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Effect of the a_1 width on $D \to \bar{K} a_1$ decays

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In this Brief Report we study $D \to \bar{K} a_1$ decays in a factorization model with neglect of the annihilation term. We find that a mass averaging over the width of a_1 using a Breit-Wigner measure does not lead to an enhancement of the rate. We discuss the missing physics that would bridge the gap between theory and experiment.

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I. INTRODUCTION

The factorization [1,2] model has been quite successful in describing D and D_s^+ decays into two pseudoscalar, a vector and a pseudoscalar, and two vector mesons, with the inclusion [3,4] of final-state interactions. It has had only questionable success [1,5] in describing decays of D mesons into an axial vector and a pseudoscalar meson.

In this Brief Report we have studied the decays $D^0 \to K^- a_1^+$, $\bar{K}^0 a_1^0$, and $D^+ \to \bar{K}^0 a_1^+$ because data now exist [5] on these decays and to highlight the problems that are particular to these decay channels. If one uses the central value of the a_1 mass, then the mass of the final state is very close to the D-meson mass, and the decay involving a P wave is strongly suppressed. If we use the parameters of Ref. [1], we get

$$
\begin{aligned}
&B(D^0 \to K^- a_1^+) = 1.46\% \quad (\text{expt}[5]\text{: } (9.0 \pm 0.9 \pm 1.7)\%) , \\
&B(D^+ \to \bar{K}^0 a_1^+) = 3.75\% \quad (\text{expt}[5]\text{: } (7.1 \pm 0.8 \pm 1.1)\%) , \\
&B(D^0 \to \bar{K}^0 a_1^0) = 0.0 \quad (\text{expt}[5]\text{: } (0.43 \pm 0.99)\%) . \qquad (1)
\end{aligned}
$$

In calculating the above branching ratios we have ignored final-state-interaction phases. However, it is clear that one needs to boost $B(D^0 \to K^- a_1^+)$ by a factor of 5–6 and $B(D^+ \to \bar{K}^0 a_1^+)$ by a factor of 2. At first sight one may think that these calculated rates could be raised due to the fact that a_1 has quite a large width which increases the phase space available. The threshold suppression could thus be obviated. In fact in the literature [5] one finds $B(D^0 \to K^- a_1^+)$ quoted [6] as 5% in the Bauer-Stech-Wirbel (BSW) model [1].

We have done a study of the effect of the finite width of a_1 on the decay rates for $D^0 \to K^- a_1^+$, $\bar{K}^0 a_1^0$, and $D^+ \to \bar{K}^0 a_1^+$ in the factorization model to investigate if an enhancement in the rates could be obtained as a result of an averaging of these rates over the a_1 mass.

II. THE FACTORIZATION MODEL AND METHOD

The relevant weak Hamiltonian needed to describe $D \to \bar{K} a_1$ decays is the Cabibbo-angle-favored effective Hamiltonian which we write down in the notation of Ref. [1]:

$$
H_w(\Delta C = \Delta S = -1) = \frac{G_F \cos^2\theta_C}{\sqrt{2}} [a_1 (\bar{u}d)_H (\bar{s}c)_H + a_2 (\bar{u}c)_H (\bar{s}d)_H] , \qquad (2)
$$

where θ_C is the Cabibbo angle ($\sin\theta_C = 0.23$). The notation $(\bar{q}q)$ is a shorthand for a color-singlet combination $\bar{q}\gamma_\mu(1-\gamma_5)q$ and the subscript H reminds us to treat the parentheses as an effective hadronic field. a_1 and a_2 are the QCD coefficients which we take as Ref. 1 $a_1 = 1.2$, $a_2 = -0.5$.

