PHYSICAL REVIEW D VOLUME 45, NUMBER 3 1 FEBRUARY 1992

Resolving ambiguity in *CP*-violation parameters

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(Received 9 September 1991; revised manuscript received 15 October 1991)

Using present data on charmless B-meson decays, B-$\bar{B}$ mixing, and CP-violating K-$\bar{K}$ mixing, and the constraint $m_t = 140 \pm 40$ GeV/c^2 derived from a fit to electroweak data, we extract a value of f_B (the B-meson decay constant), and obtain corresponding regions of parameters for elements of the Cabibbo-Kobayashi-Maskawa matrix. Two solutions are found: one with $f_B \approx f_\pi$ (similar to values obtained in potential models) and the other with $f_B \approx 1.7 f_\pi$ (closer to the large values favored in recent lattice calculations). Prospects for resolving the ambiguity via measurements of the $B \to \tau\nu$ and $K^+ \to \pi^+ \nu\bar{\nu}$ branching ratios, B_s-$\bar{B}_s$ mixing, CP violation in $B \to J/\psi K_S$ and $B \to \pi\pi$, and ϵ'/ϵ in the kaon system are discussed.

PACS number(s): 12.15.Ff, 11.30.Er, 13.20.Eb, 14.40.Jz

I. INTRODUCTION

Until now, CP violation has been observed in the laboratory only in systems of neutral kaons. The favored source of this phenomenon—a phase in the Cabibbo-Kobayashi-Maskawa [1] (CKM) quark-mixing matrix—entails many other effects, whose magnitude can depend sensitively on CKM parameters not yet well specified. Definitive tests of the CKM theory of CP violation must await more precise information about these parameters.

One source of ambiguity in extracting CKM information from known experimental quantities has been the unknown value of the top-quark mass. The analyses of Refs. [2] and [3], for example, found two preferred ranges, separated by a slightly less likely region around 160–200 GeV/c^2. It now appears that the absence of large radiative corrections to electroweak processes [4] suggests $m_t \lesssim 200$ GeV/c^2. On the other hand, recent lattice-gauge-theory calculations [5] suggest that the B-meson decay constant f_B may be considerably larger than the value $f_B \approx f_\pi$ assumed in Refs. [2] and [3] and many other analyses. As a result, there have appeared some recent evaluations of CKM matrix elements [6–8] in which m_t is limited to some value below about 200 GeV/c^2 but the value of f_B is left unspecified. Reference [9] presents results for specific values of m_t and a rather generous range of f_B.

Treating f_B as unknown, two favored ranges of values for it emerge from fits to the data. One, $f_B \approx f\pi$ is in agreement with values obtained from heavy-quark symmetry and extrapolation from the D-meson system [10–12], while the other, $f_B \approx (1.7-2) f_\pi$, is more like what has been obtained from the recent lattice calculations [6,13]. Thus, both experiment and theory speak at present with two voices on the question. The ambiguity is translated into an ambiguity for ranges of CKM parameters, whose resolution offers some interesting experimental challenges.

It is the purpose of the present work to investigate ways of resolving the ambiguities in f_B and CKM parameters so that incisive tests of our present understanding of CP violation can go forward. These means include measurements of the $B \to \tau\nu$ and $K^+ \to \pi^+ \nu\bar{\nu}$ branching ratios, B_s-$\bar{B}_s$ mixing, CP violation in $B \to J/\psi K_S$ and $B \to \pi\pi$, and ϵ'/ϵ in the kaon system. While none of these processes is new, we regard a unified discussion as helpful in focusing experimental attention on the problem. The present article is also intended as an updated version of Ref. [2] in the context of narrower m_t and wider f_B ranges.

In Sec. II we recapitulate the ambiguous nature of the solution for CKM parameters. Possible ways to resolve the ambiguity are discussed in Sec. III. We conclude in Sec. IV.

II. CKM PARAMETERS

The ways in which CKM matrix elements are determined from the data have been summarized at length [2,3,6–9,14], so we mention only the main points. We shall assume three-generation unitarity, so that the CKM matrix V is 3×3, satisfying $V^\dagger V = 1$. Errors on the elements in the upper 2×2 submatrix may be neglected: $V_{us} \equiv \lambda \approx -V_{cd} \approx 0.22$, $V_{ud} \approx V_{cs} \approx 1 - \lambda^2/2$. One can write [15] $V_{cb} \equiv A\lambda^2 \approx -V_{ts}$ where, from the lifetime of hadrons containing b quarks and details of semileptonic b decays to charm, one infers [16] $A = 0.85 \pm 0.09$. The relative rates for b decays to final states without and with charm [17] imply [9] $|V_{ub}/V_{cb}| = 0.11 \pm 0.05$. The statistical precision of the last quantity is better than would be inferred from the error, which is dominated by theoretical model dependence.

