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Decoupling confinement and chiral-symmetry breaking: An explicit model

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Using a covariant model of mesons developed previously, we obtain new numerical solutions for the light-quark sector, and show explicitly how the small mass of the pion emerges as a natural consequence of chiral-symmetry breaking. Then we generalize the model, and show how chiral-symmetry breaking and confinement could be realized through completely independent mechanisms with different mass scales. In particular, the confining potential can be chosen to be purely scalar, as suggested by lattice studies and phenomenology, and the remaining part of the interaction can be chosen to be chirally invariant. In a symmetry-breaking mode, this new model can still generate quark mass and a massless-pion bound state.

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I. INTRODUCTION

The two most salient features of low-energy QCD related to the hadronic spectrum are confinement and chiral-symmetry breaking. Although these mechanisms might ultimately be related at some deeper level (beyond, of course, that they both presumably follow from the QCD Lagrangian) there appears phenomenologically to be distinct scales associated with each ($\Lambda_{QCD} \sim 200$ MeV in the case of confinement and $4\pi f_\pi \sim 1$ GeV as the natural scale coming from chiral perturbation theory [1]) thus suggesting that they might be independent infrared effects. Further indications come from lattice studies which suggest that the confining potential is purely scalar [2]. Indeed, in the case of a quark-gluon plasma, it has become commonplace to discuss the possibility of two separate phase transitions, one associated with deconfinement and one with chiral restoration [3]. One naturally wonders if these two dual features of low-energy QCD can be independently and simultaneously included into a model of the hadronic spectrum. Early models generally focused on one or the other of these two defining features: confinement in the case of the bag and nonrelativistic potential models [4], and chiral-symmetry breaking in the case of the various effective-Lagrangian approaches [1,5]. In addition, many of these models suffer from a lack of Lorentz covariance. More recent approaches have attempted to include both ingredients [6], but generally in a fashion that closely weds the confinement mechanism with the chiral-breaking one. (One early attempt which, however, does not can be found in Ref. [7].) And of these, only one, to our knowledge, is covariant [8].

We recently proposed a new model of mesons as quark-antiquark bounds states that is covariant, confining, and chirally symmetric [9]. The equations which emerge from this approach give an analytic solution for a zero-mass pseudoscalar bound state in the case of exact chiral symmetry, and also reduce to the familiar, highly successful nonrelativistic linear-potential models in the limit of heavy-quark mass and lightly bound systems. The approach is therefore suitable for a unified description of all the mesons from the pion through the Υ. In this paper we extend and further develop this approach by (i) presenting new solutions in the light-quark sector which show that the physical pion can be described by the model, and (ii) showing how it can be generalized so that chiral symmetry and confinement can be realized in a completely decoupled fashion. In particular, we show that the confinement mechanism could be taken as arising from a purely scalar interaction, and that, as long as the remaining interaction is chirally invariant, dynamical quark mass and a zero-mass Goldstone boson (the pion) can still emerge through symmetry breaking. Independent of the ultimate correctness of this decoupling, a model in which this separation is explicitly realized shows that at least in principle we could be discussing two separate, independent manifestations of low-energy QCD.

The paper is divided into five sections. In Sec. II, the model is briefly reviewed. Much of the formalism has already been presented in Ref. [9] and we will refer back to it, when necessary, for details of the approach. However, in that first paper solutions for the case of light quarks and mesons were not given, and these solutions are now presented in Sec. III. We find that a form factor which depends on the off-shell quark mass must be added to the kernel in order to obtain solutions which have the correct chiral limit, and with this addition the model can describe a realistic pion. In Sec. IV it is shown how to decouple the confinement mechanism from dynamical chiral-symmetry breaking. Conclusions are presented in Sec. V.

