PHYSICAL REVIEW D
PARTICLES AND FIELDS

THIRD SERIES, VOLUME 45, NUMBER 3 1 FEBRUARY 1992

RAPID COMMUNICATIONS

Rapid Communications are intended for important new results which deserve accelerated publication, and are therefore given priority in editorial processing and production. A Rapid Communication in **Physical Review D** *should be no longer than five printed pages and must be accompanied by an abstract. Page proofs are sent to authors, but because of the accelerated schedule, publication is generally not delayed for receipt of corrections unless requested by the author.*

τ decay puzzle

William J. Marciano
Brookhaven National Laboratory, Upton, New York 11973
(Received 2 October 1991)

Directly measured τ decay branching ratios $B(\tau\rightarrow e\nu\bar{\nu})$, $B(\tau\rightarrow\mu\nu\bar{\nu})$, $B(\tau\rightarrow\nu_\tau\pi/K)$, and $B(\tau\rightarrow\nu_\tau\pi^-\pi^0)$ are compared with standard-model predictions parametrized in terms of the τ mass and lifetime. For current averages, $m_\tau=1784.1^{+2.7}_{-3.6}$ MeV and $\tau_\tau=3.04\pm0.07\times10^{-13}$ s, the experimental branching ratios are found to be systematically smaller than theory, thereby suggesting that significant reductions in the τ lifetime or mass (perhaps both) are likely. If τ_τ and m_τ are correct, a heavy fourth-generation neutrino with $m_{\nu_4}\gtrsim45.3$ GeV and mixing $\sin^2\theta_{34}\simeq0.06$ could be the source of the discrepancy.

PACS number(s): 14.60.−z, 12.15.Ff, 13.35.+s

Recently, τ lepton studies have been initiated at the CERN e^+e^- collider LEP [1]. The combination of good detector efficiency and low backgrounds make that facility an ideal laboratory for examining τ decays. Among the interesting results that have already emerged from those studies is a confirmation of lower than expected leptonic branching ratios for the τ, a long-standing puzzle. Since the expected branching ratios are based on standard-model theory, the measured τ lifetime, and τ mass [2],

$$\tau_\tau=3.04\pm0.07\times10^{-13}\,\text{s}$$
$$=(2.165\pm0.050\times10^{-12}\,\text{GeV})^{-1}\,, \tag{1}$$

$$m_\tau=1784.1^{+2.7}_{-3.6}\,\text{MeV}\,, \tag{2}$$

the implication is that theory, τ_τ, or m_τ is wrong.

In this paper, I scrutinize and extend the above discrepancy by combining several well-measured τ branching ratios with theoretical predictions. The novelty of my comparison is the inclusion of radiative corrections where they are known and an estimate of the uncertainty for cases where they have not been completely computed.

Comparing the new LEP results [1,3] with recent findings by the CELLO [1], ARGUS [4], and CLEO [5] Collaborations and older measurements found in the Particle Data Group compilation, one obtains the new world averages

$$B(\tau\rightarrow e\nu\bar{\nu})_{\text{ave}}=0.1780\pm0.0023\,, \tag{3}$$

$$B(\tau\rightarrow\mu\nu\bar{\nu})_{\text{ave}}=0.1743\pm0.0024\,, \tag{4}$$

$$B(\tau\rightarrow\nu_\tau\pi/K)_{\text{ave}}=0.1209\pm0.0032\,, \tag{5}$$

$$B(\tau\rightarrow\nu_\tau\pi^-\pi^0)_{\text{ave}}=0.2305\pm0.0055\,. \tag{6}$$

I have singled out those four branching ratios because their uncertainties are relatively small and the theoretical underpinnings are very good, as we shall see.

For $\tau\rightarrow l\nu\bar{\nu}$, $l=e$ or μ, the standard model (including electroweak radiative corrections) predicts [6]

$$\Gamma(\tau\rightarrow l\nu\bar{\nu})=\frac{G_\mu^2 m_\tau^5}{192\pi^3}f\left(\frac{m_l^2}{m_\tau^2}\right)\left(1+\frac{3}{5}\frac{m_\tau^2}{m_W^2}\right)\times\left[1+\frac{\alpha(m_\tau)}{2\pi}\left(\frac{25}{4}-\pi^2\right)\right]\,, \tag{7}$$

