Hamiltonian model for the Higgs resonance

Charles B. Chiu and E. C. G. Sudarshan
Center for Particle Physics and Department of Physics, University of Texas, Austin, Texas 78712

G. Bhamathi
Department of Physics, University of Texas, Austin, Texas 78712
and North Texas Research Institute, University of North Texas, Denton, Texas 76203
(Received 2 August 1991)

Starting from a field-theoretic expression for the effective potential between two strongly interacting longitudinal components of the vector bosons in the standard electroweak theory, we compute the full amplitude in the two-particle sector based on a relativistic generalization of the Hamiltonian formalism. The Higgs-resonance trajectories are studied as a function of an energy scale and a dimensionless renormalization parameter in the effective potential. An intriguing possibility of a narrow-width Higgs resonance in the 1-TeV region cannot be ruled out from general considerations.

PACS number(s): 12.15.Cc, 11.10.Qr, 12.40.Qq, 14.80.Gt

I. INTRODUCTION

The interaction between the longitudinal components w^+, z, w^- of the "weak" bosons W^+, Z, W^- through the spontaneously broken symmetry via the Higgs boson becomes the dominant interaction at TeV energies; and to a good approximation the gauge interactions may be considered to be secondary [1]. Many authors have investigated the implications on the position of the Higgs resonance in this framework [2]. By virtue of these strong interactions the Higgs boson decays rapidly and in the process acquires a substantial width. Methods useful for narrow-width resonances are inadequate to deal with this situation. A more systematic strong-coupling theory involving the effects of rescattering corrections is necessary.

We construct a Hamiltonian model [3] of the two-meson system. Since total angular momentum ("spin") and weak isospin are constants of motion, the various spin-isospin channels do not mix with each other. For the two-meson system the problem thus reduces to a single-channel problem which is easy to handle. We will primarily be interested in the w^+w^- and zz channels. The weak isospins possible are $I=0$, 2 for the S waves (more generally for even spin) and $I=1$ for the P waves (more generally for odd spin). The Higgs resonance should occur in the $I=0$, S-wave channel.

In nonrelativistic physics an interacting two-particle system is represented by a Hamiltonian in the center-of-mass frame, which consists of the free-particle (reduced) kinetic energy together with an interaction which depends on the relative coordinate, and hence mixes states with different free energies. While local potentials are often used to describe the interaction, velocity-dependent potentials such as the separable potential [4] can also be used. The scattering states are labeled by the energy and are continuum normalized.

When we consider relativistic two-particle systems, the energy function is not an appropriate quantity to consider, since it is not an invariant expression. Bakamjian and Thomas and many other authors [5] have shown that the natural generalization is to consider the effective mass squared, i.e., the square of the energy in the center-of-mass frame, as the counterpart of the nonrelativistic Hamiltonian. The importance of the center of mass in relativistic interactions was emphasized by Eddington [6]. Lorentz invariance of the theory is guaranteed since the little group of the center of mass is the rotation group; and the interaction contribution to the mass operator is taken as a scalar. The angular momentum in the center-of-mass frame is preserved by the modified squared mass operator. So, we could separate out the various spin channels and deal with them one by one.

In many ways, then, the squared mass operator serves the role of the Hamiltonian in nonrelativistic theories; and the problem separates into each (total) angular momentum channel. For spinless particles the effective spin is simply the orbital angular momentum. So the system can be, without any loss of generality, treated in terms of a series of "Hamiltonians" with the states labeled by the center-of-mass energy squared, s. The model we will employ for this purpose will be the separable potential with an interaction obtained from perturbation diagrams which are two-particle irreducible in the direct channel. We choose the separable potential for two reasons: first, the structure of the wave functions and the scattering amplitudes is relatively simple [7] even with the natural constraint of unitarity; second, no generality is lost by using a separable potential since any one channel scattering amplitude can be reproduced by a suitable separable potential [8]. For a strong-coupling problem in which the potential is "known," this Hamiltonian model is most appropriate; we can also perform analytic continuation of the formalism to locate the second-sheet singularities.

II. THE HAMILTONIAN MODEL

We use the center-of-mass energy-squared variable s to label the spin-isospin definite channel states with continuum normalization. The free "Hamiltonian" is then

$$H(s,s')=s\delta(s-s') \ . \tag{1}$$

The interaction is

$$H_1(s,s')=\mp g(s)g(s') \ , \tag{2}$$

where, without loss of generality, $g(s)$ may be chosen real and positive and the minus or plus signs designate the potential to be, respectively, attractive or repulsive. For the S wave, an attractive potential may lead to a resonance while a repulsive potential does not. The continuum wave functions $\phi_\gamma(s)$ satisfy

$$(\gamma-s)\phi_\gamma(s)=\mp g(s)\int g(s')\phi_\gamma(s')ds' \ , \tag{3}$$

with the "in" solutions

$$\phi_\gamma(s)=\delta(\gamma-s)\mp\int g(s')\phi_\gamma(s')ds'\frac{g(s)}{\gamma-s+i\epsilon} \ . \tag{4}$$