The remaining uncertainties in the CKM matrix may be visualized by writing the elements in its upper right and lower left corners as [15] $V_{ub} = A\lambda^3(\rho - i\eta)$, $V_{td} = A\lambda^3(1 - \rho - i\eta)$. The four free parameters of the CKM matrix may then be regarded as λ, A, ρ, and η, with the major uncertainties confined to the last two. As we shall see, two main regions of (ρ, η) emerge from fits to data.

Additional information is provided by the contribution of box diagrams to B-$\bar{B}$ mixing and CP-violating $K\bar{K}$ mixing. Internal top-quark lines dominate the first process almost exclusively, while they provide the leading contribution to the second for large ($\gtrsim 100$ GeV$/c^2$) top-quark masses.

From B-$\bar{B}$ mixing, with [18] $x_d \equiv (\Delta m/\Gamma)|_{B_d} = 0.67 \pm 0.15$, $\tau_B = 1.23 \pm 0.09$ ps [8], and other parameters chosen as in Ref. [2], we find (cf. the analogous expression in Ref. [2]) that

$$m_t A \sqrt{F(m_t^2/M_W^2)}[(1-\rho)^2+\eta^2]^{1/2}(f_B/f_\pi) = 134 \pm 16 \text{ GeV}/c^2 , \quad (1)$$

where $f_\pi = 131$ MeV, while from the value of ϵ in the kaon system we would find the limit of large m_t that

$$m_t A^2 \sqrt{F(m_t^2/M_W^2)} \sqrt{\eta(1-\rho)} \approx 54 \text{ GeV}/c^2 \left[\frac{0.8}{B_K} \right]^{1/2} . \quad (2)$$

Here $F(x)$ is the function [19]

$$F(x) \equiv \frac{1}{4} \left[1 + \frac{3-9x}{(x-1)^2} + \frac{6x^2 \ln x}{(x-1)^3} \right] . \quad (3)$$

With $B_K = 0.8 \pm 0.2$, which we shall assume here (see Sec. V F of Ref. [2]), the right-hand side of (2) contains an error of ± 7 GeV$/c^2$. We have assumed $B_B = 1$ for the vacuum-saturation factor associated with the matrix element of the loop diagram for B-$\bar{B}$ mixing. Since only the product $f_B^2 B_B$ is determined from the mixing, one should rescale f_B accordingly for any other choice of B_B. Other parameters not mentioned explicitly will be taken as in Ref. [2].

If $f_B = f_\pi$, the quotient of Eqs. (1) and (2) would lead to $\mathrm{Arg}(V_{td}) = -(14 \pm 6)^\circ$, independent of the top-quark mass. This would correspond to a band of approximately fixed slope in the (ρ,η) plane, intersecting the circular band $(\rho^2+\eta^2)^{1/2} = 0.50 \pm 0.23$ associated with the constraint on $|V_{ub}/V_{cd}|$ in two regions. One region, with $\rho < 0$, corresponds to top quark masses below 200 GeV$/c^2$, while the other corresponds to heavier top quarks.

Inclusion of charmed-quark contributions to Eq. (2) does not change these qualitative conclusions [2]. In our subsequent analysis, as in Ref. [2], we include such contributions, taking account of $c\bar{c}$, $c\bar{t}+t\bar{c}$, and $t\bar{t}$ intermediate states in loop diagrams for ϵ.

It has now become likely, on the basis of fits to electroweak data [4], that the top quark indeed lies below 200 GeV$/c^2$, permitting in principle a choice between the two regions mentioned above. On the other hand [6–8], our knowledge of f_B may be considerably less precise than earlier estimates. We thus can turn the above analysis into a means of determining f_B, given only rather weak constraints on the top quark mass [6–8]. We shall take present electroweak constraints to imply $m_t = 140 \pm 40$ GeV$/c^2$; significant improvements on these bounds may be possible if the top quark is discovered within the next few years.

If we fix the top quark mass, Eq. (2) specifies a hyperbola in the (ρ,η) plane. This curve cuts a semicircle of constant $\rho^2+\eta^2$ in two points: one (in general) for $\rho < 0$ and the other for $\rho > 0$. Given Eq. (1), these points then imply very different values for f_B. This qualitative argument is borne out by a more precise analysis.

