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# Lattice study of semileptonic decays of charm mesons into vector mesons

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We present our lattice calculation of the semileptonic form factors for the decays $D \to K^*$, $D_s \to \phi$, and $D \to \rho$ using Wilson fermions on a $24^3 \times 39$ lattice at $\beta = 6.0$ with 8 quenched configurations. For $D \to K^*$, we find for the ratio of axial form factors $A_2(0)/A_1(0) = 0.70 \pm 0.16^{+0.20}_{-0.15}$. Results for other form factors and ratios are also given.

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Continuing our study of exclusive semileptonic decays of mesons on the lattice, we focus here on the form factors for pseudoscalar decays to final states with vector mesons. We have previously reported, in detail, on semileptonic decays into pseudoscalar mesons [1-3]. Here we concentrate on the decays $D \to K^*$, $D_s \to \phi$, and $D \to \rho$. The decay $D \to K^*$, in particular, has recently received considerable attention. There appears to be some disagreement among experimental results as well as among theoretical calculations.

Some of our preliminary results for vector-meson final states were described in Ref. [2]. Lattice calculations of semileptonic form factors have also been reported by the European Lattice Collaboration (ELC) group for pseudoscalar [4,5] and vector [5] final states.

With the exception of the spin of the final-state particle, the analysis here follows that of our previous work. We therefore emphasize only those aspects that are different from Ref. [1]. Taking as an example the decay $D \to K^*$, we can parametrize the matrix element in question in terms of (Euclidean space) form factors [2,6]:

$$\langle K^*, \lambda | (V-A)_\nu | D \rangle = \epsilon_\alpha^{(\lambda)} T_{\nu\alpha} , \tag{1}$$

$$\begin{aligned} T_{\nu\alpha} = {} & \frac{2}{m_D + m_{K^*}} V(q^2) \epsilon_{\nu\alpha\rho\sigma} p_D^\rho p_{K^*}^\sigma \\ & - A_1(q^2)(m_D + m_{K^*}) \delta_{\nu\alpha} \\ & + A_2(q^2) \frac{1}{m_D + m_{K^*}} (p_D + p_{K^*})_\nu p_{D\alpha} \\ & - \frac{2 m_{K^*}}{q^2} A(q^2) (p_D - p_{K^*})_\nu p_{D\alpha} , \end{aligned} \tag{2}$$

$$\begin{aligned} & A(q^2) \equiv A_0(q^2) - A_3(q^2) , \\ & A_3(q^2) \equiv \frac{m_D + m_{K^*}}{2 m_{K^*}} A_1(q^2) - \frac{m_D - m_{K^*}}{2 m_{K^*}} A_2(q^2) , \end{aligned} \tag{3}$$

with $A_0(0) = A_3(0)$. $\epsilon_\alpha^{(\lambda)}$ is the polarization vector of the $K^*$ meson with helicity $\lambda = 0, +, -$, and as usual $q \equiv (p_D - p_{K^*})$ is the four-momentum transfer. We have written $T_{\nu\alpha}$ in the helicity basis, in which the form factors are related to resonances with definite quantum numbers. Pole dominance [6] then implies

$$\begin{aligned} & V(q^2) = \frac{V(0)}{1 - q^2/m_{1^-}^2} , \quad A_0(q^2) = \frac{A_0(0)}{1 - q^2/m_{0^-}^2} , \\ & A_i(q^2) = \frac{A_i(0)}{1 - q^2/m_{1^+}^2} , \quad i = 1,2,3 . \end{aligned} \tag{4}$$

For completeness we note that under the above pole-dominance assumption one can express the rates for longitudinally and transversely polarized $K^*$'s in terms of the form factors at $q^2 = 0$ [7]:

$$\begin{aligned} & \Gamma(D \to K^* l \nu) = \Gamma_L + \Gamma_T , \\ & \Gamma_L = |V_{cs}|^2 [C_L A_1^2(0) + C_2 A_2^2(0) - C_{12} A_1(0) A_2(0)] , \\ & \Gamma_T = |V_{cs}|^2 [C_T A_1^2(0) + C_V V^2(0)] . \end{aligned} \tag{5}$$

The $C$'s arise from the phase-space integration and have been calculated in Ref. [7] in the limit of zero lepton mass for various decays.