The integral on the right-hand side satisfies the algebraic equation

$$\int g(s)\phi_\gamma(s)ds=g(\gamma)\mp\langle g^2(\cdot)\rangle\int g(s')\phi_\gamma(s')ds' \ , \tag{5}$$

so that

$$\int g(s')\phi_\gamma(s')ds'=\frac{g(\gamma)}{1\pm\langle g^2(\cdot)\rangle}=\frac{g(\gamma)}{\beta(\gamma)} \ , \tag{6}$$

where

$$\beta(\gamma)=1\pm\langle g^2(\cdot)\rangle=1\pm\int\frac{g^2(s')ds'}{\gamma-s'+i\epsilon} \ . \tag{7}$$

The scattering amplitude is [7]

$$T(s)=-\pi(\gamma-s)\phi_\gamma(s)|_{s=\gamma}=\pm\frac{\pi g^2(s)}{\beta(s)}$$
$$=\frac{\beta^\dagger(s)-\beta(s)}{2i\beta(s)} \ . \tag{8}$$

The wave functions are complete and normalized except for the possibility of a bound state.

Our primary interest is in the study of the Higgs particle in interaction with the "scalar" mesons w^+w^- and zz. We want to assure ourselves of a model in which relativistic invariance, weak isospin symmetry, and unitarity are incorporated. Our model is defined by Eqs. (7) and (8). We compute the effective separable potential (in the center of mass) using the simplest field-theoretic two-particle "direct channel irreducible" diagrams. This effective potential is the numerator of the exact scattering amplitude, and enters the denominator through its dispersion integral. Given the effective potential calculated to any order in the coupling, the scattering amplitude is computed to all orders in the effective potential. The approximation can be improved by including higher-order diagrams in the cross channel to compute the effective potential.

Note that a consistent computation of the scattering amplitude by the above method must treat the direct and cross channel differently. All two-particle "direct channel irreducible" diagrams are computed up to some prescribed order. Direct channel reducible diagrams enter only by virtue of the denominator function through dispersion integrals; and the entire set of iterations are automatically included. This is somewhat in the spirit of computing the wave functions and energies of the Dirac electron in a Coulomb field. The Coulomb potential is computed to the lowest order in the cross channel, but then the problem is solved exactly. To the lowest order in the potential, by setting $V(s)=H_1(s,s)$, we get

$$T_1(s)=-\pi V(s)=\pm\pi g^2(s) \ , \tag{9}$$

and this can be used to determine $g(s)$. An attractive potential corresponds to a negative V, or $T_1>0$; while for a repulsive potential $V>0$, or $T_1<0$. The integrals $\langle g(\cdot)\rangle$ may be formally divergent. In the spirit of dimensional regularization, we will use the minimally subtracted expression for the integrals.

The present Hamiltonian model furnishes a unitary scattering amplitude generated from a perturbatively computed effective potential. There are other methods of generating unitary scattering amplitudes, such as the K matrix and the Padé approximant [2]. They give different expressions from what we have obtained.

The essence of unitarity is that the inverse of the dimensionless scattering amplitude has the imaginary part -1. The K-matrix unitarization accomplishes this by taking the real part of the inverse K-matrix amplitude T_K to be the inverse of the real part of the starting amplitude T and augmenting it with an imaginary part -1. In this approach, no change is made in the real part of the inverse amplitude, i.e., $\mathrm{Re}(1/T_K)=\mathrm{Re}(1/T)$. Because of this, the K-matrix amplitude is an unreliable approximation though it satisfies unitarity:

$$T\to T_K=[(\mathrm{Re}T)^{-1}-i]^{-1} \ . \tag{10}$$

In contrast, the Padé approximation starts with a perturbative amplitude which satisfies unitarity nontrivially to some order and then innovates on the amplitude. For definiteness let us take the simplest case of the $(1,1)$ Padé approximant starting with the amplitude up to the second order,

$$T=T_1+T_2 \ , \tag{11}$$

with unitarity satisfied to first order,

$$\mathrm{Im}T_1=0, \quad \mathrm{Im}T_2=|T_1|^2 \ . \tag{12}$$

Then the (1,1) Padé approximant

$$T^{(1,1)}=T_1\left[1-\frac{T_2}{T_1}\right]^{-1} \tag{13}$$

is exactly unitary. In this case

$$\mathrm{Re}T^{(1,1)}=\frac{T_1-\mathrm{Re}T_2}{1-2\,\mathrm{Re}(T_2/T_1)+|T_2|^2/T_1^2} \ , \tag{14}$$

which in general, as is known, differs from

$$\mathrm{Re}T=T_1+\mathrm{Re}T_2 \tag{15}$$

in higher-order terms.

In the Hamiltonian scheme

$$T=-\pi V\rightarrow\frac{-\pi V}{1-\langle V\rangle}\ , \tag{16}$$

where $-(1/\pi)\langle V\rangle$ is the dispersion integral with an imaginary part V and a real part which is the Hilbert transform of V. Needless to say, this amplitude is the exact result for the effective potential V.