In Table I we summarize the inputs X_i to a χ^2 fitting program. We represent these inputs schematically as $X_i(m_t, A, \rho, \eta) = D_i \pm \sigma_i$, where D_i denote central values and σ_i denote errors. The results for χ^2 using the first five inputs of Table I are shown in Fig. 1 by the solid curve. The two exact solutions correspond (in a convention in which $f_\pi = 131$ MeV) to $f_B = 130.9$ MeV [with $(\rho,\eta) = (-0.43, 0.26)$] and $f_B = 225$ MeV [with $(\rho,\eta) = (0.27, 0.42)$]. The region around $f_B = 180$ MeV is slightly disfavored with respect to lower or higher values. This dichotomy would be accentuated, but only slightly, if the error on the top quark mass were reduced, as indicated by the dashed lines in Fig. 1. An increase in the central value of m_t would correspond to a decrease in the overall f_B scale, and vice versa.

Two regions of f_B were also obtained in the fits of Refs. [7] and [8]. In both works, the errors on $|V_{ub}/V_{cb}|$ were taken to be smaller than here, but in Ref. [8] a distinction was drawn among values of $|V_{ub}/V_{cb}|$ extracted from different models [20] of hadronization in $b \to u l \nu$ decay. We have chosen to include the systematic error for $|V_{ub}/V_{cb}|$ in our total error. The dichotomy between low-f_B and high-f_B regions is accentuated for larger values of $|V_{ub}/V_{cb}|$ and reduced for smaller values [8].

With the parameters we have chosen, there is not a crisp separation between the low-f_B and high-f_B solutions. This separation is enhanced in Refs. [7] and [8] for two main reasons. First, smaller errors are taken there on $|V_{ub}/V_{cb}|$. Second, and more importantly, a larger

TABLE I. Inputs to a fit for parameters of the CKM matrix.

Experiment	Quantity	Value
Top-quark mass	m_t (GeV$/c^2$)	140 ± 40
b-quark decays to charm	$A \equiv \|V_{cb}\|/\|V_{us}\|^2$	0.85 ± 0.09
Charmless b decays	$(\rho^2+\eta^2)^{1/2}$	0.50 ± 0.23
B-$\bar{B}$ mixing	Eq. (1) LHS (GeV$/c^2$)	134 ± 16
CP-violating K-$\bar{K}$ mixing	Eq. (2) LHS (GeV$/c^2$)[a]	54 ± 7
B_s-$\bar{B}_s$ mixing[b]	$(x_s/x_d)\lambda^2[(1-\rho)^2+\eta^2]$	1.6 ± 0.4

[a]Corrected for contributions of charmed quarks in loop diagrams (See Ref. [2].)
[b]This input used only for determining the contours in Fig. 4.

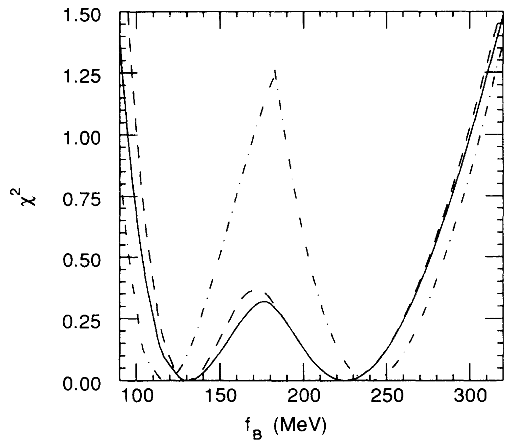

FIG. 1. Behavior of χ^2 as function of f_B for a simultanous fit to m_t, A, ρ, and η with parameters as chosen in Table I (solid curves); as chosen in Table I except $m_t=140\pm20$ GeV/c^2 (dashed curve), and as chosen in Table I except $A=0.93\pm0.10$ and $|V_{ub}/V_{cb}|=0.11\pm0.03$ (dot-dashed curve).

