Model-independent connections between pion photoproduction and Compton scattering in the $\Delta(1232)$ region

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We exploit Grushin *et al.*'s *independent determinations* of the real and the imaginary parts of the pion photoproduction multipoles in the $\Delta(1232)$ region, without the use of an approximate form of Watson's theorem, and obtain model-independent estimates for the Compton amplitudes for the resonance excitation off nucleons. We compare these with the results obtained from other analyses of photoproduction multipoles. We find the lower bound for the differential cross section using unitarity, and compare our results with available experiments, both the older-generation experiments, primarily at Bonn, and the preliminary results from the recent studies at the Saskatchewan Accelerator Lab. We explore consistency of photoproduction multipoles with the forward Compton scattering amplitude extracted from the measured total hadronic cross sections of photons in hydrogen, using the optical theorem. Finally, we discuss implications for future precision Compton studies of the $\Delta(1232)$ excitation, in particular, attempts to measure the $E2$ to $M1$ amplitude ratio, in the nucleon-to-Δ electromagnetic transition, which will be feasible at new photon facilities such as the Brookhaven Laser Electron-Gamma Source, exploring the photon polarization observables. These amplitudes contain valuable information on the structure of nucleon and Δ baryons, of great topical interest. We vividly demonstrate here the inadequacy of the older generation of Compton scattering experiments, as their poor photon energy resolution and counting statistics limit the quality of physics extractable from the data. This urgently calls for newer-generation *high-statistics* experiments with *high* photon energy resolution, using the photon polarization as a powerful tool.

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I. INTRODUCTION

The Compton scattering (CS) process

$$\gamma+N\rightarrow\gamma+N \tag{1}$$

is a classic probe of the structure [1] of the nucleon N, and whatever hadronic states that can be excited at the expense of the incident photon. In the so-called *intermediate-energy* domain, defined for the purpose of this paper by the photon lab energy E_γ lying roughly between the pion photoproduction threshold and 2 GeV, one can study the excitation of the nucleon resonances, of basic interest to any understanding of hadronic structure via quantum chromodynamics (QCD). In this energy regime, an especially interesting region is the one spanning the excitation for the $\Delta(1232)$ resonance because of the splendid isolation of this resonance from others. One advantage of studying the Δ resonance by CS is the absence of complications from the probe particle itself, as direct manifestations of any hadronic interaction by the photon (say, that implied by the vector dominance model [2]) are absent in the photon interactions as these energies. By unitarity, CS is related to the processes [3]

$$\pi+N\rightarrow\pi+N\ , \tag{2a}$$

$$\gamma+N\rightarrow\pi+N\ , \tag{2b}$$

wherein the pion can be produced *with* or *without* excitation of the Δ resonance. One of the issues that we shall examine in this paper is the application of unitarity at a level of precision allowed by the current data. In CS, the complex intermediate hadron dynamics must finally yield the simple initial state with which one starts, viz., a photon and a nucleon, neatly hiding all of the hadronic violence, and yet containing its full implications on the observables. Among the many applications of the CS, of potential interest to the objectives of this paper, are its relationships [4] to nucleon-antinucleon annihilation via crossing symmetry, and applications involving dispersion relations and the optical theorem [5]. Recently, attempts have also been made to examine the single-nucleon CS process as a probe of hadron models [6], and as an input to the study of CS in complex nuclei [7]. Finally, there is the important issue of the validity of the Drell-Hearn-Gerasimov sum rule and its connections to deep-inelastic scattering [8].

Despite the fundamental significance of CS, touched on

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above, experimental data on it *in the* Δ *region* have been very scarce due to the relative smallness of the cross section of the process (1), proportional to the square of the fine-structure constant α. Also neutral-pion photoproduction, and the subsequent decay of π^0 to two photons, provides much more intense competition in this energy regime. Older data of interest here come from experiments done in the 1960s, and have been reviewed by Genzel, Joos, and Pfeil [9]. The best available data, so far, come from the Bonn group [10] some 15 years ago. To this, more recent data, of limited quantity, have been added (above the Δ peak) by Wada *et al.* and by Ishii *et al.* [11]; most recently, Delli Carpini, Booth, and Miller [12] have measured angular distributions for E_γ ranging from 200 to 290 MeV below the $\Delta(1232)$ peak. Theoretical understanding of this process has been hampered by this paucity of data, and only one critical theoretical study [13] is available to bear on this region. The importance of further experimental and theoretical studies has been underscored by Genzel *et al.* in their experimental report [10]:

"While most of the [Bonn] experimental data lie well above the unitarity limit (of Pfeil *et al.* [13]), a critical situation becomes visible at the $\Delta(1232)$ resonance energy. Here the data points are equal to or even somewhat lower than the limit. This fact reveals a basic problem of the theoretical understanding of the Compton scattering."

Further examination of this important point, in the context of our present knowledge of the pion photoproduction multipoles, will be one of the foci of this paper. We shall also make use of the optical theorem and check if the extracted photopion multipoles are consistent with the measured total photon-hadron cross section.

Fortunately, the experimental situation is about to change. Several electron and photon "factories" are under construction or development around the world. At energy somewhat below the $\Delta(1232)$ peak, the University of Saskatchewan Accelerator Laboratory (SAL) operates a tagged photon facility [14] that is beginning to produce precise CS data. A particularly interesting facility, which has recently come into operation, is the Brookhaven Laser Electron-Gamma Source (LEGS) [15]. It has intense medium-energy photon beams, generated by the CS of laser-produced photons by the accelerated electrons (at $\sim$2–3 GeV); the resultant photons can be highly polarized at will. This is likely to revolutionize the experimental art of studying hadrons with CS, particularly in the Δ region, wherein difficult polarization experiments would become feasible. This would be extended to the higher-resonance region with emerging and improving facilities like those at Bates, Bonn, and Mainze, and the Continuous Electron Beam Accelerator Facility (CEBAF) now under construction. This exciting prospect provides a topical motivation for this work.

One interesting experiment proposed for LEGS by Sandorfi *et al.* [15], is the measurement of the difference of cross sections:

$$\mathcal{S}=\frac{d\sigma_\parallel}{d\Omega}-\frac{d\sigma_\perp}{d\Omega}\ , \tag{3}$$

where $d\sigma_\parallel/d\Omega$ and $d\sigma_\perp/d\Omega$ are the CS cross sections in the c.m. frame, with the incoming photon polarized parallel or perpendicular to the scattering plane. It has been argued [15] that this measurement would yield the ratio of the Compton amplitudes,

$$R=\sqrt{3}\frac{\mathrm{Im} f_{ME}^{1+}}{\mathrm{Im} f_{MM}^{1+}}\ , \tag{4}$$

where the CS amplitudes are in the standard notation [16]. This, in turn, is related to the ratio of the electric quadrupole to the magnetic dipole amplitudes (EMR) in the resonant excitation [17] in the electromagnetic process (2). We recall here for the reader the importance of this ratio to models of hadrons: A nonzero value of this ratio could be due to the "deformed" configurations [18] induced by the color hyperfine interaction [19] in nonrelativistic quark models. It can also be nonzero due to other reasons. Thus, in relativistic quark models, it may not vanish even in "spherical" configurations [20]. In the deformed bag model [21], Skyrmion [22], and other soliton models [23], it is also nonzero. Presently, the magnitude of the EMR is estimated [24] to be about 1% from the analysis of the pion photoproduction data. The success of the proposed experiment at LEGS depends on a number of favorable situations for which the expression for $\mathcal{S}$ simplifies greatly. These include the additional demand [15] that

$$\mathrm{Re} f_{MM}^{1+}=0\ , \tag{5a}$$

$$\mathrm{Re} f_{ME}^{1+}=0\ , \tag{5b}$$

and $\mathrm{Im} f_{MM}^{1+}$ must be accurately known, for some photon energy in the Δ region. This last item, in particular, provides us with a good motivation for examining the photo-pion multipole data sets. We shall also examine the validity of relation (5a).

Our analysis reported in this paper is inspired by a novel determination of multipole amplitudes for the pion photoproduction process by Grushin *et al.* [25] for E_γ from 300 to 420 MeV covering the Δ region. This work is, to our knowledge, ***unique*** among various multipole analyses in that the real and imaginary parts of the multipole amplitudes are *independently* determined, unlike all other analyses [26] preceding it. In these earlier analyses, an approximate form of the exact unitarity theorem, commonly called Watson's theorem [27], has to be invoked, and only pion-nucleon strong phase shifts are used to fix the phase between real and imaginary parts of a given multipole amplitude. In so doing, the Compton phase shift δ^c is neglected compared with the corresponding strong pion-nucleon phase shift δ^π:

$$\delta^c+\delta^\pi\approx\delta^\pi\ . \tag{6}$$

In the present work, we shall *not* make this approximation in discussing magnetic CS, thanks to the opportunity provided by the analysis of Grushin *et al.* [25], where the information on the Compton phase shift is implicitly contained and preserved in their multipole amplitudes.

Before concluding this section, let us define what we mean by "model-independent" connections between pion photoproduction and Compton scattering. By this, we refer to the results we shall derive primarily by exploiting unitarity. We shall also take advantage of the fact that CS in the $\Delta(1232)$ resonance region will be dominated by the magnetic dipole amplitude. In addition, dispersion-theoretic and other phenomenological indications would allow us to take the real parts of the CS amplitudes to be zero in this resonance region, simplifying considerably many observables. We shall also pay attention to the prospect of testing this last criterion directly from the CS experiments. The remainder of this paper is organized as follows. Section II deals with a precise determination of the dominant Compton amplitude relevant to this paper, that in the spin-isospin $\frac{3}{2}$ channel. Section III explores the unitarity relations for Compton scattering, discussing, in particular, the problem with the Bonn data and examines the recent SAL data at lower energies. Section IV elucidates the use of the optical theorem to determine the forward Compton amplitude and its usefulness. Section V analyzes the prospect of the determination of the resonant electric quadrupole amplitude in the $N \Longleftrightarrow \Delta(1232)$ excitation by photons. Finally, Sec. VI summarizes our conclusions, and poses some further research problems in this area.