We now recapitulate the experimental situation. For the decay $D \to K^*$, there is considerable uncertainty in the experimental status of the form factors. Some time ago Fermilab experiment E691 [8] reported the interesting result that $A_2(0)$ is consistent with zero. This contradicted the expectation from various quark-model calculations [6,9-11] or from QCD sum rules [12,13] which predict $A_2(0)$ to be $\sim 1$ (see Table I). Since $A_2$ affects the polarization of the $K^*$, there is also a (mild) discrepancy between model predictions and the E691 result for the ratio $\Gamma_L/\Gamma_T$. However, the experimental situation is not yet completely clear: a comparison of E691 with the result by Mark III [14] and also with the preliminary result by E653 [15] shows a large spread in the central value for this ratio. The Fermilab experiment E653 has studied the semimuonic decay $D \to K^* \mu \nu$. In an analysis similar to E691 they have also extracted the form-factor ratios $A_2/A_1$ and $V/A_1$, which appear to be in good agreement

 

TABLE I. The form factors for $D \to K^*$ from various experiments and model calculations.

| Group | $A_1(0)$ | $A_2(0)$ | $A_2/A_1(0)$ | $V(0)$ | $V/A_1(0)$ |
|---|---|---|---|---|---|
| E691 [8] | $0.46 \pm 0.05$ | $0.0 \pm 0.2$ | $0.0 \pm 0.5$ | $0.9 \pm 0.3$ | $2.0 \pm 0.6$ |
| Syst. error | $\pm 0.05$ | $\pm 0.1$ | $\pm 0.2$ | $\pm 0.1$ | $\pm 0.3$ |
| E653 [15] | | | $0.82^{+0.22}_{-0.23}$ | | $2.0^{+0.34}_{-0.32}$ |
| Syst. error | | | $\pm 0.11$ | | $\pm 0.16$ |
| BSW [6] | 0.88 | 1.15 | | 1.27 | |
| KS [9] | 0.82 | 0.82 | 1.0 | 0.82 | 1.0 |
| AW/GS [11] | 0.8 | 0.6 | | 1.5 | |
| BBD [13] | $0.50 \pm 0.15$ | $0.60 \pm 0.15$ | $1.2 \pm 0.2$ | $1.10 \pm 0.25$ | $2.2 \pm 0.2$ |
| ELC [5] | $0.52 \pm 0.07$ | $0.05 \pm 0.35$ | | $0.85 \pm 0.08$ | |
| This work | $0.83 \pm 0.14$ | $0.59 \pm 0.14$ | $0.70 \pm 0.16$ | $1.43 \pm 0.45$ | $1.99 \pm 0.22$ |
| Syst. error | $\pm 0.28$ | ${}^{+0.24}_{-0.23}$ | ${}^{+0.20}_{-0.15}$ | ${}^{+0.48}_{-0.49}$ | ${}^{+0.31}_{-0.35}$ |

with some model predictions. Note that all three experiments are still consistent with each other within $1.5\sigma$ because of the rather large experimental errors.

The form factor $A_1(0)$ is measured by E691 to be $\approx 0.5$, smaller than most model predictions. Note that the ELC (lattice) results [5] for $A_2(0)$ and $A_1(0)$ are close to that of E691 and tend to disagree with the quark-model calculations. There also would appear to be some disagreement between the ELC results and the E653 value of the ratio $A_2(0)/A_1(0)$.

For the form factor $V(0)$, unlike $A_2(0)$ [and, to a lesser extent, $A_1(0)$], the experimental and theoretical results are in good agreement with each other. However, this form factor is phase-space suppressed [7] and thus is not important for the decay rate. $A_0(0)$ is weighted with the lepton mass in the decay rate and is therefore experimentally not measurable.