III. THE HIGGS RESONANCE

In the minimal standard theory, the interaction Lagrangian for the Higgs-scalar sector is given by

$$\mathcal{L}_{\text{Higgs}}=-\frac{\lambda}{4}[2w^+w^-+z^2+(h+v)^2-v^2]^2\ , \tag{17}$$

where the "bare" mass of the Higgs scalar is given by $m^2=2\lambda v^2$.

For the two-scalar-meson system with the w^+w^- and zz channels, the lowest-order t-channel diagrams are indicated in Figs. 1(a) and (b). Together they contribute to the Feynman amplitude:

$$-2\lambda\left[1+\frac{m^2}{t-m^2}\right]=-\frac{m^2}{v^2}\frac{t}{t-m^2}\xrightarrow{m^2\rightarrow\infty}\frac{t}{v^2}\ . \tag{18}$$

We denote the sum of the two by the diagram in Fig. 1(c). There are the corresponding u-channel and s-channel diagrams, respectively, proportional to u and s. The lowest-order effective potential is therefore linear in s,t,u. When the partial waves are projected the t and u dependences become transmuted to s dependences.

The next-order contributions to the effective potential come from the one-loop exchange in the t channel (and/or the u channel) but not from the s channel. The diagrams up to one-loop order contributing to the effective potential are illustrated in Fig. 2.

The one-loop cross-channel diagrams shown in Figs. 2(d) and 2(e) are formally divergent. The loop contribution may be regulated, say through a cutoff, and the subsequent cutoff-dependent polynomials removed through the appropriate counterterms. Finally this one-loop contribution to the s-channel $I=0$, S-wave partial-wave amplitude is

$$T_{1\text{ loop}}\propto s^2\left[\ln\frac{s}{\nu}+R'\right]\ . \tag{19}$$

Here ν is an energy scale parameter and R' is a renormalization constant. Later on ν will be set to some common scale. We will consider both cases, i.e., treating R' as a dependent parameter and treating it as an independent parameter. The s-channel loop is not a part of the effective potential but arises out of rescattering and will appear in the exact solution through a dispersion integral.

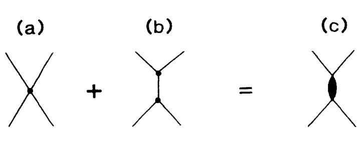

FIG. 1. The t-channel tree diagrams.

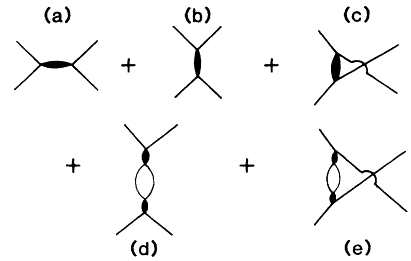

FIG. 2. Perturbation diagrams up to one-loop order contributing to the effective potential.

The contribution of these diagrams for the $I=0$, S-wave amplitude is [9]

$$T_1(s)=\pi g^2(s)=\frac{s}{16\pi v^2}+\frac{s^2}{64\pi v^4}\left[R-\frac{1}{4\pi^2}\frac{7}{18}\ln\frac{s}{\nu}\right]\ , \tag{20}$$

where ν is a scale mass squared which cannot be chosen to be 0 or infinity. The bare Higgs-boson mass m has been taken to be infinite. We use $v=0.246$ TeV as the vacuum expectation value of the Higgs field. For large finite bare mass m, this is modified to

$$T_1(s)=\frac{sm^2}{16\pi v^2(m^2-s)}+\frac{s^2}{64\pi v^4}\frac{m^4}{(m^2-s)^2}\left[R-\frac{1}{4\pi^2}\frac{7}{18}\ln\frac{s}{\nu}\right]\ . \tag{21}$$

Both expressions satisfy the low-energy theorem [10] that $T_1(s)\rightarrow 0$ as $s\rightarrow 0$. Below we will only consider the infinite-m case. Identifying $T_1(s)$ with $-\pi V$, or $+\pi g^2(s)$, we can fix the separable potential and compute the full scattering amplitude according to

$$T(s)=\frac{\pi g^2(s)}{\beta(s)}\ , \tag{22}$$

with

$$\beta=1+\langle g^2(\cdot)\rangle\ . \tag{23}$$

As noted earlier, the s-channel loop is not a part of the effective potential. It arises out of rescattering and appears in the exact solution through a dispersion relation. The number R will be referred to as the renormalization constant, which is related to R' by

$$R=-\frac{1}{4\pi^2}\frac{7}{18}R'\ . \tag{24}$$

At this stage R is a free parameter, but the requirement of the positivity of the imaginary part of the propagator would constrain R to be limited in its absolute value if it is negative. (For small negative values of R the width of the Higgs resonance becomes narrower: see the discussion below.)