II. MAGNETIC COMPTON AMPLITUDE IN THE SPIN-ISOSPIN $\frac{3}{2}$ CHANNEL: DETERMINATION FROM PION PHOTOPRODUCTION

Let us first consider photon-induced processes (1) and (2b), along with the πN elastic scattering (2a), in the dominant spin-isospin $\frac{3}{2}$ channel in which $\Delta(1232)$ is resonant. For the electromagnetic processes, we shall first consider only the dominant $M1$ (magnetic dipole) contribution, ignoring the corrections due to other small, but possibly nonzero, amplitudes. The general 2×2 **S** matrix, manifestly unitary and time-reversal invariant is

$$\mathbf{S}=\begin{bmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{bmatrix} , \quad (7)$$

where η is the elasticity parameter, and δ_1 and δ_2 are the strong and Compton phase shifts in the $I=J=\frac{3}{2}$ (33) channel. The above form displays explicitly the Fermi-Watson theorem: the off-diagonal amplitudes are proportional to the sum of the strong *and* Compton phases. All previous multipole analyses [26] of the pion photoproduction data, other than that of Grushin *et al.* [25], ignore the phase δ_2 in the off-diagonal amplitude. The **T** matrix, defined in the usual fashion,

$$\mathbf{T}=i(1-\mathbf{S})/2 , \quad (8)$$

has matrix elements given by

$$T_{kk}==\frac{i}{2}(1-\eta e^{2i\delta_k}) , \quad (9a)$$

$$T_{12}=\frac{\sqrt{1-\eta^2}}{2}e^{i(\delta_1+\delta_2)} . \quad (9b)$$

It is now useful to recall the relationship between **T** matrix elements and experimental photoproduction multipoles available [25,26] in the literature:

$$T_{12}=\sqrt{2qk}\,M_{1+}^{(3)} , \quad (10)$$

with q and k being the meson and photon c.m. momenta, $M_{1+}^{(3)}$ being the magnetic dipole amplitude in the 33 channel.

Let us write the expression for $\mathrm{Re}T_{22}$ and $\mathrm{Im}T_{22}$ explicitly. From (9a), we have

$$\mathrm{Re}T_{22}=\tfrac{1}{2}\eta\sin2\delta_2 , \qquad \mathrm{Im}T_{22}=\tfrac{1}{2}(1-\eta\cos2\delta_2) . \quad (11)$$

From the work of Grushin *et al.*, we can estimate the angle $\Psi=\delta_1+\delta_2$ and the quantity η, since they give $\mathrm{Re}T_{12}$ and $\mathrm{Im}T_{12}$ separately. Thus,

$$\tan\Psi=\mathrm{Im}T_{12}/\mathrm{Re}T_{12} , \quad (12a)$$

$$\eta=\sqrt{1-4|T_{12}|^2} . \quad (12b)$$

Knowing δ_1 from the phase-shift analyses [28] of the pion-nucleon scattering data, δ_2 can be estimated using Eq. (12). Equations (11) then give us the dominant piece of the magnetic Compton scattering amplitude. From Eqs. (11), we can obtain the commonly defined form of the Compton amplitude for the proton, by the relations

$$f_{MM}^{1+(3)}=\frac{T_{22}}{2k} , \quad (13a)$$

$$f_{MM}^{1+}\simeq\frac{2}{3}f_{MM}^{1+(3)} , \quad (13b)$$

ignoring the small isospin-$\frac{1}{2}$ contribution for the proton target. Using the VPI [28] phase shift δ_{33}, the quantity $\mathrm{Im}f_{MM}^{1+}$ is most accurately known from the Grushin *et al.* pion photoproduction multipoles at $E_\gamma^0=348$ MeV, corresponding to c.m. energy $W_0=1239$ MeV. In units of $10^{-4}/m_{\pi^+}$, this is

$$\mathrm{Im}f_{MM}^{1+}=15.4\pm0.2 . \quad (14)$$

Using the Karlsruhe phase shifts [28], on the other hand, the most accurate value of $\mathrm{Im}f_{MM}^{1+}$ is found from Grushin *et al.*'s photoproduction multipoles at $E_\gamma^0=343$ MeV, corresponding to $W_0=1235$ MeV. In the same units as before, this is

$$\mathrm{Im}f_{MM}^{1+}=15.7\pm0.2 . \quad (15)$$

Several points are in order here. First, the multipoles from the photoproduction of pions can yield the magnetic Compton amplitude to a precision better than 2% only at a particular energy E_γ^0. Second, this energy is sensitively determined by the πN phase shifts: For the Karlsruhe and VPI phase shifts, this energy value is not identical, but separated by 5 MeV. Finally, away from this "magic" energy value, the precision of $\mathrm{Im}f_{MM}^{1+}$, determined from the photomultipoles, decreases drastically (Fig. 1). Given the relatively improved resolution of the photon beam energy currently achievable at emerging facilities such as the Brookhaven LEGS ($\sim\pm3$ MeV at $E_\gamma=300$ MeV), our ability to determine rather precisely ($\leq2\%$) the Compton magnetic amplitude at one energy with an uncertainty of ±2.5 MeV using the data on pho-

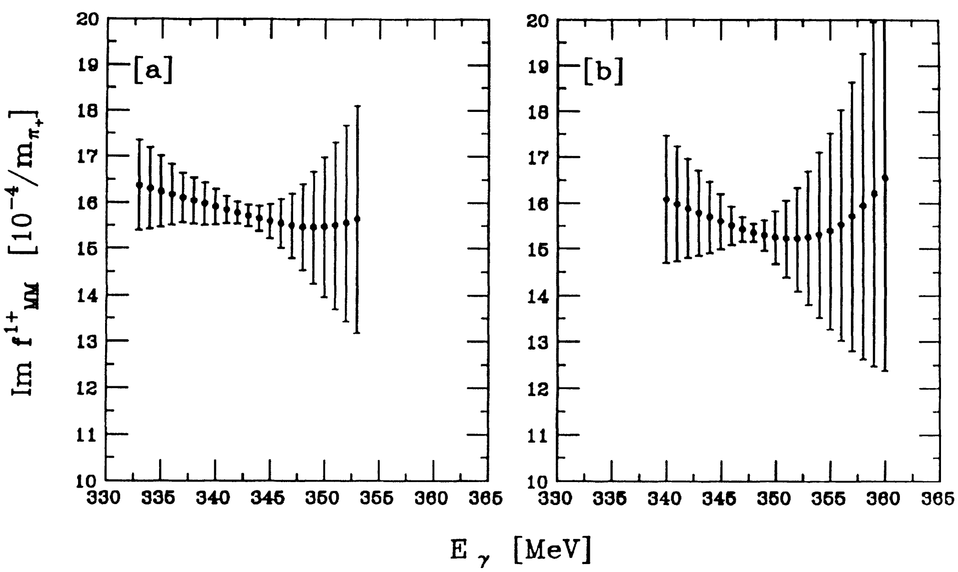

FIG. 1. The imaginary part of the dominant magnetic Compton amplitude f^{1+}_{MM} from the analysis of the 2×2 **S** matrix (Sec. II). (a) and (b) correspond, respectively, to our use of the Karlsruhe (KH80) and VPI (SP89) sets of the pion-nucleon 33 phase shifts [28] in the analysis.

toproduction of pions demonstrated here, would not be wasted. We can now exploit at such facilities the precision we have reached here in extracting the magnetic Compton amplitude from the pion photoproduction data.

We conclude this section by noting that our treatment of the magnetic Compton amplitude for the proton, presented above, is approximate in two ways: we have neglected the small roles of the $I=\frac{1}{2}$ multipoles in arriving at Eq. (13b), and ignored the relatively tiny contribution of the $E2$ photon. A complete treatment requires a consideration of the 4×4 **S** matrix, which takes into account all hadron charge channels relevant and $M1$ and $E2$ photon multipoles. Ignoring the $E2$ contribution, it is at least a 3×3 matrix. This has some consequences that we discuss in the next section.

III. UNITARITY AND COMPTON SCATTERING

We now focus here on the question of unitarity and CS, in particular, the question, raised in Sec. I, as to whether the data of Genzel *et al.* [10] at $E_\gamma=320$ MeV, $\theta_{\text{c.m.}}=90°$ violate the unitarity bound [13]. To make our discussion complete, we shall also examine other available data on CS in the Δ region, viz., those from Tokyo experiments [11] and the new preliminary data from SAL at lower energies [14]. In all, we shall examine the photon lab energy region from 180 to 420 MeV. For a survey of the formalism, we refer the reader to the work of Pfeil *et al.* [13], wherein the relevant formulas have been collected. The most important difference of our work in relation to the analyses of Pfeil, Rollnik, and Stankowski [13] and Sandorfi *et al.* [15] is that we shall make use of the new pion photoproduction multipole data base provided by Grushin *et al.* [25], where the Compton phase has not been implicitly neglected. The unitarity relation for the **T** matrix is

$$i[T^\dagger_{lm}-T_{lm}]=\sum_f \langle l|\mathbf{T}^\dagger|f\rangle\langle f|\mathbf{T}|m\rangle \ , \tag{16}$$

where one can safely ignore, in the intermediate state in the $\Delta(1232)$ region, relatively small contributions due to multiple pion photoproduction and production of heavier mesons. From this, it is easy to verify that isospin-breaking effects at the level of strong scattering amplitudes must be included, in order to achieve consistency in unitarity relations contained in Eq. (16), if we include effects of order e^2 in the amplitude.

To demonstrate this last point, let us discuss the 3×3 problem in which the physical channels are

$$(1)\ \pi^+n\ ,\quad (2)\ \pi^0p\ ,\quad (3)\ \gamma(M)p\ , \tag{17}$$

where $\gamma(M)$ represents a magnetic-dipole photon in this instance [we can extend this problem to 4×4, by including an electric-quadrupole photon $\gamma(E)$, but the essence of our remarks here would also apply to that case]. Ignoring isospin-breaking corrections, we can write the **T** matrix elements for the physical channels as [29]

$$\begin{aligned} T_{11}&=\tfrac{1}{3}[2t_1+t_3]\ ,\\ T_{22}&=\tfrac{1}{3}[t_1+2t_3]\ ,\\ T_{12}&=\frac{\sqrt{2}}{3}[t_1-t_3]\ ,\\ T_{13}&=\frac{\sqrt{2}}{3}[3m_1-m_3]\ ,\\ T_{23}&=\tfrac{1}{3}[3m_1+2m_3]\ , \end{aligned} \tag{18}$$

where t_i and m_i are the strong and electrostrong amplitudes in the isospin basis. Rewriting (16) in the form

$$\mathrm{Im}T_{ij}=\sum_k T_{ik}T_{jk}^* \ , \tag{19}$$

we can determine $\mathrm{Im}(t_3-t_1)$ in two ways, starting from (19): using $i=1$, $j=2$, or combining the two equations obtained with $i=j=1$ and $i=j=2$. The resulting equations *do not agree* at the level of interference of the magnetic terms proportional to $m_1 m_3^*$. Thus, isospin breaking at the strong-interaction level *must be included* in order to have complete consistency with the unitarity requirement (19) if we wish to include electromagnetic corrections. This is easily understood: the electromagnetic interaction is responsible for isospin violation at the strong-interaction level. The former cannot be properly taken care of without the latter.