Very little is known about the decay $D_s \to \phi l \nu$; its branching ratio relative to $D_s \to \phi\pi$ is measured as

$$\frac{\Gamma(D_s \to \phi l \nu)}{\Gamma(D_s \to \phi\pi)} = \begin{cases} 0.49 \pm 0.10^{+0.10}_{-0.14}, & \text{CLEO [16]}, \\ 0.57 \pm 0.15 \pm 0.15, & \text{ARGUS [17]}. \end{cases} \tag{6}$$

For the (Cabibbo-suppressed) decay $D \to \rho l \nu$ there only exists an upper limit for the branching fraction at 90% confidence level [14]:

$$B(D \to \rho l \nu) < 0.37\% . \tag{7}$$

We now briefly describe how these form factors are extracted on the lattice. Defining the two-point function for the vector meson ($K^*$) as follows, one finds, in the large-time limit (under the usual lattice assumptions),

$$G_{K^*}(\mathbf{p},t;\mu,\alpha) \equiv \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}} \langle 0 | \chi^{\mu}_{K^*}(x) \chi^{\alpha\dagger}_{K^*}(0) | 0 \rangle \to \frac{1}{2E_{K^*}} e^{-E_{K^*}t} C_{K^*} \sum_{\lambda} \epsilon^{(\lambda)}_{\mu} \epsilon^{(\lambda)}_{\alpha} , \tag{8}$$

$$|\langle 0 | \chi^{\mu}_{K^*}(0) | K^*, \lambda \rangle| = C^{1/2}_{K^*} \epsilon^{(\lambda)}_{\mu} .$$

For the three-point function, one has, similarly,

$$G_3(\mathbf{p},t_{K^*},t_D;\mu,\nu) \equiv \sum_{\mathbf{x},\mathbf{y}} e^{-i\mathbf{p}\cdot\mathbf{x}} \langle 0 | \chi^{\nu}_{K^*}(x) (V-A)_{\nu}(0) \chi^{\dagger}_{D}(y) | 0 \rangle$$

$$\to \frac{1}{2E_{K^*} 2m_D} e^{-E_{K^*}t_{K^*} - m_D t_D} (C_{K^*} C_D)^{1/2} \sum_{\lambda} \epsilon^{(\lambda)}_{\mu} \langle K^*, \lambda | (V-A)_{\nu} | D \rangle . \tag{9}$$

With the ratio $R$ defined by

$$R(\mathbf{p};\mu,\nu,\beta) \equiv \left| \frac{G_3(\mathbf{p},t_{K^*},t_D;\mu,\nu)}{G_{K^*}(\mathbf{p},t_{K^*};\mu,\beta) G_D(0,t_D)} \right| (C_{K^*} C_D)^{1/2} , \tag{10}$$

we find, in the large-time limit,

$$\sum_{\lambda} \epsilon^{(\lambda)}_{\mu} \epsilon^{(\lambda)}_{\beta} R(\mathbf{p};\mu,\nu,\beta) \to \sum_{\lambda} \epsilon^{(\lambda)}_{\mu} \epsilon^{(\lambda)}_{\alpha} T_{\nu\alpha} \equiv M_{\mu\nu} , \tag{11}$$

with a sum on $\alpha$ but not on $\beta$ or $\mu$. The lattice calculation of the ratio $R$ enables us to use this equation to solve for the form factors [2]. One can also extract $M_{\mu\nu}$ directly from Eq. (9).

For the renormalization of the vector and axial-vector currents we take the values from perturbation theory [18], using a renormalized coupling as suggested in Ref. [19], $g^2 \approx 1.75$ at $\beta = 6.0$:

$$Z^{\rm loc}_{V} \approx 0.70, \quad Z^{\rm loc}_{A} \approx 0.77, \quad Z^{\rm loc}_{V}/Z^{\rm loc}_{A} \approx 0.91 . \tag{12}$$

Note that with the Wilson action the *free* lattice quark propagator is normalized[1] with respect to the continuum propagator by

$$\int d^3\mathbf{x}\langle 0|\psi(x)\bar{\psi}(0)|0\rangle^{\rm cont}=2\kappa e^{ma}\sum_{\mathbf{x}}\langle 0|\psi(x)\bar{\psi}(0)|0\rangle^{\rm latt} \tag{13}$$

with

$$ma=\ln\left[1+\frac{1}{2\kappa}-\frac{1}{2\kappa_c}\right]. \tag{14}$$

Equations (13) and (14) are also approximately valid in the interacting case if $\kappa_c$ is replaced by the renormalized $\kappa_c$. The factor $2\kappa$ in Eq. (13) is included in the renormalization constants in Eq. (12); we include $e^{ma}$ through Eq. (14). Note that keeping the $e^{ma}$ factor results in a quark propagator which is correctly normalized in both the large mass (static) and small mass limits. The systematic error associated with the normalization in the intermediate regime is therefore expected to be reduced; it is estimated below.