Now consider the effective potential at the tree level:

$$T_{1\,\mathrm{tree}}=\frac{s}{16\pi v^2},\quad \beta_{\mathrm{tree}}=1+\frac{1}{(4\pi v)^2}\langle s\rangle \ . \tag{25}$$

The dispersion integral $\langle s\rangle$ may be regulated by a cutoff and the cutoff-dependent polynomial removed through appropriate counterterms. This leads to

$$\beta=1+\frac{s}{(4\pi v)^2}\left[\ln\frac{-s}{v}+R''\right], \tag{26}$$

where R'' is a renormalization constant associated with the evaluation of the dispersion relation.

For the theory defined in Eq. (17), it is natural to relate R'' in Eq. (26) to R' of Eq. (19), since both constants originated from the same loop integrals evaluated in different channels. In particular, notice that for $V-T_{1\,\mathrm{tree}}$, in Eq. (16), the quantity which is second order in the expansion of V, $-\pi V\langle V\rangle$, is the s-channel loop contribution to the S-wave partial-wave amplitude. The parameter R'', which appears as a subtraction constant in the dispersion relation, may also be identified as the renormalization constant in the evaluation of this s-channel loop diagram. It is natural that the same renormalization constant should also appear in the evaluation of the t-channel loop of Fig. 2(d) and the u-channel loop in Fig. 2(e). In other words, in the full amplitude, the logarithm factors which arise from loop integrals in different channels should take the form

$$\ln(-s/v)+R''\ ,$$
$$\ln(-t/v)+R''\ ,$$
$$\ln(-u/v)+R''\ .$$

The t-channel and the u-channel loop contributions to the $I=0$ amplitude may be determined through the application of the present renormalization condition to the general form of the $I=0$ amplitude given in the Appendix. For the present case, the sum of the t-channel and the u-channel loop contributions is given by

$$T_{1\,\mathrm{loop}}(s,t,u)=-\frac{1}{(4\pi v^2)^2}\left[\left[\frac{3t^2}{2}+\frac{u^2-s^2}{6}\right]\left[\ln\frac{-t}{v}+R''\right]+\left[\frac{3u^2}{2}+\frac{t^2-s^2}{6}\right]\left[\ln\frac{-u}{v}+R''\right]\right]. \tag{27}$$

The corresponding S-wave partial-wave amplitude is

$$T_{1\,\mathrm{loop}}(s)=-\frac{1}{64\pi v^4}\frac{s^2}{4\pi^2}\frac{7}{18}\left[\ln\frac{s}{v}+R''-\frac{11}{42}\right]. \tag{28}$$

Comparing Eqs. (19) and (28), we have

$$R'=R''-\tfrac{11}{42}\ . \tag{29}$$

So here R' and R'' should be counted as one independent parameter. Incidentally, as a consistency check, using Eqs. (24) and (29), one sees that the loop contribution to Eq. (28) is the same as that given in Eq. (20).

On the other hand, on general grounds, the renormalization constant associated with a direct channel dispersion relation need not be correlated with specific parameters in the effective potential. In the Appendix we show that in the effective chiral Lagrangian theory the renormalization constant associated with the s-loop contributing to the $I=0$ amplitude, and that associated with the cross-channel loop, are related to different combinations of coefficients in the Lagrangian; i.e., they are independent parameters.

For the sake of generality, we will continue to treat R'' and R' as independent parameters, and at appropriate points we will discuss the implication when they are related through Eq. (29).

The positivity of $T_{1\,\mathrm{tree}}$ implies that the corresponding effective potential is attractive [see Eq. (9)]. Therefore a resonance may be expected. We choose the scale parameter v such that the resonance occurs "near" $s=v$ (i.e., when narrow-width approximation is valid). More specifically, we assume that at $s=v$, $\mathrm{Re}\beta_{\mathrm{tree}}=0$, which gives

$$R''=-\frac{(4\pi v)^2}{v}\ ,$$
$$\beta_{\mathrm{tree}}=1-\frac{s}{v}+\frac{s}{(4\pi v)^2}\ln\left[\frac{-s}{v}\right]. \tag{30}$$

We proceed to evaluate the remainder of the dispersion integral in the denominator function $\beta(s)$. We assume that the real (dispersive) part of each of the remainder contributions to $\beta(s)$ continues to vanish at $s=v$. This implies

$$\mathrm{Re}\langle s^2\rangle\rightarrow s^2\ln\frac{s}{v}\ ,$$
$$\mathrm{Re}\left\langle s^2\ln\frac{s}{v}\right\rangle\rightarrow\frac{s^2}{2}\left[\ln\frac{s}{v}\right]^2. \tag{31}$$

Notice that, in principle, the scaling squared mass in the various integrals could be different; and changing them to a common scale could introduce additional terms according to

$$\ln\frac{s}{v}\rightarrow\ln\frac{s}{v'}+\ln\frac{v'}{v}\ . \tag{32}$$

We have taken the position that all of these integrals have the same common scale v. No additional parameters are then introduced.