Thus, the analysis presented in Sec. II is only approximately correct: the exact procedure would require going at least to the 3×3 level, and would necessitate the consideration of isospin violation at the strong-interaction level on one hand, and investigating the 4×4 **S** matrix including the electric-quadrupole photon on the other. We do not believe we have experimental and theoretical precisions at hand to do this here. For example, the isospin-breaking effects at the level of pion-nucleon scattering are not fully understood in extracting the strong phase shifts. Henceforth, we shall continue with the approximate analysis, wherein we shall take into account effects of smaller multipoles, wherever possible, but continue to ignore the electromagnetic effects at the level of strong interactions.

A. Unitarity relations for Compton scattering amplitude

We now return to the discussion of the unitarity bound for CS. Starting from Eq. (16), it is straightforward to derive the unitarity relations for the partial-wave Compton amplitudes:

$$\mathrm{Im}f^{L+}_{\substack{MM\\EE}}=q\sum_c\begin{Bmatrix}|M^c_{L+}|^2\\|E^c_{(L+1)-}|^2\end{Bmatrix}+k(L+1)\{L|f^{L+}_{\substack{MM\\EE}}|^2+(L+2)|f^{L+}_{\substack{ME\\EM}}|^2\} \ , \tag{20a}$$

$$\mathrm{Im}f^{(L+1)-}_{\substack{MM\\EE}}=q\sum_c\begin{Bmatrix}|M^c_{(L+1)-}|^2\\|E^c_{L+}|^2\end{Bmatrix}+k(L+1)\{(L+2)|f^{(L+1)-}_{\substack{MM\\EE}}|^2+L|f^{(L+1)-}_{\substack{ME\\EM}}|^2\} \ , \tag{20b}$$

$$\mathrm{Im}f^{L+}_{\substack{ME\\EM}}=\mp q\,\mathrm{Re}\sum_c\begin{Bmatrix}M^c_{L+}E^{c*}_{L}\\M^c_{(L+1)}-E^{c*}_{(L+1)-}\end{Bmatrix}+k(L+1)\times\mathrm{Re}\{[Lf^{L+}_{\substack{MM\\EE}}+(L+2)f^{(L+1)-}_{\substack{EE\\MM}}]f^{L+*}_{\substack{ME\\EM}}\} \ , \tag{20c}$$

where the lower line corresponds to the second amplitude on the left-hand side of Eqs. (20), L is the total angular momentum of the photon, and the notation $L\pm$ means that the total angular momentum of the γ-N system is given by $J=L\pm\frac{1}{2}$, the summation on c is over the two possible hadron charge states π^+n and π^0p in the photoproduction of pions from the proton. Note that we have correction terms in Eq. (20a), to take one example, due to the small Compton contributions to the photoproduction contribution embodied in the first term on the right-hand side of Eq. (20a). These small corrections have been ignored in our 2×2 unitarity equations discussed in the last section. We prefer to include them here to make the unitarity discussion theoretically a bit more accurate, though it will turn out to have no *real* numerical significance for the dominant magnetic CS amplitude beyond the 2×2 unitarity relation, which is recovered here by dropping the Compton corrections on the right-hand side of Eqs. (20). Effects of these small corrections can be estimated by taking the value of $\mathrm{Im}f^{1+}_{MM}$ from Eq. (20a), ignoring these corrections. This gives, for the Grushin *et al.* multipoles at $E_\gamma=320$ MeV,

$$[\mathrm{Im}f^{1+}_{MM}]_A\simeq 16.813\frac{10^{-4}}{m_{\pi+}} \ . \tag{21}$$

We can now use this estimate on the right-hand side of Eq. (20a). This yields the corrected value of $\mathrm{Im}f^{1+}_{MM}$ to be

$$[\mathrm{Im}f^{1+}_{MM}]_B\simeq 16.913\frac{10^{-4}}{m_{\pi+}} \ . \tag{22}$$

Given the error on f^{1+}_{MM} (see Table I), the difference between (21) and (22) is negligible. One can easily show that similar conclusions hold for the other Compton amplitudes. Thus, Pfeil *et al.*'s estimating procedure, ignoring the CS corrections on the right-hand side of Eq. (20a), was quite reasonable.

B. Compton scattering observables

In order to get the unitary lower bound for the differential cross section, one expresses the so-called

TABLE I. Imaginary parts of various Compton amplitudes at $E_\gamma=320$ MeV in units of $10^{-4}/m_{\pi^+}$. Experimental pion photoproduction multipole data sets are from Refs. [25,26], and are abbreviated as Gru (Grushin *et al.*), BD (Berends and Donnachie), and PS (Pfeil and Schwela) in the "input" column.

Input	f^{1+}_{MM}	f^{2-}_{EE}	f^{1+}_{ME}	f^{1-}_{EE}	f^{1-}_{MM}
Gru	16.813±0.766	0.052±0.012	0.565±0.054	2.750±0.161	0.159±0.069
BD	16.560±0.134	0.084±0.008	0.385±0.074	3.082±0.170	0.446±0.059
PS	16.581±0.191	0.063±0.019	0.360±0.079	3.172±0.230	0.247±0.145

Hearn-Leader [5] amplitudes ϕ_i in terms of the CS multipoles $f_{ab}^{L\pm}$. Assuming that only s- and p-wave contributions are important, the ϕ_i reduce to

$$\begin{aligned}
\phi_1 &= \{2(f_{EE}^{1-}+f_{MM}^{1-})+\tfrac{1}{2}(3x-1)A\}\cos\frac{\theta}{2} ,\\
\phi_2 &= -\{2(f_{EE}^{1-}-f_{MM}^{1-})+\tfrac{1}{2}(3x-1)A\}\sin\frac{\theta}{2} ,\\
\phi_3 &= \tfrac{1}{2}(1-x)B\cos\frac{\theta}{2} ,\\
\phi_4 &= \tfrac{1}{2}(1+x)B\sin\frac{\theta}{2} ,\\
\phi_5 &= \tfrac{1}{2}(1+x)C\cos\frac{\theta}{2} ,\\
\phi_6 &= -\tfrac{1}{2}(1-x)C\sin\frac{\theta}{2} ,
\end{aligned} \tag{23}$$

with $x=\cos\theta$, $\theta\equiv\theta_{\text{c.m.}}$, and

$$\begin{aligned}
A &= 9f_{EE}^{2-}+f_{MM}^{1+}-6f_{ME}^{1+} ,\\
B &= 9f_{EE}^{2-}-3f_{MM}^{1+}+6f_{ME}^{1+} ,\\
C &= 3f_{EE}^{2-}+3f_{MM}^{1+}+6f_{ME}^{1+} .
\end{aligned} \tag{24}$$

We thus have

$$\frac{d\sigma}{d\Omega}=\frac{1}{2}\sum_i \eta_i|\phi_i|^2 , \tag{25}$$

with $\eta_{3,4}=2$ and $\eta_{1,2,5,6}=1$. For completeness, let us enumerate here other observables of interest to us in this paper. Thus, $\mathcal{S}$, defined in (3), becomes

$$\mathcal{S}=2\frac{d\sigma}{d\Omega}\Sigma=2\,\text{Re}\{(\phi_6-\phi_2)\phi_4^*-(\phi_5+\phi_1)\phi_3^*\} , \tag{26}$$

where Σ is the photon asymmetry [30]. Another quantity [31] obtained as the ratio of differential cross sections for different photon polarizations is

$$\frac{d\sigma_\parallel}{d\sigma_\perp}=\frac{2(d\sigma/d\Omega)+\mathcal{S}}{2(d\sigma/d\Omega)-\mathcal{S}}=\frac{1+\Sigma}{1-\Sigma} . \tag{27}$$

The polarization of the recoil nucleon is given by

$$\frac{d\sigma}{d\Omega}\mathcal{P}=-\text{Im}\{(\phi_6-\phi_2)\phi_3^*+(\phi_5+\phi_1)\phi_4^*\} . \tag{28}$$

Reexpressing $d\sigma/d\Omega$, $\mathcal{S}$, and $\mathcal{P}$ in terms of $f_{ab}^{L\pm}$, we obtain

$$\begin{aligned}
\frac{d\sigma}{d\Omega}=&\,2(|f_{EE}^{1-}|^2+|f_{MM}^{1-}|^2)+\frac{(3x^2+7)}{2}|f_{MM}^{1+}|^2+\frac{9(3x^2+7)}{2}|f_{EE}^{2-}|^2+18(1+x^2)|f_{ME}^{1+}|^2\\
&+\text{Re}\{4xf_{EE}^{1-}f_{MM}^{1-*}+2xf_{EE}^{1-}A^*+(3x^2-1)f_{MM}^{1-}A^*\}+3(3x^2-1)\text{Re}\{3f_{EE}^{2-}f_{MM}^{1+*}-6f_{EE}^{2-}f_{ME}^{1+*}+2f_{MM}^{1+}f_{ME}^{1+*}\} ,
\end{aligned} \tag{29}$$

$$\mathcal{S}=3(1-x^2)\{|f_{MM}^{1+}-3f_{EE}^{2-}|^2-12|f_{ME}^{1+}|^2+2\,\text{Re}\{(f_{MM}^{1+}-3f_{EE}^{2-})(2f_{ME}^{1+}+f_{MM}^{1-})^*-2f_{ME}^{1+}f_{MM}^{1-*}\}\} , \tag{30}$$

and

$$\frac{d\sigma}{d\Omega}\mathcal{P}=-\sin\theta\,\text{Im}\left\{\left[f_{EE}^{1-}+xf_{MM}^{1-}+\frac{x}{2}(A+C)\right]B^*\right\} . \tag{31}$$

We can now go back to the unitarity relations (20a)–(20c) and rewrite them, ignoring the small Compton correction for the proton target:

$$\text{Im}f_{MM}^{1+}=q\{3|M_{1+}^{(1)}|^2+\tfrac{2}{3}|M_{1+}^{(3)}|^2\} , \tag{32}$$

and so on, for the other imaginary pieces of the Compton multipoles. Thus, using the photoproduction data, we can compute the $\text{Im}f_{ab}^{L\pm}$'s, but not the $\text{Re}f_{ab}^{L\pm}$'s in this fashion. The results are given in Table I for $E_\gamma=320$ MeV, to give one example for the procedure. We obtain the unitary *lower bound* of the differential cross section by putting all the real parts of the amplitudes equal to zero, i.e., $d\sigma/d\Omega\geq d\sigma/d\Omega|_{\text{Re}\phi_i=0}$. For other observables, discussed in this section, this procedure of neglecting the real parts of the Compton amplitudes is not always justifiable and can be totally misleading, since it does not yield, in general, any bound on these observables. However, the neglect of the real parts of the Compton amplitudes does not appear to be a bad approximation around $E_\gamma=320$ MeV, and we shall make use of that fact to compute these observables using this approximation only at $E_\gamma=320$ MeV. Clearly, further theoretical and experimental work on the real parts of the Compton amplitudes is needed to do better than this, and to extend the calculations at other energies.