The results presented here are from our $24^3\times 39$ lattice at $\beta=6.0$ with 8 quenched gauge configurations generated under the DOE Grand Challenge Program as described in Ref. [1]. Our previous estimate for the scale parameter

TABLE II. The form factors for $D$ decays to vector-meson final states on the $24^3\times 40$ lattice at $\beta=6.0$. (The subscript sp in $\kappa_{\rm sp}$ stands for spectator.)

| $\kappa_i$ | $\kappa_f$ | $\kappa_{\rm sp}$ | $p_f$ | $q^2/m_i^2$ | $A_1(q^2)$ | $A_2(q^2)$ | $A_2/A_1(q^2)$ | $V(q^2)$ | $V/A_1(q^2)$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.118 | 0.152 | 0.152 | 0.0 | 0.299 | 0.99(15) | | | | |
| | | | 1.0 | 0.206 | 0.69(12) | 0.71(20) | 1.03(20) | 1.53(27) | 2.20(16) |
| | | | $\sqrt{2}$ | 0.125 | 0.58(13) | 0.66(23) | 1.15(17) | 1.22(37) | 2.12(38) |
| | | | 2.0 | −0.032 | 0.62(15) | 0.50(18) | 0.81(15) | 1.03(29) | 1.68(25) |
| 0.118 | 0.154 | 0.154 | 0.0 | 0.360 | 1.01(17) | | | | |
| | | | 1.0 | 0.249 | 0.70(14) | 0.87(44) | 1.24(51) | 1.59(39) | 2.27(26) |
| | | | $\sqrt{2}$ | 0.160 | 0.46(17) | 0.33(35) | 0.72(49) | 1.00(53) | 2.19(65) |
| | | | 2.0 | −0.030 | 0.47(23) | 0.05(34) | 0.12(66) | 0.85(47) | 1.8(11) |
| 0.118 | 0.155 | 0.155 | 0.0 | 0.391 | 1.03(22) | | | | |
| | | | 1.0 | 0.274 | 0.69(16) | 0.91(87) | 1.3(11) | 1.61(54) | 2.34(45) |
| | | | $\sqrt{2}$ | 0.180 | 0.31(21) | −0.15(49) | −0.5(19) | 0.75(64) | 2.4(14) |
| | | | 2.0 | −0.037 | 0.42(47) | −0.32(72) | −0.8(22) | 1.1(10) | 2.6(31) |
| 0.118 | 0.152 | 0.154 | 0.0 | 0.321 | 1.04(16) | | | | |
| | | | 1.0 | 0.217 | 0.73(14) | 0.82(33) | 1.13(37) | 1.61(34) | 2.22(18) |
| | | | $\sqrt{2}$ | 0.131 | 0.54(16) | 0.50(29) | 0.93(29) | 1.16(46) | 2.16(48) |
| | | | 2.0 | −0.043 | 0.65(21) | 0.53(26) | 0.82(21) | 1.17(47) | 1.80(42) |
| 0.118 | 0.152 | 0.155 | 0.0 | 0.331 | 1.07(16) | | | | |
| | | | 1.0 | 0.221 | 0.75(14) | 0.90(53) | 1.20(62) | 1.74(40) | 2.33(19) |
| | | | $\sqrt{2}$ | 0.133 | 0.50(17) | 0.37(35) | 0.74(48) | 1.14(53) | 2.28(52) |
| | | | 2.0 | −0.052 | 0.66(26) | 0.51(36) | 0.78(35) | 1.40(70) | 2.13(58) |
| 0.135 | 0.152 | 0.152 | 0.0 | 0.135 | 1.08(14) | | | | |
| | | | 1.0 | 0.006 | 0.77(11) | 0.64(11) | 0.83(12) | 1.22(34) | 2.03(14) |
| | | | $\sqrt{2}$ | −0.109 | 0.65(11) | 0.62(12) | 0.95(09) | 1.22(34) | 1.87(33) |
| | | | 2.0 | −0.324 | 0.69(14) | 0.44(08) | 0.64(10) | 1.04(24) | 1.50(19) |
| 0.135 | 0.154 | 0.154 | 0.0 | 0.187 | 1.10(17) | | | | |
| | | | 1.0 | 0.031 | 0.79(14) | 0.80(25) | 1.02(25) | 1.55(33) | 1.97(17) |
| | | | $\sqrt{2}$ | −0.101 | 0.57(15) | 0.47(18) | 0.83(16) | 1.00(44) | 1.77(48) |
| | | | 2.0 | −0.355 | 0.79(29) | 0.58(18) | 0.74(18) | 1.07(56) | 1.36(45) |
| 0.135 | 0.155 | 0.155 | 0.0 | 0.215 | 1.12(21) | | | | |
| | | | 1.0 | 0.047 | 0.79(18) | 0.90(53) | 1.14(53) | 1.53(43) | 1.95(26) |
| | | | $\sqrt{2}$ | −0.094 | 0.45(18) | 0.21(28) | 0.48(48) | 0.79(50) | 1.76(72) |
| | | | 2.0 | −0.376 | 0.93(63) | 0.73(42) | 0.78(30) | 1.2(13) | 1.24(80) |
| 0.135 | 0.152 | 0.154 | 0.0 | 0.148 | 1.13(15) | | | | |
| | | | 1.0 | 0.002 | 0.81(13) | 0.73(18) | 0.90(20) | 1.58(28) | 1.95(13) |
| | | | $\sqrt{2}$ | −0.124 | 0.64(14) | 0.56(15) | 0.87(12) | 1.15(39) | 1.79(39) |
| | | | 2.0 | −0.361 | 0.77(22) | 0.50(14) | 0.65(14) | 1.14(40) | 1.48(30) |
| 0.135 | 0.152 | 0.155 | 0.0 | 0.154 | 1.15(15) | | | | |
| | | | 1.0 | −0.001 | 0.84(14) | 0.77(31) | 0.92(34) | 1.65(33) | 1.97(13) |
| | | | $\sqrt{2}$ | −0.132 | 0.62(16) | 0.51(20) | 0.82(21) | 1.13(43) | 1.80(42) |
| | | | 2.0 | −0.383 | 0.83(29) | 0.55(24) | 0.66(21) | 1.31(60) | 1.57(39) |

