## PHYSICAL REVIEW D VOLUME 45, NUMBER 3

## Oblique electroweak corrections and new physics

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Oblique electroweak parameters  $\overline{S}$ ,  $\overline{T}$ , and  $\overline{U}$ , defined so as to be nonvanishing only for physics beyond the standard model, are determined by direct use of high-statistics data from the CERN  $e^+e^$ collider LEP at different energy points around the Z peak. Additional information from related electroweak measurements are used as constraints. The results are  $\overline{S} = -0.76 \pm 0.71$ ,  $\overline{T} = -0.70 \pm 0.49$ , and  $\tilde{U} = -0.11 \pm 1.07$ . The consequent restrictions on extra fermion generations and an extra neutral gauge boson are discussed.

PACS number(s): 12.15.Cc, 12.15.3i, 14.80.Er

The "oblique" [1] radiative parameters<sup>1</sup> [2-7] S, T, and U, which enter the electroweak theory through vector-boson propagators, can be used optimally and in a model-independent way to probe physics beyond the standard model. Recent months have seen major efforts towards ascertaining their values within minimum possible error bars. In this Rapid Communication we present an improved determination of those values and focus on the consequent constraints on certain types of new physics.

In all investigations reported so far, only certain global quantities have been fitted in terms of  $S$ ,  $T$ , and  $U$ . These consist of the ones measured on the Z peak [energy scale consist of the ones measured on the Z peak tenergy scale<br> $(q^2)^{1/2} \approx M_Z$  as well as those determined at much lowe energies  $\left[\left(q^2\right)^{1/2} \ll M_Z\right]$ . The former comprise the total Z width  $\Gamma_Z$ , the Z mass  $M_Z$ , the peak value of the total cross section  $\sigma_P$  (e<sup>+</sup>e<sup>-</sup>  $\rightarrow$  Z  $\rightarrow$  visible), the charged leptonic plus b-quark forward-backward asymmetries  $(A_f^h)$ , and the r polarization asymmetry  $(A_{FB}^{r}$ . The latter include<sup>2</sup>  $R_v = \sigma$  ( $v_\mu$  nucleus  $\rightarrow v_\mu X)/\sigma(v_\mu$  nucleus  $\rightarrow \mu X$ ) and the weak charge  $Q_W$  associated with atomic parity violation.

We constrain a slightly different version of those oblique parameters by directly confronting them with the following data at sixteen different energy points around the Z peak: (i) the cross sections ( $\sigma$ ) for  $e^+e^- \rightarrow$  hadrons,  $e^+e^-, \mu^+\mu^-, \tau^+\tau^-,$  and (ii)  $A_{FB}^l$  ( $l=e,\mu,\tau$ ) from the four experimental groups at the CERN  $e^+e^-$  collider LEP [8]. Because of greater sensitivity to the errors of experimental measurements due to the full shape and energy dependence of  $\sigma$  and  $A_{FB}$ , the central values and ranges of those parameters are now more reliable in probing any possible new physics. As an extension of our analysis we also study the effect on our fitted parameters of constraints from the measurements of  $A_{FB}^{b}$  ( $M_Z^2$ ) [9],  $A_{FB}^{t}$ <sup>tool</sup>  $(M_Z^2)$  [9,10],  $R_v$  [11], and  $Q_W$  [12].