II. THE MODEL

The model given in Ref. [9] is a covariant generalization of nonrelativistic linear-potential models [10], that includes chiral-symmetry breaking by dynamically generating a constituent quark mass. The light mesons are viewed as bound states of these dynamically generated, massive quark-antiquark pairs. A self-consistency condition then ensures that in the chiral limit when the current quark mass is zero, the pion appears as a zero-mass pseudoscalar Goldstone boson. Our effective quark interaction $V_{\rm eff}(k)$, contains two components: one piece is a covariant generalization of a linear potential and provides confinement, and a second piece is a covariant generalization of a nonrelativistic constant potential:

$$V_{\rm eff}(k)=V_L(k)\sum_i O_1^i O_2^i + V_C(k)\sum_i \tilde{O}_1^i \tilde{O}_2^i \ . \tag{1}$$

The Dirac matrices O and $\tilde{O}$ operate on the Dirac indices of quarks 1 and 2, and describe the spin-dependent structure of each of the two pieces of the effective interaction. In Ref. [9], $O=\tilde{O}$ was assumed, but they are, in general, distinct. The covariant scalar functions $V_L(k)$ and $V_C(k)$ contain the momentum dependence of the two pieces of the effective interaction.

We start from the self-consistent equations for the quark self-energy and bound-state vertex function, Figs. 1(a) and 1(b). The heavy dashed lines schematically represent the quark potential, modeled as an exchange interaction (as would occur in a simple boson-exchange picture), involving two three-point vertices with the exchanged momentum determined by energy-momentum conservation. In both equations, the kernel is further defined by restricting some of the quarks to their mass shell. In the vertex equation, two channels are created, one with the *quark* restricted to its positive-energy mass shell, and one with the *antiquark* restricted to its negative-energy mass shell. The resulting two-channel bound-state equation is shown in Fig. 2. These restrictions mean that even though the equations are exactly covariant, they depend, like nonrelativistic equations, on the relative three-momentum only, and have a smooth nonrelativistic limit. The second (antiquark) channel is necessary for a consistent description of deeply bound states, as discussed in Ref. [9], but is negligible for loosely bound heavy-quark systems. Finally, restricting both the internal and external quarks to their mass shell reduces the self-energy equation [Fig. 1(a)] to an algebraic self-consistency condition between the bare (current) and

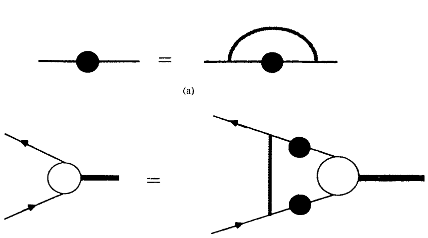

FIG. 1. The self-consistent Dyson equations for the quark self-energy and vertex function.

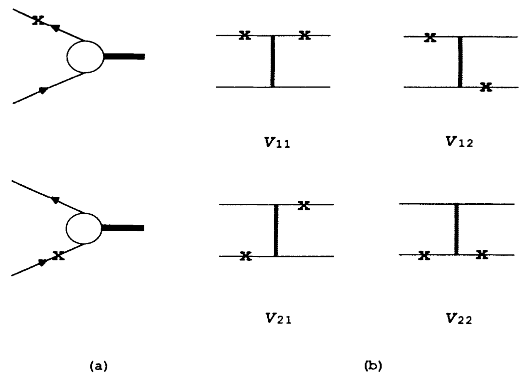

FIG. 2. The representation of the two-channel bound-state equation with four potentials. A quark line with an × is on shell.

dynamical (constituent) quark masses, and the parameters of the same kernel appearing in the bound equation.

An essential feature which makes these equations tractable is the infrared-regularized Fourier-transformed linear potential $V(r)=\sigma r$. In momentum space, the linear potential behaves as $1/q^4$ plus an infrared subtraction that regulates the potential at $q^2=0$ and ensures that $V(r=0)=0$ [9]. The covariant generalization of this condition satisfied by the confining potential $V_L(k)$ is

$$\int \frac{d^3k}{(2\pi)^3}\left[\frac{m}{E_k}\right]V_L(k)=0\ , \tag{2}$$

where $E_k=\sqrt{m^2+k^2}$, and m is the quark constituent mass. (Note that, in the limit $m\to\infty$, the factor $[m/E(k)]\to 1$, and expression (2) is precisely the statement $V(r=0)=0$.) We likewise define a covariant generalization of a nonrelativistic constant potential, $V_C(k)$, which satisfies:

