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Bremsstrahlung and zero-energy Rindler photons

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The bremsstrahlung effect is analyzed in the frame coaccelerating with the charge. In particular, it is shown that the usual rate of photon emission with a given transverse momentum computed in the inertial frame can be interpreted as the combined rate of emission (absorption) of zero-energy Rindler photons into (from) the thermal bath as calculated in the particle rest frame.

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The issues related to the excitation of accelerated quantum detectors have been the subject of much interest in recent years. As is well known, a detector with a constant proper acceleration a acts as if it were in a thermal bath with a temperature $a/2\pi$ (the Fulling-Davies-Unruh effect [1]) in the units where $k_B=c=\hbar=1$, which will be used throughout this paper. The close connection between this effect and Hawking radiation by black holes [2] is an example of how the description of physics from the point of view of accelerated observers can lead to insight into phenomena occurring in curved spacetimes. A comprehensive study of this effect including an analysis from the inertial point of view has been given by Unruh and Wald [3]. More recently, specific calculations of emission rates corresponding to the Fulling-Davies-Unruh effect from the inertial point of view have been given by Kolbenstvedt [4], who also considered in this frame the scalar bremsstrahlung.

On the other hand, the question of whether or not an observer comoving with an accelerated charge sees radiation has been the subject of some controversy. Recently, this issue has been clarified by Boulware [5], in the classical context, who showed that all the radiation goes into a region of spacetime inaccessible to the coaccelerating observer.

The aim of this paper is to show by explicit computations that the usual QED bremsstrahlung from a charged source with a constant proper acceleration can be reproduced in the coaccelerated frame if one takes into account the thermal bath mentioned above. We compare the emission rate of photons computed in the inertial frame and the rate of absorption (emission) of photons from (into) the thermal bath in which the charge is immersed as seen in the coaccelerated frame. (A similar computation in de Sitter spacetime can be found in Ref. [6].) Both rates are computed for photons with a fixed value of transverse momentum, i.e., the component of the momentum perpendicular to the acceleration of the charge.

It must be emphasized that since we will be working in the lowest order of perturbation theory, the event of emission of a photon in the inertial frame will correspond to *either* the emission *or* the absorption of a Rindler photon in the accelerated frame. This is the result of the fact that observers in both frames must agree on whether or not there is a change in the state of the quantum field.

Let us consider a point source with charge q, and constant proper acceleration a along the z direction. In the corresponding Rindler wedge, the line element has the form (see, e.g., [7])

$$ds^2=e^{2a\xi}(d\tau^2-d\xi^2)-dx^2-dy^2\,, \tag{1}$$

where the Rindler coordinates are related to the usual Minkowski coordinates by

$$t=\frac{e^{a\xi}}{a}\sinh a\tau,\ \ z=\frac{e^{a\xi}}{a}\cosh a\tau\,. \tag{2}$$

The world line of our accelerated charged source is given by $\xi=x=y=0$ in this coordinate system. Hence, the components of the corresponding current are

$$j^\tau=q\delta(\xi)\delta(x)\delta(y),\ \ j^\xi=j^x=j^y=0\,. \tag{3}$$

The static source represented by current (3) can only excite zero-energy Rindler modes, and usually the corresponding rate would be zero. However, since the density of zero-energy Rindler photons in the Fulling-Davies-Unruh thermal bath is infinite, we must regularize the resulting undefined expression. For this purpose we consider (3) as the $E\to 0$ limit of the current $j^\tau=\sqrt{2}q\cos E\tau \times\delta(\xi)\delta(x)\delta(y)$, $j^\xi=j^x=j^y=0$, which corresponds to an oscillating charge. The factor $\sqrt{2}$ ensures that the time average, associated with the square of this oscillating charge, coincides with the one obtained with our original current (3). In order to guarantee current conservation, we will consider an oscillating dipole arrangement given by the current

$$j^\tau=\sqrt{2}q[\delta(\xi)-e^{-2a\xi}\delta(\xi-L)]\delta(x)\delta(y)\cos E\tau\,, \tag{4}$$

$$j^\xi=\sqrt{2}qEe^{-2a\xi}\theta(\xi)\theta(L-\xi)\delta(x)\delta(y)\sin E\tau\,, \tag{5}$$