In calculating the decay matrix elements we use the following normalizations ($P \equiv 0^-$ meson, $V \equiv 1^-$ meson, and $a \equiv 1^+$ meson):

$$
\begin{aligned}
&\langle P(k) | A_\mu(0) | 0 \rangle = -i f_p k_\mu , \\
&\langle V(k) | V_\mu(0) | 0 \rangle = \epsilon_\mu^* m_V f_V , \qquad (3) \\
&\langle a(k) | A_\mu(0) | 0 \rangle = \epsilon_\mu^* m_a f_a .
\end{aligned}
$$

We introduce the form factors $A(q^2)$ and $V_i(q^2)$,

$$
\langle a(k) | A_\mu(0) | D(p) \rangle = \frac{2}{m_D + m_a} \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p^\rho k^\sigma A(q^2) , \qquad (4)
$$

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$$\langle a(k)|V_\mu(0)|D(p)\rangle = i\left[\epsilon^*_\mu(m_D+m_a)V_1(q^2) - \frac{\epsilon^*\cdot(p-k)}{m_D+m_a}(p+k)_\mu V_2(q^2) - \frac{\epsilon^*\cdot(p-k)}{q^2}(2m_a)(p-k)_\mu V_3(q^2) + \frac{\epsilon^*\cdot(p-k)}{q^2}(2m_a)(p-k)_\mu V_0(q^2)\right], \quad (5)$$

where $q=(p-k)$, $V_0(0)=V_3(0)$, and the following constraint is satisfied by $V_i(q^2)$:

$$2m_a V_3(q^2) = (m_D+m_a)V_1(q^2) - (m_D-m_a)V_2(q^2) . \quad (6)$$

The divergence of (4) vanishes and that of (5) is proportional to $V_0(q^2)$ only. Following Ref. [1], we also define the form factors F_0 and F_1:

$$\langle P(k)|V_\mu(0)|D(p)\rangle = \left[p+k-\frac{m_D^2-m_P^2}{q^2}q\right]_\mu F_1(q^2) + \frac{m_D^2-m_P^2}{q^2}q_\mu F_0(q^2) . \quad (7)$$

The divergence of (7) involves $F_0(q^2)$ only. With the definitions introduced in (3)–(7) we get the following decay amplitudes in the factorized spectator model (annihilation term is neglected and a factor $\epsilon^*\cdot p$ is suppressed):

$$A(D^0\to K^-a_1^+) = \frac{G_F}{\sqrt{2}}\cos^2\theta_C(2m_a)f_aF_1(m_a^2)a_1 ,$$
$$A(D^0\to \bar{K}^0a_1^0) = \frac{G_F}{\sqrt{2}}\cos^2\theta_C\frac{1}{\sqrt{2}}(2m_a)f_KV_0(m_K^2)a_2 , \quad (8)$$
$$A(D^+\to \bar{K}^0a_1^+) = \frac{G_F}{\sqrt{2}}\cos^2\theta_C(2m_a)\times[f_aF_1(m_a^2)a_1+f_KV_0(m_K^2)a_2] .$$

The decay rate is given by

$$\Gamma(D\to Ka_1) = \frac{p^3}{8\pi m_a^2}|A(D\to Ka_1)|^2 , \quad (9)$$

where p is the three-momentum of the final-state particles in the rest frame of the D meson. Note also that the mass dependence in the denominator of (9) cancels against the mass dependence of $A(D\to \bar{K}a_1)$ as seen in (8).

The form factor $F_1(q^2)$ is assumed to have the form [1]

$$F_1(q^2) = h_1\Big/\left[1-\frac{q^2}{m_{1^-}^2}\right] , \quad (10)$$

where [1] $h_1=0.76$ for $D\to\bar{K}$ transition and $m_{1^-}=2.11$ GeV. We also take [1] $f_a=221$ MeV and $V_0(0)=0$ due to orthogonality of the wave functions. We first quote the rates treating a_1 as a sharp resonance and ignoring final-state interactions:

$$\Gamma(D^0\to K^-a_1^+) = 2.40a_1^2\times10^{10}\ \text{sec}^{-1} ,$$
$$B(D^0\to K^-a_1^+) = 1.46\% ,$$

or (11)

$$\Gamma(D^0\to \bar{K}^0a_1^0) = 0 ,$$
$$\Gamma(D^+\to \bar{K}^0a_1^+) = 2.45a_1^2\times10^{10}\ \text{sec}^{-1} ,$$

or

$$B(D^+\to \bar{K}^0a_1^+) = 3.75\% .$$

The first question we ask is: Can final-state interactions bridge the gap between experiment and theory? The answer is very likely "no." The reason for this is best illustrated by using the isospin decomposition