Consequently,

$$\beta(s)=1-\frac{s}{\nu}+\frac{1}{(4\pi v)^2}\left\{s\ln\left[\frac{-s}{\nu}\right]+\frac{Rs^2}{4v^2}\ln\left[\frac{-s}{\nu}\right]-\frac{7}{36}\left[\frac{s}{4\pi v}\right]^2\left[\ln^2\left[\frac{-s}{\nu}\right]+\pi^2\right]\right\}, \tag{33}$$

with $\ln(-s/\nu)=\ln(s/\nu)-i\pi$. For the case where R' is related to R'', from Eqs. (24), (29), and (30),

$$R=\frac{14}{9}\frac{v^2}{\nu}+\frac{1}{(4\pi)^2}\frac{11}{27}. \tag{34}$$

We are interested in the s value where the resonance occurs. At this point the denominator function vanishes. A real zero of the denominator function can occur only outside of the spectrum of the free Hamiltonian [7]. In the narrow-width approximation we could locate the approximate position of this zero by looking for the zero of the real part of $\beta(s)$, which occurs at $s=\nu$. However, for the present problem where a large width is involved, we directly search for the complex zero of $\beta(s)$ in the second sheet by means of a numerical method.

IV. NUMERICAL RESULTS

We rewrite the two-parameter expression of (33) as

$$z=1+\sigma z\ln(-z)+4\pi^2R(\sigma z)^2\ln(-z)$$
$$-\tfrac{7}{36}(\sigma z)^2[\ln^2(-z)+\pi^2], \tag{35}$$

where $\sigma=c\nu$, $c=1/(4\pi v)^2$, $z=s/\nu$. The second term on the right-hand side is the dispersion contribution of the lowest-order diagrams. The last two terms are from the cross-channel loops.

To lowest order

$$z=1+\sigma z\ln(-z). \tag{36}$$

This can be rewritten in the form

$$\frac{1}{c\nu}+\ln(c\nu)=\frac{1}{cs}+\ln(cs)-i\pi=\kappa. \tag{37}$$

If $s=re^{i\theta}$, this implies

$$\frac{\cos\theta}{cr}+\ln(cr)=\kappa, \tag{38}$$

$$\frac{\sin\theta}{cr}=\theta-\pi. \tag{39}$$

So

$$\kappa=f(\theta)=(\theta-\pi)\cot\theta+\ln\left[\frac{\sin\theta}{\theta-\pi}\right]. \tag{40}$$

the function $f(\theta)$ is displayed in Fig. 3(a). The relation between κ and $c\nu$ is displayed in Fig. 3(b).

In Fig. 4, the dashed curve shows the solution for the lowest-order equation (36) in the W plane, where $W=\sqrt{s}$, as a function of the square-energy scale ν. [Compare the resonance trajectory of Einhorn [11], which was obtained in the $1/N$ expansion of the O($2N$) model.]

In the narrow-width approximation, the resonance width, defined by $W_{\rm res}=m_{\rm res}-i\Gamma/2$, is given by

$$\Gamma=\frac{m_{\rm res}^3}{16\pi v^2}. \tag{41}$$

This is reminiscent of the expression obtained by Brown and Goble [12] for the $\pi\pi$ system.

We now turn to the results, based on Eq. (35), where the effective potential is computed up to the one-loop level.

Case 1. One independent parameter ν. For this case, R' is related to R'', and the renormalization constant R is completely determined by the parameter ν [see Eq. (34)]. The resonance location as a function of ν is shown in Fig. 4 as the solid curve. Notice that the trend of this solid curve is similar to that of the dashed curve, and at each ν value, the resonance width of the solid curve is persistently larger than that of the dashed curve.

Case 2. Two independent parameters ν and R. This is the case where R' and R'' are not correlated, i.e., ν and R are two independent parameters. The resonance trajectories as a function of ν for the different values of R are shown as the solid curves in Fig. 5. They are to be com-

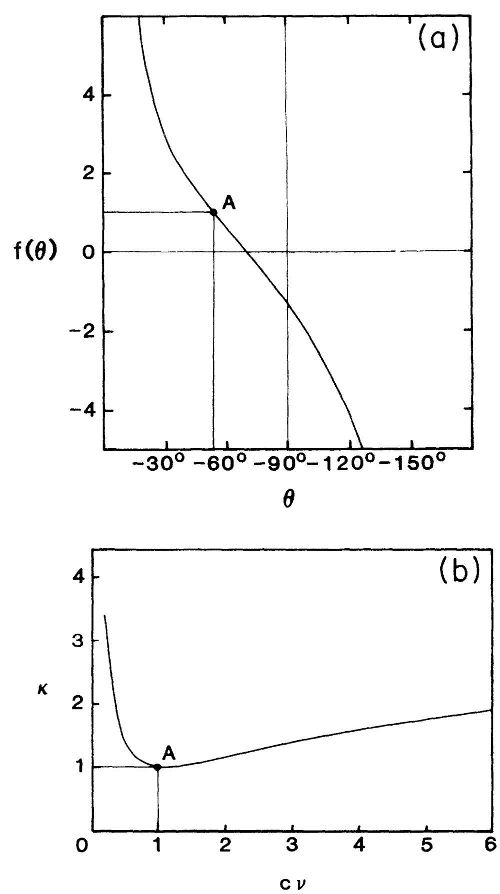

FIG. 3. (a) The θ dependence of the $f(\theta)$ function. Point A is at $f(\theta)=1$. (b) The ν dependence of κ. Point A is at $c\nu=1$.