We first define a slightly modified set of electroweak parameters  $\tilde{S}$ ,  $\tilde{T}$ , and  $\tilde{U}$  which solely indicate physics beyond the standard model; i.e., in the absence of such physics for a specified set of standard-model parameters used as a reference point, they vanish. These are introduced within one-loop calculations without recourse to any further approximations concerning the energy scale  $(q^2)^{1/2}$  which have sometimes reduced the generality of previous analyses. (For instance, we do not need to assume in general that the onset of new physics must be at a scale much larger than  $M_Z$ .) They are also defined in a way that they are manifestly divergence-free so that there is no ambiguity in extracting a finite part. (Throughout we work with the on-shell renormalization scheme [13].) We then fit the cross section and lepton asymmetry data of the four LEP experimental groups (269 data points at 16 different energy values around  $M<sub>Z</sub>$  comprising over 650000 Z decays) with S, T, and  $M_Z$  as free parameters. We also use the other relevant experimental quantities mentioned above as constraints. We incorporate the standard-model radiative corrections in the region  $(q^2)^{1/2} \approx M_Z$  (for given values of the QCD coupling  $\alpha_s$  and the top-quark and Higgs-boson masses) using the program package ZFITTER [14] which adopts an analytic approach based on the improved Born approximation and takes into account the initial-state radiative corrections as well. This leads first to the determination of the fitted parameters S and T (along with  $M_Z$ ), while U is subsequently obtained by making use of the collider measurement of  $M_W$  [15].

The oblique parameters of our interest emerge from the (generally divergent)  $\gamma$ , Z, and W self-energies and the  $\gamma$ -Z mixing amplitudes  $\Pi_{rr}(q^2)$ ,  $\Pi_{ZZ}(q^2)$ ,  $\Pi_{WW}(q^2)$ , and  $\Pi_{\gamma Z}(q^2)$ , respectively, defined as functions of the energy scale of the gauge boson. Electromagnetic gauge invariance implies  $\Pi_{rr}(0) = \Pi_{rZ}(0) = 0$ . We denote the weak isospin currents as  $J_{1,3}^{\mu}$  and the electromagnetic current as  $J_0^{\mu} = J_3^{\mu} + J_1^{\mu}$  so that the Z current is  $(e/sc)(J_3^{\mu} - s^2J_0^{\mu})$ ,  $J_0 = J_3 + J_1$  so that the Z current is  $\frac{F}{C}$ ,  $\frac{F}{J_3} = \frac{F}{J_1}$ <br>where  $c = (1 - s^2)^{1/2} = M_W/M_Z$  and  $e^2 = 4\pi a$  with

 ${}^{1}S$  and T were introduced in Ref. [2], U in Ref. [7]. Different but equivalent definitions appear in Refs. [3]-[6].

<sup>&</sup>lt;sup>2</sup>Some authors also take the ratio  $\sigma(v_{\mu}e \rightarrow v_{\mu}e)/\sigma(\bar{v}_{\mu}e)$  $\rightarrow \bar{v}_u e$ ) as well as the asymmetries in deep-inelastic *eD*, *eC* scattering. Because of the larger experimental errors in these quantities, we prefer to exclude them.

 $\alpha \equiv \alpha(0)$ . Hence<sup>3</sup> (as in Ref. [16]),

$$
\Pi_{\gamma\gamma} = e^2 \Pi_{QQ},\tag{1a}
$$

$$
\Pi_{ZZ} = \frac{e^2}{c^2 s^2} (\Pi_{33} - 2s^2 \Pi_{3Q} + s^4 \Pi_{QQ}), \qquad (1b)
$$

$$
\Pi_{WW} = \frac{e^2}{s^2} \Pi_{11} \,, \tag{1c}
$$

$$
\Pi_{\gamma Z} = \frac{e^2}{cs} (\Pi_{3Q} - s^2 \Pi_{QQ}).
$$
 (1d)

Equations (1) apply at all values of  $q^2$ . We now define<sup>4</sup> S, T, and U as [5]

$$
S = \frac{16\pi}{M_Z^2} [\Pi_{33}(M_Z^2) - \Pi_{33}(0) - \Pi_{3Q}(M_Z^2)]
$$
  
= 
$$
\frac{16\pi}{M_Z^2} [\Pi_{3Y}(0) - \Pi_{3Y}(M_Z^2)]
$$
, (2a)

$$
T \equiv \frac{4\pi}{s^2 c^2 M_Z^2} [\Pi_{11}(0) - \Pi_{33}(0)] \,, \tag{2b}
$$

$$
U = \frac{16\pi}{M_W^2} \left[\Pi_{11}(M_W^2) - \Pi_{11}(0)\right] - \frac{16\pi}{M_Z^2} \left[\Pi_{33}(M_Z^2) - \Pi_{33}(0)\right].
$$
\n(2c)