[1]We are grateful to Paul Mackenzie for his remarks concerning the normalization of the propagator in the free and interacting cases.

TABLE III. The form factors for $D \rightarrow K^*$ and various systematic errors. The errors for the "SU(3) limit" show what change would occur by enforcing $m_s = m_{\mathrm{light}}$; they are not included in the total since we do *not* work in that limit.

| | $A_1(0)$ | $A_2(0)$ | $A_2/A_1(0)$ | $V(0)$ | $V/A_1(0)$ | $A_0(0)$ | $A_0/A_1(0)$ |
|---|---|---|---|---|---|---|---|
| Result | 0.83 | 0.59 | 0.70 | 1.43 | 1.99 | 0.71 | 0.94 |
| Stat. error | 0.14 | 0.14 | 0.16 | 0.45 | 0.22 | 0.16 | 0.09 |
| Extrapolation | 0.03 | 0.11 | 0.12 | 0.06 | 0.20 | 0.10 | 0.17 |
| $a^{-1}$ | −0.04 | −0.08 | −0.06 | +0.05 | +0.12 | +0.03 | +0.11 |
| | +0.03 | +0.11 | +0.14 | −0.14 | −0.20 | −0.05 | −0.15 |
| Scaling | 0.25 | 0.17 | 0.07 | 0.43 | 0.20 | 0.21 | 0.09 |
| Quark normal | 0.11 | 0.07 | | 0.19 | | 0.09 | |
| [SU(3) limit | −0.13 | +0.03 | +0.14 | −0.23 | −0.02 | −0.04 | +0.19] |
| Total | −0.28 | −0.23 | −0.15 | +0.48 | +0.31 | +0.25 | +0.22 |
| | +0.28 | +0.24 | +0.20 | −0.49 | −0.35 | −0.25 | −0.24 |

was $a^{-1} = 1.7$ GeV, based on earlier string tension determinations [20]. However, we now take a higher central value at $\beta = 6.0$ of $a^{-1} = 2.0$ GeV, which is roughly at the center of the range used by various lattice groups, and thus simplifies comparison with other work. Note that an even higher value ($a^{-1} = 2.3$ GeV) is suggested by a recent string tension computation [21]; we assume below a 20% uncertainty in the scale.

