depicted in Fig. 2. The total branching ratio, neglecting the small effect of $g_{E1}$, is

$$B(K_L\to\pi^+\pi^-e^+e^-)$$
$$=(1.3\times10^{-7})_{BR}+(1.8\times10^{-7})_{M1}+(0.04\times10^{-7})_P \ .$$

If a cut $\sqrt{y}>30$ MeV is imposed on the invariant mass of the $e^+e^-$ pair, the branching ratios for bremsstrahlung, direct $M1$, and pole contributions become $0.80\times10^{-8}$, $3.7\times10^{-8}$, and $0.4\times10^{-8}$, respectively.

## IV. CP-VIOLATING ASYMMETRY

From the differential cross section $d\Gamma/dx\,dy\,d\phi$, we derive a $CP$-violating asymmetry

$$A(x,y)=\frac{\int_D d\phi\, d\Gamma/dx\, dy\, d\phi}{\int_S d\phi\, d\Gamma/dx\, dy\, d\phi}\,, \tag{22}$$

where

$$\int_D d\phi \left[\int_S d\phi\right] = \left[ \left[\int_0^{\pi/2} + \int_\pi^{3\pi/2}\right] -(+) \left[\int_{\pi/2}^{\pi} + \int_{3\pi/2}^{2\pi}\right]\right] d\phi\,,$$

The asymmetry in the cross section integrated over the $\pi^+\pi^-$ invariant mass is

$$A_y=\frac{\int_D d\phi \int dx\, d\Gamma/dx\, dy\, d\phi}{\int_S d\phi \int dx\, d\Gamma/dx\, dy\, d\phi} = A_1(y)\cos\Theta_1 + A_2(y)\cos\Theta_2 \left|\frac{g_{E1}}{g_{M1}}\right|\,, \tag{23}$$

where

$$\Theta_1=\arg(g_{M1}g_{\rm BR}^*)=\Phi_{+-}+\delta_0-\bar{\delta}_1 \bmod \pi\,,$$
$$\Theta_2=\arg(g_{M1}g_{E1}^*)=\Phi_{+-} \bmod \pi \tag{24}$$

(here $\bar{\delta}_1$ denotes an average phase in the $\pi\pi$ $p$-wave $I=1$ channel). The functions $A_{1,2}(y)$ are plotted in Fig. 3(a) and are quite substantial over the whole range of $y$. The analogous asymmetry in the cross section integrated over $y$ is

$$A_x=\frac{\int_D d\phi \int dy\, d\Gamma/dx\, dy\, d\phi}{\int_S d\phi \int dy\, d\Gamma/dx\, dy\, d\phi} = A_3(x)\cos\Theta_1 + A_4(x)\cos\Theta_2 \left|\frac{g_{E1}}{g_{M1}}\right|\,, \tag{25}$$

where the functions $A_{3,4}(x)$ are shown in Fig. 3(b). Finally, the asymmetry in the cross section integrated over the whole domain of $x$ and $y$ is

$$\langle A \rangle = 15\% \cos\Theta_1 + 38\% \cos\Theta_2 \left|\frac{g_{E1}}{g_{M1}}\right|\,. \tag{26}$$

Inserting $\Phi_{+-}=43°$, $\delta_0=40°$, $\bar{\delta}_1\approx 10°$ (assuming an average $\pi\pi$ mass of approximately 0.4 GeV), and $|g_{E1}/g_{M1}|=0.05$, we have $\cos\Theta_1\approx\pm0.29$, $\cos\Theta_2\approx\pm0.73$, implying an integrated asymmetry of the order of $|\langle A\rangle|=3.8\pm1.4\,\%$.

