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Evanescent black holes

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A renormalizable theory of quantum gravity coupled to a dilaton and conformal matter in two spacetime dimensions is analyzed. The theory is shown to be exactly solvable classically. Included among the exact classical solutions are configurations describing the formation of a black hole by collapsing matter. The problem of Hawking radiation and back reaction of the metric is analyzed to leading order in a $1/N$ expansion, where N is the number of matter fields. The results suggest that the collapsing matter radiates away all of its energy before an event horizon has a chance to form, and black holes thereby disappear from the quantum-mechanical spectrum. It is argued that the matter asymptotically approaches a zero-energy "bound state" which can carry global quantum numbers and that a unitary S matrix including such states should exist.

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Following his ground-breaking work [1] on black-hole evaporation, Hawking [2] argued that the process of formation and subsequent evaporation of a black hole is not governed by the usual laws of quantum mechanics: Rather, pure states evolve into mixed states [3]. This conjecture is hard to check in detail because of the many degrees of freedom and inherent complexity of the process in four spacetime dimensions. It would be useful to have a toy model in which greater analytic control is possible.

In this paper we investigate such a model. It is a consistent, renormalizable theory of quantum gravity in two spacetime dimensions coupled to conformal matter. It contains black-hole solutions as well as Hawking radiation, and is exactly soluble at the classical level. As we shall see, the theory is just complicated enough to enable one to ask the interesting questions concerning black-hole evaporation, yet simple enough to obtain some answers.

We begin with the action in two spacetime dimensions:

$$S=\frac{1}{2\pi}\int d^2x\sqrt{-g}\{e^{-2\phi}[R+4(\nabla\phi)^2+4\lambda^2]-\tfrac{1}{2}(\nabla f)^2\}\,, \tag{1}$$

where g, ϕ, and f are the metric, dilaton, and matter fields, respectively, and λ^2 is a cosmological constant. This action arises as the effective action describing the radial modes of extremal dilatonic black holes in four or higher dimensions [4–6];[1] it is also closely related to the spacetime action for $c=1$ noncritical strings. However, these connections need not concern us here; the theory defined by the action (1) is of interest in its own right as a renormalizable theory of two-dimensional (2D) "dilaton gravity" coupled to matter.

The quantization of related theories of 2D gravity has been considered in [7]. Gravitational collapse in related theories has been studied in [8]. The black-hole solution of (1) in the absence of matter has appeared previously [9] as a low-energy approximation to an exact solution of string theory.

The classical theory described by (1) is most easily analyzed in the conformal gauge:

$$g_{+-}=-\tfrac{1}{2}e^{2\rho}\,,\quad g_{--}=g_{++}=0\,, \tag{2}$$

where $x^{\pm}=(x^0\pm x^1)$. The metric equations of motion then reduce to

$$\begin{aligned}
T_{++}&=e^{-2\phi}(4\partial_+\rho\partial_+\phi-2\partial_+^2\phi)+\tfrac{1}{2}\partial_+f\partial_+f=0\,,\\
T_{--}&=e^{-2\phi}(4\partial_-\rho\partial_-\phi-2\partial_-^2\phi)+\tfrac{1}{2}\partial_-f\partial_-f=0\,,\\
T_{+-}&=e^{-2\phi}(2\partial_+\partial_-\phi-4\partial_+\phi\partial_-\phi-\lambda^2e^{2\rho})=0\,.
\end{aligned} \tag{3}$$

The dilaton and matter equations are

$$-4\partial_+\partial_-\phi+4\partial_+\phi\partial_-\phi+2\partial_+\partial_-\rho+\lambda^2e^{2\rho}=0\,, \tag{4}$$

$$\partial_+\partial_-f=0\,. \tag{5}$$

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[1]In the context of superstrings f does not have the usual dilaton coupling because it arises from a Ramond-Ramond field.