We used two different hopping parameters for the charmed quark: $\kappa_{\mathrm{ch}} = 0.135$ and $\kappa_{\mathrm{ch}} = 0.118$. With $a^{-1} = 2.0$ GeV, we interpolate to $\kappa_{\mathrm{ch}} = 0.128$ to get the $D$ meson mass $m_D = 1.87$ GeV in the chiral limit. The hopping parameter for the strange quark is taken to be $\kappa_s = 0.152$. The $D$ is always at rest, while the $K^*$ is given three momentum 0, $\mathbf{p}_{\min}$, $\sqrt{2}\mathbf{p}_{\min}$, or $2\mathbf{p}_{\min}$, with $ap_{\min} = \pi/12$. Imposing the discrete symmetries improves the signal (see [1]).

Table II shows our results for all the different hopping parameters before any extrapolations are made. For the decay $D \rightarrow K^*$, Table III lists the (physical) form factors and ratios of form factors at zero momentum transfer, and the contributions of various systematic errors. Figure 1 shows our results for $A_1(q^2)$, extrapolated to physical light-quark mass. On the lattice the form factors are calculated at various values of the momentum transfer (as shown in Fig. 1). Thus assumptions about the $q^2$ dependence are, in principle, not needed. However, numerical limitations do not allow us to extract quantitatively the $q^2$ dependence at present. The form factors at $q^2 = 0$ are therefore obtained using the pole-dominance assumption with limited consistency checks. The resonance masses for this are chosen as described in Ref. [1]. The extrapolation error in Table III includes different methods of extrapolation (see Ref. [1]) as well as different methods of extracting the form factors as described after Eq. (11).

Table III also shows the effect of the overall scale uncertainty (taken to be 20%) on the form factors and their ratios. The scale uncertainty has varying effects on the dimensionless form factors since it acts indirectly through their mass (hence $q^2$) dependence when the physical hopping parameters of the quarks are determined. In particular, $A_2$ and $A_2/A_1$ vary by $\approx 20\%$, whereas $V$ and $A_0$ are only affected at the few percent level.

The error due to scaling violation (i.e., variation of dimensionless quantities with $\beta$) is considered separately. It was estimated in [1] (by comparison of results from $\beta = 5.7$ and 6.0) as a 30% effect for the form factors and a 10% effect for the form-factor ratios.

For the quark field normalizations described in Eqs. (13) and (14), we have estimated an uncertainty of 13% by comparison with an alternative evaluation of $e^{ma}$ in Eq. (14) taking the physical charm-quark mass from potential models. Note that the ratios of form factors do not depend on the quark normalization and are thus not affected by this uncertainty.

We have done this calculation using nondegenerate quarks, i.e., $m_s \neq m_{\mathrm{light}}$. A comparison with the SU(3) limit for this decay shows appreciable deviations in many cases.

Note that we have not included an estimate of the systematic error associated with the quenched approxima-

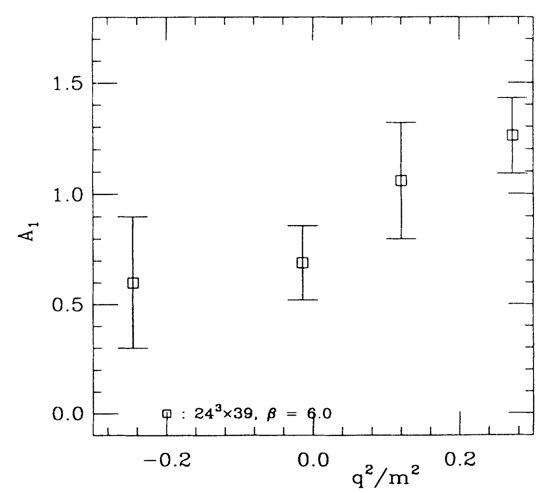


FIG. 1. The form factor $A_1(q^2)$ vs $q^2/m_D^2$. The errors shown are statistical only.

tion. By comparison of our results for $K \to \pi$ and $D \to K$ [1] with experiment, one could place a weak limit of $\sim 30\%$ on this effect. Our expectation is that the actual error is considerably smaller, but there is no hard evidence for this.