T and U receive nonzero contributions from the violation of weak isospin [18] and are finite on account of the weak isospin symmetric nature of the divergence terms.  $S$  originates from the mixing between weak hypercharge and the third component of weak isospin being a consequence of the spontaneous symmetry breakdown mechanism. The latter involves soft operators and does not affect the leading divergences on account of Symanzik's theorem [19]. Consequently, 5 possesses no divergences and hence is finite. The expressions given in (2) are complete in the sense of including contributions from the standard model as well as any possible new physics.

To one loop, the  $\Pi$  functions receive contributions from different sources additively. This fact enables us to define<br>for every  $\Pi$ , a  $\overline{\Pi} = \Pi - \Pi^{SM}$  where  $\Pi^{SM}$  is the contribution to that 11 from one-loop terms within the standard model. Correspondingly,  $\tilde{S}$ ,  $\tilde{T}$ ,  $\tilde{U}$  are obtained by replacing  $\Pi$  by  $\tilde{\Pi}$  in (2). The values of  $S^{SM}$ ,  $T^{SM}$ , and  $U^{SM}$  are to be calculated analytically. They, of course, depend on the yet unknown top-quark mass  $m<sub>t</sub>$  and the Higgs-boson mass  $M_H$ . Direct experimental searches as well as requirements of theoretical consistency demand that  $89 \le m<sub>t</sub>$  $\leq$  200 GeV while  $M_H$  is allowed to be anywhere between 48 GeV and <sup>1</sup> TeV. We choose the standard-model reference point at  $m_l = 140$  GeV,  $M_H = 100$  GeV, and  $\alpha_s = 0.120$  [20]. The dependence of the radiative corrections on  $M_H$  is logarithmic in one-loop terms owing to the Veltman screening theorem [21] while their leading  $m_l$ behavior is quadratic. We shall indicate later the trend of variations of the results of our analysis by shifting the standard-model reference point.

In order to determine  $\overline{S}$ ,  $\overline{T}$ , and  $\overline{U}$  from available experimental data, let us initially concentrate on the first two. T is simply related to the  $\rho$  parameter which measures the ratio of the neutral- and charged-current amplitudes at vanishing momentum transfer.  $\rho$  can be defined to one loop as [22]

$$
\rho \equiv 1 + \frac{e^2}{c^2 s^2 M_Z^2} [\Pi_{11}(0) - \Pi_{33}(0)]. \tag{3}
$$

(3) and (2b) trivially imply that

$$
\rho = \rho^{SM} + a\tilde{T}, \qquad (4)
$$

where  $\rho^{\text{SM}}$  is the one-loop standard-model contribution to  $\rho$ . On the other hand, the one-loop corrected sin<sup>2</sup> $\bar{\theta}_w$ , which appears in the effective vector coupling of the onshell Z to fermions, relates both  $\overline{S}$  and  $\overline{T}$  to physics. First, we can write

$$
\sin^2 \bar{\theta}_W = \frac{1}{2} \left[ 1 - \left( 1 - \frac{4\pi a (M_Z^2)}{\sqrt{2} G_\mu M_Z^2} \right)^{1/2} \right] + \frac{e^2}{(c^2 - s^2) M_Z^2}
$$
  
×[ $\Pi_{33}$ ( $M_Z^2$ ) -  $\Pi_{3Q}$ ( $M_Z^2$ ) -  $\Pi_{11}$ (0)] , (5)

where  $G_{\mu}$  is the muon decay constant. Now (5), (2a), and (2b) lead to the result

$$
\sin^2 \bar{\theta}_W = (\sin^2 \bar{\theta}_W)^{SM} + \frac{\alpha}{4(c^2 - s^2)} (\tilde{S} - 4c^2 s^2 \tilde{T}), \qquad (6)
$$

where  $(\sin^2 \bar{\theta}_W)^{SM}$  is the effective  $\sin^2 \theta_W$  to one loop on the Z peak in the standard model.