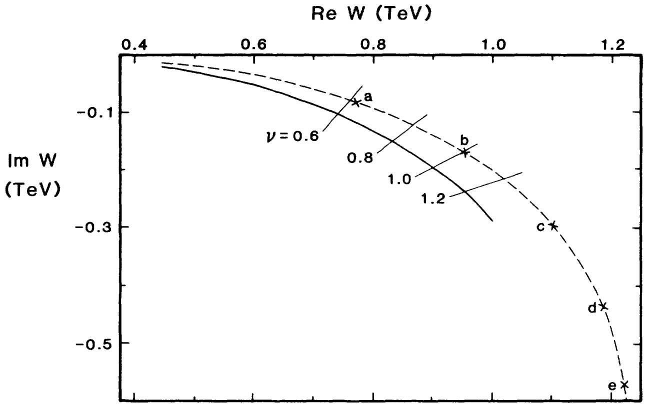

FIG. 4. Resonance trajectories in the complex W plane as a function of the scale ν. The dashed trajectory is for the tree-level effective potential. The solid trajectory is for the loop-level effective potential. For the definition of R see Eq. (34). $\nu=0.6$, 0.8, 1.0, and 1.2 correspond to $\sigma=c\nu=0.06$, 0.08, 0.11, and 0.13. Points a through e will be referred to in Fig. 6.

pared with the tree-level prediction, the dashed curve, which is the same as that shown in Fig. 4.

We note that the $R=0$ curve differs little from the dashed curve. This shows that the last term in Eq. (35) makes only insignificant modification. From the curves with various values of R, one may discern some trends. For fixed $\operatorname{Im} W$, as R decreases $\operatorname{Re} W$ increases. For fixed $\operatorname{Re} W$, as R decreases $\operatorname{Im} W$ increases. Note that as R becomes more and more negative the magnitude of the imaginary part tends to become smaller. For example, at $\operatorname{Re} W=1$ TeV for $R=0, -0.05, -0.1$, and -0.2,

$$\operatorname{Im} W = -0.20, -0.15, -0.11, \text{ and } 0.03\,\text{TeV} ,$$

respectively.

The fact that a Higgs particle could appear with a relatively narrow width in the TeV region is surprising. We arrive at this possibility only when ν and R are taken to be independent parameters. Still, we believe such a possibility should not be ruled out. Of course there is a restriction on how negative R can be. As mentioned earlier, from general considerations R must take on a value which leads to a positive-definite width. It is particularly interesting that with negative R there is the reduction of $\operatorname{Im} W$, that is, the width of the Higgs resonance. This may make it easier to detect this resonance experimentally.

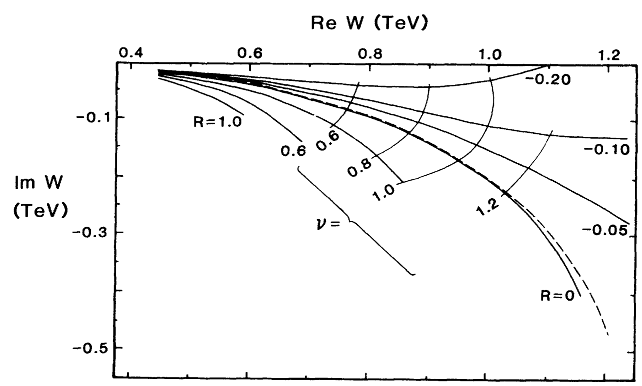

FIG. 5. A family of resonance trajectories in the complex W plane for various values of the renormalization parameter R. The dashed trajectory is for the first-order effective potential. The solid curves are for the effective potential computed up to the second order. $\nu=0.6$, 0.8, 1.0, and 1.2 correspond to $\sigma=c\nu=0.06$, 0.08, 0.11, and 0.13.

V. CONCLUSIONS

In this work we propose a model for the strongly interacting longitudinal vector bosons in the two-boson sector at high energy. We start from an effective potential and compute the exact scattering amplitude that corresponds to this potential. The potential is obtained from the two-particle "s-channel irreducible" diagrams up to one-loop order in the perturbation theory. It has one independent parameter, which can be either the dimensionless renormalization parameter R, or the scale ν.

In the Hamiltonian model based on this potential, the full scattering amplitude is evaluated exactly. The resonance position is determined by the complex zero of the denominator function $\beta(s)$. We consider two cases, one with one parameter and the other with two parameters. Our results for the one-parameter case are displayed in Fig. 4 and for the two-parameter case in Fig. 5. We point out that the intriguing possibility of narrow-width Higgs resonance cannot be ruled out *a priori* for the latter case.