In the expressions (23)–(31), we have ignored the Compton amplitudes $f_{EE}^{1+}, f_{EM}^{1+}, f_{MM}^{2-}$ due to the d-wave pion photoproduction multipoles (E_{2-}, M_{2-}). These could be as large as some of the smaller Compton amplitudes included, but they do not influence significantly the bounds we discuss below. We shall return to their roles in Sec. IV.

Before discussing the test of the unitary lower bound on the differential cross section, we examine two important model-independent results on CS observables. First is a powerful conclusion for the recoil-nucleon polarization in the limiting case of vanishing real parts of all CS amplitudes:

$$\mathcal{P}=0 , \quad \text{with } \text{Re}f_{ab}^{L\pm}=0 . \tag{33}$$

There are no experimental tests available so far for this simple prediction. It would be very helpful to measure $\mathcal{P}$ as a function of the photon energy, in determining the importance of the real parts of the Compton amplitudes, a topic to which we shall return later. Second is a set of conclusions that depend on the dominance of the magnetic-dipole Compton amplitude, f_{MM}^{1+}. In the limit of all Compton amplitudes other than f_{MM}^{1+} tending to zero, we have very simple expressions for the CS observables, for example,

$$\frac{d\sigma}{d\Omega} \simeq \frac{(3x^2+7)}{2}|f_{MM}^{1+}|^2 , \quad \mathcal{S} \simeq 3(1-x^2)|f_{MM}^{1+}|^2 . \tag{34}$$

In particular, we get, for polarized photons,

$$\frac{d\sigma_{\parallel}}{d\sigma_{\perp}} \simeq \frac{5}{3x^2+2} , \tag{35}$$

which simplifies, for $\theta_{\rm c.m.}=90°$ (or, $x=0$) to

$$\frac{d\sigma_{\parallel}}{d\sigma_{\perp}}(\theta_{\rm c.m.}=90°) \simeq \tfrac{5}{2} . \tag{36}$$

Though not exact, this approximation of $M1$ dominance is very good in the $\Delta(1232)$ excitation region, and helps experimentalists in the design of their experiments.

We divide somewhat arbitrarily our discussion on the unitarity test into three photon energy regimes: (a) low energy ($E_\gamma < 300$ MeV); (b) medium energy ($300 \le E_\gamma \le 350$ MeV); (c) photon lab energy above 350 MeV but below 430 MeV. The upper-energy cutoff is dictated by our consideration of the $\Delta(1232)$ excitation region, by the restriction of the number of channels in our unitarity equations, and by the fact that all the photoproduction multipoles are determined below 450 MeV. In the energy region (a), we shall only consider very recent high-quality preliminary data from Saskatchewan. In the energy region (b), around $E_\gamma \sim 320$ MeV, the Bonn data would be considered, along with some discussion of the older data overlapping with this region. In the energy region (c), most of the data are from Japan, with some from Bonn. Genzel, Joos, and Pfeil [9] should be consulted for the complete survey of the older data.

1. $E_\gamma < 300$ MeV: The Saskatchewan Accelerator Laboratory preliminary data set [14]

Experimentalists at SAL have recently completed CS measurements at $E_\gamma = 170$, 181, 200, 240, and 293 MeV, giving us the opportunity to test the existing photopion multipoles against this preliminary data, using unitarity to generate the lower bound, as before. Only the multipoles [26] of Pfeil and Schwella (PS), and Berends and Donnachie (BD) cover the range of energies needed here. Closest to the experiments at SAL are $E_\gamma = 180$, 206, 240, 280, and 300 MeV for PS and $E_\gamma = 240$, 290, and 300 MeV for the BD multipole sets. In Table II, we display the computed lower bounds for the PS and BD data sets. In Fig. 2, we display these (PS case) against the measured SAL preliminary data at different energies. Everywhere the measured values are well above the expected lower bounds from the PS and the BD multipole data sets and are thus consistent with these bounds. This also suggests

TABLE II. The angular distribution coefficients for the lower bound on the differential cross section $d\sigma/d\Omega = U_0 + U_1 x + U_2 x^2$ for the range of photon lab energies (E_γ) close to where workers at SAL have recently completed CS measurements [14], $x = \cos\theta_{\rm c.m.}$. The units are nb/sr. The errors on the coefficients are not shown.

E_γ (MeV)	Input	U_0	U_1	U_2
180	PS	5.74	0.48	0.15
206	PS	6.97	0.92	2.25
240	BD	13.06	3.17	14.23
	PS	13.44	3.15	13.34
280	BD	71.21	10.13	72.27
	PS	75.49	11.79	68.17
290	BD	101.15	12.86	91.51

that the contributions of the real parts of the CS amplitudes should be sizable in all these cases, in contrast to our inference earlier from the Bonn data at $E_\gamma = 320$ MeV. Understanding this energy dependence of the CS amplitude from the underlying hadron dynamics is an interesting open problem.

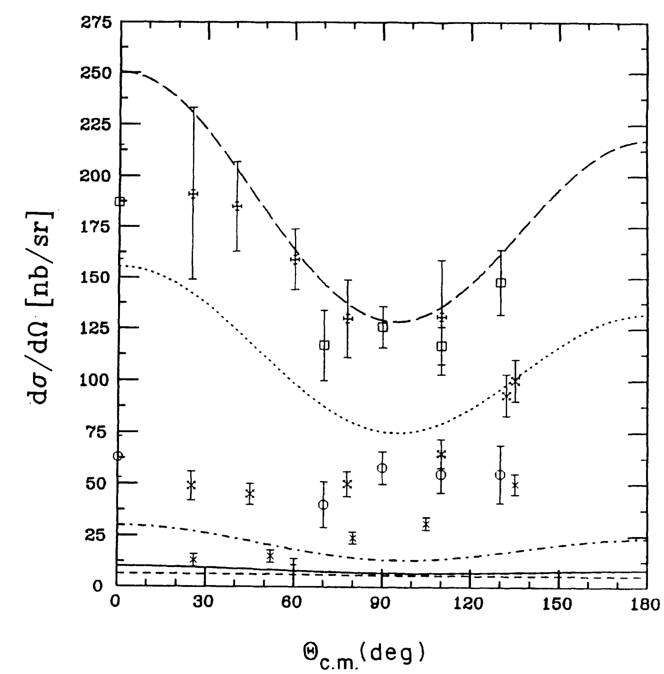

FIG. 2. The unitary lower bound for the differential cross section from the Pfeil-Schwela multipole data set [26]. The dashed curve corresponds to $E_\gamma = 180$ MeV, the solid curve to $E_\gamma = 206$ MeV, the dot-dashed curve to $E_\gamma = 240$ MeV, the dotted curve to $E_\gamma = 280$ MeV, and finally the long-dashed curve corresponds to $E_\gamma = 300$ MeV. The experimental data points are indicated by circles ($E_\gamma = 240$ MeV) and squares ($E_\gamma = 280$ MeV) from Genzel *et al.* [10]; + ($E_\gamma = 181$ MeV), × ($E_\gamma = 200.2$ MeV), fancy × ($E_\gamma = 239.9$ MeV), and fancy + ($E_\gamma = 293.4$ MeV) from Saskatchewan Accelerator Laboratory [14].

2. $300 \leq E_\gamma \leq 350$ MeV: The Bonn data [10] and related older experiments

In this energy region, there exist four data sets on differential cross sections: The three oldest data sets are from the Cornell group [32], the Tokyo group [33], and the Illinois group [34]. Of these, the Tokyo group had the best photon energy resolution to data (± 5 MeV), but their work suffers from very poor statistics. On the other hand, the Cornell and the Illinois group had very poor photon energy resolution, but somewhat better, though still poor, statistics. In this regard, the Bonn experiment by Genzel *et al.* was an improvement in energy resolution and statistics taken together, though the energy resolution was poorer than that of the earlier Tokyo experiment. In judging quality of unitarity tests, these factors should be all kept in mind, particularly the relatively poor energy resolution. In Fig. 3(a), the experimental differential cross sections from all the available sources are displayed in this energy region. Hereafter we shall emphasize the Bonn data only in the context of the test of the unitarity lower bound on the differential cross sec-