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The general solution of the dilaton, matter, and T_{+-} equations (which do not involve f) may be expressed in terms of two free fields

$$w = w_+(x^+) + w_-(x^-)\,, \quad u = u_+(x^+) + u_-(x^-) \tag{6}$$

as

$$e^{-2\phi} = u - h_+ h_-\,, \quad e^{-2\rho} = e^{-w}(u - h_+ h_-)\,, \tag{7}$$

where

$$h_\pm(x^\pm) = \lambda \int^{x^\pm} e^{w_\pm}\,. \tag{8}$$

The matter equation, of course, implies

$$f = f_+(x^+) + f_-(x^-)\,. \tag{9}$$

The remaining constraint equations $T_{++} = T_{--} = 0$ may then be solved for u in terms of $f_\pm$ and $w_\pm$. The general solution is

$$u_\pm = \frac{M}{2\lambda} - \frac{1}{2}\int e^{w_\pm} \int e^{-w_\pm} \partial_\pm f \partial_\pm f\,, \tag{10}$$

where M is an integration constant.

We now consider solutions with $f=0$, which implies that one can set $u = M/\lambda$. The conformal gauge leaves the conformal subgroup of diffeomorphisms unfixed. This gauge freedom can be fixed (on shell) by setting $w=0$. The general $f=0$ solution is then

$$e^{-2\phi} = \frac{M}{\lambda} - \lambda^2 x^+ x^- = e^{-2\rho} \tag{11}$$

up to constant translations of x. It is readily seen [6] that for $M \neq 0$ this corresponds to the $r-t$ plane of the higher-dimensional black holes of [4,5] near the extremal limit, or to the two-dimensional black-hole solution of [9], with M the black-hole mass. It is not immediately apparent that the parameter M corresponds to the black-hole mass. This can be verified by a calculation of the Arnowitt-Deser-Misner (ADM) mass for this configuration as described in [9] or by a calculation of the Bondi mass as is done later in this paper. For $M=0$ one can introduce coordinates in which the metric is flat and the dilaton field ϕ is linear in the spatial coordinate. This "linear dilaton" vacuum has appeared in previous studies of lower-dimensional string theories and also corresponds to extremal higher-dimensional black holes.

From the above we may expect that any matter perturbation of the linear dilaton vacuum will result in the formation of a black hole. To see that this is indeed the case consider the example of an f shock wave traveling in the x^- direction with magnitude a described by the stress tensor

$$\tfrac{1}{2}\,\partial_+ f \partial_+ f = a\delta(x^+ - x_0^+)\,. \tag{12}$$

One then finds in the gauge $w=0$ that

$$e^{-2\rho} = e^{-2\phi} = -a(x^+ - x_0^+)\Theta(x^+ - x_0^+) - \lambda^2 x^+ x^-\,. \tag{13}$$

For $x^+ < x_0^+$, this is simply the linear dilaton vacuum,

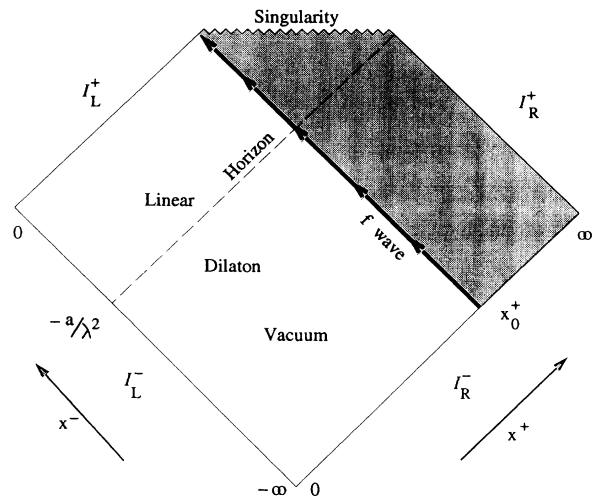

FIG. 1. An incoming f wave in classical dilaton gravity produces a black-hole metric (shaded region) with a horizon and singularity.

while for $x^+ > x_0^+$ it is identical to a black hole of mass $a x_0^+ \lambda$ after shifting x^- by a/λ^2. The two solutions are joined along the f wave. The Penrose diagram for this spacetime is depicted in Fig. 1.