$$\int \frac{d^3k}{(2\pi)^3}\left[\frac{1}{E_k}\right]V_C(k)=C\ . \tag{3}$$

For the initial numerical studies, we chose to work with a particularly convenient form for the Dirac matrices, O and $\tilde{O}$:

$$\sum_i O_i O_i=\sum_i \tilde{O}_i\tilde{O}_i=\tfrac{1}{2}(1-\gamma_1^5\gamma_2^5-\gamma_1^\mu\gamma_{2\mu})\ . \tag{4}$$

This form was chosen because it is invariant under U(1) chiral transformations, and because it simplifies the equations for the vertex function $\Gamma(p,P)$ (where P is the bound-state momentum and p the relative momentum of the two quarks) allowing a bound-state solution which is a pure isovector pseudoscalar: $\Gamma(p,\mu)=\Gamma_0(p)\tau\gamma^5$, where μ is the bound-state rest mass. While the choice (4) is convenient, it is certainly not best from a phenomenological point of view, and the optimal form for O and $\tilde{O}$ will be deferred to a later work when we use this model to fit the physical spectrum. This will be discussed further in Sec. IV.

Using the form (4), and placing both quark legs on shell in the self-energy diagram Fig. 1(a), gives the following relation [9] between the constituent quark mass m, the bare quark mass m_0, and the strength of the constant potential C:

$$\left[1-\frac{m_0}{m}\right]=-\int\frac{d^3k}{(2\pi)^3E_k}[V_L(k)+V_C(k)]=-C\ . \qquad (5)$$

Notice that, because of the constraint (2), the strength of the linear confining potential does not enter this relation. The linear confining potential thereby makes no contribution to the generation of quark mass, and it is the decoupling which permits the confining potential to be purely scalar. (This is discussed further in Sec. IV.) In nonrelativistic models, the constant piece provides an overall mass shift. This is also true in our relativistic model, but now this shift comes about through the dynamical breaking of chiral symmetry, and is therefore associated with the structure of the vacuum.

Assuming that the quark self-energy can be approximated by a constant mass shift in the effective quark propagator, Fig. 1(a), as occurs in the model of Nambu and Jona-Lasinio [5], we obtain the following two-channel bound-state equation [9] for the wave functions $\Psi_1(p)=\Gamma_1(p,P)/(2E_p-\mu)$, and $\Psi_2(p)=\Gamma_2(p,P)/(2E_p-\mu)$:

$$\begin{aligned}(2E_p-\mu+E_pC)\Psi_1(p)+E_pC\Psi_2(p)&=-\int\frac{d^3k}{(2\pi)^3}V_{11}(p,k)\left[\Psi_1(k)-\left[\frac{E_p}{E_k}\right]\Psi_1(p)\right]-\int\frac{d^3k}{(2\pi)^3}V_{12}(p,k)\Psi_2(k)\ ,\\ (2E_p+\mu+E_pC)\Psi_2(p)+E_pC\Psi_1(p)&=-\int\frac{d^3k}{(2\pi)^3}V_{22}(p,k)\left[\Psi_2(k)-\left[\frac{E_p}{E_k}\right]\Psi_2(p)\right]\\ &\quad-\int_{k<k_1}\frac{d^3k}{(2\pi)^3}V_{21}(p,k)\left[\Psi_1(k)-\left[\frac{E_{k_1}}{E_k}\right]\Psi_1(k_1)\right]\\ &\quad-\int_{k<k_1}\frac{d^3k}{(2\pi)^3}V_{21}(p,k)\left[\Psi_1(k)-\left[\frac{E_{k_2}}{E_k}\right]\Psi_1(k_2)\right]\ .\end{aligned} \qquad (6)$$

The potentials V_{11}, V_{12}, V_{22}, and V_{21} are schematically defined in Fig. 2(b). The off-diagonal potentials V_{12} and V_{21} depend on the bound-state mass μ and have been regulated with an infrared subtraction analogous to that for the diagonal potentials, Eq. (3). In the limit of zero pion mass, they reduce to the diagonal elements V_{11} and V_{22}. Observe that in the zero-mass-pion limit with zero bare quark mass, $m_0=0$, because of the constraint Eq. (3) we obtain the analytic solution $\Psi_1(p)=\Psi_2(p)=N/E_p$ and $C=-1$, as required by chiral symmetry, Eq. (5). The linear potential again completely decouples in the chiral limit.

