PHYSICAL REVIEW D
PARTICLES AND FIELDS

THIRD SERIES, VOLUME 45, NUMBER 9 — 1 MAY 1992

RAPID COMMUNICATIONS

Rapid Communications are intended for important new results which deserve accelerated publication, and are therefore given priority in editorial processing and production. A Rapid Communication in **Physical Review D** *should be no longer than five printed pages and must be accompanied by an abstract. Page proofs are sent to authors, but because of the accelerated schedule, publication is generally not delayed for receipt of corrections unless requested by the author.*

Theoretical prediction for $D^* \to D$ decays

Lisa Angelos and G. Peter Lepage
Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853
(Received 3 February 1992)

We have predicted that $B(D^{*+} \to D^+\gamma)$ is between 0.4% and 2% using data from the similar decays of D^{*0} and K^* mesons. Our calculation is based on an effective field theory with expansions in external momentum and heavy-quark mass. We also have calculated the D^{*0} branching fractions and found reasonable agreement between theory and experiment. Our calculation suggests that nonleading terms in the quark-mass expansion may still be quite important for both s and c quarks.

PACS number(s): 13.25.+m, 11.30−Hv, 13.40−Hq

I. INTRODUCTION

The 1990 Particle Data Group compilation lists $B(D^{*+} \to D\gamma) = (18 \pm 4)\%$ [1], suggesting that this decay mode gives a way to reconstruct D^* mesons (which might then be used to study $B \to D^* l\nu$). However, recent CLEO II measurements of D^* branching fractions [2], along with several theoretical calculations [3], cast doubt on this value. In this paper we present a new theoretical analysis of D^* branching fractions based on an extrapolation from K^* data.

The K^* and D^* decays are spin-flip decays of mesons with one heavy and one light quark. Each has two decay modes, $D^*(K^*) \to D(K)\pi$ and $D^*(K^*) \to D(K)\gamma$, with a final pion momentum of about 40 MeV (290 MeV) and a final photon momentum of about 140 MeV (310 MeV). Since these momenta are small relative to typical scales of hadron structure (of order m_ρ, as in the proton form factor), we expect that these mesons can be described accurately as pointlike particles. That is, these decay rates can be written in terms of simple meson interactions with coupling constants. Furthermore, using the ideas of heavy-quark symmetry, we can extrapolate from K^* data to find the coupling constants for D^* decays. Here we combine these two ideas to create a simple formalism for relating the various decay rates of K^* and D^* mesons.

Making use of the measurement of D^{*0} branching fractions, a calculation of the D^{*+} branching fractions reduces to finding the ratio $\Gamma(D^{*0} \to D^0\gamma)/\Gamma(D^{*+} \to D^+\gamma)$. This ratio often is given in terms of the meson constituents' magnetic moments [3,4], which usually have been found by requiring, within some model, consistency among many additional measurements [4]. Given the model, uncertainty comes from the range of magnetic-moment values consistent with experimental data. In our analysis, we avoid much of the model dependence by focusing only on the decays of D^* and K^* mesons. Our uncertainty comes from higher-order terms of expansions in external momentum and inverse heavy-quark mass.

We will calculate $B(D^{*+} \to D^+\gamma)$ using measurements of D^{*0} branching fractions and K^* decay rates along with an effective field theory that describes the interactions of photons with these mesons. Then we will extend the formalism to include pions. This allows us to calculate D^{*0} branching fractions and discuss, in terms of our two expansions, the level of agreement between theory and experiment.

II. D^{*+} BRANCHING FRACTIONS

First we turn our attention to the D^{*+} branching fractions. By manipulating $(B^0_\pi/\Gamma^0_\pi)(\Gamma^0_\gamma/B^0_\gamma) = 1 =$