The branching ratio of $K_L\to\pi^+\pi^-e^+e^-$ predicted by the model is $3.1\times10^{-7}$. The current round of experiments (e.g., E799 at Fermilab) expects to gather 25–50 events of this decay [2]. These statistics will be increased by a factor of 20 in the next phase of this experiment. Still higher statistics may be expected from a dedicated $K$-meson factory. There is a reasonable prospect that the $CP$-violating asymmetry calculated in this paper will be

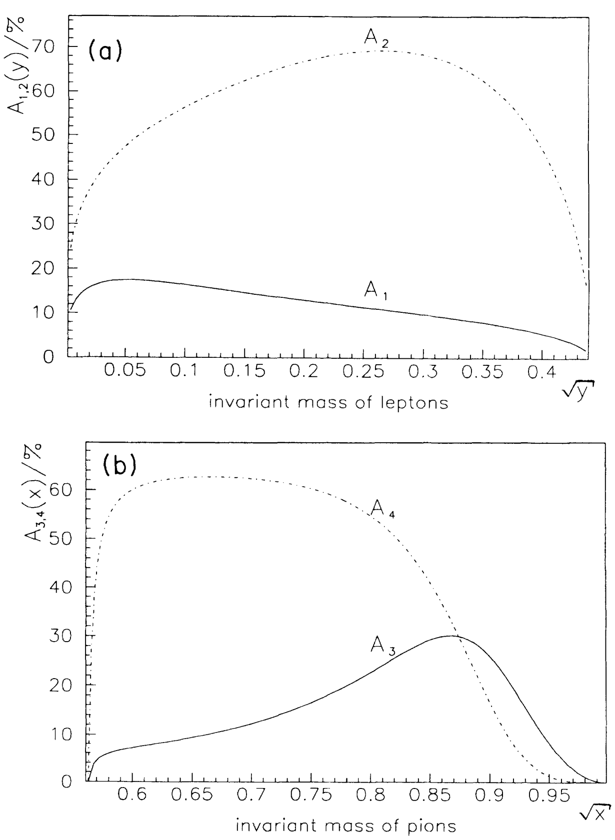


FIG. 3. (a) Functions $A_{1,2}(y)$ defining $CP$-violating asymmetry as function of $y$ [see Eq. (23)]. (b) Functions $A_{3,4}(x)$ defining $CP$-violating asymmetry as function of $x$ [see Eq. (25)].

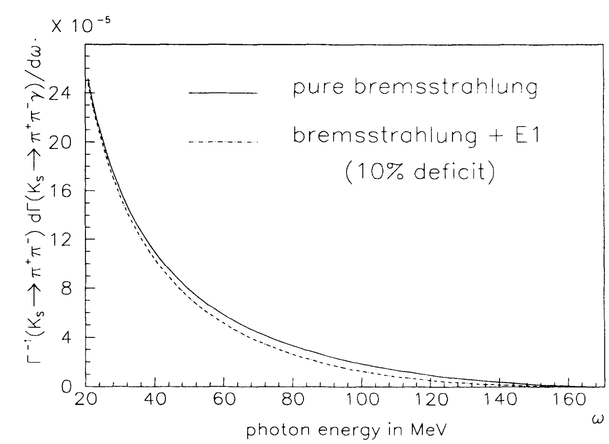


FIG. 4. Photon energy spectrum in the reaction $K_S\to\pi^+\pi^-\gamma$, normalized to $\Gamma(K_S\to\pi^+\pi^-)$, according to Eq. (A3). Here it was assumed that the bremsstrahlung amplitude interferes destructively with the direct $E1$ amplitudes; the dashed curve shows the spectrum for a deficit of 10%.

experimentally accessible.

Although the asymmetry calculated above is as large as 4%, the effect is essentially an $\epsilon$-related phenomenon. The question of a direct $CP$-violating term in the amplitude (7) and its possible impact on the above asymmetry remains to be investigated. A related question is the possible size of a $\pi^+/\pi^-$ or $e^+/e^-$ asymmetry in the Dalitz plot. We hope to return to these issues in a future publication.


