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How large is the total cross section at supercollider energies?

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Although the value of the total cross section is critical for the operation and physics exploitation of supercolliders, we have until now been unable to anticipate its magnitude. Extrapolations of low-energy data patterned after models with a varying degree of dynamical justification, and invariably a too large number of free parameters to be truly predictive, led to a wide range of predictions. We point out that a series of new measurements at the Fermilab Tevatron Collider on forward-scattering parameters dramatically narrows the range of extrapolations and we anticipate that $\sigma_{\rm tot} = 107 \pm 4$ mb at $\sqrt{s} = 16$ TeV, and $\sigma_{\rm tot} = 121 \pm 5$ mb at $\sqrt{s} = 40$ TeV, using a QCD-inspired parametrization. More surprisingly, the model dependence of the extrapolations is reduced by the new data to the point that a wide range of models investigated converge on the above values.

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I. INTRODUCTION

The commissioning of the Fermilab Tevatron has extended the kinematic range over which QCD can be confronted with experiment. Recent measurements [1, 2] of $\sigma_{\rm tot}$, B, and the ρ parameter at $\sqrt{s} = 1800$ GeV have basically completed the information we will have on forward-scattering parameters before the supercolliders are in operation. On the theoretical side there are now three main approaches to interpret this information: (i) the Regge pole model, (ii) analytic asymptotic amplitude analysis, and (iii) QCD-inspired models. Although these models can all accommodate the data, they differ in significant aspects. Most importantly, the three models ascribe the rise of the total cross section as due to (i) a Regge power s^{α}, (ii) an asymptotic term which the data now pinpoints to behave as $\ln s$ [3, 4] and (iii) the dramatic increase of the number of soft partons, respectively.

In actual fits the three approaches give virtually indistinguishable results in the energy region in which data are available. However, this similarity disappeares at sufficiently high energy leading to widely varying predictions for total cross sections at energies of the CERN Large Hadron Collider (LHC) and the Superconducting Super Collider (SSC). A study of the high energy predictions of such models is timely and important, given the critical impact of the forward-scattering parameters on the operation and physics exploitation of hadron colliders. Our main conclusion will be that all approaches converge on the value $\sigma_{\rm tot}({\rm LHC}) = 107 \pm 4\,{\rm mb}$ and $\sigma_{\rm tot}({\rm SSC}) = 121 \pm 5\,{\rm mb}$ once the new data is included. The errors are specific to an analysis performed with the models of type (iii). In the future these predictions can still be sharpened by more accurate measurements of the high energy ρ values. This latter statement is model independent, to the extent that ρ (at a lower energy) is connected to the higher energy behavior of $\sigma_{\rm tot}$ by analyticity.

II. GENERAL DISCUSSION OF THE MODELS

What we have learned from Tevatron measurements is that even at $\sqrt{s} = 1.8\,{\rm GeV}$ we are not witnessing the asymptotic behavior of hadron collisions, despite the fact that $\sigma_{\rm tot}$ has risen by almost a factor of 2 from a nominal 40-mb low-energy value. Instead, as we will show below, we are in a transitional energy region in which competing contributions are at play, and the energy is still too low to decipher experimentally the true asymptotics. We will now make several arguments supporting this thesis.

(i) At the highest energy (1800 GeV) $\sigma_{\rm tot} \approx 70\,{\rm mb}$. Clearly the rising component does not yet dominate over whatever mechanism is responsible for low-energy cross sections in the 40 mb range.

(ii) The minijet phenomenon is one of many indications that the properties of hadronic interactions are changing in the Tevatron Collider energy region. It is difficult to believe that this physics is not going to influence the total cross section via unitarity.

(iii) We already pointed out [5] that the behavior of the slope of the differential elastic scattering cross section shows a transition from the region in which the features of the forward peak are dominated by the hadronic matter form factor of the nucleon to a region in which the nucleon seems to develop a disklike structure [5], due to the gluon-gluon interaction. The logarithmic curvature C, the second logarithmic derivative of the differential cross section, is observed to become zero [6] at $\sqrt{s} = 1.8\,\text{TeV}$, whereas for lower energies, it was substantially positive. We remind the reader that a disk has a *negative* curvature, given by $C = -\frac{1}{192}R^4$, where R is the radius of the disk.

III. SPECIFIC MODELS

We next turn our attention to the detailed asymptotic behavior of these three models.