Our best results are obtained for ratios of form factors as opposed to the form factors themselves. This is due to the partial cancellation of scaling errors, the reduction of statistical fluctuations (the form factors are correlated), the aforementioned cancellation of the quark normalizations, and the likely reduction of uncertainties due to the nonperturbative renormalizations of the currents. However, note that the $O(a)$ effects could be different for different form factors, so we are not guaranteed a complete cancellation of nonperturbative renormalization effects even for the ratio of two axial-vector form factors. Note also that for the ratio $A_2/A_1$ the assumption of pole dominance is not needed; one only has to assume that the two form factors have the same $q^2$ dependence.

Our results for $D \to K^*$ are

$$
\begin{aligned}
A_1(0) &= 0.83 \pm 0.14^{+0.28}_{-0.28}, \\
A_2(0) &= 0.59 \pm 0.14^{+0.24}_{-0.23}, \\
V(0) &= 1.43 \pm 0.45^{+0.48}_{-0.49}, \\
A_0(0) &= 0.71 \pm 0.16^{+0.25}_{-0.25}, \\
A_2/A_1(0) &= 0.70 \pm 0.16^{+0.20}_{-0.15}, \\
V/A_1(0) &= 1.99 \pm 0.22^{+0.31}_{-0.35}, \\
A_0/A_1(0) &= 0.94 \pm 0.09^{+0.22}_{-0.24}.
\end{aligned}
\tag{15}
$$

The first error is the statistical uncertainty (computed using the jackknife method); the second is the systematical uncertainty (see Table III). Our results for the form factors for $D \to K^*$ are in rough agreement with quark-model calculations. They also do not appear to be inconsistent with the results reported by E691 [8] within the (rather large) uncertainties. Our values for $A_2(0)/A_1(0)$ and $V(0)/A_1(0)$ seem to be in good agreement with the results reported by E653 [15].

The ELC Collaboration has performed a similar lattice calculation [5] on a $20\times 10^2 \times 40$ lattice, also at $\beta=6.0$ (see Table I). Our results for $A_1(0)$, $A_2(0)$, and $V(0)$ are consistent with theirs within $\sim 1.5\sigma$. However, we suspect that their would be more significant disagreement were they to compute the ratio $A_2(0)/A_1(0)$ directly (see below). In any case the present results give a very different qualitative impression since $A_2(0)$ here is

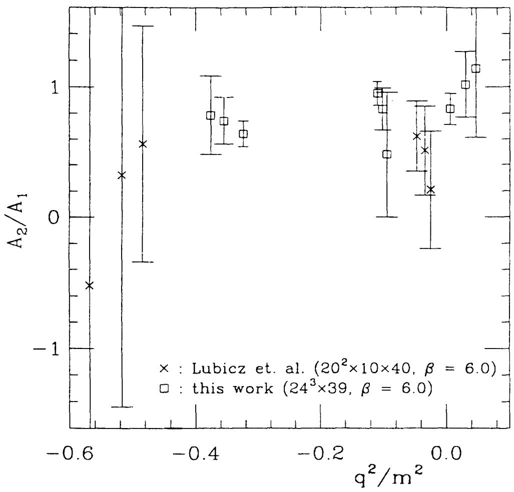


FIG. 2. The ratio $A_2/A_1(q^2)$ vs $q^2/m_D^2$ in comparison. Statistical errors only are shown; the errors for the results by Lubicz, Martinelli, and Sachrajda [5] are computed by us from their results for $A_2$ and $A_1$, and are therefore likely to be an overestimate. Each group of three points corresponds to a particular value of $\mathbf{p}_{K^*}$. Within each of the three groups of points near $q^2/m^2 \approx 0.0$, light-quark mass decreases to the right; within each of the two groups of points near $q^2/m^2 \approx -0.4$, light-quark mass decreases to the left.

significantly different from zero. In order to trace down the difference, we show together in Fig. 2 the two results for the ratio $A_2/A_1$ in the SU(3) limit before any extrapolations are made and before any renormalization constants are put in. The lattice operators, couplings, and heavy-quark hopping parameters are the same; the volumes (and therefore $\mathbf{p}_{K^*}$) and the light hopping parameters differ. We used $\kappa_{\text{light}}=0.152, 0.154, 0.155$ and a $24^3$ spatial volume; whereas the ELC choices are $\kappa_{\text{light}}=0.1515, 0.153, 0.1545$ and $20\times 10^2$. Because the ELC group has not calculated the ratio directly in their simulation, the errors on their values are assigned by us and are presumably overestimated because of the correlations. Note first that, for both calculations, the ratio at fixed light-quark mass shows little $q^2$ dependence. This is not unexpected since it is certainly true in the pole-dominance approximation. The major difference between the calculations comes in the extrapolation to the chiral limit. While the ELC results for both momentum values ($\mathbf{p}_{\min}$ and $2\mathbf{p}_{\min}$) drop as the light-quark mass deceases, our results show no universal trend. Indeed, for two out of the