III. NUMERICAL SOLUTIONS FOR LIGHT QUARKS

For finite pion masses, we solve the equations numerically by expanding the wave functions in a set of basis functions $b_i(p)$, and then creating a generalized eigenvalue problem by acting upon the bound-state equation (6) with the covariant operator $\int d^3p\, b_i(p)/E_p$. For convenience, we choose to fix the bound-state mass μ, and then solve for the constant C.

The linear-potential term $V_L(q)$ requires an ultraviolet form factor not needed in its nonrelativistic counterpart. This arises because the $1/q^4$ potential loses two powers of k when the quarks are restricted to their mass shell, thereby generating a logarithmic divergence in the subtracted pieces of Eq. (6) that contain the wave function evaluated at some fixed point (p, k_1, or k_2). In our first paper [9] we chose a form factor that depended only on q^2, the argument of the linear potential. At that time we could only obtain reliable solutions for heavy-quark masses (i.e., when the constituent quark mass is taken to be much larger than the strength of the linear potential: $m^2 \gg \sigma$). We also ignored the off-diagonal matrix elements V_{12} and V_{21}. In the case of light-quark masses, we were obtaining values for C less than -1, in apparent contradiction to the mass-shift relation Eq. (5), which requires $C \geq -1$. We subsequently discovered that a form factor which depends only on q^2 does not provide sufficient convergence to ensure consistency with chiral symmetry. With such a form factor, the residue of the principal value integration at $q^2=0$ remains very large, even at very large momenta, generating terms which behaved like an additional positive constant potential, driving C to values less than -1. This effect presumably occurs also for the heavy-quark cases studies in Ref. [8], but the effect depends strongly on the scaled strength of the linear potential, $\sigma_0=\sigma/m^2$, and is very small in the heavy-quark case. To obtain light-quark solutions consistent with chiral symmetry, we must therefore choose a form factor with a more general momentum dependence. We have tried two distinct (covariant) choices for this dependence: one which depends on the sum of the quark momenta entering a vertex, the so-called sideways form factor [11], and one which can be factored into functions which depend only on one of the invariant masses of each of the legs entering a vertex (the factorized form factor).

They both give the same qualiative results. Here we will present results only for the factorized form factor, anticipating future applications when issues of electromagnetic gauge invariance will be a concern [12].

Our results are shown in Figs. 3–5. which take the constituent quark mass $m=350$ MeV, the strength of the linear potential $\sigma=0.2$ GeV^2, and both masses in the form factor are $\Lambda=\Lambda_1=\Lambda_2=600$ MeV. The form factor has the factorized form

$$F(k_1^2,p_1^2,q^2=(k_1-p_1)^2)=f_q(k_1^2)f_q(p_1^2)f_L(q^2) \tag{7a}$$

where

$$f_q(k_1^2)=\left[\frac{(\Lambda_2^2-m^2)^2}{(\Lambda_2^2-m^2)^2+(m^2-k_1^2)^2}\right]^2 , \tag{7b}$$

$$f_L(q^2)=\frac{\Lambda^4}{(\Lambda^4+q^4)} .$$

In Fig. 3 we plot our results for the constant C as a function of the bound-state mass. Notice how close the value of C is to the chiral limit at the physical pion mass ($\mu/2m\sim0.2$). Inserting numbers, ($C\sim-0.996$), and using the mass-shift relation, Eq. (5), gives a bare quark mass $m_0\sim1.5$ MeV, in rough agreement with sum-rule estimates. By adjusting the parameters (e.g., $\Lambda\rightarrow490$ MeV) we can obtain a more canonical value for $m_0\sim4.6$ MeV. It seems clear that this approach can give an account of the physical pion, provided the parameters are properly chosen.

