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Chiral perturbation theory for hadrons containing a heavy quark

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An effective Lagrangian that describes the low-momentum interactions of mesons containing a heavy quark with the pseudo Goldstone bosons π, K, and η is constructed. It is invariant under both heavy-quark spin symmetry and chiral $SU(3)_L \times SU(3)_R$ symmetry. Implications for semileptonic B and D decays are discussed.

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The interactions of a heavy quark Q (i.e., $m_Q \gg \Lambda_{\rm QCD}$) are simplified by going over to an effective theory where the heavy-quark mass goes to infinity with its four-velocity fixed [1–13]. The effective theory has new symmetries that are not manifest in the full theory of QCD [1,2]. For N heavy quarks the effective theory has an $SU(2N)$ spin-flavor symmetry because the interactions of heavy quarks are independent of their mass and spin. Heavy-quark symmetry has been used to predict many properties of hadrons containing a single heavy quark. For example, it implies that all the form factors for $B \rightarrow De\bar{\nu}_e$ and $B \rightarrow D^* e\bar{\nu}_e$ can be expressed in terms of a single universal function [1] and that the value of this function at zero recoil (where in the rest frame of the B the D and D^* are at rest) is known [1,10,11]. These results are expected to play an important role in determining the value of the Cabibbo-Kobayashi-Maskawa matrix element V_{cb}.

The strong interactions have an approximate $SU(3)_L \times SU(3)_R$ chiral symmetry that is spontaneously broken to the vector $SU(3)_V$ subgroup. This symmetry arises because the light u, d, and s quarks have masses that are small compared with the QCD scale $\Lambda_{\rm QCD}$. Associated with the spontaneous breaking of the approximate chiral symmetry are the light pseudoscalar mesons π, K, and η. The interactions of these pseudo Goldstone bosons, at low momentum, are described by an effective Lagrangian that contains the most general couplings consistent with chiral symmetry. Much of the predictive power arises because at low momentum terms with the fewest number of derivatives and insertions of the $(\bar{3}_L, 3_R) + (3_L, \bar{3}_R)$ symmetry-breaking light-quark mass matrix dominate.

The pseudo Goldstone bosons are incorporated in a 3×3 unitary matrix

$$\Sigma = \exp\left(\frac{2iM}{f}\right), \tag{1a}$$

where

$$M = \begin{pmatrix} (\frac{1}{2})^{1/2}\pi^0 + (\frac{1}{6})^{1/2}\eta & \pi^+ & K^+ \\ \pi^- & -(\frac{1}{2})^{1/2}\pi^0 + (\frac{1}{6})^{1/2}\eta & K^0 \\ K^- & \bar{K}^0 & -(\frac{2}{3})^{1/2}\eta \end{pmatrix}. \tag{1b}$$

Here f is the pseudoscalar pion (or kaon) decay constant, $f \simeq 135$ MeV. Under a chiral $SU(3)_L \times SU(3)_R$ transformation

$$\Sigma \rightarrow L\Sigma R^\dagger \tag{2}$$

where $L \in SU(3)_L$ and $R \in SU(3)_R$. It is convenient when discussing the interactions of the π, K, η with other fields to introduce

$$\xi = \sqrt{\Sigma}. \tag{3}$$

Under a chiral $SU(3)_L \times SU(3)_R$ transformation,

$$\xi \rightarrow L\xi U^\dagger = U\xi R^\dagger, \tag{4}$$

where in general the special unitary matrix U is a complicated nonlinear function of L, R and the pseudo-Goldstone-boson fields. [However, for an unbroken $SU(3)_V$ transformation, $V = L = R$, the matrix U is equal to V.]

In this paper the low-momentum interactions of the π, K, and η, with the ground-state heavy mesons with $Q\bar{q}^a$ flavor quantum numbers are studied. (Here $a = 1,2,3$ and $q^1 = u$, $q^2 = d$, $q^3 = s$.) These heavy mesons have $s_l^{\pi_l} = \frac{1}{2}^-$, for the spin-parity of the light degrees of freedom. Combining the spin of the light degrees of freedom with the spin of the heavy quark gives (in the $m_Q \rightarrow \infty$ limit) three degenerate doublets consisting of an $SU(3)_V$ antitriplet of spin-zero mesons that are denoted by P_a and an $SU(3)_V$ antitriplet of spin-one mesons that are denoted by P_a^*. In the case $Q = c$ these are the spin-zero D^0, D^+, D_s mesons and the spin-one D^{*0}, D^{*+}, D_s^* mesons. It is important that the effective Lagrangian that describes the low-momentum interactions of the heavy P_a and P_a^* mesons with the pseudo Goldstone bosons be invariant under heavy-quark symmetry. For example, even if one is interested in processes involving only a real D meson (e.g., D semileptonic decay) the D^* meson will occur as a virtual particle in pole-type Feynman diagrams. The heavy-quark spin symmetry relates the couplings of the D^* to those of the D.