$$A(D^0\to K^-a_1^+) = \frac{1}{\sqrt{3}}(A_{3/2}e^{i\delta_{3/2}}+\sqrt{2}A_{1/2}e^{i\delta_{1/2}}) ,$$
$$A(D^0\to \bar{K}^0a_1^0) = \frac{1}{\sqrt{3}}(\sqrt{2}A_{3/2}e^{i\delta_{3/2}}-A_{1/2}e^{i\delta_{1/2}}) , \quad (12)$$
$$A(D^+\to \bar{K}^0a_1^+) = \sqrt{3}A_{3/2}e^{i\delta_{3/2}} .$$

Since $B(D^0\to\bar{K}^0a_1^0)$ is consistent with zero, it implies that isospin-$\frac{1}{2}$ and -$\frac{3}{2}$ amplitudes largely cancel each other. This in turn implies that in $D^0\to K^-a_1^+$ these two amplitudes largely reinforce each other with the result that $A(D^0\to K^-a_1^+)\approx A(D^+\to\bar{K}^0a_1^+)$. Thus $\Gamma(D^0\to K^-a_1^+)\approx\Gamma(D^+\to\bar{K}^0a_1^+)$. The difference in the branching ratios is entirely due to the different lifetimes τ_{D^0} and τ_{D^+}. A nonzero $(\delta_{1/2}-\delta_{3/2})$ is only likely to reduce $B(D^0\to K^-a_1^+)$. *The crucial point in this argument is the vanishing of* $B(D^0\to\bar{K}^0a_1^0)$.

The second question that arises is: What is the effect of finite, and rather large, a_1 width? Since a_1 is rather wide the effective final-state phase space is larger than the "nominal" value which uses the central mass of a_1; could this raise the rate for $D^0\to K^-a_1^+$ and $D^+\to\bar{K}^0a_1^+$? We address ourselves to this question in the following.

Let us assume a running mass m for a_1. One then has to average the decay rate over this mass using a measure $\rho(m)$, i.e.,

$$\bar{\Gamma}(D^0\to K^-a_1^+) = \int\rho(m)\Gamma(D^0\to K^-a_1^+(m))dm \quad (13)$$

with

$$\int\rho(m)\,dm = 1 . \quad (14)$$

We have used a "Breit-Wigner" measure [7]

$$\rho(m) = \frac{2mN}{\pi}\frac{m_a\Gamma_{\text{tot}}(m)}{(m^2-m_a^2)^2+m_a^2\Gamma_{\text{tot}}^2(m)} , \quad (15)$$

where m_a is the central value of a_1 mass, 1.26 GeV, and N a normalization factor to ensure that (14) is satisfied.

III. RESULTS AND DISCUSSION

In using (15) for the measure we parametrized the total width of a_1 as follows:

$$\Gamma_{tot}(m)=\rho_{3\pi}(m)\Gamma_{3\pi}\theta(m-3m_\pi)+\rho_{\rho\pi}(m)\Gamma_{tot}(1-\Gamma_{3\pi}/\Gamma_{tot})\theta(m-m_\rho-m_\pi) . \quad (16)$$

Here Γ_{tot} is the total width of a_1 and $\Gamma_{3\pi}$ the partial width. The step functions ensure that the relevant channels open at the appropriate masses. $\rho_{3\pi}$ and $\rho_{\rho\pi}$ are kinematic factors defined as

$$\rho_{3\pi}(m)=\left[\frac{m-3m_\pi}{m_{a_1}-3m_\pi}\right]^n , \quad n \text{ arbitrary but positive} , \quad (17)$$

or

$$\left[\frac{m^2-9m_\pi^2}{m_{a_1}^2-9m_\pi^2}\right]^n , \quad n \text{ arbitrary but positive} , \quad (18)$$

$$\rho_{\rho\pi}(m)=\frac{k(m)}{k(m_{a_1})},$$

$$k=\text{c.m. momentum of } \rho \text{ or } \pi \text{ in } a_1 \text{ rest frame} . \quad (19)$$