The fact that any f wave, no matter how weak, produces a black hole, of course, implies that weak-field perturbation theory breaks down. The reason for this is simple. From (1) it is evident that the weak-field expansion parameter is proportional to e^ϕ. Equation (13) shows that this parameter becomes arbitrarily large close to $\mathcal{J}_L^\pm$ or to the singularity and that the weak-field expansion diverges in this region.

This has a higher-dimensional interpretation as follows [5]. When (1) is taken as an effective-field theory for higher-dimensional dilatonic black holes, the 2D linear dilaton vacuum corresponds to the infinite throat in the extremal black-hole solutions. The center of the black hole is at $x^+ x^- = 0$. An arbitrarily small infalling matter wave then produces a nonextremal black hole with an event horizon and a singularity.

So far the discussion has been purely classical. As a first step towards including quantum effects, we now compute the Hawking radiation in the fixed background geometry (11). This can be computed exactly for the collapsing f wave because of the elegant relation [10] between Hawking radiation and the trace anomaly for 2D conformal matter coupled to gravity.

The calculation and its physical interpretation is clearest in coordinates where the metric is asymptotically constant on $\mathcal{J}_R^\pm$. We thus set

$$e^{\lambda\sigma^+} = \lambda x^+\,, \quad e^{-\lambda\sigma^-} = -\lambda x^- - a/\lambda\,. \tag{14}$$

This preserves the conformal gauge (2) and gives for the new metric

$$-2g_{+-} = e^{2\rho} = \begin{cases} [1 + (a/\lambda)e^{\lambda\sigma^-}]^{-1} & \text{if } \sigma^+ < \sigma_0^+\,, \\ \{1 + (a/\lambda)\exp[\lambda(\sigma^- - \sigma^+ + \sigma_0^+)]\}^{-1} & \text{if } \sigma^+ > \sigma_0^+\,, \end{cases} \tag{15}$$

with $\lambda x_0^+ = \exp(\lambda\sigma_0^+)$. By the standard one-loop anomaly argument, the trace T^f_{+-} of the stress tensor is proportional to the curvature scalar which is, in these coordinates, just the Laplacian of ρ. The result is[2]

$$\langle T^f_{+-}\rangle = -\tfrac{1}{12}\,\partial_+\partial_-\rho\,. \tag{16}$$

One can then integrate the equations of conservation of T^f to infer the following one-loop contributions to T^f_{++} and T^f_{--}:

$$\begin{aligned}\langle T^f_{++}\rangle &= -\tfrac{1}{12}\,[\partial_+\rho\,\partial_+\rho - \partial_+^2\rho + t_+(\sigma^+)]\,,\\ \langle T^f_{--}\rangle &= -\tfrac{1}{12}\,[\partial_-\rho\,\partial_-\rho - \partial_-^2\rho + t_-(\sigma^-)]\,.\end{aligned} \tag{17}$$

The functions of integration $t_\pm$ must be fixed by boundary conditions. For the collapsing f wave, T^f should vanish identically in the linear dilaton region, and there should be no incoming radiation along $\mathcal{J}_R^-$ except for the classical f wave at σ_0^+. Using the formula for ρ, this implies

$$t_+ = 0,\quad t_- = -\tfrac{1}{4}\lambda^2[1-(1+ae^{\lambda\sigma^-}/\lambda)^{-2}]\,. \tag{18}$$

The stress tensor is now completely determined, and one can read off its values on $\mathcal{J}_R^+$ by taking the limit $\sigma^+ \to \infty$:

$$\begin{aligned}&\langle T^f_{++}\rangle \to 0,\ \langle T^f_{+-}\rangle \to 0\,,\\ &\langle T^f_{--}\rangle \to \frac{\lambda^2}{48}\left[1-\frac{1}{(1+ae^{\lambda\sigma^-}/\lambda)^2}\right].\end{aligned} \tag{19}$$

The limiting value of T^f_{--} is the flux of f-particle energy across $\mathcal{J}_R^+$. In the far past of $\mathcal{J}_R^+$ ($\sigma^- \to -\infty$) this flux vanishes exponentially while, as the horizon is approached, it approaches the constant value $\lambda^2/48$. This is nothing but Hawking radiation. The surprising result that the Hawking radiation rate is asymptotically independent of mass has been found in other studies of two-dimensional gravity.