It is convenient to combine the P_a and $P^*_{a\mu}$ meson fields into a 4×4 matrix H_a, given by [14,15]

$$H_a = \frac{1+\not{v}}{2}(P^*_{a\mu}\gamma^\mu - P_a\gamma_5)\,. \tag{5}$$

Here $P^*_{a\mu}$ is the field operator that destroys a P^*_a meson of four-velocity v. It satisfies the constraint

$$v^\mu P^*_{a\mu} = 0\,. \tag{6}$$

The field operator P_a destroys a P_a meson of four-velocity v. Pair creation does not occur in the effective theory (where $m_Q \to \infty$) so these field operators do not create the corresponding antiparticles. Under $SU(3)_L \times SU(3)_R$ chiral symmetry,

$$H_a \to H_b U^\dagger_{ba}\,, \tag{7}$$

where the repeated index b is summed over 1, 2, 3. Here U is the 3×3 matrix introduced in Eq. (4). Under the heavy-quark spin symmetry group [i.e., $SU(2)_v$],

$$H_a \to S H_a\,, \tag{8}$$

where $S \in SU(2)_v$ (recall that [2] $[\not{v}, S] = 0$). Finally, under Lorentz transformations,

$$H_a \to D(\Lambda) H_a D(\Lambda)^{-1}\,, \tag{9}$$

where $D(\Lambda)$ is an element of the 4×4 matrix representation of the Lorentz group. It is convenient to introduce the matrix

$$\bar{H}_a = \gamma^0 H^\dagger_a \gamma^0\,. \tag{10}$$

Explicitly,

$$\bar{H}_a = (P^{*\dagger}_{a\mu}\gamma^\mu + P^\dagger_a\gamma_5)\frac{1+\not{v}}{2}\,. \tag{11}$$

For $\bar{H}_a$ the transformation laws corresponding to those in Eqs. (7), (8), and (9), respectively, become $\bar{H}_a \to U_{ab}\bar{H}_b$, $\bar{H}_a \to \bar{H}_a S^{-1}$, and $\bar{H}_a \to D(\Lambda)\bar{H}_a D(\Lambda)^{-1}$.

The strong interactions of the pseudo Goldstone bosons with the heavy mesons are described by the effective Lagrangian density

$$\mathcal{L} = -i\,\mathrm{Tr}\bar{H}_a v_\mu \partial^\mu H_a + \tfrac{1}{2} i\,\mathrm{Tr}\bar{H}_a H_b v^\mu (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)_{ba} + \tfrac{1}{2} i g\,\mathrm{Tr}\bar{H}_a H_b \gamma_\nu \gamma_5 (\xi^\dagger \partial^\nu \xi - \xi \partial^\nu \xi^\dagger)_{ba} + \cdots\,, \tag{12}$$

where the ellipsis denotes terms with more derivatives and the repeated indices a, b are summed over 1, 2, 3. Derivatives on the heavy-meson fields give factors of the small residual momentum [2]. The Lagrangian density in Eq. (12) is the most general one that is invariant under $SU(3)_L \times SU(3)_R$ chiral symmetry, Lorentz transformations, $SU(2)_v$ heavy-quark spin symmetry and parity. [Note that $\mathrm{Tr}\bar{H}_a H_b \gamma_5 v_\nu (\xi^\dagger \partial^\nu \xi - \xi \partial^\nu \xi^\dagger)_{ba}$ vanishes.] Factors of $\sqrt{m_P}$ and $\sqrt{m_{P^*}}$ have been absorbed into the P_a and $P^*_{a\mu}$ fields so that the Lagrangian is independent of the heavy-quark mass. Consequently, the heavy-meson fields have dimension $\frac{3}{2}$.

Expanding ξ in powers of the meson fields and taking the traces yields Feynman rules for the interactions of the π, K, and η with the heavy mesons. The P_a and P^*_a propagators that follow from (12) are $i\delta_{ab}/2v\cdot k$ and $-i\delta_{ab}(g_{\mu\nu} - v_\mu v_\nu)/2v\cdot k$, respectively. In the case $Q = c$ the $D^* \to D\pi$ decay width is determined by g:

$$\Gamma(D^{*+} \to D^0\pi^+) = \left(\frac{1}{6\pi}\right)\frac{g^2}{f^2}|\mathbf{p}_\pi|^3\,. \tag{13}$$

Unfortunately, because of the small value of the pion three-momentum, $|\mathbf{p}_\pi| = 40$ MeV, the D^* lifetime is so long that it is very hard to measure. [At present there is an experimental limit $\Gamma(D^{*+} \to D^0\pi^+) < 0.6$ MeV, which gives the bound $g^2 < 3$.] The coupling g is independent of the heavy-quark mass.