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## APPENDIX

(1) The function $\Delta(x,y)$ appearing in the expression for $d\Gamma/dx\,dy\,d\phi$ [Eq. (18)] is given by

$$\begin{aligned}\Delta(x,y)=&\frac{1}{18}|g_{E1}|^2\frac{1}{x^2}\lambda^{3/2}(x,\mu^2,\mu^2)2y\\&+\frac{8}{3}\mathrm{Re}(g_{E1}g^*_{\mathrm{BR}})\left[\lambda^{1/2}(x,\mu^2,\mu^2)\frac{(1-x)^2-y^2}{\lambda(1,x,y)}-2(1-x+y)\frac{x^2y+\mu^2\lambda(1,x,y)}{\lambda^{3/2}(1,x,y)}\ln(L)\right]\\&+\frac{8}{3}|g_{\mathrm{BR}}|^2\left[\frac{16x^2y\lambda^{1/2}(x,\mu^2,\mu^2)}{\lambda(1,x,y)[x^2(x+y-1)^2-\lambda(1,x,y)\lambda(x,\mu^2,\mu^2)]}-\frac{8x^2y}{(x+y-1)\lambda^{3/2}(1,x,y)}\ln(L)\right]\ .\end{aligned}\tag{A1}$$

(2) We give here an estimate of the direct $E1$ coupling $g_{E1}$, defined in Eq. (7), based on the discrepancy between the measured rate of $K_S\to\pi^+\pi^-\gamma$ and the pure bremsstrahlung expectation [see remarks after Eq. (3)]. Defining the decay amplitude of $K_S\to\pi^+\pi^-\gamma$ by

$$\mathcal{M}_S=eg_{\mathrm{BS}}\left[\frac{p_{+\mu}}{p_+\cdot k}-\frac{p_{-\mu}}{p_-\cdot k}\right]\epsilon^\mu+e\frac{g_{ES}}{m_K^4}(p_+\cdot\epsilon p_-\cdot k-p_-\cdot\epsilon p_+\cdot k)\ ,\tag{A2}$$

the decay rate can be written as

$$\begin{aligned}\frac{\Gamma(K_S\to\pi^+\pi^-\gamma)}{\Gamma(K_S\to\pi^+\pi^-)}=\frac{\alpha}{\pi}\Bigg\{&\int_{\omega_{\min}}^{\omega_{\max}}\frac{d\omega}{\omega}\left[\frac{\beta^2+1}{\beta}\ln\left[\frac{1+\beta}{1-\beta}\right]-2\right]\frac{\beta}{\beta_0}\left[1-2\frac{\omega}{m_K}\right]\\&+\mathrm{Re}\left[\frac{g_{ES}}{g_{BS}}\right]\int_{\omega_{\min}}^{\omega_{\max}}\frac{d\omega\,\omega}{m_K^2}\left[\frac{\beta^2-1}{\beta}\ln\left[\frac{1+\beta}{1-\beta}\right]+2\right]\frac{\beta}{\beta_0}\left[1-2\frac{\omega}{m_K}\right]\\&+\left|\frac{g_{ES}}{g_{BS}}\right|^2\int_{\omega_{\min}}^{\omega_{\max}}\frac{d\omega\,\omega^3}{m_K^4}\left[\frac{\beta^2}{6}\right]\frac{\beta}{\beta_0}\left[1-2\frac{\omega}{m_K}\right]\Bigg\}\\=7.00\times10^{-3}&\left[1+\mathrm{Re}\left[\frac{g_{ES}}{g_{BS}}\right]5.4\times10^{-3}+\left|\frac{g_{ES}}{g_{BS}}\right|^2 1.9\times10^{-5}\right]\ ,\end{aligned}\tag{A3}$$

where $\beta$, $\beta_0$, and $\omega_{\max}$ have the same definition as in Eq. (10) and a minimum photon energy of 20 MeV was required.