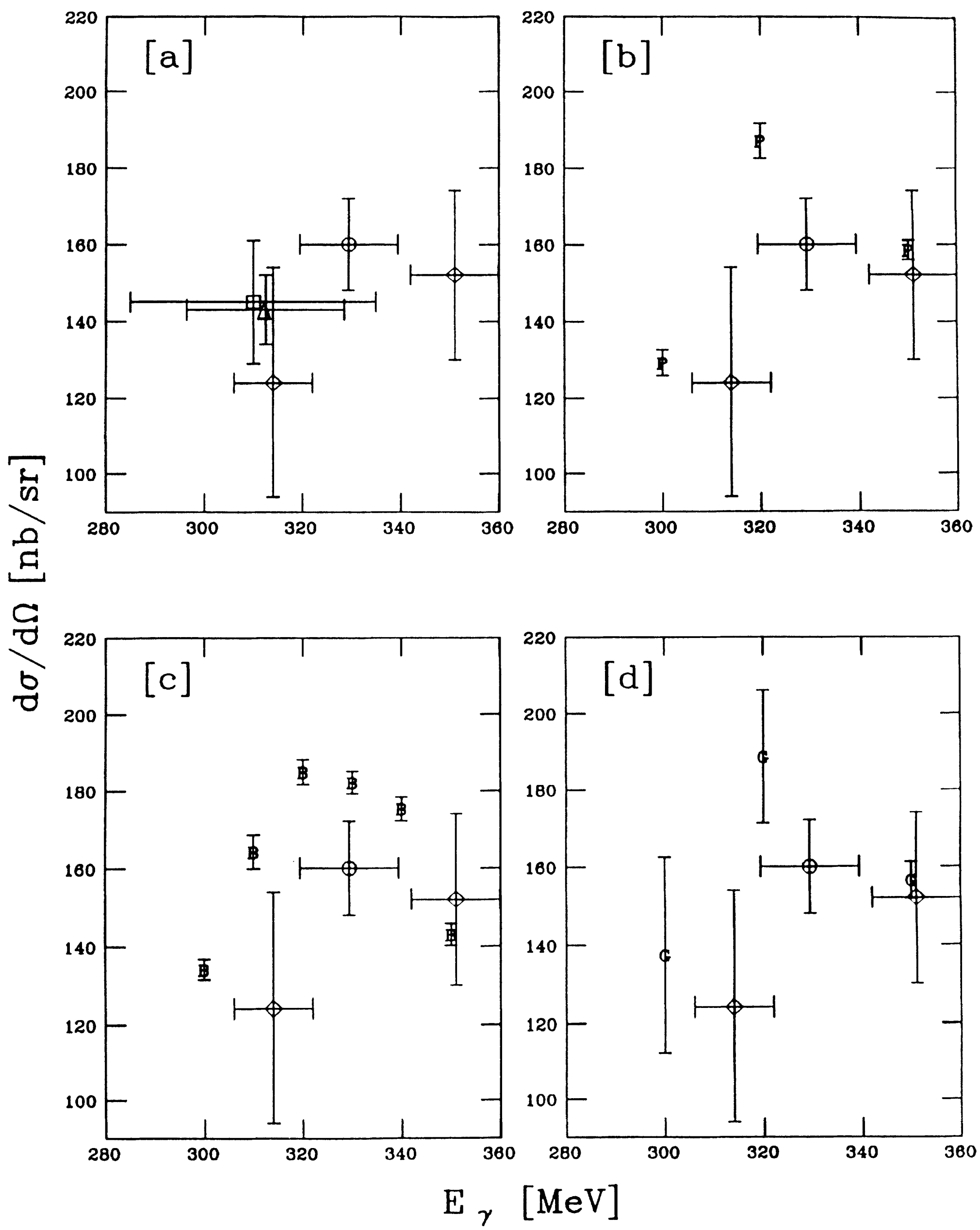

FIG. 3. (a) Experimental differential cross section for the Compton scattering on protons at $\theta_{c.m.} = 90°$ in the Δ (1232) excitation region, as measured in various laboratories. Circles, Genzel *et al.* [10]; triangles, De Wire *et al.* [32]; diamonds, Nagashima [33]; and squares, Gray and Hansen [34]. Notice the long horizontal error bar for the energy resolution. (b)–(d) are our theoretical lower bound for the PS, BD, and Gru multipoles of Refs. [26,26,25], respectively.

tion, though the graphical comparison will also include other data sets.

In the photon energy region $300 \le E_\gamma \le 335$ MeV, overlapping with the first two energy regions defined above, there exists an old measurement from Frascati [31] on the ratio $d\sigma_\parallel / d\sigma_\perp$, for the c.m. scattering angle of 90°. This experiment gave

$$\frac{d\sigma_\parallel}{d\sigma_\perp}(90°) = 2.1^{+0.5}_{-0.4}\ , \tag{37}$$

for the above broad range of energy. This, equivalently, yields

$$\Sigma(90°) = 35.5^{+10.4}_{-8.3}\ , \tag{38}$$

in percent, for the same energy range. Below we shall also comment on the significance of this experiment.

(a) Unitary lower bound on the differential cross section. Here we first recall the conclusion of Pfeil *et al.* that the 320-MeV, 90° experimental data point of the Bonn group [10] is *below* the theoretical lower bound computed using the multipole set of Pfeil and Schwela [26] for pion photoproduction. Before making a definite conclusion about this, we propose to take several steps here. First, we shall repeat the computation of Pfeil *et al.* with the PS multipole data set, and those by Berends and Donnachie [26], with particular attention to the error-propagation analysis. The analysis with the BD data set has not been previously reported before. Second, remembering the neglect of the Compton phase in the extraction of the PS and BD multipoles, we shall reexamine this with the Grushin *et al.* (Gru) multipoles. Third, we shall pay particular attention to the fact that the photon energy resolution in the Bonn and other experiments (except the old Tokyo data set, with bad statistics) is rather poor, a fact not taken into consideration in the analysis of Pfeil, Rollnik, and Stankowski [13].

In Figs. 3(b)–3(d), we plot various experimental data at 90° c.m. angle and compare them with the theoretical lower bound on the CS differential cross section at this angle, obtained with various pion photoproduction multipole data. The reader should pay attention not only to the vertical error bars of the experimental points, largely controlled by the counting statistics, but also to the horizontal error bar indicating the energy resolution of different experiments. Our conclusions here are as follows. In the case of the multipoles extracted by Grushin *et al.*, the lower bound *touches* the experimental point of the Bonn group, while the bounds obtained with the PS and BD multipoles overshoot the data point, confirming the observation of Pfeil *et al.* for the PS multipole data set. Thus, while the disagreement between the unitary lower bound and the Bonn data practically disappears for the Gru multipole set, it still persists at $E_\gamma = 320$ MeV with the older multipole data base, a conclusion highlighted in Fig. 4. Any contribution from the real parts of the CS multipole amplitudes is positive, thus adding to the discrepancy. Given the complexity of the analysis, poor resolution of photon energy in the experiments done so far, and the limitations of the multipole data bases of PS and BD in the implicit neglect of the Compton phase that we have stressed many times earlier, the *possible disagreement* in the PS and BD predictions of the unitary lower bound of the CS cross section and the experimental data *is not compelling*. Further improvements of the photon energy resolution, angular acceptance of the particle detectors improving the accuracies of measuring $\theta_{\text{c.m.}}$, and the determination of another independent observable (say, $\mathcal{S}$, which we have also plotted in Fig. 4) would be a very helpful confirmation of persistence or disappearance of the discrepancy. Various multipole bounds differ on the latter by about 5–6 %, given a precision experiment better aimed at cross checking this critical issue. We also note that no discrepancy exists between the unitary lower bound obtained from the photoproduction multipoles and experiments at other energies.

In Table III, we display the numerical values of the lower bound on the differential cross section for $E_\gamma = 320$ MeV, obtained from different multipole analyses of the data on photoproduction of pions. In this table, we also include an analysis due to Grushin (Gru′), published posthumously in 1989 [25], in which he attempted to analyze (γ, π^+) and (γ, π^0) experimental results separately. This analysis, like all others except the set Gru published earlier in 1983 [25], shows a disagreement with the Bonn data at $\theta_{\text{c.m.}} = 90°$. Grushin himself has considered this analysis to be less reliable than his previously published results, so no further reference will be made to it. The important point that stands out is the small contribution for the real parts of the Compton amplitudes at $E_\gamma = 320$ MeV, implied by the data, since the lower bounds, generated by the imaginary pieces of the CS amplitudes, are saturated, if not exceeded, by the experiment. There is

TABLE III. Derived lower bound on the differential cross section at photon lab energy $E_\gamma = 320$ MeV from various multipole data sets [25,26] compared with the experiment of Genzel *et al.* [10]. Gru′ refers to the 1989 multipole set of Grushin [25]. The units are nb/sr.

$\theta_{\text{c.m.}}$ (deg)	Gru	Gru′	BD	PS	Experiment
0	330±29	332±13	322±6	319±8	341±45
60	228±21	229±8	224±4	225±5	240±16
70	209±19	210±8	205±4	207±5	225±15
90	189±18	190±7	185±3	187±5	160±12
110	198±18	199±8	192±3	194±5	189±11
130	231±21	232±9	221±4	221±5	197±13

some support [35] from dispersion theory that the contributions from the real parts should, indeed, be small at this energy, but more definitive work is needed both theoretically and experimentally to test this point. We shall return to the possibility of measuring real parts later. In the meantime, the Bonn data set at $E_\gamma=320$ MeV is definitely open for a reexamination by a careful independent experiment, while the older data sets are too crude to be helpful at present, when their combined statistics and photon energy detector resolutions are considered.

We give the analytical form of our derived lower bounds in this energy region in Table IV.

(b) Photon asymmetry and other observables. Though there are no accurate predictions possible from the pion photoproduction multipoles for CS observables other than the unitary lower bound on the differential cross sec-

TABLE IV. As in Table II, for the photon lab energy range 300 $E_\gamma \leq 350$ MeV.