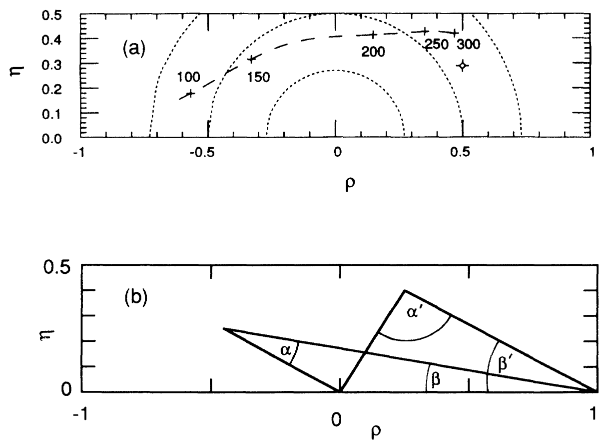

FIG. 2. (a) Regions of the (ρ,η) plane specified by χ^2 minima for specific values of f_B (labels on dashed curve). The dotted semicircles denote central value and 1σ limits associated with the $|V_{ub}/V_{cb}|$ constraint. The isolated plotted point corresponds to the conjecture of Ref. [21]. (b) Comparison of shapes of unitarity triangles for the solutions with $\chi^2=0$. The unprimed and primed angles correspond to the low- and high-f_B solutions, respectively.

value of A than ours is taken. For example, Ref. [8] assumes $V_{cb}=0.045\pm0.005$, which with $V_{us}=0.22$ corresponds to $A=0.93\pm0.10$. Because of the strong A dependence of Eq. (2), a larger value of A corresponds to a hyperbola in the (ρ,η) plane which lies closer to its focus, and hence cuts the circular band of fixed $\rho^2+\eta^2$ at lower values of η and more widely separated values of ρ.

If we take the error on V_{ub}/V_{cb} to be ±0.03, the parameter A to be 0.93 ± 0.10, and other parameters as in Table I, the result is shown in Fig. 1 by the dot-dashed line. The values of f_B corresponding to $\chi^2=0$ are somewhat farther from one another, and are separated by a region where $\chi^2>1$.

We have taken somewhat larger errors on $B_K=0.8\pm0.2$ than in Ref. [8] ($B_K=0.8\pm0.1$). Reduction of the errors on B_K also accentuates the distinction between low-f_B and high-f_B regions.

The locus of points in the (ρ,η) plane with a minimum χ^2 for specific choices of f_B is shown in Fig. 2(a). The unitarity triangles corresponding to the two points with $\chi^2=0$, shown in Fig. 2(b), are of very different shapes. The solutions with high f_B are compatible with the speculation in Ref. [21] that $\rho=\frac{1}{2}$, $\eta=1/(2\sqrt{3})$ [the isolated point plotted in Fig. 2(a)]. In contrast with the remark made in Ref. [22], one does not need a very high value of the top quark mass in order for this speculation to hold as long as f_B is high.

The confidence levels associated with the fits were evaluated via Monte Carlo calculations, loosely following the procedures suggested in Refs. [23]. In the (ρ,η) plane, for instance, the constraints describe contours of the function

$$\chi^2(\rho,\eta)=\min_{(m_t,A)}\sum_{i=1}^{5}\frac{[X_i(m_t,A,\rho,\eta)-D_i]^2}{\sigma_i^2}\ . \tag{4}$$

We find the correspondence between confidence levels and χ^2 height by first generating thousands of "experimental" data sets $(d_i)_\alpha$; these sets are Gaussian distributed about the set D_i with width σ_i. Then the values $(\rho_\alpha,\eta_\alpha)$ minimizing the function

$$\tilde{\chi}^2_\alpha(\rho,\eta)=\min_{(m_t,A)}\sum_{i=1}^{5}\frac{[X_i(m_t,A,\rho,\eta)-(d_i)_\alpha]^2}{\sigma_i^2} \tag{5}$$

are computed. These are then substituted into the origi-

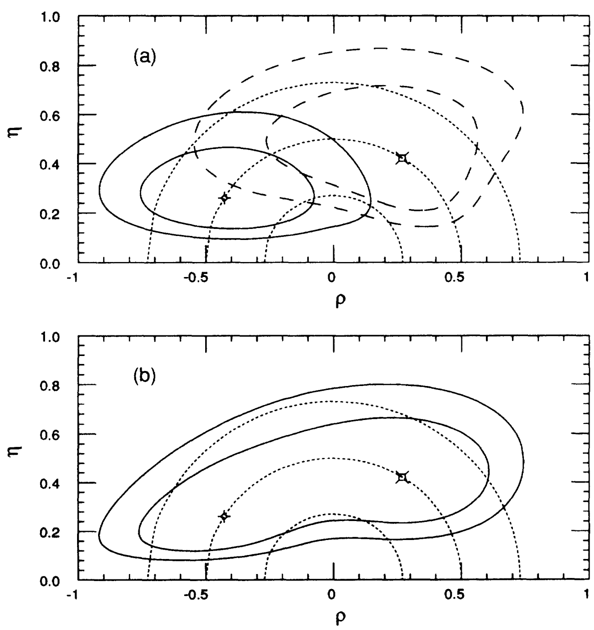

FIG. 3. Contours of 68% (inner curves) and 90% (outer curves) confidence levels for regions in the (ρ,η) plane. Dotted semicircles as in Fig. 2(a). Plotted points denote solutions with $\chi^2=0$. (a) Solid curves: $f_B=131$ MeV; dashed curves: $f_B=225$ MeV. (b) Solid curves: f_B unspecified.