with

$$f(x)=1-8x+8x^3-x^4-12x^2\ln x\,,$$

$$G_\mu=1.16637\pm0.00002\times10^{-5}\,\text{GeV}^{-2}\,,$$

$$\alpha^{-1}(m_\tau)\simeq133.3\,.$$

Based on that prediction, one expects

$$B(\tau \to e\nu\bar{\nu})_{\text{expected}} = 0.1900 \left[\frac{m_\tau}{1784.1\ \text{MeV}} \right]^5 \left[\frac{\tau_\tau}{3.04\times 10^{-13}\ \text{s}} \right], \tag{8}$$

$$B(\tau \to \mu\nu\bar{\nu})_{\text{expected}} = 0.1848 \left[\frac{m_\tau}{1784.1\ \text{MeV}} \right]^5 \left[\frac{\tau_\tau}{3.04\times 10^{-13}\ \text{s}} \right], \tag{9}$$

where the central values in (1) and (2) are used to normalize the branching ratios. The τ lifetime and mass errors translate together into about a $\pm 2.5\%$ uncertainty. (There is essentially no other uncertainty in the predictions.) Comparing the expected branching ratios with the average measurements in (3) and (4), one finds the expectation to be about 2.3σ higher in both cases. (The uncertainties in τ_τ and m_τ are included in that difference.) There are two straightforward experimental solutions to that discrepancy. Perhaps m_τ is actually about 1761 MeV, a significant downward shift, or $\tau_\tau \simeq 2.85\times 10^{-13}$ s. Those changes would correspond to 6.4σ and 2.7σ shifts, respectively. On that basis, it would appear more likely that τ_τ will come down. However, the consistency of many τ lifetime measurements over the years does not yet indicate such a shift. On the other hand, the small errors on m_τ cited in (2) mainly stem from a single DELCO [7] measurement of $\tau^+\tau^-$ production near threshold which gave $m_\tau = 1782^{+2}_{-7}$ MeV. (It was subsequently modified by the Particle Data Group [2].) Only a 3σ reduction in the actual DELCO value of m_τ would bring theory and experiment into accord. So, a 23-MeV downward shift in m_τ is not out of the question.

The decay $\tau \to \nu_\tau \pi$ or K can in principle be used to confirm an error in τ_τ or m_τ. The predicted rate for $\tau \to \nu_\tau \pi$ is given by [6]

$$\Gamma(\tau \to \nu_\tau \pi) = \frac{G_\mu^2 f_\pi^2 |V_{ud}|^2}{16\pi} m_\tau^3 \left[1 - \frac{m_\pi^2}{m_\tau^2} \right]^2 \left[1 + \frac{2\alpha}{\pi} \ln \frac{m_Z}{m_\tau} + \cdots \right], \tag{10}$$

where the leading short-distance radiative corrections have been included and the ellipsis represent $O(\alpha/\pi)$ (structure-dependent) corrections not absorbed in G_μ or $f_\pi |V_{ud}|$ which have not been calculated and probably constitute an uncertainty of roughly $\pm 1\%$. The value of $f_\pi |V_{ud}|$ can be very precisely obtained from $\pi_{\mu 2}$ decay. Indeed, including structure-dependent radiative corrections, an analysis in Ref. [8] found

$$f_\pi |V_{ud}| = 127.4 \pm 0.1\ \text{MeV}. \tag{11}$$

A similar analysis for $\tau \to \nu_\tau K$ gives [6]

$$\Gamma(\tau \to \nu_\tau K) = \frac{G_\mu^2 f_K^2 |V_{us}|^2}{16\pi} m_\tau^3 \left[1 - \frac{m_K^2}{m_\tau^2} \right]^2 \left[1 + \frac{2\alpha}{\pi} \ln \frac{m_Z}{m_\tau} + \cdots \right], \tag{12}$$

with [8]

$$f_K |V_{us}| = 35.18 \pm 0.05\ \text{MeV} \tag{13}$$

obtained from $K_{\mu 2}$ decays.

Combining (10) and (12) and again using τ_τ and m_τ parametrizations, one finds

$$B(\tau \to \nu_\tau \pi/K)_{\text{expected}} = (0.1236 \pm 0.0012) \left[\frac{m_\tau}{1784.1\ \text{MeV}} \right]^3 \left[\frac{\tau_\tau}{3.04\times 10^{-13}\ \text{s}} \right], \tag{14}$$

where a conservative $\pm 1\%$ uncertainty from uncalculated radiative corrections is included. Note the m_τ^3 dependence as compared with the m_τ^5 dependence in (8) and (9) gives, in principle, some leverage on m_τ. (I have neglected small Δm_τ sensitivities suppressed by m_π^2/m_τ^2 or m_K^2/m_τ^2.) The ratio of (14) and (5) gives