TABLE IV. The form factors for $D \to \rho$ and $D_s \to \phi$ on the $24^3\times 40$ lattice at $\beta=6.0$.

| Process | $A_1(0)$ | $A_2(0)$ | $A_2/A_1(0)$ | $V(0)$ | $V/A_1(0)$ | $A_0(0)$ | $A_0/A_1(0)$ |
|---|---|---|---|---|---|---|---|
| $D \to \phi$ | 0.73 | 0.55 | 0.78 | 1.30 | 2.00 | 0.71 | 1.14 |
| Stat. error | 0.12 | 0.10 | 0.08 | 0.32 | 0.19 | 0.13 | 0.04 |
| Syst. error | $\pm 0.24$ | $^{+0.24}_{-0.20}$ | $^{+0.17}_{-0.13}$ | $\pm 0.43$ | $^{+0.20}_{-0.25}$ | $\pm 0.23$ | $^{+0.13}_{-0.19}$ |
| $D \to \rho$ | 0.65 | 0.59 | 0.89 | 1.07 | 2.01 | 0.64 | 1.21 |
| Stat. error | 0.15 | 0.31 | 0.37 | 0.49 | 0.40 | 0.17 | 0.16 |
| Syst. error | $^{+0.24}_{-0.23}$ | $^{+0.28}_{-0.25}$ | $^{+0.22}_{-0.19}$ | $\pm 0.35$ | $^{+0.30}_{-0.33}$ | $\pm 0.21$ | $^{+0.25}_{-0.26}$ |

three momentum values (i.e., for $\mathbf{p}_{\min}$ and $2\mathbf{p}_{\min}$ but not $\sqrt{2}p_{\min}$), the ratio increases slightly as one approaches the chiral limit. Thus, whether we fit to a constant in $q^2$ for fixed light quark mass and extrapolate to the chiral limit, or extrapolate first and then fit, we get a value for the ratio which is clearly bounded away from zero. The ELC group takes the quark mass dependence seen on their lattices seriously, and gets a value for $A_2(0)$ which is consistent with zero. It would be interesting to know if the differences between the calculations are physical (a finite-volume effect?), but we cannot tell from Fig. 2 since we do not know how much we have overestimated the ELC errors. (The errors in the form factors themselves are too large in both calculations to learn anything interesting there.)

Our results for the decays $D \rightarrow \rho$ and $D_s \rightarrow \phi$ are summarized in Table IV. We have taken $\phi = s\bar{s}$. The (Zweig suppressed) disconnected graphs that appear only in the decay $D_s \rightarrow \phi l \nu$ have been neglected. The decay rates can be calculated from the form factors in Table IV and Eq. (5) and compared with Eqs. (6) and (7), but because of the large experimental and theoretical uncertainties, we learn little from this exercise at the present time. Certainly a determination of the form factors for these decays from experiments is desirable for a truly meaningful comparison with the lattice results.

In conclusion, we have evaluated the form factors for various semileptonic decays into vector mesons on the lattice. The comparison with experiments is not yet conclusive, because of large uncertainties in both lattice and experimental results. On the experimental side this will be resolved in a few years time. On the lattice side, we expect [1] that calculations at the $\leq 10\%$ precision level will be possible, especially if current efforts towards a significant increment in computing power are successful [22].

*Note added.* After this paper was completed, we received a paper [INFN Report No. 808, Southampton Report No. SHEP 90/91-27 (unpublished)] from the ELC group of V. Lubicz, G. Martinelli, M. S. McCarthy, and C. T. Sachrajda, in which they present results obtained by including an additional 15 configurations along with the original 15 used in the study quoted here [5]. They report no qualitative change in their results. In particular, they emphasize that their value for $A_2(0)$ is still consistent with zero ($0.19 \pm 0.21$). We are grateful to Guido Martinelli for discussions.

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