The total energy lost by the collapsing f wave at some value of retarded time σ^- can be estimated by integrating the outgoing flux along $\mathcal{J}_R^+$ up to σ^-. If the total radiated flux is computed by integrating along all of $\mathcal{J}_R^+$, an infinite answer is obtained, because the outgoing flux approaches a steady state at late retarded times. This is obviously nonsense—the black hole cannot lose more mass than it possesses. This nonsensical answer is, of course, a result of the fact that we have neglected the back reaction of the radiation on the collapsing f wave. As a first step toward analyzing the back reaction, it is useful to estimate, to leading order in the mass, the retarded time at which the integrated energy of the Hawking radiation on $\mathcal{J}_R^+$ equals the initial mass $ax_0^+\lambda$ of the incoming f wave. This is given by

$$e^{-\lambda\sigma^-} = \frac{\exp(-\lambda\sigma_0^+)}{24}\,. \tag{20}$$

[2]It is assumed that the functional measure for the matter fields is defined with the metric g. One could imagine using instead the (flat) metric $e^{-2\phi}g$, in which case there would be no Hawking radiation.

Unfortunately this picture cannot yet be taken seriously because the turn-around point at which all the energy has backscattered has coordinates $(\sigma_0^+, \sigma_0^+ + (\ln 24)/\lambda)$. The value of the dilaton at this point is from (13) for small mass

$$e^{-2\phi} = 1/24 \tag{21}$$

independent of σ_0^+ or a. As we have stated, e^ϕ is the loop expansion parameter for dilaton gravity. Since this parameter is not small at the turn-around point, our one-loop calculation of the Hawking flux breaks down before the f wave fully backscatters.

The situation can be remedied by proliferating the number of matter fields. This introduces a new small expansion parameter into the theory: $1/N$, where N is the (large) number of matter fields [11]. For N matter fields the Hawking flux is N times as great and one finds that the f wave has completely backscattered by $(\sigma_0^+, \sigma_0^+ + (\ln 24/N)/\lambda)$. For large N, the value of the dilaton at this point is

$$e^{-2\phi} = N/24\,, \tag{22}$$

which indeed corresponds to weak coupling. This suggests that, for large N, the essential physics of Hawking radiation back reaction takes place in a weak-coupling regime and should be amenable to a semiclassical treatment. In what follows, we will present some proposals for the development of such a fully consistent treatment of the scattering problem, along with some informed conjectures about the form of the solution.

In a systematic expansion in $1/N$, one must include the one-loop matter-induced contribution to the gravitational effective action at the same order as the classical action (1). This incorporates both Hawking radiation and back reaction. Because of the way the dilaton varies with position, there is a region in spacetime where the $O(N)$ one-loop matter-induced gravitational action is of the same order as the strictly classical part and the loop coupling constant is nonetheless small. As described above, it is precisely in this region where the essential back-reaction physics will occur and a semiclassical treatment of the proper action should give meaningful answers. To leading order in $1/N$, and in the conformal gauge, the quantum effective action to be solved is

$$\begin{aligned}S_N = \frac{1}{\pi}\int d^2\sigma\Bigg[&e^{-2\phi}(-2\partial_+\partial_-\rho + 4\partial_+\phi\,\partial_-\phi - \lambda^2 e^{2\rho})\\ &-\frac{1}{2}\sum_{i=1}^{N}\partial_+ f_i\,\partial_- f_i + \frac{N}{12}\,\partial_+\rho\,\partial_-\rho\Bigg].\end{aligned} \tag{23}$$

The last term is the Liouville term induced by the N matter fields and the conformal gauge constraints [the $T_{\pm\pm}$ equations of (3)] are modified by its presence in a way which will shortly be made explicit.[3] We have also tuned the coefficient of the possible Liouville cosmological constant (to be distinguished from the classical "dilatonic" cosmological constant λ) to zero. In a slight abuse of terminology, we nevertheless refer to the dynamics governed by the last term in (23) as Liouville gravity.