Finally we turn to the search for the Higgs resonance. We began from the fact that in the minimal standard model, the bare mass of the Higgs boson is *a priori* a free parameter. We are interested in the scenario where the bare mass is very large and the coupling is strong. Within our approach the Higgs resonance is present over a wide range of parameter values. The position of the Higgs pole on the second sheet is governed by one or two parameters. Each choice of parameter(s) corresponds to a definite line shape in the cross section. Figure 6 shows sample line shapes for the tree-level case, where the square of the normalized partial-wave amplitude is plotted versus s.

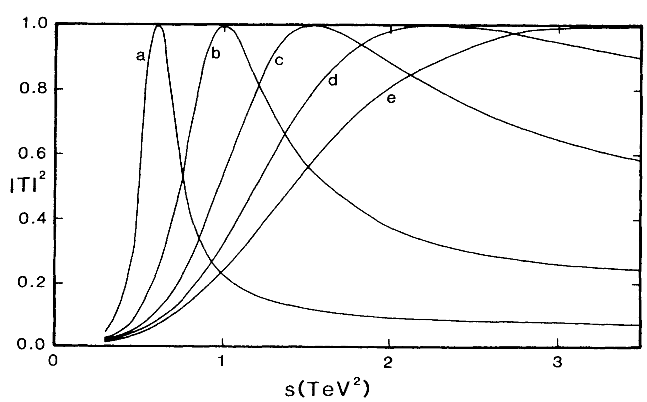

FIG. 6. Line shapes for those points along the dashed curve indicated in Fig. 4.

Our model may be used as a phenomenological guide in the search for the Higgs boson in sub-TeV and TeV regions. An optimistic scenario would be as follows. Through the interplay between model prediction and measurements, the data confirms one specific line shape of our model. This implies the discovery of the Higgs resonance and at the same time it predicts the resonance pole position on the second sheet.

ACKNOWLEDGMENTS

We would like to thank Maria Herrero and Ruiming Xu for discussions. One of us (G.B.) would like to thank Austin Gleeson and George Sudarshan for hospitality at the Department of Physics and the Center for Particle Theory where this work was done. This work was supported by the Department of Energy Grant No. DE-FG05-85ER40200.

APPENDIX

We show below that in the effective chiral Lagrangian theory, for the $I=0$ amplitude, the renormalization constant associated with the s-channel loop and that associated with the cross-channel loops, are two independent parameters. Following the basic notation of Dobado *et al.* [2], consider the effective chiral Lagrangian

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_1 , \tag{A1}$$

where

$$\mathcal{L}_0=\frac{v^2}{4}\operatorname{tr}(\partial_\mu U\,\partial^\mu U^\dagger) \tag{A2}$$

and

$$\mathcal{L}_1=\tfrac{1}{32}E\operatorname{tr}\{[(\partial_\mu U)U^\dagger,(\partial_\nu U)U^\dagger]\} + \tfrac{1}{8}G[\operatorname{tr}(\partial_\mu U\,\partial^\mu U^\dagger)]^2 , \tag{A3}$$

with $U(x)=\exp[i\mathbf{w}(x)\cdot\boldsymbol{\sigma}/v]$ and $\boldsymbol{\sigma}$ being the Pauli matrices. The coefficients E and G are the two independent parameters. The scattering amplitude for $w_i w_j \to w_k w_l$ can be written as

$$T_{ij,kl}=A(s,t,u)\delta_{ij}\delta_{kl}+A(t,s,u)\delta_{ik}\delta_{jl} + A(u,t,s)\delta_{il}\delta_{jk} . \tag{A4}$$

To one-loop level, the scattering amplitude is

$$A^{\text{loop}}(s,t,u)=\frac{s}{v^2}+\frac{E_R(\nu)}{4v^4}(-2s^2+t^2+u^2)+\frac{G_R(\nu)}{v^4}s^2 - \frac{1}{(4\pi)^2v^4}\left[\frac{1}{12}(3t^2+u^2-s^2)\ln\left[\frac{-t}{\nu}\right]+\frac{1}{12}(3u^2+t^2-s^2)\ln\left[\frac{-u}{\nu}\right]+\frac{s^2}{2}\ln\left[\frac{-s}{\nu}\right]\right] , \tag{A5}$$

where the renormalization constants defined at $s=\nu_0$ are given by

$$E_R(\nu)=E_R(\nu_0)-\frac{1}{12\pi^2}\ln\frac{\nu}{\nu_0} \quad\text{and}\quad G_R(\nu)=G_R(\nu_0)-\frac{1}{16\pi^2}\ln\frac{\nu}{\nu_0} . \tag{A6}$$

There are two independent renormalization constants defined at the scale ν_0. As expected the amplitude A^{loop} is independent of the renormalization point ν.