In our numerical work n was chosen to be $\frac{1}{2}$ or 1. For m below the $\rho\pi$ threshold the decay of a_1 to 3π proceeds via the first term in (16). The choice of $\Gamma_{3\pi}$ is largely speculative but guided by the fact that $\Gamma(\rho\to\pi\pi)$ is ≈ 150 MeV; we expect $a_1(m)\to 3\pi$ to be somewhat lower; $\Gamma_{3\pi}\approx 100$ MeV appears to be reasonable. The final results are not very sensitive to $\Gamma_{3\pi}$. We have varied Γ_{tot} in the range 300–500 MeV [8]. In evaluating $\overline{\Gamma}$, the average rate, from (13) we used the running mass m in the form factor $F_1(q^2=m^2)$ defined in (10) and scaled $f_a(m)$ by

$$f_a(m)=(m_a/m)^{1/2}f_a(m_a) . \quad (20)$$

In Table I we have listed the enhancement factor in the rate, that is, the ratio $\overline{\Gamma}(D\to\overline{K}a_1)/\Gamma(D\to\overline{K}a_1)$, where $\overline{\Gamma}$ is the average rate defined in (13) and Γ the rate calculated in (11). In this table we have also listed the result for $f_a(m)=f_a(m_a)$.

As we see from the tabulation the enhancement factor is somewhat less than unity. We also tried a parametrization with $\Gamma_{3\pi}=0$. The resulting enhancement factors were, as is expected, decreased.

Thus the dilemma remains: while $B(D^0\to K^-a_1^+)$ is well below the measured value [5], perhaps by a factor of 5, $B(D^+\to\overline{K}^0a_1^+)$ is only a factor of 2 below the central value measured [5]. $B(D^0\to\overline{K}^0a_1^0)$ is consistent with data [5]. A Breit-Wigner measure does not yield the desired enhancement.

TABLE I. Enhancement factor $\equiv\overline{\Gamma}(D\to\overline{K}a_1)/\Gamma(D\to\overline{K}a_1)$ using the Breit-Wigner measure (13)–(19). $\Gamma_{3\pi}$ is set at 100 MeV. The numbers in parentheses represent the case $f_a(m)=f_a(m_a)$.

Γ_{tot}	Using $\rho_{3\pi}(m)$ as in (17)		Using $\rho_{3\pi}(m)$ as in (18)	
(GeV)	$n=0.5$	$n=1.0$	$n=0.5$	$n=1.0$
0.3	1.20	1.09	1.15	0.97
	(0.99)	(0.88)	(0.98)	(0.86)
0.4	1.16	1.06	1.12	0.96
	(0.96)	(0.86)	(0.94)	(0.83)
0.5	1.13	1.04	1.09	0.94
	(0.93)	(0.83)	(0.91)	(0.81)

So, what missing physics could account for the discrepancy between theory and experiment? It is, of course, possible that factorization breaks down in $D\to\overline{K}a_1$ decays as the center of mass of the two quarks that finally combine to form a_1 moves relatively slowly. If so, the nonfactorizable contributions could well account for the discrepancy between theory and experiment.

Barring the failure of factorization, annihilation terms, neglected so far in this paper, could also help remove the discrepancy between theory and experiment. If we introduce an annihilation term, represented by a parameter R, formulas (8) modify as follows (using $V_0=0$):

$$A(D^0\to K^-a_1^+)=\frac{G_F}{\sqrt{2}}\cos^2\theta_C(2m_a)[f_aF_1(m_a^2)a_1-Ra_2] , \quad (21)$$

$$A(D^0\to\overline{K}^0a_1^0)=\frac{G_F}{\sqrt{2}}\cos^2\theta_C\frac{(2m_a)}{\sqrt{2}}Ra_2 ,$$

$$A(D^+\to\overline{K}^0a_1^+)=\frac{G_F}{\sqrt{2}}\cos^2\theta_C(2m_a)f_aF_1(m_a^2)a_1 .$$

Since data [5] allow $B(D^0\to\overline{K}^0a_1^0)\approx 1\%$, one can set a limit on the parameter R. We find that an annihilation term that generates $B(D^0\to\overline{K}^0a_1^0)=1\%$ will raise $B(D^0\to K^-a_1^+)$ by a factor of 4.8 and, of course, leave $B(D^+\to\overline{K}^0a_1^+)$ unaffected. This could be a mechanism that would selectively raise $(D^0\to K^-a_1^+)$ and bring theory in better agreement with experiment.