We write the radiatively corrected vector and axialvector couplings of Z to fermions in terms of  $\rho$  and  $\sin^2 \theta_w$ as

$$
v_f = \sqrt{\rho} (t_{3f} - 2Q_f \sin^2 \bar{\theta}_W), \qquad (7a)
$$

$$
a_f = \sqrt{\rho} t_{3f} , \qquad (7b)
$$

with  $t_{3f}$  and  $Q_f$  being the third weak isospin component and the charge of the fermion  $f$ , respectively. The partial width  $\Gamma(Z \to f\bar{f}) \equiv \Gamma_f$  can now be written as

$$
\Gamma_f = N_c^f \frac{G_\mu M_Z^3}{6\pi\sqrt{2}} (v_f^2 + a_f^2) , \qquad (8)
$$

where

$$
N_c^f = 1 + \frac{3\alpha}{4\pi} Q_f^2 \quad (f = \text{lepton})
$$
\n
$$
= 3 \left[ 1 + \frac{3\alpha}{4\pi} Q_f^2 \right] \left[ 1 + \frac{\alpha_s(M_Z)}{\pi} + 1.405 \frac{\alpha_s^2(M_Z)}{\pi^2} \right] \quad (f = \text{quark}) \tag{9b}
$$

<sup>&</sup>lt;sup>3</sup>Reference [16] is based on the  $\star$  scheme whose connection with the on-shell scheme of Ref. [13] has been discussed in Ref. [17].

<sup>40</sup>ur definition agrees with those of Refs. [2,4] in the limit of linear approximation concerning the scale  $(q^2)$ <sup>1</sup>

<sup>&</sup>lt;sup>5</sup>For  $f=b$ , the additional leading  $m<sub>t</sub>$  dependence from the top-mediated triangular vertex correction at the  $Zb\bar{b}$  vertex also has to be taken care of. This is done by modifying  $\rho \rightarrow \rho_b = \rho(1 - \frac{4}{3} \Delta \rho_t)$  and  $\sin^2 \overline{\theta}_w \rightarrow \sin^2 \overline{\theta}_w + \sin^2 \overline{\theta}_w (1$  $+\frac{2}{3}\Delta\rho_l$ ) where  $\Delta\rho_l \approx 3G_\mu m_l^2 (8\pi^2\sqrt{2})^{-1}$ . We have neglected the masses of all the fermions in which  $Z$  can decay.

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The radiatively corrected charged-lepton forward-backward asymmetry  $A_{FB}^I(q^2)$  in the channel  $I\bar{I}$  can similarly be written in terms of  $v_f$  and  $a_f$  [23]. For convenience, we display an approximate formula for  $A_{FB}^f(q^2)$  near the Z peak:

$$
A_{\text{FB}}^{\ell}(q^2) \approx \frac{3v_e a_e v_f a_f}{(v_e^2 + a_e^2)(v_f^2 + a_f^2)} - 6Q_f \left[ 1 - \frac{M_Z^2}{q^2} \right] \sin^2 2\bar{\theta}_W \frac{a_e a_f}{(v_e^2 + a_e^2)(v_f^2 + a_f^2)} \,. \tag{10}
$$

The first term in (10) corresponds to  $A_{FB}^{\prime}$  measured on the Z peak, which has been used in earlier analyses [2,7]. In our analysis, the second term, which is an energydependent function, provides additional information through a different combination of  $S$  and  $T$ .

