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Heavy quark symmetries and the decays $B \rightarrow$ baryon+antibaryon

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We enumerate the form factors necessary to describe the two-body baryonic decays of B mesons. We use the symmetries of the heavy quark effective theory to arrive at some relations among these form factors, when at least one of the daughter baryons is heavy.

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I. INTRODUCTION

The B meson offers a unique laboratory for the study of the interplay between weak and strong interaction dynamics, since it is the only meson that is sufficiently heavy to decay weakly into pairs of baryons. Indeed, present measurements indicate that the branching fraction for decays of this meson into $p+$ anything is $\approx 8\%$, and into $\Lambda+$ anything is $\approx 4\%$ [1].

While it may be desirable to attempt to understand these inclusive modes directly, it is instructive to begin with a study of the exclusive two-body modes, since these are (hopefully) the least complicated modes to describe. In addition, many of the multi-particle modes may be understood as cascade processes which began as two-body modes, in which one or both daughter hadrons were themselves unstable.

To this end, we attempt to understand the general Lorentz structure of the amplitudes that describe these decays, namely $B \rightarrow$ baryon antibaryon. In other words, we count the form factors necessary for these decays, with the only restriction being that both baryons are ground state ($J^P = \frac{1}{2}^+, \frac{3}{2}^+$) baryons. We then use the additional symmetries of the heavy quark effective theory (HQET) [2] to find relationships among different form factors, when at least one of the daughter baryons is heavy. It turns out that HQET does not provide much help in limiting the number of form factors, and that explicit model calculations of the form factors would be needed.

In much of what follows, we are primarily interested in decays of the B meson in which at least one of the daughter baryons is heavy. We must therefore briefly elucidate the structure of the baryons we discuss. For the light baryons (i.e., baryons consisting solely of u, d, and s quarks), the usual nomenclature and spin assignments suffice. Thus, baryons from the $J^P = \frac{1}{2}^+$ ground state octet will be described by Dirac spinors, while those from the $J^P = \frac{3}{2}^+$ decuplet are described by a Rarita-Schwinger field.

For the heavy baryons, the spin symmetry of the HQET allows us to relate some of the $J^P = \frac{1}{2}^+$ baryons to the $J^P = \frac{3}{2}^+$ baryons. It is therefore more useful to refer to these baryons as being Λ-type baryons and Σ-type baryons. In the Λ-type baryons, the light quarks and gluons have their spins coupled to give a total spin of zero, so that the total spin of the baryon is simply that of the heavy quark. The Λ-type baryons of interest here are the Λ_c ($[(ud)_0 c]_{1/2}$) and the Ξ_c ($[(ds)_0 c]_{1/2}$, $[(us)_0 c]_{1/2}$). These baryons may be represented by a Dirac spinor.

The Σ-type baryons are those in which the light quarks have a total spin of 1, so that the total spin of the baryon is $\frac{1}{2}$ or $\frac{3}{2}$. These baryons include the Σ_c ($[(ud)_1 c]_{1/2}$), the Σ_c^* ($[(ud)_1 c]_{3/2}$), the Ξ_c' ($[(us)_1 c]_{1/2}$) and the Ξ_c^* ($[(us)_1 c]_{3/2}$). To leading order in HQET, the Σ_c and Σ_c^* (or the Ξ_c' and the Ξ_c^*) are degenerate members of the same multiplet. Generically, these baryons may be represented by the spinors $\Sigma_c^{(*)}(v) \rightarrow B_\mu^{(m)}(v)$. More specifically [3],

$$\begin{aligned} &\Sigma_c(v), \Xi_c'(v) \rightarrow B_\mu^{(1)}(v) = \frac{1}{\sqrt{3}}(v_\mu + \gamma_\mu)\gamma_5 u(v) , \\ &\Sigma_c^*(v), \Xi_c^*(v) \rightarrow B_\mu^{(2)}(v) = u_\mu(v) , \end{aligned} \tag{1}$$

where $u_\mu(v)$ is the usual Rarita-Schwinger field. These objects satisfy the auxiliary conditions

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$$v^\mu B_\mu^{(m)}(v)=0\ ,$$
$$\not{v}B_\mu^{(m)}(v)=B_\mu^{(m)}(v)\ ,$$
$$\gamma^\mu B_\mu^{(2)}(v)=0\ .$$

In addition, we note that the B meson may be represented by the matrix $B\rightarrow(\sqrt{m_B}/2)\gamma_5(\not{v}-1)$.