Solving the quantum theory to leading order in $1/N$ is equivalent to solving the classical theory described by S_N.

Unlike S_0, it does not appear possible to solve S_N exactly, though it may be possible to solve the equations numerically. At present the best we can do is make the following educated guess about the evolution of an incoming f wave. Consider a quantization of the system defined on null surfaces $\Sigma(\sigma^-)$ of constant σ^-. The light-cone Hamiltonian P_- evolves the system in the direction of increasing σ^-. The changes $P_\pm$ are not separately conserved because translation invariance is spontaneously broken. The combination $H=P_++P_-$ generates an unbroken symmetry and is conserved for spacelike surfaces. In fact, there are in general two conserved quantities, given by boundary terms at the two spatial infinities. For the null surfaces Σ, the eigenvalue $M(\sigma^-)$ of H is given by a boundary term on $\mathcal{I}_R^+$ (assuming the boundary term on $\mathcal{I}_L^-$ vanishes) and is called the Bondi mass. The Bondi mass is not conserved because radiation energy can leak out onto $\mathcal{I}_R^+$.

Now consider an initial state at $\sigma^-=-\infty$ describing an incoming f_1 wave as in (12), with the other $N-1$ f's set to zero. In addition, it is useful to let this wave be characterized by the nonanomalous, left-moving global conserved charge $Q_{1L}=\int d\sigma^+\partial_+f_1$. Near $\sigma^-=-\infty$, $e^{-2\phi}$ is very large and the extra Liouville term may therefore be neglected in the description of the incoming state on $\Sigma(-\infty)$, which is essentially described by (13). As σ^- increases away from $\mathcal{I}_R^-$, $M(\sigma^-)$ will decrease. From the point of view of the quantum effective action S_N, this is not due to Hawking radiation, but is simply a consequence of the extra Liouville term. As $\sigma^-\to+\infty$, it is plausible that $M(\sigma^-)$ decreases to zero. However, the state on $\Sigma(\sigma^-)$ cannot revert to the linear dilaton vacuum on $\mathcal{I}_L^+$ because it carries the conserved charge Q_{1L}.

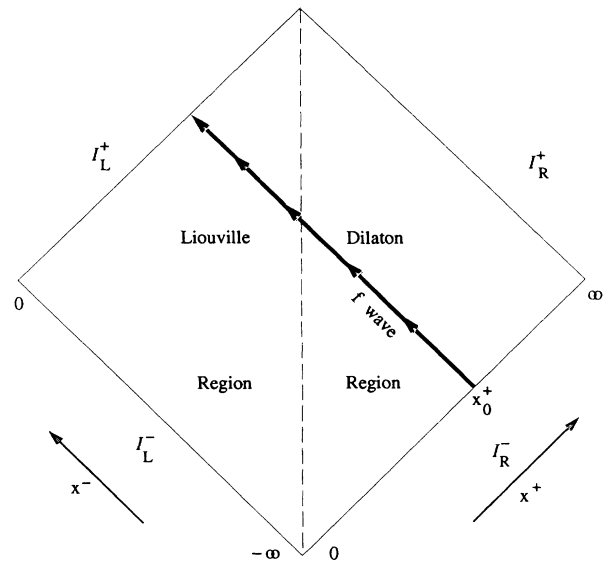

FIG. 2. An incoming f wave in quantum dilaton gravity eventually propagates into the region dominated by Liouville gravity, for which the curvature is constrained to vanish and all excitations have zero energy.

The picture can thus be summarized as follows. A state with a nonzero charge Q_{1L} and Bondi energy is incoming from $\mathcal{I}_R^-$. As it evolves it loses its energy, but retains its charge. Asymptotically it approaches a zero-energy state with charge Q_{1L} on $\mathcal{I}_L^+$. This is illustrated in Fig. 2.