nal function (4) to generate a distribution $\chi^2(\rho_\alpha,\eta_\alpha)$, from which one can determine which χ^2 values lie at the 90 and 68.3 percent confidence levels. Our assumption that the experimental errors are Gaussian distributed is not valid for all quantities; some of the uncertainties are due to systematic errors. A more detailed and sophisticated treatment of errors would presumably not change our results qualitatively.

We compare contours of fixed confidence levels when f_B is fixed [Fig. 3(a)] and when it is unspecified [Fig. 3(b)]. Even for fixed values of f_B, we find considerable overlap of allowed regions at the 90% confidence level. When f_B is not specified, an enormous region of the (ρ,η) plane is allowed.

III. RESOLVING THE AMBIGUITY

We now discuss ways, summarized in Table II, to reduce the uncertainty of parameters.

(1) *The leptonic decays of the charged B* are expected to have branching ratios

$$B(B\to\mu^+\nu_\mu)=(2.6\times10^{-7})(f_B/f_\pi)^2|V_{ub}/0.005|^2 \tag{6a}$$

and

$$B(B\to\tau^+\nu_\tau)=(5.7\times10^{-5})(f_B/f_\pi)^2|V_{ub}/0.005|^2 \ . \tag{6b}$$

The predicted branching ratios for the solutions with $f_B=f_\pi$ and $f_B=1.7f_\pi$ differ by about a factor of 3. Reduction of the systematic error in V_{ub} is needed in order to make full use of this measurement.

(2) *The rate for B_s-$\bar{B}_s$ mixing* is predicted to be

$$x_s\equiv(\Delta m/\Gamma)|_{B_s}=(\Delta m/\Gamma)]_{B_d}(f_{B_s}/f_B)^2|V_{ts}/V_{td}|^2 \ .$$

The error in the last factor is dominated by that in m_t, so reduction of the m_t error (say, by direct observation of the top quark) will have a significant impact. The values shown in Table II are based on the range $f_{B_s}^2=(1.6\pm0.4)f_B^2$ obtained in various estimates in the literature. Only direct measurements of x_s (through oscillations) will be able to distinguish between the two predictions. We show in Fig. 4 the correlation between x_s and m_t for two different choices of f_B. The parameter $\chi_s\equiv x_s^2/[2(1+x_s^2)]$, which expresses the ratio of "wrong-sign" to "right-sign" semileptonic decays, is expected to be very close to $\frac{1}{2}$ for either choice of f_B. (For some previous discussions of mixing, see Refs. [24].)

(3) *The prediction for the $K^+\to\pi^+\nu\bar{\nu}$ branching ratio* depends on m_t and V_{td}, as well as a charmed-quark contribution which is subject to QCD corrections. In Fig. 5 we show contours of values of the predicted branching ratio as functions of ρ and η for several m_t values. We have used the expression [25]

$$B(K^+\to\pi^+\nu\bar{\nu})=2.06\times10^{-6}|\eta_c D(x_c)+\eta_t D(x_t)(A\lambda^2)^2(1-\rho-i\eta)|^2 \ , \tag{7}$$

where

$$D(x)=\frac{x(2+x)}{4(1-x)}+\frac{3x(2-x)}{4(1-x)^2}\ln x \ ; \tag{8}$$

$x_i\equiv(m_i/M_W)^2$, and we take $M_W=80.14$ GeV/c^2. We have chosen a QCD correction factor [25] of $\frac{2}{3}$ for the total contribution of the charmed quark loop. We have neglected the difference between flavors of neutrinos, presenting our results for three flavors. Previous work stressing the importance of this decay appears, for example, in Refs. [26] and [27].

As seen from Table II, the branching ratio is sensitive to f_B. For larger f_B, one can reproduce the observed x_d with a smaller value of $|V_{td}|$, and hence a smaller branching ratio for $K^+\to\pi^+\nu\bar{\nu}$ is predicted.