$$\frac{B(\tau \to \nu_\tau \pi/K)_{\text{expected}}}{B(\tau \to \nu_\tau \pi/K)_{\text{ave}}} = (1.022 \pm 0.029) \left[\frac{m_\tau}{1784.1\ \text{MeV}} \right]^3 \left[\frac{\tau_\tau}{3.04\times 10^{-13}\ \text{s}} \right], \tag{15}$$

which is consistent within errors with 1. So, the decay $\tau \to \nu_\tau \pi$ or K on its own is consistent with the τ_τ and m_τ values in (1) and (2). The central value in (15) is, however, suggestive of some reduction in τ_τ or m_τ. Either a reduction in τ_τ to 2.97×10^{-13} s (only 1σ) or m_τ to 1771 MeV would bring the ratio in (15) down to 1. Of those two possibilities, the latter would have more of an impact in (8) and (9). So, to some extent, $\tau \to \nu_\tau \pi/K$ supports a reduction in m_τ more than a reduction in τ_τ, but the evidence is not overwhelming.

The decay $\tau \to \nu_\tau \pi^- \pi^0$ can be predicted using $e^+e^- \to$ hadrons data and the conserved vector current hypothesis [9–11] (CVC). The most recent analysis [11] leads to

$$B(\tau \to \nu_\tau \pi^- \pi^0)_{\text{expected}} = (0.2508 \pm 0.010 \pm 0.008) \left[\frac{m_\tau}{1784.1\ \text{MeV}} \right]^3 \left[\frac{\tau_\tau}{3.04\times 10^{-13}\ \text{s}} \right], \tag{16}$$

where the first error comes from Ref. [11] and the second error corresponds to my estimate of the uncertainty in radiative corrections which have not been included. Short-distance electroweak corrections increase [6] the ratio in (16) by about 1.9%; however, QED corrections to $e^+e^- \to \pi^+\pi^-$ data are likely to reduce (16) by (at least) several percent. So, until those effects are carefully scrutinized, it seems best not to include any radiative corrections and assign an uncertainty to their neglect. I also note that the m_τ^3 dependence in (16) is very approximate and should not be taken too literally. In the narrow-width ρ pole approximation [9], it would scale as m_τ^3, but direct studies of $e^+e^- \to$ hadrons data would probably increase the power somewhat [10].

Taking the ratio of (16) and (6) gives

$$\frac{B(\tau \to \nu_\tau \pi^- \pi^0)_{\text{expected}}}{B(\tau \to \nu_\tau \pi^- \pi^0)_{\text{ave}}} = (1.088 \pm 0.061) \left[\frac{m_\tau}{1784.1 \text{ MeV}} \right]^3 \left[\frac{\tau_\tau}{3.04 \times 10^{-13} \text{ s}} \right], \tag{17}$$

where the errors have been added in quadrature. The finding in (17) further supports the likelihood of a reduction in m_τ or τ_τ, but the errors are too large to be definitive.

The above comparisons suggest rather strongly that significant shifts in τ_τ and/or m_τ should occur as new measurements become more precise. For m_τ fixed at 1784.1 MeV, the branching ratios discussed here average to $\tau_\tau = 2.87 \pm 0.02 \times 10^{-13}$ s. New lifetime measurements at LEP and a conceivable [12] $\pm 2\%$ determination of τ_τ with the CLEO II and ARGUS detectors should provide checks on that solution to the discrepancy between theory and experiment. Alternatively, fixing τ_τ at 3.04×10^{-13} s, the above constraints suggest an average $m_\tau \simeq 1762 \pm 3$ MeV. A check on that solution will come from the Beijing e^+e^- facility which can measure the $\tau^+\tau^-$ threshold turn-on with high statistics [13].

If τ_τ and m_τ do not shift away from the current averages in (1) and (2) and the directly measured branching ratios are correct, then the standard model must be breaking down, a more interesting prospect. The simplest explanation [14] for the above discrepancy, based on "new physics," involves the introduction of a heavy fourth-generation neutrino which mixes with ν_3 such that

$$\nu_\tau = \nu_3 \cos\theta_{34} + \nu_4 \sin\theta_{34} . \tag{18}$$

The ν_τ would be mostly ν_3; so, the bound $m_{\nu_\tau} < 35$ MeV would apply to m_{ν_3}. LEP Z-width constraints require for a sequential fourth neutrino $m_{\nu_4} \gtrsim 45.3$ GeV. The effect of such a heavy neutrino would be to reduce all theoretical τ decay rates by $\cos^2\theta_{34}$ or equivalently increase the theoretical lifetime prediction by $1/\cos^2\theta_{34}$. The four branching ratio expectations given in (8), (9), (14), and (15) would thus be multiplied by $\cos^2\theta_{34}$. From the measured branching ratios and values of τ_τ and m_τ in (1) and (2), one then finds