$(B_\pi^+/\Gamma_\pi^+)(\Gamma_\gamma^+/B_\gamma^+)$ we can get the useful expression

$$B(D^{*+} \to D^+\gamma) = B_\gamma^+ = \frac{1}{1+(B_\pi^0/B_\gamma^0)(\Gamma_\pi^+/\Gamma_\pi^0)(\Gamma_\gamma^0/\Gamma_\gamma^+)} \quad (1)$$

which gives the D^{*+} branching fractions in terms of three simple ratios. [Here Γ_π^+ represents $\Gamma(D^{*+} \to D\pi)$, Γ_γ^0 represents $\Gamma(D^{*0} \to D\gamma)$, and so on.] The D^{*0} branching fractions appear explicitly, so this experimental information is incorporated in a straightforward way. The rates for $D^* \to D\pi$ appear as a ratio so the overall size, which can be quite model dependent, cancels. The ratio $\Gamma_\pi^+/\Gamma_\pi^0$ is given very accurately[1] (independent of model) by $\Gamma_\pi \propto C^2p^3$, where C is a Clebsch-Gordan coefficient and p is the pion momentum. The other quantity required to calculate B_γ^+ is $\Gamma_\gamma^+/\Gamma_\gamma^0$, the ratio of the rates for $D^* \to D\gamma$.

The lowest-order (in external momentum) interaction that contributes to $D^* \to D\gamma$ and respects Lorentz, parity, and gauge symmetry is

$$\frac{g_D e}{2}\epsilon_{\mu\nu\rho\sigma}\Psi_D^\dagger F^{\rho\sigma}(D^\nu\Psi_{D^*}^\mu), \quad (2)$$

where $\Psi_{D^*}^\mu$ and Ψ_D are the D^* and D fields, $F^{\mu\nu}$ is the QED field and $D^\mu = i\partial^\mu - eA^\mu$ is the gauge-covariant derivative. This gives the decay rate $\Gamma(D^* \to D\gamma) = \alpha g_D^2 k^3/3$. The lowest-order contribution to $K^* \to K\gamma$ comes from an interaction of the same form, so $\Gamma(K^* \to K\gamma) = \alpha g_K^2 k^3/3$.

The coupling constants g_D and g_K depend on the meson structure. In the limit of an infinitely heavy quark, we can think of the heavy quark as a static source of a gluon field that binds the light quark. In this limit, the decay rate is independent of the heavy-quark mass, aside from kinematic effects due to the mass dependence of the pseudoscalar-vector splitting. Thinking of K mesons and D mesons as bound systems of an infinitely heavy quark and a light-quark therefore implies $g_D = g_K = e_q A$, where e_q is the light-quark charge and A is a spin-flip amplitude for a light quark bound by the gluon field of an infinitely heavy quark.

Unfortunately, in this simple limit the model fails to predict the ratio $\Gamma(K^{*+} \to K^+\gamma)/\Gamma(K^{*0} \to K^0\gamma)$. This indicates an important contribution from the heavy quark. To model this we use a coupling constant of the form

$$g = e_q A + e_Q B/m_Q, \quad (3)$$

where the first term is an amplitude for a light-quark spin flip and the second term, which depends on the effective heavy-quark mass m_Q is an amplitude for a heavy-quark spin flip. The light-quark amplitude also has a $1/m_Q$ contribution due to binding, but we ignore this.

Taking the coupling constant of the form $g_K = e_q A - \overline{e_s}B/m_s$, we can find the two amplitudes from the decay rates for $K^{*+} \to K^+\gamma$ and $K^{*0} \to K^0\gamma$. This gives $A^{-1} = 480$ MeV and $m_s B^{-1} = 600$ MeV. Then for D^* decays we expect

$$\Gamma_\gamma = \frac{\alpha}{3}\left(\frac{\overline{e_q}}{480\text{ MeV}} - \frac{m_s}{m_c}\frac{e_Q}{600\text{ MeV}}\right)^2 k^3. \quad (4)$$

Notice that the heavy-quark and light-quark amplitudes have opposite signs in the charged-meson decay so these rates will be smaller. For the D^{*+} decay this cancellation is emphasized since the larger heavy-quark charge compensates for the smaller heavy-quark amplitude. This makes $\Gamma_\gamma^0/\Gamma_\gamma^+$ large as well as quite sensitive to the precise definition of m_s/m_c.

There are a variety of ways one might define the effective quark mass m_Q. These include the pseudoscalar-meson mass, the vector-meson mass, averages of these two masses, the heavy-quark mass assignment of the quark model, and a mass proportional to the inverse of the vector-pseudoscalar mass splitting. All of these are the same in the limit of infinite heavy-quark mass, but for K and, to a lesser extent, D mesons there is some variation. With the above m_Q's we find $\Gamma_\gamma^0/\Gamma_\gamma^+$ ranging between 18 and 92. Taking $B_\pi^0 = 55\%$ and $\Gamma_\pi^+/\Gamma_\pi^0 = 2.20$, we then calculate B_γ^+ between 2% and 0.4%.