The $I=0$ amplitude is defined by

$$A(I=0)=3A(s,t,u)+A(t,s,u)+A(u,t,s) . \tag{A7}$$

From Eqs. (A5) and (A7), the $I=0$ amplitude up to the one-loop level can be written as

$$A^{\text{loop}}(I=0)=\frac{2s}{v^2}+\frac{2s^2}{v^4}\left[B_R(\nu)-\frac{1}{(4\pi)^2}\ln\left[\frac{-s}{\nu}\right]\right] + \frac{1}{v^4}\left[\frac{3t^2}{2}+\frac{u^2-s^2}{6}\right]\left[C_R(\nu)-\frac{1}{(4\pi)^2}\ln\left[\frac{-t}{\nu}\right]\right] + \frac{1}{v^4}\left[\frac{3u^2}{2}+\frac{t^2-s^2}{6}\right]\left[C_R(\nu)-\frac{1}{(4\pi)^2}\ln\left[\frac{-u}{\nu}\right]\right] , \tag{A8}$$

where

$$B_R(\nu)=\tfrac{9}{20}E_R(\nu)+\tfrac{8}{5}G_R(\nu) \quad\text{and}\quad C_R(\nu)=\tfrac{3}{10}E_R(\nu)+\tfrac{3}{5}G_R(\nu) . \tag{A9}$$

A glance at (A8) reveals that $B_R^{(\nu)}$ and $C_R^{(\nu)}$ are the renormalization constants defined at $s=\nu$ for, respectively, the s-loop and the crossed-channel loops. Since E and G, or equivalently $E_R(\nu)$ and $G_R(\nu)$, are independent parameters, so $B_R(\nu)$ and $C_R(\nu)$ are also independent parameters.

[1] J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, Phys. Rev. D **10**, 1145 (1974); B. W. Lee, C. Quigg, and H. Thacker, *ibid.* **16**, 1519 (1977); M. S. Chanowitz and M. K. Gaillard, Nucl. Phys. **B261**, 379 (1985).

[2] D. A. Dicus and V. S. Mathur, Phys. Rev. D **7**, 3111 (1973); B. W. Lee, C. Quigg, and H. Thacker, *ibid.* **16**, 1519 (1977); M. B. Einhorn, Nucl. Phys. **B246**, 531 (1987); Phys. Lett. B **211**, 457 (1988); A. Dobado, M. J. Herrero, and T. N. Truong, *ibid.* **235**, 129 (1990); **235**, 134 (1990); A. Dobado, *ibid.* **237**, 457 (1990); D. A. Dicus and W. W. Repko, Phys. Rev. D **42**, 3660 (1990).

[3] T. F. Jordan, A. J. MacFarlane, and E. C. G. Sudarshan, Phys. Rev. **133**, B487 (1964).

[4] Yoshio Yamaguchi, Phys Rev. **95**, 1628 (1954); Yoshio Yamaguchi and Yoricko Yamaguchi, *ibid.* **95**, 1635 (1954).

[5] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949); B. Bakamjian and L. H. Thomas, Phys. Rev. **92**, 1300 (1953); L. H. Thomas, *ibid.* **58**, 868 (1952); B. Bakamjian, *ibid.* **121**, 1849 (1961); L. L. Foldy, *ibid.* **122**, 275 (1961); Jordan, MacFarlane, and Sudarshan [3]; E. C. G. Sudarshan, in *Relativistic Particle Interactions,* Proceedings of the 1961 Brandeis Summer Institute Lectures on Theoretical Physics (Benjamin, New York, 1962).

[6] A. S. Eddington, Proc. Cambridge Philos. Soc. **35**, 186 (1939); **38**, 201 (1942). See also P. A. M. Dirac, R. E. Peierls, and M. H. L. Pryce, *ibid.* **3**, 193 (1942).

[7] C. C. Chiang, C. B. Chiu, E. C. G. Sudarshan, and X. Tata, Phys. Rev. D **26**, 2092 (1982).

[8] Hans Bebie, Ph.D. thesis, Universitat Bern, 1966.

[9] A. Dobado and M. J. Herrero, Phys. Lett. B **228**, 495 (1989). See also H. Lehmann, Phys. Lett. **41B**, 529 (1972); K. S. Jhung and R. S. Willey, Phys. Rev. D **9**, 3132 (1974); S. Weinberg, Physica **96A**, 327 (1979); J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 42 (1984); S. Dawson and S. Willenbrock, Phys. Rev. D **40**, 2880 (1989); M. J. G. Veltman and F. J. Yndurain, Nucl. Phys. **B325**, 1 (1989).

[10] S. Weinberg, Physica **96A**, 327 (1979). See also S. Weinberg, Phys. Rev. Lett. **17**, 616 (1966).

[11] M. B. Einhorn, Phys. Lett. B **211**, 457 (1988).

[12] L. S. Brown and R. L. Goble, Phys. Rev. Lett. **20**, 2251 (1967). See also Dobado *et al.* [2].