The values quoted in Table II for $A=0.85$ and $m_c=1.5$ GeV/c^2 are sensitive to these parameters. Examples of this sensitivity are shown in Table III for the values of (ρ,η) corresponding to the low-f_B and high-f_B $\chi^2=0$ solutions.

(4) *The ratio ϵ'/ϵ* is a crude indicator of the value of η for m_t values which are not too large [3,27]. We are led to expect larger values of η for larger f_B, as one sees in Figs. 2 and 3(a). The corresponding predicted range [7] of ϵ'/ϵ includes larger values for large f_B (see Table II).

The experimental values $\epsilon'/\epsilon=(6.0\pm6.9)\times10^{-4}$ (Fermilab E731, Ref. [28]) and $(23\pm7)\times10^{-4}$ (CERN NA31, Ref. [29]) are sufficiently different from one another that it is premature to conclude which solution they favor. Moreover, it has been noted [3,30] that ϵ'/ϵ may be sub-

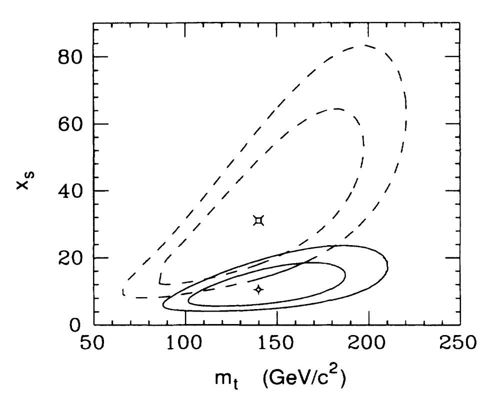

FIG. 4. Contours of 68% (inner curves) and 90% (outer curves) confidence levels for regions in the (m_t,x_s) plane, where $x_s\equiv(\Delta m/T)|_{B_s}$. Plotted points denote solutions with $\chi^2=0$. Solid curves: $f_B=131$ MeV; dashed curves: $f_B=225$ MeV.

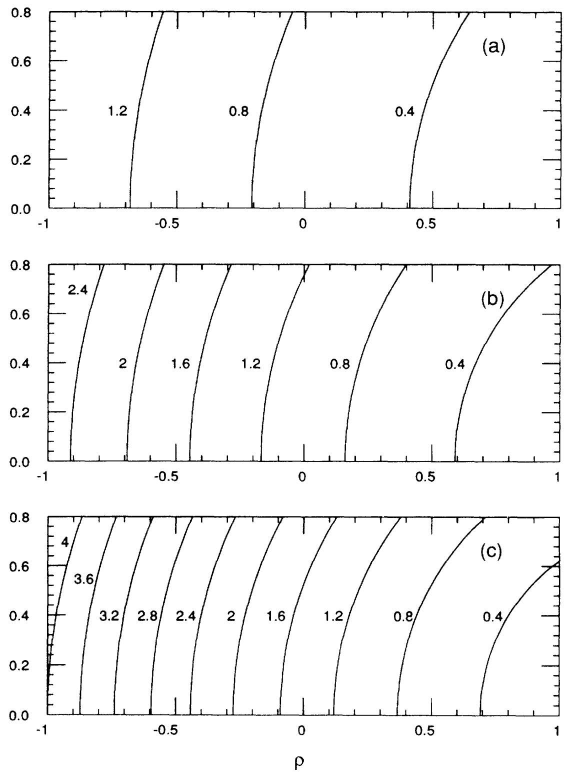

FIG. 5. Contours of predicted values $B(K^+ \to \pi^+ \nu\bar{\nu})$ (summed over neutrino species) in units of 10^{-10} (labels on curves) as functions of ρ and η for (a) $m_t = 100$ GeV/c^2; (b) $m_t = 140$ GeV/c^2; (c) $m_t = 180$ GeV/c^2. Here $A = 0.85$ and $m_c = 1.5$ GeV/c^2.

ject to large corrections in the standard model which could actually drive it negative for m_t above about 170 GeV/c^2. This point is subject to some recent dispute [31]. A naive estimate [32] in the absence of such corrections gives $\epsilon'/\epsilon = (\frac{1}{2} \text{ to } 1) \times 10^{-2}\eta$. When combined with the world average $\epsilon'/\epsilon = (15 \pm 5) \times 10^{-4}$, this would imply $\eta = (0.2) \times (2)^{\pm 1}$, which would exclude a portion of the regions in Figs. 3.

A nonzero value of ϵ'/ϵ exceeding a few parts in 10^4 could add to our confidence in the standard CKM picture of *CP* violation, but neither theory nor experiment agrees yet on the question.