II. GENERAL FORM FACTORS

There are basically three types of decay that are of interest to us. For the moment, we may classify these as decays in which (a) both daughter baryons have $J^P=\frac{1}{2}^+$, (b) one daughter baryon has $J^P=\frac{3}{2}^+$ and the other has $J^P=\frac{1}{2}^+$, and (c) both daughter baryons have $J^P=\frac{3}{2}^+$. Later on, we will look at cases in which at least one of the daughter baryons is heavy, so that we may find relations among the form factors of the group (a), (b), and (c) decays.

For the group (a) decays ($B\rightarrow\Lambda_c\bar{P}$, for example), two form factors are needed. This can be seen by noting that the most general amplitude involving two Dirac spinors requires at most two form factors:

$$M=\bar{u}(v')(A+B\gamma_5)v(p)\ . \tag{2}$$

For the group (b) decays [$B(v)\rightarrow\Lambda_c(v')\bar{\Delta}(p)$, for example], two form factors are needed, since we can write

$$M=\bar{u}(v')M^\mu v_\mu(p)\ , \tag{3}$$

where $M^\mu=Cv^\mu+Dv^\mu\gamma_5$, and $v_\mu(p)$ is the Rarita-Schwinger field describing the $\bar{\Delta}$. Treating the decay $B\rightarrow\Xi_c^*\bar{\Lambda}$ in a similar manner, we can write

$$M=\bar{u}_\mu(v')M^\mu v(p)\ , \tag{4}$$

with $M^\mu=Cv^\mu+Dv^\mu\gamma_5$

For the group (c) decays (such as $B\rightarrow\Sigma_c^*\bar{\Delta}$), Lorentz invariance allows us to write

$$M=\bar{u}_\mu(v')P^{\mu\nu}(p)\ , \tag{5}$$

with

$$P^{\mu\nu}=E_1g^{\mu\nu}+E_2v^\mu v^\nu+[E_3g^{\mu\nu}+E_4v^\mu v^\nu]\gamma_5\ . \tag{6}$$

Four form factors are therefore needed to describe these decays. Note that so far, we have used only the principles of Lorentz symmetry to enumerate the form factors, so that these results are quite general. We have not yet taken advantage of any possible simplifications allowed by the HQET. We now turn to the special cases when at least one of the daughter baryons is heavy.

III. THE DECAYS INTO HEAVY BARYONS

We begin by discussing the decays into two heavy baryons. These decays take place via the $b\rightarrow c\bar{c}s$ current (or the Cabibbo suppressed $b\rightarrow c\bar{c}d$ current). The possible final baryons therefore include Ξ_c, Ξ_c', Ξ_c^*, while one may find Λ_c's, Σ_c's, and Σ_c^*'s among the daughter antibaryons. The particular examples of decays we shall consider are $B\rightarrow\Xi_c\bar{\Lambda}_c$, $B\rightarrow\Xi_c\bar{\Sigma}_c^{(*)}$, $B\rightarrow\Xi_c^{(',*)}\bar{\Lambda}_c$, and $B\rightarrow\Xi_c^{(',*)}\Sigma_c^{(*)}$.

For the decay $B\rightarrow\Xi_c\bar{\Lambda}_c$, we may write

$$\begin{aligned}\langle\Xi_c(v_\Xi)\bar{\Lambda}_c(v_\Lambda)|\bar{s}\gamma_\mu(1-\gamma_5)c\bar{c}\gamma^\mu(1-\gamma_5)b|B(v)\rangle\\ =\bar{u}_{\Xi_c}(v_\Xi)\gamma_\mu(1-\gamma_5)\frac{\sqrt{m_B}}{2}\gamma_5(\not{v}-1)(\alpha+\not{v}'\beta)\\ \times\gamma^\mu(1-\gamma_5)v_{\Lambda_c}(v_\Lambda)\ .\end{aligned} \tag{7}$$

Here, we have used the symmetries of the HQET to express the four-quark weak current responsible for the decay as a product of two two-quark currents.

In terms of α and β, we find

$$A=2\sqrt{m_B}[\alpha(r_1-r_2)-\beta]\ ,$$
$$B=-2\sqrt{m_B}[\alpha(r_1+r_2)-\beta]\ ,$$

where $r_1=m_{\Xi_c}/m_B$, $r_2=m_{\Lambda_c}/m_B$. More generally, r_1 is the ratio of the mass of the baryon to that of the B meson, while r_2 is the ratio of the mass of the antibaryon to that of the B meson.