This picture can be corroborated by direct analysis of the Bondi energy associated with data on a null surface Σ corresponding to a charged f wave. Such data must satisfy the null constraint equations

$$\begin{aligned} 0&=T_{++}=e^{-2\phi}(4\partial_+\phi\partial_+\rho-2\partial_+^2\phi)+\tfrac{1}{2}\partial_+f\partial_+f-\frac{N}{12}[\partial_+\rho\partial_+\rho-\partial_+^2\rho+t_+(\sigma^+)]\,,\\ 0&=T_{+-}=e^{-2\phi}(2\partial_+\partial_-\phi-4\partial_+\phi\partial_-\phi-\lambda^2e^{2\rho})-\frac{N}{12}\partial_+\partial_-\rho\,. \end{aligned} \tag{24}$$

The extra function t_+ appearing in T_{++} is in agreement with (17) and is a consequence of the anomalous transformation law for T_{++}. t_+ is coordinate dependent and must be fixed by boundary conditions, as in (18). The linear dilaton configuration remains as the vacuum solution of the full leading N theory:

$$\rho=0\,,\quad f_i=0\,,\quad \phi=\tfrac{1}{2}\lambda(\sigma^+-\sigma^-)\,. \tag{25}$$

The Bondi energy may then be defined for configurations which approach (25) on $\mathcal{I}_R^+$ (i.e., the configuration must not only be asymptotically flat, but presented in an asymptotically Minkowskian coordinate system). It is given by the surface term which must be added to the integral of $T_{++}+T_{+-}$ over Σ to obtain the generator of time translations. This canonical procedure yields

$$\begin{aligned} M(\sigma^-)&=2e^{\lambda(\sigma^+-\sigma^-)}(\lambda\delta\rho+\partial_+\delta\phi-\partial_-\delta\phi)\\ &\quad+\frac{N}{12}(\partial_-\delta\rho-\partial_+\delta\rho)\,, \end{aligned} \tag{26}$$

where $\delta\rho$ and $\delta\phi$ are the asymptotically vanishing deviations of ρ and ϕ from (25), and the right-hand side is to be evaluated on $\mathcal{I}_R^+$. The first "dilaton" term was obtained in Ref. [9]. The term proportional to N, arising from matter quantum effects, actually vanishes due to the boundary conditions (25). A modified formula is required in coordinate systems [such as in (11)] for which the fields do not asymptotically approach (25).

Let us first consider the energy, evaluated on a surface Σ, of a small amplitude f_1 wave packet localized in the dilaton region where $e^{-2\phi}$ is very large, i.e., at very large $\sigma^+-\sigma^-$. Then the Liouville terms proportional to N may be neglected in solving the constraints. M will be

[3]In a systematic quantum treatment of this action one will find, at subleading order in $1/N$, that the N in (23) is shifted by the ghost and gravity measures in order to maintain a net central charge $c=26$.

given as before by the integrated value of $\frac{1}{2}\partial_+ f_1 \partial_+ f_1$ times the x^+ coordinate of the center of the wave packet.

Now, however, consider the case where the f_1 wave packet is localized on Σ in the "Liouville region" where $e^{-2\phi}$ is very small. The dilaton gravity term is then very small, and the action governing[4] ρ and f reduces to Liouville gravity coupled to conformal matter:

$$S_N(\text{large } \phi) = \frac{1}{\pi}\int d^2\sigma \left[\frac{N}{12}\partial_+\rho\partial_-\rho - \frac{1}{2}\sum_{i=1}^{N}\partial_+ f_i \partial_- f_i \right] \tag{27}$$

with constraints

$$\begin{aligned} 0 = T_{++} &= \tfrac{1}{2}\sum_{i=1}^{N}\partial_+ f_i \partial_+ f_i \\ &\quad - \frac{N}{12}(\partial_+\rho\partial_+\rho - \partial_+^2\rho + t_+(\sigma^+)), \\ 0 = T_{+-} &= -\frac{N}{12}\partial_+\partial_-\rho . \end{aligned} \tag{28}$$

The T_{+-} constraint implies that the spacetime is in fact flat. The Bondi energy of (26) reduces to its Liouville piece.

$$M(\sigma^-) = \frac{N}{12}(\partial_-\delta\rho - \partial_+\delta\rho) . \tag{29}$$

Since there is no invariant one can associate with a flat metric one would expect this expression to vanish. The fact that it does can be seen from direct evaluation of (29): If ρ approaches zero on $\mathcal{I}_R^+$ as required by the boundary conditions (25), the derivatives of ρ and, consequently, M must also vanish. Thus all asymptotically flat states of Liouville gravity plus matter have zero energy.