(5) *CP violation in B decays* is sensitive to the angles in the unitarity triangle of Fig. 2(b). Standard analyses [33] lead one to expect the following time-integrated partial rate asymmetries

$$A(f) \equiv \frac{\Gamma(B^0_{t=0} \to f) - \Gamma(\bar{B}^0_{t=0} \to f)}{\Gamma(B^0_{t=0} \to f) + \Gamma(\bar{B}^0_{t=0} \to f)} \tag{9}$$

for decays of states which are B^0 and $\bar{B}^0$ at time $t = 0$:

$$A(J/\psi K_S) = -\frac{x_d}{1+x_d^2}\sin 2\beta , \tag{10a}$$

$$A(\pi^+\pi^-) = -\frac{x_d}{1+x_d^2}\sin 2\alpha . \tag{10b}$$

TABLE II. Comparison of predictions for low-f_B and high-f_B solutions. These estimates are for the values $A = 0.85$ and $m_c = 1.5$ GeV/c^2.

Quantity	$f_B = 131$ MeV	$f_B = 225$ MeV
$B(B^+ \to \mu^+ \nu_\mu)(\times 10^{-7})$	2.6[a]	7.3[a]
$B(B^+ \to \tau^+ \nu_\tau)(\times 10^{-4})$	0.57[a]	1.7[a]
x_s	4 to 24	>8
$B(K^+ \to \pi^+ \nu\bar{\nu})(\times 10^{-10})$	1.6[b]	0.8[b]
ϵ'/ϵ $(\times 10^{-4})$	1 to 21[c]	1 to 28[c]
	0 to 13[d]	0 to 20[d]
	-2 to 9[e]	-4 to 15[e]
$A(J/\psi K_S)$	-0.17[f]	-0.41[f]
$A(\pi^+\pi^-)$	-0.29[f]	-0.04[f]
Theory of f_B	Heavy quark	Lattice
Form of CKM matrix	Cannot be symmetric	Could be symmetric

[a]For $|V_{ub}/V_{cb}| = 0.11$. Proportional to $|V_{ub}|^2$.
[b]Typical value. See Figs. 3 and 5 and Table III for possible ranges.
[c]$m_t = 100$ GeV/c^2.
[d]$m_t = 140$ GeV/c^2.
[e]$m_t = 180$ GeV/c^2.
[f]Typical value. See Figs. 3 and 6 for possible ranges.

Typical values for the two solutions are compared in Table II, and predicted contours of these asymmetries for $x_d = 0.7$ are shown in Fig. 6. The $J/\psi K_S$ asymmetry is usually much more pronounced for the high-f_B solution [7,8], while the $\pi^+\pi^-$ asymmetry tends to be negative for the low-f_B solution and near zero or positive for the high-f_B solution. The case in which the two asymmetries are equal and opposite, with locus of points $\eta = (1-\rho)[\rho/(2-\rho)]^{1/2}$, corresponds to a superweak theory [34]. One must guard against the possibility in $B \to \pi^+\pi^-$ of substantial contributions from penguin graphs, but there exist tests for such effects [35].

(6) *The process* $K_L \to \mu\mu$ (discussed recently in Refs. [36] and [37]) can in principle provide a constraint on the combination $|\mathrm{Re}(V_{ts}^* V_{td})|$; a concise expression for this constraint is given, for example, in Ref. [2]. The issue is how much short-distance contribution to this process is allowed. It is concluded in Ref. [36] by comparing present experiments with the lower bound from unitarity (based on the contribution of a real two-photon intermediate state) that there is very little room for an addi-

TABLE III. Examples of dependence of $B(K^+ \to \pi^+ \nu\bar{\nu})$ (in units of 10^{-10}) on A, m_c, and m_t.

A	m_c (GeV/c^2)	m_t (GeV/c^2)	(ρ,η) $(-0.43,0.26)$	(ρ,η) $(0.27,0.42)$
0.85	1.5	100	0.99	0.51
0.85	1.5	140	1.60	0.76
0.85	1.5	180	2.42	1.07
0.76	1.5	140	1.18	0.59
0.94	1.5	140	2.14	0.96
0.85	1.2	140	1.31	0.57
0.85	1.8	140	1.98	1.01