With  $m_t$ ,  $M_H$ , and  $\alpha_s$  as supplied parameters, we calculate  $\rho^{\text{SM}}$  and  $(\sin^2 \bar{\theta}_W)^{\text{SM}}$  by the use of ZFITTER. The inputs for our fitting analysis are  $M_Z$ ,  $\Gamma_Z$ ,  $\Gamma_{\text{had}}$ ,  $v_f$ , and  $a_f$ . One can write, near the Z peak,  $\sigma_f = (12\pi/M_Z^2)(\Gamma_e \Gamma_f)$  $\Gamma_2^2$ )F(q<sup>2</sup>,M<sub>Z</sub>, $\Gamma$ <sub>Z</sub>) where F contains the Breit-Wigner function convoluted with initial-state radiation. The matrix method [24] of handling experimental errors is then employed in our minimization of  $\chi^2$ , defined by  $\chi^2 = \Delta^T V^{-1} \Delta$ . Here  $\Delta$  is a column vector with element (th-expt), i.e., the difference between any theoretical expectation and the corresponding experimental measurement. V is an  $(N \times N)$  error-correlation matrix each of whose diagonal elements is the quadratic sum of statistical and systematic errors with off-diagonal elements being the products of common systematic errors between measurements.

The free parameters of our fit are  $M_Z$ ,  $\tilde{S}$ , and  $\tilde{T}$ . Although  $M_Z$  is a very accurately determined quantity of the standard model, the same LEP data are used to determine it. In order that the error in  $M_Z$  propagates properly to the other free parameters, we have decided to float  $M_Z$ along with  $S$  and  $T$  in our analysis. However, it turns out from all of our fits that  $M_Z$  is fairly tightly constrained at 91.175  $\pm$  0.005 GeV. All fits give a good description of the data with  $\chi^2/N_{\text{DF}}\approx 1$ , where  $N_{\text{DF}}=(\text{number of data})$ points minus number of fitted parameters). The fitted values of  $\tilde{S}$  and  $\tilde{T}$  along with their 90% confidence level (C.L.) upper (lower) bounds for  $m_l = 140$  GeV,  $M_H = 100$ GeV, and  $\alpha_s = 0.12$  are shown in Table I. We now incorporate the other constraints, mentioned earlier, in our fit. They include  $\sin^2 \overline{\theta_W}$  which is obtained from the measurement of the forward-backward asymmetry of the b quark [9] on the Z peak after necessary  $B\overline{B}$ -mixing corrections and also from the measurements of the  $\tau$  polarization

TABLE I. Fitted values of  $\tilde{S}$  and  $\tilde{T}$  at the standard-model reference point.

TURICILE DUINT.			$\leftarrow$		
<b>Parameters</b>	Fit	LEP $(\sigma, A_{FB})$ Upper bound (lower bound) $(90\% \text{ C.L.})$	Fit	LEP $(\sigma, A_{FB}^t)$ + constraints Upper bound (lower bound) $(90\% \text{ C.L.})$	- 1 -2
$\tilde{S}$	$-1.04$ ±1.06	0.70 $(-2.79)$	0.76 ± 0.71	0.40 $(-1.92)$	
Ť	$-0.78$ $\pm 0.63$	0.25 $(-1.81)$	$-0.70$ ± 0.49	0.10 $-1.50$	-2 -3 ۰
$\chi^2/N_{\rm DF}$	226/266		236/272		FIG. 1. The fitted values of $\tilde{S}$ and confidence level contour plot from the

asymmetry on the Z peak [9,10]. As before, the standard-model radiative corrections are handled using ZFITTER. Additionally, we introduce  $R<sub>v</sub>$  as measured by the CHARM group [11] retaining only the leading quadratic top dependence which suffices for the reported level of experimental accuracy. Finally, the experimentally measured [12]  $Q_W(\frac{133}{55}C_s) = -71.04 \pm 1.58 \pm 0.88$  is used to provide another constraint on  $\tilde{S}$  and  $\tilde{T}$ . In fact,  $Q_W$ leads to a direct determination of the magnitude and sign of  $\tilde{S}$  reasonably independently of  $\tilde{T}$  [4]. (For comparison, it may be noted that for the choice of  $m_l = 140$  GeV,  $M_H$  = 100 GeV, the S, T of Ref. [4] become identical to our  $\tilde{S}$ ,  $\tilde{T}$ , respectively.) The resulting fitted values of  $\tilde{S}$ and  $\overline{T}$  are also displayed in Table I. Figure 1 depicts a contour plot (90% C.L.) in the  $\tilde{S}$ - $\tilde{T}$  plane showing the allowed region, which follows from the constrained fit.