We now have a plausible global picture of the scattering process. The linear dilaton vacuum is divided into two regions characterized by $e^{-2\phi}$ large or small compared to $N/12$. This dividing line is timelike. For $e^{-2\phi} \gg N/12$, the dynamics are essentially that of classical dilaton gravity coupled to matter. For $e^{-2\phi} \ll N/12$, one has Liouville gravity coupled to matter. An incoming f_1 wave packet on $\mathcal{I}_R^-$ begins in the dilaton gravity region where it has nonzero Bondi energy. However, it eventually crosses into the Liouville region, where all excitations have zero energy. By energy conservation, all of the initial energy of the wave packet must have radiated away to $\mathcal{I}_R^+$. There is no indication of an event horizon or singularity:[5] In the region where the singularity occurs in the classical solution, the quantum dynamics are governed by Liouville gravity (with no cosmological constant) in which the curvature is required to vanish. One expects, therefore, a unitary S matrix evolving from $\mathcal{I}^-$ to $\mathcal{I}^+$. One would hope to extract information about this S matrix from a semiclassical treatment of the large-N action (23).

While we find this picture compelling, we emphasize that at present it must be regarded as speculative. We have *not* shown that an incoming f wave does not, in fact, produce a singularity, or even that the large-N equations of motion give a well-defined evolution. One might try to substantiate our speculations by doing weak-field perturbation theory in the amplitude of the f wave. However, preliminary calculations indicate that weak-field perturbation theory breaks down near the boundary of the dilaton and Liouville regions: The second-order perturbation is divergent. Thus in order to settle the question a nonperturbative analysis of the large-N theory (23) is probably needed.[6]

In conclusion, we have analyzed the process of black-hole formation and evaporation, including back reaction, in the $1/N$ expansion of a two-dimensional model. A set of equations describing the process were found, but have not been solved so far. A qualitative analysis suggests that in this model would-be black holes in fact evaporate before an event horizon or singularity has a chance to form. Thus there is no indication that pure states evolve into mixed states. The implications of our results for four-dimensional black holes remain to be explored.

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[4]The dynamics of ϕ are roughly governed by the free field $\psi = e^{-\phi}$. However, it is not clear what range should be taken for ψ.

[5]In two dimensions, unlike in higher dimensions, we know of no local notion of an apparent horizon. Global event horizons exist as usual when the spacetime is singular or otherwise incomplete.

[6]Of course, a singularity at large N does not imply a singularity of the full quantum theory since the $1/N$ expansion breaks down as soon as fields grow to order N.

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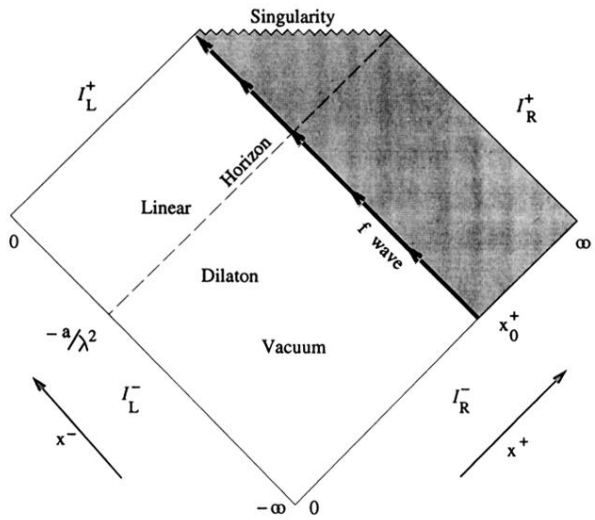

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