E_γ (MeV)	Input	U_0	U_1	U_2
300	Gru	137.41	15.56	106.27
	BD	134.10	15.76	108.02
	PD	129.25	16.52	104.59
310	BD	164.24	17.20	120.24
320	Gru	188.67	15.62	125.81
	BD	184.94	19.59	116.94
	PS	187.16	19.64	112.04
330	BD	182.19	19.47	111.45
340	BD	175.24	21.03	100.13
350	Gru	156.49	11.84	82.06
	BD	143.00	21.00	83.71
	PS	158.46	20.08	72.96

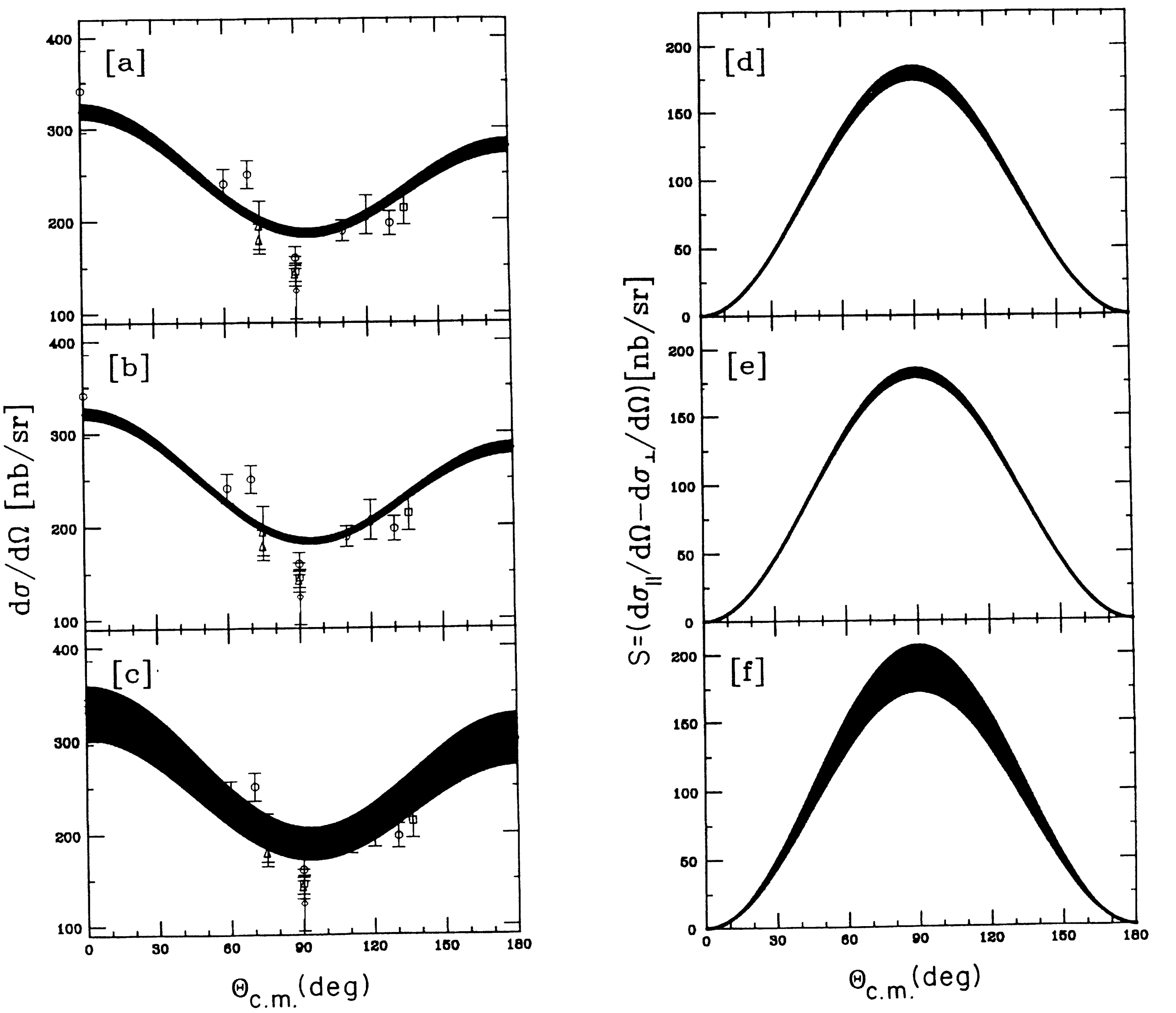

FIG. 4. $d\sigma/d\Omega$ and $\mathcal{S}$ at photon lab energy $E_\gamma=320$ MeV, evaluated by putting the real parts of the Compton amplitude equal to zero. The photo-pion multipole data sets used are Pfeil-Schwela [26] [(a), (d)], Berends-Donnachie [26] [(b),(e)], and Grushin *et al.* [25] [(c),(f)]. The dark bands are due to the uncertainties in the photo-pion multipoles. The differential cross-section data in (a)–(c) are indicated by circles [10], triangles [32], diamonds [33], and squares [34].

TABLE V. Observables computed assuming real parts of the Compton amplitudes are zero, at $\theta_{c.m.}=90°$, for photon lab energies where the photo-pion multipole data sets [25,26] overlap most.

E_γ (MeV)	Input	$\frac{d\sigma}{d\Omega}$ (nb/sr)	$\mathcal{S}$ (nb/sr)	Σ (%)	$\frac{d\sigma_\parallel}{d\sigma_\perp}$
300	Gru	137.4±25.1	143.6±25.9	52.3±0.8	3.19±0.05
	BD	134.1±2.7	142.8±2.9	53.3±0.6	3.28±0.06
	PS	129.3±3.3	136.9±4.0	53.0±1.2	3.25±0.11
320	Gru	188.7±17.5	189.6±17.2	50.3±0.7	3.02±0.04
	BD	184.9±3.3	181.8±3.8	49.1±1.0	2.93±0.08
	PS	187.2±4.6	179.0±5.6	47.8±1.3	2.83±0.10
350	Gru	156.5±4.7	145.3±5.1	46.4±1.1	2.73±0.06
	BD	143.0±2.8	141.8±3.4	49.6±1.2	2.97±0.09
	PS	158.5±2.5	139.8±3.3	44.1±1.0	2.58±0.07
380	Gru	91.9±7.2	76.5±6.2	41.6±0.8	2.42±0.03
	BD	75.4±2.5	63.6±2.7	42.1±1.6	2.46±0.09
	PS	90.0±2.7	73.7±4.7	40.6±2.9	2.36±0.17
400	Gru	62.5±6.2	47.3±5.0	37.9±1.1	2.22±0.05
	BD	48.1±1.5	38.5±1.6	40.1±1.6	2.34±0.09
	PS	56.6±1.4	47.7±2.5	42.2±2.8	2.46±0.17

tion, the smallness of real parts of Compton amplitudes around $E_\gamma=320$ MeV encourages us to discuss photon asymmetry and related observables, assuming the absence of the real parts of Compton amplitudes. In Sec. V, we shall explore the nucleon-to-$\Delta(1232)$ electromagnetic transition amplitudes, particularly, the small $E2$ amplitude. In Table V, we display the values of $\mathcal{S}$, Σ, and $d\sigma_\parallel/d\sigma_\perp$ for the c.m. scattering angle of 90° computed from the multipole data sets of pion photoproduction assuming the real parts of the Compton amplitudes $\mathrm{Re}f_{ab}^{L\pm}$ to be all zero. Though this assumption seems to be best for E_γ, around 320 MeV, we have given the values for a broad band of energies overlapping the three regions into which we have divided our discussions. Given the uncertainties from the multipoles and poor energy resolution of the measured values in Eqs. (37) and (38), we can say that there is no sharp disagreement between the CS observables in (37) and (38) and the expectations from pion photoproduction, though the measured values of Σ is on the low side of this expectation. Clearly, a lot better needs to be done in experimental precision, including a test via (33). This would give us an idea of possible nonzero values of $\mathrm{Re}f_{ab}^{L\pm}$.

TABLE VI. As in Table II, for photon lab energy $E_\gamma>350$ MeV.

E_γ (MeV)	Input	U_0	U_1	U_2
360	BD	120.22	19.59	60.86
	PS	132.58	18.22	57.90
370	BD	87.59	12.93	66.67
380	Gru	91.95	10.56	39.24
	BD	75.45	17.75	36.72
	PS	90.91	15.91	34.17
390	BD	58.56	15.57	28.51
400	Gru	62.53	8.03	22.98
	BD	48.05	15.24	23.99
	PS	56.55	12.03	25.67
410	BD	36.03	14.35	18.65
420	Gru	34.36	12.35	16.44
	BD	32.98	14.02	15.75

3. $E_\gamma \geq 350$ MeV: The Bonn and Tokyo data sets

In this energy region, the data sets contain only a small number of data points. The Bonn data [10] are available

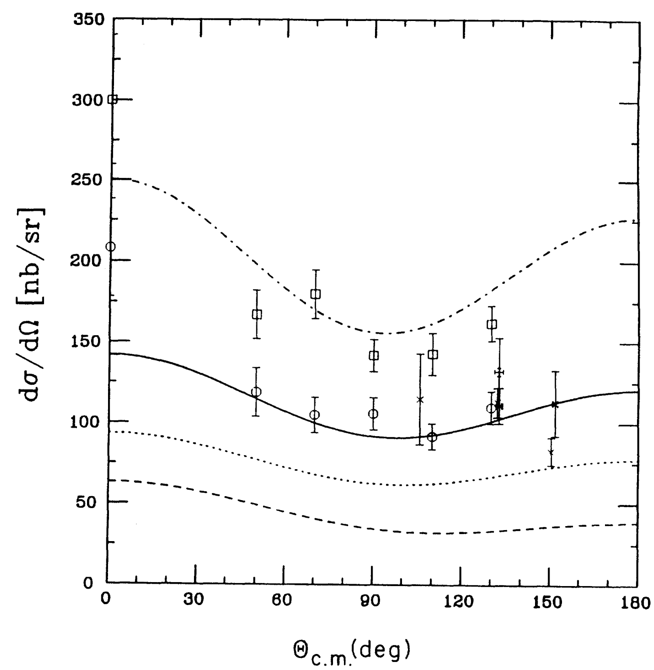

FIG. 5. The unitary lower bound for the differential cross section above $E_\gamma=350$ MeV, using the Grushin *et al.* [25] multipole data set. The dashed curve corresponds to $E_\gamma=420$ MeV, dotted curve to $E_\gamma=400$ MeV, solid curve to $E_\gamma=380$ MeV, and the dot-dashed curve to $E_\gamma=350$ MeV. The experimental data points are indicated by squares ($E_\gamma=360$ MeV) and circles ($E_\gamma=400$ MeV) from Genzel *et al.* [10] and fancy + ($E_\gamma=375$ MeV), fancy × ($E_\gamma=400$ MeV), and × ($E_\gamma=425$ MeV) from Wada *et al.* [11].

for $E_\gamma = 360$ and 400 MeV, while the Tokyo data sets [11] cover the photon energies of 375, 400, and 425 MeV, thereby giving us some opportunity for cross checks. In Fig. 5, we display these data sets against the unitary lower bound extracted from the Grushin *et al.* pion photoproduction multipoles at $E_\gamma = 350$, 380, 400, and 420 MeV. There is clearly no problem with the bounds for these data sets. The reader can refer to Table V for other observables in this energy region computed assuming real parts of the CS amplitudes to be zero. Experiments on these observables would be helpful tests on the crudity of this assumption.