Calculations involving quark magnetic moments [3, 4] use

$$\frac{\Gamma_\gamma^0}{\Gamma_\gamma^+} = \left(\frac{\mu_u + \mu_c}{\mu_d + \mu_c}\right)^2\left(\frac{k_0}{k_+}\right)^3. \quad (5)$$

Magnetic moments consistent with data such as baryon magnetic moments and meson masses [4] set this ratio between approximately 8 and 17 (B_γ^+ between approximately 4% and 2%). This range is due to uncertainty in assigning quark magnetic moments, but it does not include uncertainties about the accuracy of the quark model itself. Our wider range for $\Gamma_\gamma^0/\Gamma_\gamma^+$ reflects our more model-independent view of this ratio.

III. D^{*0} DECAYS

We can extend this framework to find the D^{*0} branching fractions. We use our expression for $\Gamma(D^* \to D\gamma)$ from the previous section along with an expression for $\Gamma(D^* \to D\pi)$ that we will find in a similar way. The lowest-order (in external momentum) interaction that contributes to $D^* \to D\pi$ and respects Lorentz, parity, and isospin symmetry is

$$m_Q h \Psi_{D^*}^{\dagger\mu}(\partial_\mu \boldsymbol{\tau}\cdot\boldsymbol{\pi})\Psi_D, \quad (6)$$

[1] Corrections include contributions from isospin breaking, form factors, and radiative corrections. Mass differences within isospin multiplets suggest that isospin breaking is only about 1%. Form factors are almost certainly even less important since the decay momentum changes only from 37 to 48 MeV; corrections here might be of order 0.1%. Radiative corrections also are unimportant. Photon emission is strongly suppressed by a factor $\alpha\beta_\pi^2$; we estimate corrections of order 0.1%. The final-state Coulomb interaction can be large sometimes, but not here since $D^* \to D\pi$ does not have two charged final-state particles. For comparison, experimental uncertainties in p^3 lead to a 5–10% uncertainty in $\Gamma_\pi^+/\Gamma_\pi^0$.

where Ψ_{D^*} and Ψ_D are the D^* and D isodoublet fields and $\boldsymbol{\tau}$ is the Pauli isospin matrices. This gives the decay rate

$$\Gamma_\pi = \frac{1}{8\pi}\left(\frac{m_Q}{M_*}\right)^2 h^2 C^2 \mid \mathbf{p}_\pi \mid^3, \tag{7}$$

where C is a Clebsch-Gordan coefficient. We have written the coupling constant as $m_Q h$ since the decay rate should, aside from kinematic factors, be independent of heavy-quark mass. Again we use a variety of definitions for the effective heavy-quark mass m_Q.

If we assume no further heavy-quark dependence (that there is only a light-quark spin flip), we can fix h from the measured rate of $K^* \to K\pi$. We find the D^* decay rate

$$\Gamma_\pi = \frac{1}{8\pi}\left(\frac{1}{140\text{ MeV}}\frac{M_{K^*}}{m_s}\frac{m_c}{M_{D^*}}\right)^2 C^2 \mid \mathbf{p}_\pi \mid^3 . \tag{8}$$

The rates Γ_γ and Γ_π as determined from three K^* decay measurements give, using a range of interpretations of the effective heavy-quark mass m_Q, $B(D^{*0} \to D^0\pi^0)$ between 72 and 90%. It is measured to be $(55 \pm 6)\%$.

The discrepancy between our model and the data is not particularly large. However, it is important to consider how the model might be modified to remove the discrepancy.

The effects of interactions higher order in external momentum would appear as meson form factors. A typical form factor has the momentum dependence

$$\frac{1}{1+\mathbf{p}^2/m_\rho^2} \approx 1- \mid \mathbf{p} \mid^2 / m_\rho^2. \tag{9}$$

The external momenta in $K^* \to K\pi$ and $K^* \to K\gamma$ are about the same so the form factor should not significantly influence the coupling-constant ratio h/g determined by K^* decays. Since $D^* \to D\pi$ is close to threshold, form factors can affect $D^* \to D\pi$ and $D^* \to D\gamma$ differently, but the effect is much too small (e.g., $p_\gamma^2/m_\rho^2 \approx 3\%$) and in the wrong direction to explain the discrepancy between our calculation and the measurement of $B(D^{*0} \to D\pi)$.