As a matter of curiosity we repeated the mass-averaging procedure defined in (13) with a Breit-Wigner form for $\rho(m)$ with a constant width:

$$\text{(a)}\quad \rho(m)=\frac{2mN}{\pi}\frac{m_a\Gamma_{tot}}{(m^2-m_a^2)^2+m_a^2\Gamma_{tot}^2} , \quad (22)$$

$$\text{(b)}\quad \rho(m)=\frac{2m_aN}{\pi}\frac{m\Gamma_{tot}}{(m^2-m_a^2)^2+m^2\Gamma_{tot}^2} , \quad (23)$$

where Γ_{tot} is a constant which we choose to be 300, 400, or 500 MeV, and N a normalization constant to ensure that (14) is satisfied. For a narrow width, mass averaging with (22) or (23) will result in a $\overline{\Gamma}$ very close to one with (15). However, since Γ_{tot} is quite large, use of (22) or (23) could result in a significantly different $\overline{\Gamma}$. Indeed, one ex-

TABLE II. Enhancement factor $\equiv\bar{\Gamma}(D\to\bar{K}a_1)/\Gamma(D\to\bar{K}a_1)$ using the constant-width Breit-Wigner measures (22) and (23). The range of integration used in calculating $\bar{\Gamma}$ is identified. The numbers in parentheses represent the case $f_a(m)=f_a(m_a)$.

Γ_{tot} (GeV)	$\bar{\Gamma}/\Gamma$ Range of integration: $3m_\pi \le m \le (m_D - m_K)$		Range of integration: $(m_\rho+m_\pi)\le m \le (m_D-m_K)$	
	Using Eq. (22)	Using Eq. (23)	Using Eq. (22)	Using Eq. (23)
0.3	2.18	2.36	1.24	1.35
	(1.60)	(1.73)	(1.09)	(1.18)
0.4	2.39	2.70	1.20	1.36
	(1.68)	(1.90)	(1.04)	(1.17)
0.5	2.53	2.99	1.15	1.35
	(1.73)	(2.04)	(0.99)	(1.15)

pects a larger value for $\bar{\Gamma}$ using (22) or (23) since these forms do not have a threshold factor in the width to damp out the contribution from the low-mass region of the integral in (13). Note that it is the low mass region that is responsible for an enhancement of the branching ratio through two factors, a larger phase space and a low mass enhancement through $f_a(m)\propto(m)^{-1/2}$.

In doing the calculation for $\bar{\Gamma}$ using (22) and (23) we have confined ourselves to the following cases: (i) $f_a(m)\propto(m)^{-1/2}$ and $f_a(m)=f_a(m_a)$; (ii) range of m: $3m_\pi \le m \le (m_D-m_K)$ and $(m_\rho+m_\pi)\le m \le (m_D-m_K)$. The results are shown in Table II. It is clear that the use of (22) and (23) yields a larger value of $\bar{\Gamma}$. From Table II we also conclude that due to the lack of a threshold factor in the width, the constant-width Breit-Wigner forms (22) and (23) lead to a value of $\bar{\Gamma}$ which is quite sensitive to the mass range used in the averaging procedure and does in fact result in a larger $\bar{\Gamma}$ than the "correct" form with appropriate threshold factors.

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[1] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987).
[2] A. N. Kamal, Phys. Rev. D **33**, 1344 (1986).
[3] R. C. Verma and A. N. Kamal, Phys. Rev. D **43**, 829 (1991).
[4] A. N. Kamal, R.C. Verma, and N. Sinha, Phys. Rev. D **43**, 843 (1991).
[5] Don Fredric DeJongh, Ph.D. thesis, California Institute of Technology, 1990; Mark III Collaboration, D. Coffman *et al.*, Report No. SLAC-PUB-5447, 1991 (unpublished).
[6] Quoted as "Private communication from B. Stech" in Ref. [5].
[7] See, for example, H. Kolanoski, Z. Phys. C **39**, 543 (1988).
[8] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).