In Fig. 4 we plot our solutions for the wave functions $\Psi_1(p)$ (solid curves) and $\Psi_2(p)$ (dashed curves) for a family of values of the bound-state mass ($\mu/2m=0.8$, 0.5, 0.3, and 0.15). Notice how the wave functions grow in momentum space as the bound-state mass decreases, and how the second channel smoothly approaches the first. We have found that, as the bound-state mass μ approaches zero, the solutions tend to approach the limiting analytic form $1/E_p$, deviating from this form at a value of p which increases as μ decreases. These solutions have been obtained using the full coupled-channel bound-state equation with the off-diagonal matrix elements included. As Fig. 5 shows, there are only the smallest changes in these curves when only the diagonal matrix elements, V_{11} and V_{22}, are kept. This occurs for two reasons. In the case of lightly bound systems, the second channel is small, as is its mixing with the first channel, and the bound-state equation reduces to effectively a relativistic, one-channel Schrodinger equation. In the deeply bound limit, the confining potential is decoupling from the problem, and so once again the contributions from V_{12} and

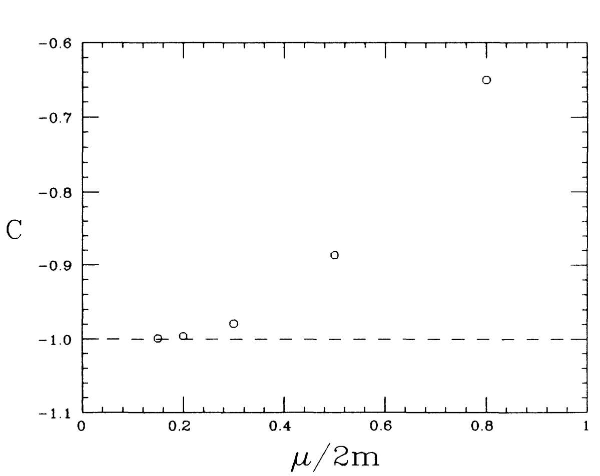

FIG. 3. Solutions for the constant C as a function of $\mu/2m$.

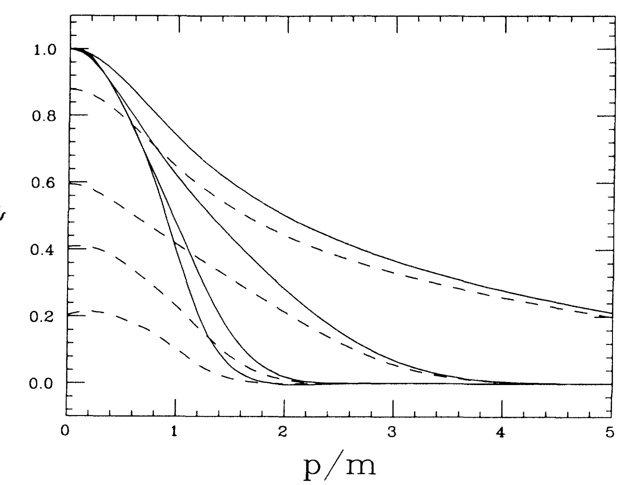

FIG. 4. Solutions for the wave functions Ψ_1 (solid curves) and Ψ_2 (dashed curves) for a family of bound-state masses: $\mu/2m=0.8$, 0.5, 0.3, and 0.15. The states with a smaller mass μ are more spread out in p.