The light-quark mass terms in the QCD Lagrangian transform as $(\bar{3}_L, 3_R) + (3_L, \bar{3}_R)$ under chiral $SU(3)_L \times SU(3)_R$. To incorporate the leading effects of explicit chiral-symmetry breaking from light-quark masses,

$$\delta\mathcal{L}^{(1)} = \lambda_1 \mathrm{Tr}\bar{H}_b H_a (\xi m_q \xi + \xi^\dagger m_q \xi^\dagger)_{ab} + \lambda_1' \mathrm{Tr}\bar{H}_a H_a (m_q \Sigma + \Sigma^\dagger m_q)_{bb} + \cdots \tag{14}$$

is added to the Lagrangian density in Eq. (12). In Eq. (14) the ellipsis denotes terms with derivatives and

$$m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \tag{15}$$

is the light-quark mass matrix. It is also possible to include deviations from the $m_Q \to \infty$ limit that violate heavy-quark spin symmetry. At order $\Lambda_{\rm QCD}/m_Q$ the heavy-quark spin symmetry is broken by the color-magnetic-moment operator [6]. To include its effects,

$$\delta\mathcal{L}^{(2)} = \frac{\lambda_2}{m_Q}\mathrm{Tr}\bar{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu} + \cdots \tag{16}$$

is added to the Lagrangian density in Eq. (12). (Note that λ_1 and λ_1' are dimensionless but λ_2 has dimension 2. The coupling constants λ_1 and λ_1' are independent of the heavy-quark mass, while λ_2 has a calculable logarithmic dependence on the heavy-quark mass [6] from perturbative QCD.) In Eq. (16) the ellipsis denotes terms with derivatives. The leading term in Eq. (16) does not involve the pseudo Goldstone bosons. Its only effect is to change, respectively, the P_a and P^*_a propagators [16] to $i\delta_{ab}/2(v\cdot k + \frac{3}{4}\Delta)$ and $-i\delta_{ab}(g_{\mu\nu} - v_\mu v_\nu)/2(v\cdot k - \frac{1}{4}\Delta)$, where $\Delta = m_{P^*} - m_P$. (In terms of the coupling constant λ_2, $\Delta = -2\lambda_2/m_Q$.) Now in the rest frame $v = (1,\mathbf{0})$, an on-shell P_a meson has the residual energy $-\frac{3}{4}\Delta$ and an on-shell P^*_a meson has the residual energy $\frac{1}{4}\Delta$. It is convenient when dealing with situations involving a real P_a meson and a virtual P^*_a meson to redefine the heavy meson fields by $\exp(i\frac{3}{4}\Delta v\cdot x)$ so that the P_a and P^*_a propagators become $i\delta_{ab}/2v\cdot k$ and $-i\delta_{ab}(g_{\mu\nu} - v_\mu v_\nu)/2(v\cdot k - \Delta)$, respectively. For $Q = c$ and $Q = b$, the mass difference Δ is of the order of the pion mass and so in power counting it is considered to be as important as one derivative.

There are many applications of chiral perturbation theory for hadrons containing a single heavy quark. For weak semileptonic B decay to noncharmed final states with low momentum, matrix elements of the operator $L_a^\nu = \bar{q}^a \gamma^\nu (1-\gamma_5) Q$ are needed (with $Q=b$ and $a=1$). This operator transforms under chiral $SU(3)_L \times SU(3)_R$ as $(\bar{3}_L, 1_R)$ and its hadronic matrix elements are given by those of

$$L_a^\nu = \left[\frac{i\alpha}{2}\right] \mathrm{Tr}\gamma^\nu(1-\gamma_5) H_b \xi_{ba}^\dagger + \cdots , \qquad (17)$$

where the ellipsis denotes terms with derivatives, factors of the light quark mass matrix m_q, or factors of $1/m_Q$. The constant α is related to the B-meson-decay constant which defined by $\langle 0|\bar{u}\gamma^\nu\gamma_5 b|B^-(v)\rangle = i f_B p_B^\nu$, where $p_B = m_B v$. Taking the $B^- \to$ vacuum matrix element of the left-handed current in Eq. (17) and noting that the vector part does not contribute (because of parity invariance of the strong interactions) gives $\alpha = f_B\sqrt{m_B}$. The parameter α has logarithmic dependence on the heavy-quark mass from perturbative QCD [7,8]. For $\bar{B}^0 \to \pi^+ e \bar{\nu}_e$ the needed matrix element is

$$\langle \pi^+(p_\pi)|\bar{u}\gamma^\nu(1-\gamma_5)b|\bar{B}^0(v)\rangle = f_+(p_B+p_\pi)^\nu + f_-(p_B-p_\pi)^\nu . \qquad (18)$$