We have studied the effects of shifting the standardmodel reference point on the fitted parameters. We change only one of  $(m_t, M_H,$  and  $\alpha_s$ ) at a time while the other two are fixed to their central values. Defining, in general,

$$
\delta \tilde{S} = \tilde{S}(m_t, M_H, \alpha_s) - \tilde{S}(140 \text{ GeV}, 100 \text{ GeV}, 0.120),
$$
\n(11a)

$$
\delta \tilde{T} \equiv \tilde{T}(m_t, M_H, a_s) - \tilde{T}(140 \text{ GeV}, 100 \text{ GeV}, 0.120) ,
$$
\n(11b)

we have evaluated these two quantities over the ranges  $100 < m<sub>t</sub> < 180$  GeV,  $100$  GeV  $< M<sub>H</sub> < 1$  TeV, and  $0.113 < \alpha_s < 0.127$ . Table II displays the shifts  $\delta \tilde{S}$  and  $\delta T$  at the extreme points of these ranges.

In order to determine  $\tilde{U}$  we make use of the measured  $M_W/M_Z =c$ .  $\tilde{U}$  enters [7] our considerations only through



FIG. 1. The fitted values of  $\tilde{S}$  and  $\tilde{T}$  along with their 90% confidence level contour plot from the constrained fit.

the relation (retaining only leading quadratic top dependence)

$$
2c^{2}\left[1+\left(1-\frac{4\pi a(M_{Z}^{2})}{\sqrt{2}G_{\mu}M_{Z}^{2}}\right)^{1/2}\right]^{-1} = 1+\frac{3a}{16\pi(c^{2}-s^{2})s^{2}}\left(\frac{m_{t}}{M_{Z}}\right)^{2}+\frac{a}{4(c^{2}-s^{2})s^{2}}[4c^{2}s^{2}\tilde{T}-2s^{2}\tilde{S}+(c^{2}-s^{2})\tilde{U}].
$$
 (12)

For  $M_W$  =80.14  $\pm$  0.31 GeV as measured experimentally [15],  $U = -0.11 \pm 1.07$  at the standard-model reference point  $m<sub>l</sub> = 140$  GeV,  $M<sub>H</sub> = 100$  GeV,  $\alpha<sub>s</sub> = 0.12$  using both the high-statistics data around the Z peak and the additional constraints.

Let us now discuss the restrictions which our analysis imposes on possible physics beyond the standard model. In this brief paper, as an illustration, we take two types of new physics: namely, an extra heavy fermionic generation and an extra neutral gauge boson Z'. (A more detailed analysis with left-right symmetry, supersymmetry, and technicolor is in progress. ) We take each type in isolation, i.e., as the only new physics possibly present. First, we take the case of a fourth generation in terms of an additional mass-degenerate heavy- (i.e., with mass greater than  $\frac{1}{2}M_Z$ ) quark (Q) doublet and a similar lepton (L) doublet. Since  $\overline{T}$  and  $\overline{U}$  receive contributions only from weak-isospin breaking, in this scenario,  $T = U = 0$  but  $\dot{S} \neq 0$ :

$$
\tilde{S} = \frac{1}{\pi} \sum_{f} n_c^f x_f \{ 2 - 4(x_f - \frac{1}{4})^{1/2} \arccot \{ 2(x_f - \frac{1}{4})^{1/2} \} \},
$$
\n(13)