For the decay $B\rightarrow\Xi_c^{(',*)}\bar{\Lambda}_c$ (with similar arguments for $B\rightarrow\Xi_c\bar{\Sigma}_c^{(*)}$), we may write

$$\begin{aligned}\mathcal{A}(B\rightarrow\Xi_c^{(',*)}\bar{\Lambda}_c)=\frac{\sqrt{m_B}}{2}\bar{B}_\lambda^{(m)}(v_\Xi)\gamma_\mu(1-\gamma_5)\\ \times\gamma_5(\not{v}-1)M^\lambda\gamma^\mu(1-\gamma_5)u(v_\Lambda)\end{aligned} \tag{8}$$

where $B_\lambda^{(m)}$ has one of the forms shown in Eq. (1), depending on whether the antibaryon is the Ξ_c' or the Ξ_c^*. The Dirac matrix M_λ is a vector which must have the form

$$M^\lambda=v^\lambda\alpha+v^\lambda\not{v}_\Lambda\beta+\gamma^\lambda\gamma+\gamma^\lambda\not{v}_\Lambda\delta\ . \tag{9}$$

After some simplification, we write

$$\begin{aligned}\mathcal{A}(B\rightarrow\Xi_c^{(',*)}\bar{\Lambda}_c)\\ =4\sqrt{m_B}\bar{B}_\nu^{(m)}(v_\Xi)(y_1v_\Lambda^\nu+y_2\gamma^\nu+y_3\gamma_5v_\Lambda^\nu\\ +y_4\gamma^\nu\gamma_5)u_\Lambda(v_\Lambda)\ ,\end{aligned} \tag{10}$$

with

$$\begin{aligned}y_1&=-r_1[\alpha(r_1-r_2)+\gamma]\ ,\\ y_2&=\beta+\delta[r_1+r_2(2v_\Lambda\cdot v_\Xi-1)]\ ,\\ y_3&=-r_1[\alpha(r_1+r_2)+\gamma]\ ,\\ y_4&=-\{\beta+\delta[r_1+r_2(2v_\Lambda\cdot v_\Xi+1)]\}\ .\end{aligned} \tag{11}$$

When we specialize to the decay $B\rightarrow\Xi_c^*\bar{\Lambda}_c$, the γ^ν terms of Eq. (10) vanish, so that only the terms y_1 and y_3 contribute to this decay. We may thus make the correspondence of y_1 with C and y_3 with D of Eq. (4), modulo factors of $4\sqrt{m_b}$. For the decay $B\rightarrow\Xi_c'\bar{\Lambda}_c$, after some simplification, we find that, in terms of the form factors of Eq. (2),

$$A=-\frac{4\sqrt{m_b}}{3}[3y_4+y_3(v_\Lambda\cdot v_\Xi+1)]\ ,$$
$$B=-\frac{4\sqrt{m_b}}{3}[3y_2+y_1(v_\Lambda\cdot v_\Xi-1)]\ .$$

Thus, all four form factors contribute to this decay.

At this point one may question whether anything has been gained here. For the general decays, without consideration of the HQET, we saw that there was a total of four form factors describing the two decays we considered. Now, using the spin symmetry of the HQET, we find that four form factors are still required, but the form factors for the decay into $\Xi_c' \overline{\Lambda}_c$ are in some way related to those for the decay into $\Xi_c^* \overline{\Lambda}_c$.

A similar situation arises when we consider the decay $B \rightarrow \Xi_c^{(',*)} \overline{\Sigma}_c^{(*)}$. Ten form factors are needed to describe these four decays. With spin symmetry arguments, we will see that ten form factors are still needed, but new relations among the form factors from different decay modes arise. The decays in question are described by the amplitude

$$\mathcal{A}(B \rightarrow \Xi_c^{(',*)} \overline{\Sigma}_c^{(*)}) = \frac{\sqrt{m_b}}{2} \overline{B}_\lambda^{(m)}(v_\Xi) \gamma_\mu (1-\gamma_5) \gamma_5 (\not{v} - 1) \times P^{\lambda\kappa} \gamma^\mu (1-\gamma_5) B_\kappa^{(m')}(v_\Sigma) , \tag{12}$$

with

$$P_{\mu\nu} = g_{\mu\nu}(A + \not{v}_\Xi B) + \sigma_{\mu\nu}(C + \not{v}_\Xi D) + v_\mu v_\nu (E + \not{v}_\Xi F) + v_\mu \gamma_\nu (G + \not{v}_\Xi H) + \gamma_\mu v_\nu (I + \not{v}_\Xi J) . \tag{13}$$