$$\cos^2\theta_{34} = 0.943 \pm 0.008 \pm 0.023^{+0.008}_{-0.010} , \tag{19}$$

where the errors come from the measured branching ratios, τ_τ, and m_τ, respectively. The implied mixing is roughly the size of Cabibbo mixing in the d-s quark sector, which is somewhat large considering the small m_{ν_3}/m_{ν_4} ratio and the required smallness of m_τ/m_L, where L is the fourth-generation charged lepton.

The phenomenology of a fourth generation has been extensively discussed in the literature and will not be repeated here. A few comments about a heavy ν_4 are, however, in order. Mixing of ν_4 with the first and second generation must be very small; otherwise, it would have led to an already observable rate [15] for $\mu \to e\gamma$ or $\mu N \to eN$. If $m_{\nu_4} \lesssim 78$ GeV, one can search for it in the decay $W \to \nu_4 + \tau$. The ν_4 would decay into $\tau e \nu$, $\tau \mu \nu$, $\tau^+\tau^-\nu$, or τ + hadrons with relative branching ratios 1:1:1:6. A signature of ν_4 would therefore be multilepton (particularly τ) events with in some cases considerable missing p_T from neutrinos.

The τ decay puzzle has been around for some time and now appears to be confirmed by recent LEP results. Its resolution will probably involve a reduction in τ_τ or m_τ (perhaps some combined movement). New high-precision measurements of τ_τ and m_τ are clearly warranted. Should the problem persist after such measurements, it may be the harbinger of a fourth generation, an exciting possibility. τ decays may not be boring after all.

This manuscript has been authored under Contract No. DE-AC02-76CH00016 with the U.S. Department of Energy.

[1] For an up-to-date review of τ decay results, see M. Danilov, in Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Geneva, Switzerland, 1991 (unpublished).

[2] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990). I have updated τ_τ with more recent results from Ref. [5].

[3] L3 Collaboration, B. Adeva *et al.*, Phys. Lett. B **265**, 451 (1991); Opal Collaboration, G. Alexander *et al.*, *ibid.* **266**, 201 (1991).

[4] ARGUS Collaboration, H. Albrecht *et al.*, DESY report, 1991 (unpublished).

[5] A. Weinstein, in Proceedings of the Meeting of the APS Division of Particles and Fields, 1991 (unpublished).

[6] W. Marciano and A. Sirlin, Phys. Rev. Lett. **61**, 1815 (1988).

[7] DELCO Collaboration, W. Bacino *et al.*, Phys. Rev. Lett. **41**, 13 (1978).

[8] W. Marciano, Annu. Rev. Nucl. Part. Sci. **41**, 469 (1991); see also B. Holstein, Phys. Lett. B **244**, 83 (1990).

[9] Y. S. Tsai, Phys. Rev. D **4**, 2821 (1971).

[10] F. Gilman and D. Miller, Phys. Rev. D **17**, 1846 (1978).

[11] J. Kühn and A. Santamaria, Z. Phys. C **48**, 443 (1990).

[12] R. Kass, in *Proceedings of the Tau-Charm Factory Workshop*, Stanford, California, 1989, edited by Lydia V. Beers (SLAC Report No. 343, Stanford, 1989).

[13] V. Luth (private communication).

[14] B. C. Barish and R. Stroynowski, Phys. Rep. **157**, 1 (1988); M. Shin and D. Silverman, Phys. Lett. B **213**, 379 (1988); S. Rajpoot and M. Samuel, Mod. Phys. Lett. A **3**, 1625 (1988).

[15] A. Acker and S. Pakvasa, University of Hawaii report, 1991 (unpublished). Those authors find $|U_{4e}^{*}U_{4\mu}|^2 \lesssim 7 \times 10^{-6}$ from $\mu \rightarrow e\gamma$ constraints. The constraint from $\mu N \rightarrow eN$ conversion [see W. Marciano and A. Sanda, Phys. Rev. Lett. **38**, 1512 (1977)] is expected to give a more restrictive constraint $|U_{4e}^{*}U_{4\mu}|^2 \lesssim 10^{-8}$. A careful analysis of that process is in progress.