where  $x_f = m_f^2/M_Z^2$  for  $f = Q, L, m_f$  being the multipletual to the multipletual to the multipletual to the multipletual to the multipletual of the multipletual to the multipletual of the multipletual to the multipletual of where  $x_f = m_f / M_{\zeta}$  for  $f = Q, L, m_f$  being the multiple<br>mass and  $n_c^{Q(L)} = 3(1)$ . In the limit when  $m_f \gg M_{\zeta}$  $\tilde{S} = \sum_{i} n_{i}^{2}/6\pi = 2/3\pi$ . To compare with experiment, we have to further restrict  $\tilde{T}=0$ . For the specified standardmodel reference point, the result is  $\tilde{S}=0.04\pm 0.44$  from the constrained fit. This means that more than three very heavy fermionic generations are ruled out at the 90% C.L. [This conclusion is valid even if these extra fermions are of the mirror type, i.e., with  $SU(2)_R \times U(1)$  assignments, since S cannot distinguish between chiralities [5].] For one extra generation of heavy fermions, one could assume  $m_Q = m_l = m$  and derive a lower bound on m from S. However, the present uncertainty in  $\tilde{S}$  is unable to push the lower bound on m (at 90% C.L.) beyond  $M_Z/2$ . However, more data on the  $Z$  peak are likely to result in a reduced upper bound on  $\dot{S}$  and lead to an interesting lower bound on  $m$ . If we introduce mass differences between such quarks and leptons and within each doublet, the corresponding  $T$  and  $U$  will be nonzero; however, there will be four unknown mass parameters so that no constraint

TABLE II. Variations in  $\tilde{S}$  and  $\tilde{T}$  with shifts from the standard-model reference point.

				$m_i$ (GeV) $M_H$ (GeV)	$\alpha$ .	
<b>Variations</b>		100		$1000$ 0.113 0.127		
$\delta \tilde{S}$ $\delta T$	LEP $(\sigma, A_{FB}^{\ell})$ -0.04 0.02 -0.24 -0.01		$0.31 - 0.41$	$0.24$ $0.14 - 0.14$		0.02
$\delta \tilde{S}$ $\delta \tilde{T}$	LEP $(\sigma, A_{FB}^{\ell})$ -0.17 0.20 $+$ constraints $0.27 - 0.34$			$-0.15$ 0.27		$0.05 - 0.05$ $0.16 - 0.16$

obtains.

The formalism developed above is specific to the  $SU(2)_L \times U(1)$  gauge group. However,  $\rho$  and  $\sin^2 \theta_W$ , as defined in (4) and (6), respectively, can sometimes be used more generally. For instance, the tree-level mixing effect of an extra  $Z'$  [25] with the weak neutral boson of the standard model can be treated on a general ground by modifying  $\rho$  and  $\sin^2 \overline{\theta}_W$  and defining  $\overline{T}$  and  $\overline{S}$  by (4) and (6). Because of mixing as well as the possible extended Higgs structures accompanying such models,  $\rho$  deviates from unity, by  $\Delta \rho$ , at the tree level itself so that<sup>6</sup>  $\rho^{\text{SM}} \rightarrow \rho^{\text{SM}}(1+\Delta\rho)$  and

$$
\bar{S}_W^2 \equiv (\sin^2 \bar{\theta}_W)^{\text{SM}} \rightarrow \bar{\bar{S}}_W^2 \equiv \bar{S}_W^2 - [\bar{C}_W^2 \bar{S}_W^2 / (\bar{C}_W^2 - \bar{S}_W^2)] \Delta \rho.
$$

The vector and axial-vector couplings of the lighter physical  $Z$  to a charged lepton  $l$  now become (leptonic universality is valid even in presence of an extra  $Z'$ )

$$
v_l = (\rho^{SM})^{1/2} (1 + \frac{1}{2} \Delta \rho) [(-\frac{1}{2} + 2\overline{S}_{W}^{2}) \cos \xi_0 + v'_{l} \sin \xi_0], \qquad (14a)
$$

$$
a_l = (\rho^{SM})^{1/2} (1 + \frac{1}{2} \Delta \rho)(-\frac{1}{2} \cos \xi_0 + a'_l \sin \xi_0), \qquad (14b)
$$

where  $\xi_0$  is the mixing angle and  $v_1$ ,  $a_1$  are, respectively, the vector and axial-vector couplings of  $Z'$  to *l*. Identifying  $\rho$  and  $\Gamma_l$ , modified due to mixing, with their respective expressions obtained in  $(S, \tilde{T})$  parametrization, we find (keeping terms only linear in  $\xi_0$ ,  $\Delta \rho$ ,  $\alpha S$ , and  $\alpha T$ )

$$
\alpha \tilde{T} = \Delta \rho \,, \tag{15a}
$$

$$
a\tilde{S} = \xi_0 \left[ 2(c^2 - s^2)v'_1 + 2\frac{c^2 - s^2}{1 - 4s^2}a'_1 \right].
$$
 (15b)