with the other components being zero. In the end of the calculations, we will take the limit where one of the charges is moved to infinity, i.e., $L\to+\infty$, in addition to the limit $E\to 0$. Here, j^τ corresponds to the oscillating charges, and j^ξ to the current flow between them. We shall find that the second charge at $\xi=L$ and the current flow between the two charges do not contribute to the final result. In order to quantize the Maxwell field, we use the Lagrangian $\mathcal{L}=-\sqrt{-g}[\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+(2\alpha)^{-1}(\nabla^\mu A_\mu)^2]$, which leads in the Feynman gauge ($\alpha=1$) to the field equation

$$\nabla_\mu\nabla^\mu A_\nu=0\,. \tag{6}$$

The physical modes are those which satisfy the Lorenz condition $\nabla_\mu A^\mu = 0$ in addition to (6), and are not pure gauge.

Since the Rindler metric (1) possesses ∂_τ, ∂_x, ∂_y as Killing fields, it is sufficient to look for solutions of Eq. (6) of the form

$$A_\mu^{(\lambda,\omega,k_x,k_y)}(x^\nu) = f_\mu^{(\lambda,\omega,k_x,k_y)}(\xi)\exp[i(k_x x + k_y y - \omega\tau)] , \tag{7}$$

where λ specifies the polarization degrees of freedom. The quantum field $\hat{A}_\mu$ is expanded in the usual way:

$$\hat{A}_\mu(x^\nu) = \int_{-\infty}^{+\infty} dk_x \int_{-\infty}^{+\infty} dk_y \int_0^{+\infty} d\omega \sum_{\lambda=1}^{4} \{a^{(\lambda,\omega,k_x,k_y)} A_\mu^{(\lambda,\omega,k_x,k_y)}(x^\nu) + \text{H.c.}\} . \tag{8}$$

We define the generalized Klein-Gordon inner product between any two modes $A_{(i)}$, $A_{(j)}$ as

$$(A_{(i)}, A_{(j)}) \equiv \int_\Sigma d\Sigma_\mu W^\mu[A_{(i)}, A_{(j)}] . \tag{9}$$

Here, Σ is a Cauchy surface for the Rindler wedge, e.g., the hypersurface with a constant τ, and

$$W^\mu[A_{(i)}, A_{(j)}] \equiv \frac{i}{\sqrt{-g}} (A^*_{(i)\nu}\pi^{\mu\nu}_{(j)} - A_{(j)\nu}\pi^{\mu\nu *}_{(i)}) , \tag{10}$$

with $\pi^{\mu\nu}_{(i)} \equiv \partial\mathcal{L}/\partial_\mu A_\nu|_{A_{(i)\mu}}$. The field equations guarantee that the current W^μ and the inner product (9) are conserved (see, e.g., Ref. [8]). Note that (i) and (j) stand for (λ,ω,k_x,k_y). As pointed out in [9], the canonical commutation relations among fields and momenta lead to

$$[a_{(i)}, a^\dagger_{(j)}] = (M^{-1})_{(i)(j)} , \tag{11}$$

where $M_{(i)(j)} \equiv (A_{(i)}, A_{(j)})$, with the other commutators being zero. Now, the two physical modes can be given by letting

$$f_\mu^{(\mathrm{I},\omega,k_x,k_y)} = C^{(\mathrm{I},\omega,k_x,k_y)}(0,0,k_y\phi,-k_x\phi) \tag{12}$$

and

$$f_\mu^{(\mathrm{II},\omega,k_x,k_y)} = C^{(\mathrm{II},\omega,k_x,k_y)}(\partial_\xi\phi,-i\omega\phi,0,0) \tag{13}$$

in (7) (see Ref. [10]), where $C^{(i)}$ are normalization constants, which will be determined later. [The components labeled by the index μ above are ordered as in (1).] The function ϕ is given by

$$\phi = K_{i\omega/a}\left[\frac{k_\perp}{a}e^{a\xi}\right] , \tag{14}$$

where $K_\nu(z)$ is the Bessel function of imaginary argument [11], and $k_\perp = (k_x^2 + k_y^2)^{1/2}$. Note that the pure gauge and nonphysical modes can be chosen to be orthogonal to these solutions with respect to the inner product (9). Therefore, in order to obtain the usual creation-annihilation commutation relations

$$[a^\lambda_{\omega,k_x,k_y}, a^{\lambda'\dagger}_{\omega',k_x',k_y'}] = \delta^{\lambda\lambda'}\delta(\omega-\omega')\delta(k_x-k_x')\delta(k_y-k_y') \tag{15}$$

for λ, λ' corresponding to the physical modes (i.e., I and II), we impose the normalization condition

$$(A^{(\lambda,\omega,k_x,k_y)}, A^{(\lambda',\omega',k_x',k_y')}) = \delta^{\lambda\lambda'}\delta(\omega-\omega')\delta(k_x-k_x')\delta(k_y-k_y') \tag{16}$$