Assuming a discrepancy (deficit) of $(4\pm4)\%$ in the decay rate of $K_S\to\pi^+\pi^-\gamma$, we obtain

$$\mathrm{Re}\left[\frac{g_{ES}}{g_{BS}}\right]=-(8\pm8)\ .$$

This fixes also the ratio $g_{E1}/g_{\mathrm{BR}}$ of the constants in Eq. (7), i.e.,

$$\mathrm{Re}\left[\frac{g_{E1}}{g_{\mathrm{BR}}}\right]=\mathrm{Re}\left[\frac{g_{ES}}{g_{BS}}\right]=-(8\pm8)\ .\tag{A4}$$

This result, together with Eqs. (9) and (12), is the basis of the estimate of $|g_{E1}/g_{M1}|$ given in Eq. (13). (As may be seen from Fig. 4, even a deficit of 10% does not appreciably affect the shape of the photon energy spectrum.)

[1] A. Carroll *et al.*, Phys. Rev. Lett. **44**, 529 (1980).
[2] E731 Collaboration, E. Ramberg, Fermilab Report No. Fermilab-Conf-91/258, 1991 (unpublished).
[3] L. M. Sehgal and L. Wolfenstein, Phys. Rev. **162**, 1362 (1967).
[4] G. Costa and P. K. Kabir, Nuovo Cimento A **61**, 564 (1967).
[5] T. D. Lee and C. S. Wu, Annu. Rev. Nucl. Sci. **16**, 511 (1966).
[6] K. Hiida and Y.-Y. Lee, Phys. Rev. **167**, 1403 (1968).
[7] Y. C. R. Lin and G. Valencia, Phys. Rev. D **37**, 143 (1986).
[8] *CP* violation can manifest itself in the decay $K_L \to \pi^+\pi^-\gamma$ as a $\pi^+$-$\pi^-$ asymmetry in the Dalitz plot. This, however, requires a *CP*-violating $E2$ multipole interfering with the bremsstrahlung amplitude, and the expected effect is very small [5]. Another manifestation is $K_L$-$K_S$ interference in the time dependence of the decay into a $\pi^+\pi^-\gamma$ final state [3]. This effect has recently been observed [2].
[9] A. D. Dolgov and L. A. Ponomarev, Yad Fiz. **4**, 367 (1966) [Sov. J. Nucl. Phys. **4**, 262 (1967)].
[10] D. P. Majumdar and J. Smith, Phys. Rev. **187**, 2039 (1969).
[11] H. Chew, Nuovo Cimento **26**, 1109 (1962).
[12] In the coefficient of $g_{E1}$ in Eq. (7), we have used $(\mathcal{P}\cdot k)p_{+\mu} - (p_+ \cdot k)\mathcal{P}_\mu$ instead of the form given in Eq. (6), in order to conform with the notation of Ref. [10]. This makes no difference for a real photon, but introduces an uncertainty of order $k^2$ when the photon is virtual.
[13] L. A. Kondratyuk, L. A. Ponomarev, and V. I. Zakharov, Phys. Lett. **27B**, 655 (1968).
[14] E.g., B. DeWitt and J. Smith, *Field Theory in Particle Physics* (North-Holland, Amsterdam, 1986), Vol. 1.
[15] E. Celeghini and R. Gatto, Nuovo Cimento **28**, 3013 (1963).
[16] C. Jarlskog and H. Pilkuhn, Nucl. Phys. **B1**, 264 (1967).
[17] The following corrections should be made in Eqs. (2.3) and (2.5) of Ref. [10]: (a) The $\alpha$ in (2.3) should be replaced by $4\pi\alpha^2$; also, the subscript $S$ on the left-hand side of that equation should be dropped; (b) the overall sign of the $B_1 C$ term has to be changed; in addition, the term $(1-x-y)$ proportional to $B_1 C$ has to be replaced by $(-1)(1-x+y)$; (c) the first term of $B_1^2$ (inside the brackets) should be divided by $x$; (d) the second term in $C^2$ should have a factor of 4; (e) the couplings $A1$, $B1$, and $B2$ should be multiplied by $m^4$ for dimensional reasons.