For the general amplitude, one may write

$$M = \overline{B}_\lambda^{(m)}(v_\Xi) R^{\lambda\kappa} B_\kappa^{(m')}(v_\Sigma) \tag{14}$$

with

$$R^{\mu\nu} = x_1 g^{\mu\nu} + x_2 \sigma^{\mu\nu} + x_3 v^\mu \gamma^\nu + x_4 \gamma^\mu v^\nu + x_5 v^\mu v^\nu + \gamma_5 (x_1' g^{\mu\nu} + x_2' \sigma^{\mu\nu} + x_3' v^\mu \gamma^\nu + x_4' \gamma^\mu v^\nu + x_5' v^\mu v^\nu) . \tag{15}$$

In terms of these, one finds

$$\begin{aligned}
x_1 &= 2\sqrt{m_B}[A(r_1 - r_2) - B] , \\
x_1' &= 2\sqrt{m_B}[A(r_1 + r_2) - B] , \\
x_2 &= -2\sqrt{m_B}[C(r_1 + r_2) + D] , \\
x_2' &= 2\sqrt{m_B}[C(r_1 - r_2) + D] , \\
x_3 &= -2\sqrt{m_B}[H(r_1 + r_2) + G] , \\
x_3' &= -2\sqrt{m_B}[H(r_1 - r_2) + G] , \\
x_4 &= -2\sqrt{m_B}[J(r_1 + r_2) + I + 2ir_1 C] , \\
x_4' &= 2\sqrt{m_B}[J(r_1 - r_2) + I + 2ir_1 C] , \\
x_5 &= -2\sqrt{m_B}[F - 2r_1 H - E(r_1 - r_2)] , \\
x_5' &= 2\sqrt{m_B}[F - 2r_1 H - E(r_1 + r_2)] .
\end{aligned} \tag{16}$$

If we now examine each of the four decays separately, we would find that the form factor A of Eq. (2) that describes the decay $B \rightarrow \Xi_c' \overline{\Sigma}_c$ is expressible in terms of the five form factors x_1, x_2, x_3, x_4, and x_5, while B is expressible in terms of the five primed form factors. Similary, for the decay $B \rightarrow \Xi_c' \overline{\Sigma}_c^*$, C and D of Eq. (3) are linear superpositions of x_1, x_3, x_5, and x_1', x_3', x_5', respectively, while for the decay $B \rightarrow \Xi_c^* \overline{\Sigma}_c$, C and D of Eq. (4) depend on x_1, x_4, x_5, x_1', x_4' and x_5'. Finally, for the decay $B \rightarrow \Xi_c^* \overline{\Sigma}_c^*$, E_1, E_2, E_3, and E_4 of Eq. (5) are expressible in terms of x_1, x_1', x_5 and x_5'. In fact, in the manner we have written things, $E_1 = x_1$, $E_2 = x_5$, $E_3 = x_1'$ and $E_4 = x_5'$.

One can perform the same kind of analysis when only one of the daughter baryons in the decay is heavy. As one would expect from the above discussion, spin symmetry does not decrease the numbers of form factors required for such decays, and the relationships among form factors are even less encouraging than for the decays into two heavy baryons.

IV. CONCLUSION

The preceding discussion has shown us that it is quite simple to enumerate the maximum number of form factors necessary to describe the two-body baryonic decays of the B meson. It has also shown us that the use of the HQET when at least one of the daughter baryons is heavy leads to some relationships among these general form factors. However, these relationships are of limited usefulness without further input, since they do not decrease the number of form factors.

For further input, one may turn, for example, to SU(3) flavor symmetry, which would relate the $\Sigma_c^{(*)}$ and the $\Xi_c^{(',*)}$, for instance. This has been done in Ref [4]. Other possibilities for further input are explicit model calculations such as the diquark model [5], or the pole model [6]. As an example of the possible usefulness of such models, note that the pole model tells us that none of the baryons (as opposed to antibaryons) can have spin $\frac{3}{2}$. This would immediately place restrictions on the form factors of Eqs. (3), (4), and (5). Such considerations are left as possible extensions of this work.

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