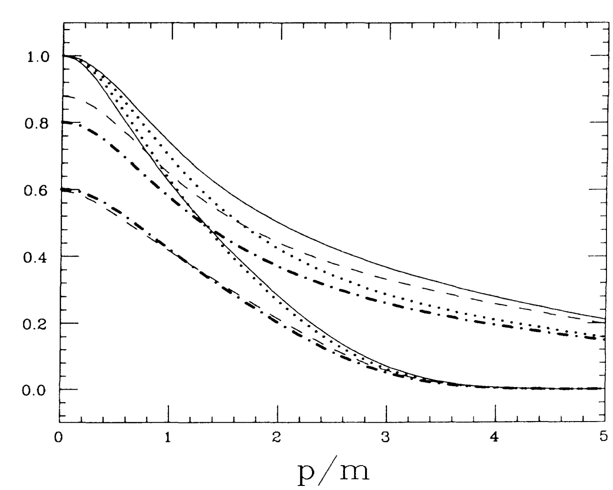

FIG. 5. Solutions for the case $\mu/2m=0.3$ and 0.15, with and without the off-diagonal potentials, V_{12} and V_{21}. In both cases, the wave functions Ψ_1 (solid curves) and Ψ_2 (dashed curves) are the full solutions (identical to Fig. 4), while Ψ_1 (dotted curves) and Ψ_2 (dashed-dotted curves) are the solutions with V_{12} and $V_{21}=0$.

V_{21} are very small. These observations, which justify ignoring the off-diagonal potentials in all cases, will prove useful in future work. When the off-diagonal potentials can be ignored, it is easy to include the one-gluon-exchange interaction in a gauge-invariant manner.

With these results, the presentation begun in Ref. [9] is now complete. In the next section we show how the model can be generalized.

IV. DECOUPLING CONFINEMENT FROM CHIRAL-SYMMETRY BREAKING

Lattice studies and recent phenomenological fits to the upsilon spectrum suggest that the linear potential is purely scalar. In this section we show that, as long as the constant part of the interaction is chirally invariant, a zero-mass pion arises in the chiral limit ($m_0=0$), and that an analytic solution for this state can be found. To show this we will demonstrate that (i) the scalar part of the mass equation is unaffected by the presence of the scalar linear potential, and (ii) the scalar confining potential gives no contribution to the pion bound-state equation in the chiral limit, $\mu=0$.

First, the Dyson equation for the self-energy for an on-shell quark (with the internal quark also restricted to its mass shell), is

$$\Sigma^S=\int\frac{d^3k}{(2\pi)^3 2E_k}V_L(p-k)\frac{m_0+\Sigma^S}{(1-\Sigma^V)^2}\ ,$$
$$m\Sigma^V=\int\frac{d^3k}{(2\pi)^3 2E_k}V_L(p-k)\frac{E_k}{1-\Sigma_V}\ , \tag{8}$$

where Σ^S and Σ^V are the scalar and vector parts of the self-energy, at the constituent quark pole. Since these are constants, the constraint (2) guarantees that the scalar term, Σ^S, is zero. (Note that the vector part is not zero, but can be removed by wave-function renormalization [9].) Hence the linear potential does not contribute to the quark mass, as anticipated in Sec. II. It is clear that this result will hold for many choices of the spin invariants $\tilde{O}$; only ones which contain momentum-dependent factors can upset this general result.

Next, from Fig. 2, the effect of a scalar linear potential on a pure pseudoscalar bound state $\Gamma=\gamma_5\Gamma_0$, where Γ_0 is a scalar function, is

$$\Gamma_{11}^{SC}(p,P)=\int\frac{-d^3k}{(2\pi)^3 2E_k}[V_L(p-k)]_{11}\Gamma_0^1(k^2,P^2)\times\frac{(\not k+\tfrac{1}{2}\not P+m)\gamma_5(\not k-\tfrac{1}{2}\not P+m)}{[m^2-(k-\tfrac{1}{2}P)^2]}\ , \tag{9a}$$

where both the external and internal quarks are on shell (denoted by the subscript 1), and by

$$\Gamma_{12}^{SC}(p,P)=\int\frac{-d^3k}{(2\pi)^3 2E_k}[V_L(p-k)]_{12}\Gamma_0^2(k^2,P^2)\times\frac{(\not k+\tfrac{1}{2}\not P+m)\gamma_5(\not k-\tfrac{1}{2}\not P+m)}{[m^2-(k+\tfrac{1}{2}P)^2]}\ , \tag{9b}$$