In Table VI, we give the analytical forms of our unitary lower bound estimates for the Grushin *et al.* multipole data sets in this energy range.

IV. UNITARITY AND OPTICAL THEOREM FOR THE TOTAL PHOTOHADRON CROSS SECTION

Thanks to the existence [36] of some fairly old data on the total hadronic cross section of photons on hydrogen as a function of the photon lab energy, we can make another independent test of the quality of the pion photoproduction multipole data base, making use of the optical theorem. This bypasses the CS cross-section data entirely, and provides a very useful check on the reliability of the photopion multipole data base. We shall examine this for the multipole data of Grushin *et al.*

The basic theoretical quantity relevant here is the forward scattering amplitude f_1, first discussed in this context by Gell-Mann, Goldberger, and Thirring [1]. Following Pfeil, Rollnik, and Stankowski [13], we use

$$f_1 = \frac{E_\gamma}{k}\{f_{EE}^{1-} + f_{MM}^{1-} + 2(f_{MM}^{1+} + 3f_{EE}^{2-}) + 2(f_{EE}^{1+} + 3f_{MM}^{2-})\} \; . \tag{39}$$

We are considering s and p waves in our calculation and the last term (due to the d wave) on the right-hand side of Eq. (39) is ignorable, as shown below by making an estimate of error introduced by the neglect of the last term. The optical theorem is

$$\sigma_T = \frac{4\pi}{E_\gamma}\operatorname{Im} f_1 \; , \tag{40}$$

where σ_T is the total hadronic cross section of photons on hydrogen, measured by Armstrong *et al.* [36].

As an illustration of this procedure, let us consider the photon lab energy $E_\gamma = 320$ MeV and the photopion multipoles of Grushin *et al.* [25]. Using Eq. (39), and ignoring the d-wave term, we get

$$\operatorname{Im} f_1 \approx 13.3 \pm 0.6 \; \mu\text{b GeV} \; . \tag{41}$$

From the experiments of Armstrong *et al.*, we find that the photo-hadron measurements yield the following values of $\operatorname{Im} f_1$:

$$\operatorname{Im} f_1 = 13.2 \pm 0.2 \; \mu\text{b GeV} \quad \text{at } E_\gamma = 315 \text{ MeV} \; , \tag{42a}$$

$$\operatorname{Im} f_1 = 13.0 \pm 0.2 \; \mu\text{b GeV} \quad \text{at } E_\gamma = 340 \text{ MeV} \; . \tag{42b}$$

The experimental numbers have a precision of better than 2%. The Grushin *et al.* photopion multipoles imply the photo-hadron cross section for $E_\gamma = 320$ MeV to be

$$\sigma_T = 523 \pm 22 \; \mu\text{b} \; . \tag{43}$$

Directly measured experimental numbers at 315 and 340 MeV, respectively, are

$$\sigma_T = 527 \pm 8 \; \mu\text{b} \; , \quad \sigma_T = 478 \pm 8 \; \mu\text{b} \; . \tag{44}$$

Thus, there is excellent agreement between (43) and (44), providing an independent unitarity test of the photo-pion multipoles of Grushin *et al.* More numerical results on the total hadronic cross section, as obtained from this multipole data base, are displayed in Table VII for photon lab energies between 300 and 400 MeV.

Several interesting points regarding the use of the optical theorem can now be made. As shown in Table VII,

TABLE VII. Total photon-hadron cross section and the imaginary part of the forward Compton scattering amplitude f_1. The second set of numbers in columns 3 and 4 are due only to f_{MM}^{1+}. Experimental photopion multipoles in the "input" column are from [25,26]. The quantity χ is defined in Eq. (45).

E_γ (MeV)	Input	$\operatorname{Im} f_1$ (μb GeV)	σ_T (μb)	χ (μb GeV)
300	Gru	11.75±0.96, 10.29±0.95	492.2±40.0, 431.0±39.8	0.57±0.04
	BD	11.75±0.15, 10.21±0.10	492.1±4.8, 427.5±4.1	0.60±0.02
	PS	11.63±0.14, 9.99±0.12	487.1±6.1, 418.6±5.2	0.64±0.03
320	Gru	13.33±0.57, 12.17±0.56	523.5±22.2, 477.7±22.0	0.45±0.03
	BD	13.44±0.12, 11.98±0.10	527.9±4.6, 470.6±3.8	0.56±0.03
	PS	13.37±0.17, 12.00±0.14	525.1±6.8, 471.1±5.4	0.53±0.04
350	Gru	12.05±0.19, 11.13±0.16	432.6±6.9, 399.5±5.9	0.35±0.04
	BD	12.39±0.11, 10.64±0.10	444.7±4.0, 381.9±3.5	0.66±0.02
	PS	12.34±0.11, 11.01±0.08	443.0±4.0, 395.2±3.0	0.50±0.03
380	Gru	9.45±0.35, 8.52±0.34	312.6±11.5, 281.9±11.2	0.35±0.03
	BD	9.40±0.14, 7.65±0.12	310.8±4.6, 253.0±4.1	0.65±0.02
	PS	9.68±0.15, 8.33±0.11	320.0±5.1, 275.6±3.6	0.50±0.04
400	Gru	7.88±0.36, 7.04±0.35	247.5±11.5, 221.1±11.1	0.31±0.03
	BD	7.87±0.11 6.08±0.09	247.4±3.4, 191.1±2.9	0.66±0.02
	PS	7.98±0.11, 6.58±0.05	250.8±3.4, 206.8±1.7	0.51±0.04

the overwhelming contribution to the total hadronic cross section comes from the term $\mathrm{Im} f_{MM}^{1+}$. Thus, one can rewrite Eq. (40) with the help of Eq. (39) as

$$\mathrm{Im} f_{MM}^{1+} = \frac{k}{8\pi}\sigma_T - \chi \ , \tag{45}$$

in the $\Delta(1232)$ region, where χ is a small correction due to CS multipoles other than $\mathrm{Im} f_{MM}^{1+}$. This can be reliably estimated from the existing multipole data base (see Table VII). Thus, a very precise determination of the total photon-hadron cross section, along with *existing* knowledge of the pion photoproduction multipoles, would allow a rather precise determination of the magnetic Compton amplitude. This, in turn, would provide an accurate check of the CS experiments in the Δ region. This powerful tool can also be helpful in the future determination of the EMR. The other point of interest here is the possible importance of the *d*-wave terms in Eq. (39). Using an old work, Berends, Donnachie, and Weaver [37], we have been able to estimate its contribution, in the Δ region, to σ_T to be about 2%. So these should be *included* in χ, in order to make the relation (45) useful in a very precise determination of $\mathrm{Im} f_{MM}^{1+}$. Finally, Armstrong *et al.* [36] confirm the fact that $\mathrm{Re} f_1$ is between a tenth and a third of $\mathrm{Im} f_1$ in magnitude, as E_γ goes from 315 to 340 MeV, *independently confirming* the smallness of the real parts of CS amplitudes at $E_\gamma = 320$ MeV. This fact has been anticipated in obtaining the lower bound in Sec. III, using unitarity. It also shows that the real part increases in importance as the value of E_γ becomes much lower or goes higher, compared with $E_\gamma = 320$ MeV.

V. EXPLORATION OF THE NUCLEON-TO-Δ ELECTRIC QUADRUPOLE AMPLITUDE IN COMPTON SCATTERING

As stated in the Introduction, the reaction

$$N_\gamma \Longrightarrow \Delta(1232) \tag{46}$$

is an interesting window for the study of the deformation in nucleon and Δ configurations of valence quarks, which results in a small, but nonvanishing, $E2$ amplitude in the above process [17]. Sandorfi *et al.* [15] wish to get a handle on this from the CS experiment. We examine this possibility further in this section.

Let us define the ratio

$$\rho = \frac{\mathrm{Im} E_{1+}^{(3)}}{\mathrm{Im} M_{1+}^{(3)}} \ , \tag{47}$$

which we have called the EMR, for the resonance contribution alone. Since the phases of $M_{1+}^{(3)}$ and $E_{1+}^{(3)}$ are the same, the ratio of the real parts is the same as the ratio of the imaginary parts given above. In Table VIII, we display this ratio as a function of E_γ for the three different sets of multipoles PS, BD, and Gru. We note that these ratios vary with energy, and even change sign. This is expected, because ρ contains both the resonance and background contributions. In order to separate the resonance piece from the background, the unitarized amplitude can be written in the form [25,38,39]

TABLE VIII. The parameter ρ (in %) as defined in the text [Eq. (47)], computed at different photon lab energies for the different multipole data sets [25,26] considered.

E_γ (MeV)	Gru	BD	PS
300	-4.41 ± 1.13	-4.20 ± 0.34	-4.55 ± 0.47
320	-3.08 ± 1.28	-1.67 ± 0.45	-1.72 ± 0.42
350	-1.24 ± 0.52	$+0.26\pm0.46$	-0.37 ± 0.22
380	$+0.32\pm0.32$	$+3.15\pm0.57$	$+2.05\pm0.79$
400	$+1.83\pm0.36$	$+4.18\pm0.61$	-0.75 ± 0.92

$$A = (A_B \cos\delta_{33} + N \sin\delta_R) e^{i\delta_{33}} \ , \tag{48}$$

where $\delta_{33} = \delta_1 + \delta_2$, δ_R is the resonant phase, A represents either $M_{1+}^{(3)}$ or $E_{1+}^{(3)}$ multipoles, A_B is the projection of the nonresonant contribution in these multipoles, and N is the resonance contribution. Using Eq. (48), we can extract the ratio N^E/N^M

$$\xi(E_\gamma) = \frac{N^E}{N^M} = \rho + \frac{\rho A_B^M - A_B^E}{\mathrm{Re} M_{1+}^{(3)} + \mathrm{Im} M_{1+}^{(3)^2} / \mathrm{Re} M_{1+}^{(3)} - A_B^M} \ . \tag{49}$$

Grushin [23] has used this method to calculate the variation for the $E2/M1$ ratio as a function of energy. We follow the same procedure by choosing for the background the well known pseudovector (PV) Born terms and reproduce Grushin's result that, within the uncertainties of the experimental multipoles, ξ does not change significantly in the energy range from $E_\gamma = 300$ to 420 MeV. The values for ξ that we obtain, using the Grushin *et al.* multipole data set, are presented in Table IX. For comparison, we also quote Grushin's results [25] which overlap with ours. One can also extract this information in the effective Lagrangian approach, as has been done by Davidson, Mukhopadhyay, and Wittman. In this case, the resonance contribution N can be readily calculated [38]

$$\xi_{\mathrm{EL}}(W) = \frac{N^E}{N^M} = (M - W)\left[\frac{2Mg_{1\Delta} - Wg_{2\Delta}}{2M(3W+M)g_{1\Delta} - W(W-M)g_{2\Delta}}\right], \tag{50}$$

TABLE IX. Energy variation of the parameter ξ [Eq. (49)] as compared with the prediction ξ_{EL} of the effective Lagrangian model (Sec. V). Differences between Grushin [25] and this work could be attributed to possible differences in model assumptions. All averages except that in the last column are weighted averages.