The importance of a heavy-quark amplitude in Γ_γ might indicate that Γ_π has a non-negligible dependence on the effective heavy-quark mass. We have assumed that the combination $h^{-1}M_*/m_Q$ is the same for D^* and K^* mesons; however, fitting to K^* data we find that $h_K^{-1}M_{K^*}/m_s = 140$ MeV while fitting to D^{*0} data gives $h_D^{-1}M_{D^{*0}}/m_c \approx 210$ MeV. The nonasymptotic contributions are significant, but not surprisingly so in light of the large corrections in photon decays.

Other calculations of D^{*0} decays [3] use Γ_γ^0 as determined by quark magnetic moments: $\Gamma_\gamma^0 = 4\alpha(\mu_u + \mu_c)^2 k_0^3/3 \approx 35$ keV. To find branching fractions, this is combined with a calculation of $\Gamma_\pi^0 = A^2C^2p^3$, where C is a Clebsch-Gordan coefficient and p is the pion momentum in the D^* rest frame. The overall size, A^2, is quite model dependent. We find A^{-1} between 400 MeV and 700 MeV. The results of Eichten *et al.* [3] are typical of several other analyses. Their calculation, like ours, makes use of K^* data, but, due to the nonrelativistic analysis used to model the coupling, an extra factor of $\sqrt{2E_\pi E_M}$ multiplies the coupling constant h used in our calculation. From this they find $A^{-1} \approx 850$ MeV, which, combined with $\Gamma_\gamma^0 \approx 35$ keV, gives $B_\pi^0 \approx 53\%$

IV. CONCLUSION

In this paper, we have predicted that $B(D^{*+} \to D^+\gamma)$ is between 0.4% and 2%, that is, $\Gamma(D^{*0} \to D^0\gamma)/\Gamma(D^{*+} \to D^+\gamma)$ between 18 and 92, using data from the similar decays of D^{*0} and K^* mesons. This range for $\Gamma_\gamma^0/\Gamma_\gamma^+$ is wider and slightly higher than that given by quark magnetic moment calculations. Our calculation is based on an effective field theory with expansions in external momentum and heavy-quark mass. We also have calculated the D^{*0} branching fractions and found reasonable agreement between theory and experiment, although our calculation would favor a higher branching fraction $B(D^{*0} \to D^0\pi^0)$.

We believe that for these decays it works well to treat mesons as pointlike particles. However, a striking aspect of our analysis is the strong dependence on the effective heavy-quark mass in the decays of K^* and D^* mesons. Assuming no heavy-quark dependence in the photon amplitude gives poor results for K^* decays and also is incompatible with recent D^* data. The $1/m_Q$ correction that gives agreement between theory and experiment is of order one for $K^* \to K\gamma$ and 30 to 100% for $D^* \to D\gamma$. The same seems to be true for pion decays, assuming that the branching fraction $B(D^{*0} \to D^0\pi^0)$ remains as low as 55%. Perhaps this analysis provides an indication of the magnitude of such corrections in the more general context of D physics.

ACKNOWLEDGMENTS

We thank D. Cassel, P. Drell, B. Geiser, and B. Gittelman for many useful discussions. This work was supported in part by a grant from the National Science Foundation.

[1] Unless otherwise specified, all properties of observed particles are quoted from Particle Data Group, J.J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).

[2] P. Drell, Cornell Report No. CLNS 92/1143 (unpublished).

[3] For example, S. Ono, Phys. Rev. Lett. **37**, 655 (1976); E. Eichten *et al.*, Phys. Rev. D **21**, 203 (1980); R.L. Thews and A.N. Kamal, *ibid.* **32**, 810 (1985).

[4] For example, L. Brekke and J.L. Rosner, Comments Nucl. Part. Phys. **18**, 83 (1988); G.A. Miller and P. Singer, Phys. Rev. D **37**, 2564 (1988).