when the external quark is on shell and the internal antiquark is on its negative-energy mass shell (2). In the first case $k^\mu=(E_k-\tfrac{1}{2}\mu,\mathbf{k})$, and in the second $k^\mu=(E_k+\tfrac{1}{2}\mu,\mathbf{k})$. Now, for a zero-mass bound state, $P=(\mu,0)\to 0$, and this limit may be studied for each of the terms (9) using the relation

$$(m+\not k+\tfrac{1}{2}\not P)\gamma_5(m+\not k-\tfrac{1}{2}\not P)=[m^2-k^2+\tfrac{1}{4}P^2]\gamma_5-m\gamma_5\not P+\tfrac{1}{2}\gamma_5(\not k\not P-\not P\not k)\ . \tag{10}$$

In this limit, $\Gamma_0^1=\Gamma_0^2$ and $[V_L]_{11}=[V_L]_{12}$ (because they are identical), and the remaining terms approach

$$\lim_{P\to 0}\frac{(m+\not k+\tfrac{1}{2}\not P)\gamma_5(m+\not k-\tfrac{1}{2}\not P)}{m^2-(k-\tfrac{1}{2}P)^2}=\gamma_5\left[\frac{1}{2}-\frac{m}{2E_k}\gamma_0+\frac{\not k\gamma_0-\gamma_0\not k}{4E_k}\right]\ , \tag{11a}$$

$$\lim_{P\to 0}\frac{(m+\not k+\tfrac{1}{2}\not P)\gamma_5(m+\not k-\tfrac{1}{2}\not P)}{m^2-(k+\tfrac{1}{2}P)^2}=\gamma_5\left[\frac{1}{2}+\frac{m}{2E_k}\gamma_0-\frac{\not k\gamma_0-\gamma_0\not k}{4E_k}\right]\ , \tag{11b}$$

where the first relation applies to $\Gamma_{11}^{SC}(p,P)$, and the second to $\Gamma_{12}^{SC}(p,P)$. Note that these terms are regular in this limit. The total contribution from the linear potential is the sum of these two terms, and is therefore

$$\Gamma^{SC}(p,0)=\gamma_5\int\frac{-d^3k}{(2\pi)^3 2E_k}V_L(p-k)\Gamma_0(k^2,0) \tag{12}$$

because the other spin invariants cancel.

Equation (12) shows that, in the $P\to 0$ limit, the scalar linear gives a vertex function with a pure γ_5 structure. Furthermore, since the k^2 in the argument of Γ_0 is the square of the *four*-vector (because Γ_0 is a Lorentz-invariant function), when $P=0$, $k^2=m^2$, and Γ_0 is necessarily a constant. In this case, (12) reduces to the constraint (2), proving that the scalar potential does not contribute to the pion equation in the limit $P=0$. The decoupling is hence complete.

V. CONCLUSIONS

This paper completes the model study begun in Ref. [9], and discusses some generalizations of that work. We find that solutions for light quarks (or a large linear potential, which is by scaling the same thing) require a form factor dependent on the mass of the off-shell quark in order to converge sufficiently rapidly to be consistent with chiral symmetry. With this added convergence, we are able to map out the light-quark solutions just as we did for heavy quarks in Ref. [9]. Illustrative numbers show that it is easy in this model to explain why the symmetry breaking represented by a current quark mass of a few MeV can generate a pion bound state with a mass of 140 MeV.

We also show that the model is sufficiently flexible to permit the linear confining potential to completely break chiral symmetry without changing the connection between the generation of dynamical quark mass and the emergence of an almost-zero-mass Goldstone boson (the pion) which is an essential feature of this model. In particular, we show that the linear part of the confining potential could be pure scalar without altering the chiral properties of the model. The reason for this surprising result is that the constraint which controls the infrared behavior of the linear potential also ensures that it does not contribute in the chiral limit.

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