E_γ (MeV)	ξ (Grushin) (%)	ξ (This work) (%)	ξ_{EL} (%)
300	-2.09 ± 0.87	-2.36 ± 0.82	-1.87
320	-1.84 ± 1.22	-2.00 ± 1.22	-1.91
350	-1.86 ± 0.83	-1.86 ± 0.75	-1.95
380	-1.54 ± 0.38	-1.42 ± 0.53	-1.99
400	-1.91 ± 0.33	-1.60 ± 0.42	-2.00
420	-2.83 ± 1.03	-2.23 ± 1.04	-2.01
Avg.	-1.83 ± 0.22	-1.73 ± 0.27	-1.96

where M is the nucleon mass, W the c.m. energy, and $g_{1\Delta}, g_{2\Delta}$ are the gauge couplings in the $\gamma N \Delta$ vertex defined for the matrix element of process (46):

$$M_{fi} = \frac{e}{2M} \bar{u}_n \tilde{\tau}_3 \gamma_5 \left[g_{1\Delta}(\gamma \cdot k\, \varepsilon_\mu - \gamma \cdot \varepsilon\, k_\mu) + \frac{g_{2\Delta}}{2M}(P_n \cdot \varepsilon\, k_\mu - P_n \cdot k\, \varepsilon_\mu) \right] u_\Delta^\mu , \quad (51)$$

where k and P_n are the photon and nucleon four-momentum respectively, ε_μ the photon polarization, $\tilde{\tau}_3$ the $\frac{3}{2} \leftrightarrow \frac{1}{2}$ isospin transition matrix, u_n the nucleon spinor, and u_Δ^μ the Δ vector spinor. Davidson, Mukhopadhyay, and Wittman [38] have found $g_{1\Delta}, g_{2\Delta}$ to be dependent on the unitarization method used to combine background and resonant contributions to the processes $\gamma N \to \pi N$ and also on the multipole data set used. Their ranges of values for the Gru multipole data set are $g_{1\Delta} = 4.82-5.93$, $g_{2\Delta} = 5.04-6.39$. Using their values for the Noelle unitarization procedure [39], $\xi_{\rm EL}$ does not vary much over the photon energy range between 300 and 420 MeV, which is consistent with the results obtained using (49), as shown in Table IX. This should be kept in mind in exploring the ratio ξ (=EMR at $W = M_\Delta$) from the experiments on CS.

Now we want to see how CS observables are sensitive to the change in $E_{1+}^{(3)}$ or equivalently to ρ, by keeping all other multipoles fixed to their values given by the Grushin *et al.* analysis [25]. A better determination of $E_{1+}^{(3)}$ would result in a more accurate ξ, which would lead to a better precision in the value for $E2/M1$. Thus, if we neglect the errors due to $E_{1+}^{(3)}$, the weighted average of ξ becomes -1.71 ± 0.05, compared to -1.73 ± 0.27 when they are not neglected. In Table X, we display the sensitivity of the CS differential cross section $d\sigma/d\Omega$, the cross-section difference $\mathcal{S}$ and the ratio $d\sigma_\parallel/d\sigma_\perp$, as we vary ρ between +6% and −6% at $E_\gamma = 320$ MeV. We note here that all these observables are evaluated with the assumption that the real parts of the Compton amplitudes $f_{ab}^{L\pm}$ are zero. It is clear that $\mathcal{S}$ and $d\sigma_\parallel/d\sigma_\perp$ are mostly sensitive to the variation of ρ at $\theta_{\rm c.m.} = 90°$. On the other hand, at $\theta_{\rm c.m.} = 90°$, $d\sigma/d\Omega$ is not as sensitive to the variation of ρ as it is at forward and backward angles. Thus, some complementarity can be achieved by observing all these observables simultaneously.

We end this section with a caution. The observables in Eqs. (37) and (38) cannot be immediately converted into the ratio for $E2/M1$ using Table X. This is because of our neglect of the real parts of the Compton amplitudes, which might play significant roles in this extraction. This remains to be explored, but is beyond the scope of this work.

VI. SUMMARY AND CONCLUSIONS

Given the theoretical interest arising from the prospect of testing QCD in the nonperturbative domain by computing hadron properties, and the experimental possibilities of exploring many of these properties in the novel electron-photon facilities now under development, we have studied Compton scattering in the $\Delta(1232)$ resonance region, with a view to help understand structures of the nucleon and the Δ resonance. Both the magnetic-dipole and electric-quadrupole excitation amplitudes, resonant in the Δ channel, are accessible in this reaction, the former quite accurately. In this paper, we have studied existing knowledge on this, using the recent photopion multipoles extracted by Grushin *et al.* and comparing them with others. These authors have avoided using the Watson theorem in its restricted form, which neglects the Compton phase, and have determined the real and imaginary parts of the amplitudes directly from the data. Though less precise overall, compared to other extant multipole data sets, their work has allowed us to ask questions in a model-independent fashion about the CS process in many areas of current interest. Our conclusions are as follows.

TABLE X. Sensitivity of the Compton scattering observables to the variation of the parameter ρ (Sec. V) at $E_\gamma = 320$ MeV using the Grushin *et al.* [25] multipole data set and assuming real parts of the Compton amplitudes to be zero.

ρ (%)	ξ (%)	$\frac{d\sigma}{d\Omega}(0°)$ (nb/sr)	$\frac{d\sigma}{d\Omega}(90°)$ (nb/sr)	$\frac{d\sigma}{d\Omega}(180°)$ (nb/sr)	$\mathcal{S}(90°)$ (nb/sr)	$\frac{d\sigma_\parallel}{d\sigma_\perp}(90°)$
−6.1	−5.3	355.8	180.4	330.3	200.6	3.50
−5.0	−4.2	346.6	183.1	319.3	197.6	3.35
−4.0	−3.1	338.1	185.8	308.9	194.0	3.19
−3.0	−2.0	330.1	188.7	298.9	189.6	3.02
−2.0	−0.9	322.7	191.7	289.4	184.6	2.86
−1.0	0.2	316.1	194.8	280.7	179.1	2.70
0.0	1.3	310.0	198.0	272.4	172.8	2.55
1.0	2.4	304.6	201.4	264.7	165.8	2.40
2.0	3.5	299.9	204.8	257.7	158.4	2.26
3.0	4.6	295.8	208.3	251.2	150.3	2.13
4.0	5.6	292.4	212.0	245.3	141.6	2.00
5.0	6.8	289.8	215.6	240.2	132.5	1.89
6.1	7.9	287.9	219.4	235.6	122.6	1.78

(1) From the photoproduction of ions, it is possible to extract the imaginary part of the Compton amplitude f^{1+}_{MM} rather accurately, using the information on the pion-nucleon phase shifts [Eqs. (14) and (15)]. However, this procedure works only for selected values of energy (Sec. II).

(2) The new unpublished preliminary data [14] from the Saskatchewan Accelerator Laboratory at lower ($E_\gamma < 300$ MeV) energies and older Bonn-Tokyo data at $E_\gamma > 350$ MeV show no discrepancy with the extant photo-pion multipole data sets, when the test of the unitarity-driven lower bound is employed. These experiments suggest large contributions from the real parts of the CS amplitudes (Secs. III B 1 and III B 3).

(3) The discrepancy reported by the Bonn group [10] between the unitary lower bound of the CS cross section at a photon lab energy of 320 MeV and c.m. angle 90° disappears when the multipoles from the Grushin *et al.* analysis [25] are used and the poor resolution of photon energy in the Bonn experiment is taken into account. This is an important conclusion of our work (Sec. III B 2).

(4) The test of the optical theorem [1] in determining the forward Compton amplitude from the total photon-hadron cross section [36] is nicely satisfied by the amplitudes derivable from the multipoles of Grushin *et al.* [Eqs. (43) and (44), Sec. IV].

(5) Projected experiments, such as those proposed [15] for the Brookhaven LEGS, are sensitive to the theoretically interesting electric-quadrupole resonant amplitude in the $N \Longleftrightarrow \Delta$ electromagnetic transition, in particular, the observable $\mathcal{S}$, dependent on the polarized photons (Sec. V). However, extraction for such an amplitude is going to be hard and model dependent.

On the experimental side, a new challenge on the existing, now classic, Bonn experiment at $E_\gamma = 320$ MeV is urgently needed with better photon energy resolution and counting statistics. Given the inaccuracies of the multipoles of Grushin *et al.*, new work is needed in this difficult art, in order to make further quantitative progress in comparing photoproduction of pions with the Compton scattering in the Δ region. The optical theorem provides an opportunity to push for higher accuracies in the determination of the forward Compton scattering amplitude from the photon-hadron measurements. This, in turn, would allow a more precise determination of the $\mathrm{Im} f^{1+}_{MM}$ amplitude. Finally, the laser-driven polarized photon facilities such as LEGS open up new possibilities through measurement of quantities such as the cross-section difference $\mathcal{S}$, sensitive to the $N \Longleftrightarrow \Delta$ electric-quadrupole transition amplitude.

On the theory side, solving the 4×4 unitarity equations in full glory is very difficult, due to the required precision of experimental data needed to extract new physics. However, understanding the energy variation of the CS amplitude, particularly around the resonance peak, is an urgent task. We are also at a theoretical infancy in deriving the Compton amplitudes from the underlying quark-gluon structure of hadrons. There is also the work of relating information [40] on the magnetic polarizability of nucleons, extracted from the CS at lower photon energies, to that obtained here at the peak of the Δ resonance. Finally, there is the related subject of the testing of the Drell-Hearn-Gerasimov sum rule [8] and its implications for deep-inelastic scattering.

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