We now consider the specific case where the extra  $Z'$  originates as the low-energy manifestation of a grand unifying  $E_6$  symmetry possibly of superstring origin. In such a  $\epsilon_6$  symmetry possibly of superstring origin. In such a<br>scenario,  $v'_1 = -(s/2) [\cos \theta_2 + (5/3)^{1/2} \sin \theta_2]$  and a  $=s\left[\frac{1}{6}\cos\theta_2-\frac{1}{2}(5/3)^{1/2}\sin\theta_2\right]$ . The mixing angle of the two extra neutral generators of  $E_6$  is parametrized by  $\theta_2$ , which depends on the symmetry-breaking chain. Feeding back these  $v_i$  and  $a_i$  in the general equation (15b), it follows  $\alpha \tilde{S} \approx (0.90 \cos \theta_2 - 4.84 \sin \theta_2) \xi_0$ . Thus for  $\theta_2 = 0^\circ$ (the  $Z_n$  model [26]),  $|\xi_0|$  < 0.016; for  $\theta_2$  =52.24° (the  $Z_n$ ) model [26]),  $|\xi_0| < 0.004$ ; for  $\theta_2 = -52.24^\circ$ ,  $|\xi_0| < 0.003$ ; and for  $\theta_2 = -82.76^{\circ}$ ,  $|\xi_0| < 0.003$  ( $\xi_0$  are in radians) at the 90% C.L. which follow from the constrained fit analysis for the specified standard-model reference point. In the  $\theta_2 = 0^{\circ}$  model the mixing angle is relatively weakly constrained. This is so because in this model  $\tilde{S}$  is much

<sup>&</sup>lt;sup>6</sup>Here we do not take into account loop effects due to the  $Z'$ .

<sup>&</sup>lt;sup>7</sup>In our formalism  $a_1'$  has to be identified with  $-g_A^{\prime e}$  of Ref. I2SI.

less sensitive to the variation of  $\xi_0$ .

To summarize, we have determined the oblique parameters  $\tilde{S}$  and  $\tilde{T}$  (which along with  $\tilde{U}$  are indicators of new physics) by directly fitting them with the high statistics data on  $\sigma(q^2)$  and  $A f f f^{(r)}(q^2)$  from LEP around the Z peak along with additional constraints from  $A_{FB}^{b}(M_Z^2)$ ,  $A_{FB}^{r}Po1(M_Z^2)$ ,  $R_v$ , and  $Q_w$ . Our values are  $\tilde{S} = -0.76$  $\pm 0.71$ ,  $\tilde{T} = -0.70 \pm 0.49$ . We have obtained  $\tilde{U} = -0.11$  $\pm$  1.07 using the measured  $M_W$  as an input. To this end, we have chosen a standard-model reference point  $m_l$ =140 GeV,  $M_H$  =100 GeV, and  $\alpha_s$  =0.120. From these we have obtained specific constraints on certain kinds of new physics: namely, the loop effects of an extra fermion

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generation and the tree-level mixing between Z and a possible  $Z'$ , considered separately. The effects of shifting the standard-model reference point on the fitted values of S and  $\overline{T}$  have also been studied. In this framework the possibility of new physics can be further constrained as the data from LEP improve and additionally when the top quark is found.

We thank Rajesh Gopakumar and W. J. Marciano for discussions. G. B. acknowledges financial support from the Council of Scientific and Industrial Research (Government of India) and the hospitality of the Tata Institute of Fundamental Research.

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