[see Eq. (11)]. Since the current given by (4) and (5) clearly will not excite the physical mode I [see Eq. (12), and Eq. (18) below], we just need to evaluate the normalization constant $C^{(\mathrm{II},\omega,k_x,k_y)}$ in (13). Using (7), (13), and (14) in (16) for $\lambda = \lambda' = \mathrm{II}$ we obtain

$$|C^{(\mathrm{II},\omega,k_x,k_y)}| = \frac{[\sinh(\pi\omega/a)]^{1/2}}{2\pi^2 k_\perp \sqrt{a}} . \tag{17}$$

Now, the interaction between the charged particle and the Maxwell field is described by the Lagrangian

$$\mathcal{L}_{\text{int}} = \sqrt{-g}\, j^\mu \hat{A}_\mu . \tag{18}$$

Therefore, the amplitude for the absorption of a Rindler photon by the accelerated charge is

$$\mathcal{A}^{\text{abs}}_{(\omega,k_x,k_y)} = {}_R\langle 0| i\int d^4x \sqrt{-g}\, j^\mu(x)\hat{A}_\mu(x)|\mathrm{II},\omega,k_x,k_y\rangle_R , \tag{19}$$

where the subscript R indicates Rindler states. Using (4), (5), and (8) in (19), and performing the corresponding integrations, we obtain

$$\mathcal{A}^{\text{abs}}_{(\omega,k_x,k_y)} = \frac{iq}{\pi(2a)^{1/2}}\delta(E-\omega)[\sinh(\pi E/a)]^{1/2}\left\{K'_{iE/a}(k_\perp/a) - e^{aL}K'_{iE/a}(k_\perp e^{aL}/a) - \frac{E^2}{ak_\perp}\int_{k_\perp/a}^{(k_\perp/a)e^{aL}} \frac{dz}{z}K_{iE/a}(z)\right\} , \tag{20}$$

where the prime corresponds to derivatives with respect to the argument of the function. Next, taking the limit $L \to +\infty$ in (20), we evaluate the probability of absorption per unit time, for a fixed transverse momentum (k_x,k_y) [i.e., $dW^{\text{abs}}(\omega,k_x,k_y) = |\mathcal{A}^{\text{abs}}|^2 d\omega/T$], for small E as

$$dW^{\text{abs}}(\omega,k_x,k_y) = \left\{\frac{q^2E}{4\pi^2a^2}|K'_{iE/a}(k_\perp/a)|^2 + O(E^3)\right\}\delta(E-\omega)d\omega , \tag{21}$$

where we have identified $T = \delta(0)/2\pi$. Now, recall that the charged source is immersed in a thermal bath characterized by the temperature $a/2\pi$. So, the total absorption rate of photons with given (k_x,k_y) is

$$P^{\rm abs}_{(k_x,k_y)}=\int_0^{+\infty}dW^{\rm abs}(\omega,k_x,k_y)\frac{1}{e^{2\pi\omega/a}-1}\,. \qquad (22)$$

Taking the limit $E\to 0$ after performing the integration in (22) yields

$$P^{\rm abs}_{(k_x,k_y)}dk_x dk_y=\frac{q^2}{8\pi^3 a}|K_1(k_\perp/a)|^2 dk_x dk_y\,. \qquad (23)$$

Similarly, the total emission rate with a given (k_x,k_y) is

$$P^{\rm em}_{(k_x,k_y)}=\int_0^{+\infty}dW^{\rm em}(\omega,k_x,k_y)\left[\frac{1}{e^{2\pi\omega/a}-1}+1\right], \qquad (24)$$

where the two terms inside the parentheses are related to induced and spontaneous emission, respectively. Since $dW^{\rm em}(\omega,k_x,k_y)=dW^{\rm abs}(\omega,k_x,k_y)$ by unitarity, one can integrate (24) using (21). We note that only the induced emission contributes in the limit $E\to 0$, and therefore we obtain

$$P^{\rm em}_{(k_x,k_y)}=P^{\rm abs}_{(k_x,k_y)} \qquad (25)$$

in this limit. The reason why $P^{\rm em}_{(k_x,k_y)}$ and $P^{\rm abs}_{(k_x,k_y)}$ do not vanish as $E\to 0$, in spite of the overall factor E in (21), is the presence of an infinite number of zero-energy Rindler photons in the thermal bath.