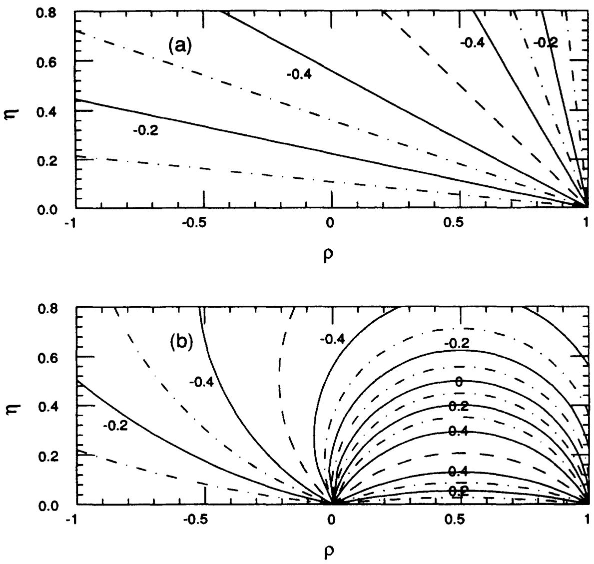

FIG. 6. Contours of predicted partial rate asymmetries $A(f)$ (labels on solid curves) for final states $f=$(a) $J/\psi K_S$; (b) $\pi^+\pi^-$, with $x_d=0.7$. Dashed curves correspond to maximum asymmetries $A(f)=\pm 0.47$. Dot-dashed curves correspond to interpolating values of asymmetries.

tional short-distance contribution, so that restrictive bounds can be placed. The investigation of Ref. [37], however, finds that the dispersive part of the two-photon contribution can be about half (in amplitude) of the absorptive (real two-photon) part, and moreover, that the short-distance contribution is likely to be of opposite sign to the dispersive two-photon contribution. It is the conclusion of Ref. [37] that no useful bounds on the short-distance contribution can be placed as of now. This situation could change if one were able to understand the dispersive two-photon contribution better; a proposal to do this by improved study of the decay $\eta \to \mu\mu$ is under consideration [38].

(7) *Further theoretical information on f_B may be forthcoming from lattice-gauge-theory calculations.* In particular, self-consistency requires that one understand in more detail why the heavy-quark scaling behavior $(f_B/f_D)^2 \approx M_D/M_B$ seems so badly violated by the present lattice calculations [39].

(8) *A genuine theory of quark masses and CKM elements* may provide guidance as to whether, for example, the *CKM* matrix should be representable in symmetric form, as occurs (for example) in Ref. [21]. The large-f_B solution would then be preferred.

IV. CONCLUSIONS

The most important ambiguity in the parameters of the Cabibbo-Kobayashi-Maskawa matrix appears now to be due to uncertainty in the B meson decay constant f_B. One class of solutions is associated with $|V_{td}| > |V_{ub}|$ and values of f_B comparable to f_π, as suggested by extrapolation from f_D using heavy-quark symmetry. The other class of solutions is consistent with $|V_{td}| \approx |V_{ub}|$ and larger values of f_B (nearly twice f_π). Experiments that can shed light on the question in the next few years without further theoretical input include measurements of the branching ratios for $B \to \tau\nu$ and $K^+ \to \pi^+ \nu\bar{\nu}$, B_s-$\bar{B}_s$ mixing, and *CP*-violating asymmetries in $B \to J/\psi K_S$. Similar asymmetries in $B \to \pi^+\pi^-$ may also provide information but must be interpreted with some care. Reduction of the experimental errors on m_t and $|V_{cb}|$ is needed in order to make full use of the impending $K^+ \to \pi^+ \nu\bar{\nu}$ measurement. If the branching ratio turns out to be less than about 10^{-10}, additional effort to resolve the charmed-quark mass dependence of this process will be especially important. More theoretical work is needed to interpret values of ϵ'/ϵ and to resolve directly the discrepancy among different calculations of f_B. Finally, one might hope for an overall theory of the CKM matrix.

Note added in proof We have recently noticed that the predicted ratio $A(\pi^+\pi^-)/A(J/\psi K_S)$ lies between $-\infty$ and 1 for $\rho > 0$, $\eta > 0$, $\rho^2+\eta^2 < 1$, and between 1 and ∞ for $\rho < 0$, $\eta > 0$, $\rho^2+\eta^2 < 1$.

ACKNOWLEDGMENTS

We would like to thank E. Eichten, J. Flynn, G. Martinelli, P. Meyers, R. Peccei, B. Winstein, S. Wojcicki, and J. Yoh for helpful discussions. B. Winstein spotted an error in an earlier version of this manuscript. This work was supported in part by the United States Department of Energy under Grant No. DE FG02 90ER 40560.

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