In chiral perturbation theory this matrix element of L_1^ν follows from the Feynman diagrams in Fig. 1. They yield

$$f_+ + f_- = -(f_B/f)[1 - g v\cdot p_\pi/(v\cdot p_\pi + \Delta)] , \qquad (19)$$

and

$$f_+ - f_- = -g f_B m_B/[f(v\cdot p_\pi + \Delta)] , \qquad (20)$$

for the form factors $f_\pm$ at small $v\cdot p_\pi$. Experimentally, $\Delta = m_{B^*} - m_B \simeq 50$ MeV. Note that $f_+ - f_-$ is enhanced

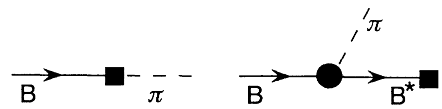

FIG. 1. Feynman diagrams for the matrix element in Eq. (18). The solid square denotes the left-handed current and the solid circle is the $BB^*\pi$ coupling that follows from the term proportional to g in Eq. (12).

[17] by (m_B/f) over $f_+ + f_-$, so $f_+ \simeq -f_-$ and

$$f_+ \simeq -g f_B m_B/[2f(v\cdot p_\pi + \Delta)] . \qquad (21)$$

This is the same result as Ref. [18] which previously considered the influence of the B^* pole on $B \to \pi e \bar{\nu}_e$, and found that it dominates f_+ for small $v\cdot p_\pi$. Of course, with the methods developed here, multipion final states can also be considered. Equations (19) and (20) hold for $D \to \pi \bar{e} \nu_e$ if the substitutions $f_B \to f_D$, $m_B \to m_D$, and $\Delta = m_{D^*} - m_D$ are made. In principle they can also be applied to $D \to K \bar{e} \nu_e$. In that case $\Delta = m_{D_s^*} - m_D$. However, it is important to remember that the kaon mass is not very small so in the $D \to K \bar{e} \nu_e$ case significant corrections to Eqs. (19) and (20) are expected from terms in Eq. (17) with one derivative.

Semileptonic decays $B \to D X e \bar{\nu}_e$ and $B \to D^* X e \bar{\nu}_e$, where X is a low-momentum state of one or more pseudo Goldstone bosons, can also be studied using chiral perturbation theory. Hadronic matrix elements of $\bar{c}\gamma_\mu(1-\gamma_5)b$ are needed for these decays. This operator is a singlet under $SU(3)_L \times SU(3)_R$ and in chiral perturbation theory, its hadronic matrix elements are given by those of

$$\bar{c}\gamma_\mu(1-\gamma_5)b = -\beta(v\cdot v')\mathrm{Tr}\bar{H}_a^{(c)}(v')\gamma_\mu(1-\gamma_5)H_a^{(b)}(v) + \cdots , \qquad (22)$$

where the ellipsis denotes terms with derivatives, factors of m_q, or factors of $1/m_Q$. From the $B \to D$ (or D^*) transition it follows that $\beta(v\cdot v')$ is the universal function relevant for semileptonic $B \to D e \bar{\nu}_e$ and $B \to D^* e \bar{\nu}_e$ decay [1]. It has a logarithmic dependence on the heavy quark masses from perturbative QCD [7–9]. The leading term in Eq. (22) is independent of the π, K, and η fields. Consequently, in chiral perturbation theory, it is pole-type diagrams that dominate the amplitudes for semileptonic $b \to c$ transitions that have low-momentum pseudo Goldstone bosons in the final state. These pole-type diagrams are determined by the function $\beta(v\cdot v')$ and the coupling g. An interesting aspect of the application of chiral perturbation theory to these decays is that its validity is not restricted to $v\cdot v'$ very near unity. For example, if $X=\pi$, then chiral perturbation theory is valid for $v\cdot p_\pi$ and $v'\cdot p_\pi$ small. This can occur when $v\cdot v'$ is not particularly close to one.

A more complete discussion of the application of the methods developed in this paper to semileptonic B and D decays will be presented in a further publication.

Note added. After completion of this work I received a paper by A. Burdman and J. Donoghue [Report No. UMHEP-365 (unpublished)]. It contains similar work to that presented here.

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[15] This is a shorthand notation. In cases where the type of heavy quark Q and its four-velocity v are important the 4×4 matrix is denoted by $H_a^{(Q)}(v)$.

[16] The terms in Eq. (14) alter the P_a^* and P_a masses, giving a common mass shift that depends on a.

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