Finally, as mentioned in the beginning, we must add the absorption rate and the emission rate of Rindler photons [see (23) and (25)] to obtain the total response rate:

$$P^{\rm tot}_{(k_x,k_y)}k_x dk_y=\frac{q^2}{4\pi^3 a}|K_1(k_\perp/a)|^2 dk_x dk_y\,. \qquad (26)$$

This result should be compared directly with the calculation in the inertial frame, since the transverse momentum (k_x,k_y) is invariant under the boost connecting the accelerated and the inertial frames.

The amplitude of emission by an accelerated charged point source of a photon with momentum $\mathbf{k}$, and (physical) polarization λ in the Minkowski vacuum is

$$\mathcal{A}^{(\mathbf{k},\lambda)}={}_M\langle \mathbf{k},\lambda|i\int d^4x\, j^\mu(x)\hat{A}_\mu(x)|0\rangle_M\,, \qquad (27)$$

where the subscript M indicates Minkowski states, and j^μ is the current given in (3), which takes the form

$$\begin{aligned} &j^t=qaz\delta(\xi)\delta(x)\delta(y)\,,\\ &j^z=qat\delta(\xi)\delta(x)\delta(y),\ \ j^x=j^y=0 \end{aligned} \qquad (28)$$

in the inertial coordinates. The states $|\mathbf{k},\lambda\rangle$ are normalized according to the conventions of Ref. [12].

One can express the total rate of emission of photons in Minkowski spacetime with transverse momentum (k_x,k_y), but with arbitrary k_z, divided by total proper time T as

$$P^{\rm tot}_{(k_x,k_y)}=\sum_{\lambda=1}^{2}\int_{-\infty}^{+\infty}d\tilde{k}_z|\mathcal{A}^{(\mathbf{k},\lambda)}|^2/T\,, \qquad (29)$$

where $d\tilde{k}_z=dk_z/[2(2\pi)^3k_0]$, $k_0\equiv(k_z^2+k_\perp^2)^{1/2}$. Substituting (27) in (29), we obtain

$$P^{\rm tot}_{(k_x,k_y)}=-\frac{1}{T}\int d\tilde{k}_z\int d^4x\int d^4x' j^\mu(x)j_\mu(x')\exp[i\omega(t-t')-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')]\,, \qquad (30)$$

where j^μ is given in (28), and we have used the standard expansion of the operator $\hat{A}_\mu(x)$ (see Ref. [12]). Finally we evaluate (30), turning off the interaction at $t=\pm\infty$ by letting $\tau(t)\to\tau(t)+i\epsilon$ and $\tau(t')\to\tau(t')-i\epsilon$, where $\tau(t)$ is the proper time of the charged source, and ϵ is an infinitesimal positive real number. The result thus obtained is

$$P^{\rm tot}_{(k_x,k_y)}dk_x dk_y=\frac{q^2}{4\pi^3 a}|K_1(k_\perp/a)|^2 dk_x dk_y\,, \qquad (31)$$

which coincides with (26).

The agreement between the results (26) and (31) indicates that the ordinary QED bremsstrahlung can be interpreted by an observer coaccelerated with the charge as the emission (absorption) of zero-energy Rindler photons into (from) the Rindler thermal bath corresponding to the Minkowski vacuum.

The issue of the detectability of these zero-energy Rindler photons by Rindler observers is not trivial since they carry nonzero momentum, and will be carefully analyzed in a detailed version of this work. However, the fact that the number of zero-energy Rindler photons in the thermal bath diverges and the equality of the emission and absorption rates are indications that a Rindler observer will not be able to distinguish the emitted photons from the ones already present in the thermal bath. This view is in agreement with the classical electrodynamics analysis by Boulware [5]. Finally, we think that this result can be generalized for arbitrary spaces with a Killing horizon. This and related questions are currently under investigation.

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