Regge pole. The Regge pole model [7], in which the scattering amplitude grows as a power of s, i.e., $s^{0.086}$, violates unitarity and thus cannot be applicable in this form at sufficiently high energies. Hence, the cross sections $\sigma_{\text{tot}} \approx 115\,\text{mb}$ at $\sqrt{s} = 16$ TeV and $\sigma_{\text{tot}} \approx 135\,\text{mb}$ at $\sqrt{s} = 40$ TeV predicted by this model should be regarded as an upper limit. In an analysis implementing unitarization in a minimal way by including the exchange of two Regge poles, the latter value is reduced to 120 mb, thus anticipating [7] the value on which our analysis of the new Tevatron data will converge.

Amplitude analysis. Block and White [3] have made an analytic asymptotic amplitude analysis [4] to the existing experimental data. They used an even amplitude whose cross section contribution went as a constant term, a Regge term that vanished as $1/\sqrt{s}$, and, *asymptotically*, a term in $\ln s$. They were *unable* to get a satisfactory fit using a term in $\ln^2 s$, however. Using a $\ln s$ fit, they obtained a SSC cross section $\sigma_{\text{tot}} = 118 \pm 1.2\,\text{mb}$ (where the error is statistical, and results from the errors in the fitted parameters) and a LHC cross section $\sigma_{\text{tot}} = 105 \pm 1.2\,\text{mb}$.

These predictions probably should be regarded as a lower limit. The authors have carefully pointed out that they consider this analysis more an exercise in fitting than in fundamental physics, since they have strong doubts about the validity of applying *any* asymptotic amplitude analysis at energies less than or the order of the Tevatron Collider. They do *not* feel that, from present data, a cross section varying *asymptotically* as $\ln^2 s$ (at *energies much higher* than those for which data is now available) is excluded. Thus, the $\ln s$ value of 118 mb might be expected to be a *lower limit.*

Eikonalized QCD models. Cheng and Wu [8] argued that eikonalization should properly unitarize Regge models. We have previously used a QCD-inspired, eikonalized model [5,9]. As we will show, this model predicts a cross section of 121 mb value, a value *smaller* than the Regge prediction, as expected.

QCD-inspired models allow one to reformulate the Froissart bound born in axiomatic field theory. We found that, asymptotically [5],

$$\sigma_{\text{tot}} = 2\pi b_c^2 = 2\pi \left(\frac{J-1}{\mu_{gg}}\right)^2 \ln^2 \frac{s}{s_0}\,. \qquad (1)$$

The coefficient of the $\ln^2 s$ term is given in terms of parameters describing the gluon density of the nucleon, rather than the pion mass which sets the scale of the $\ln^2 s$ coefficient in the original bound. The origin of the rising cross section in Eq. (1) is the increasing number of soft gluons at small x, where the gluon structure function behaves as x^{-J}. The large number of gluons turns the proton into a disk with radius $\mu^{-1} \simeq (0.8\,\text{GeV})^{-1}$. In this model the eikonal behaves *asymptotically* as s^{J-1}. From fitting to experiment, we find $J-1 \approx 0.05$–0.06, not very different from the power behavior of $s^{0.086}$ as given by the Donnachie-Landshoff Pomeron amplitude [7].

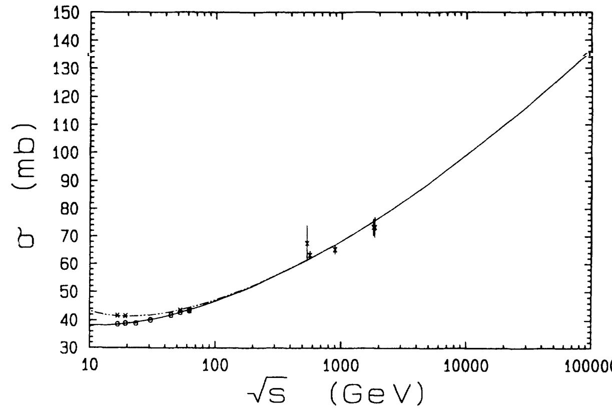

FIG. 1. σ_{tot}, the calculated and measured total cross sections in mb, for $\bar{p}p$ (dashed curve and crosses) and pp (full curve and circles), vs $\sqrt{s}$, the energy, in GeV.

It is fair to say that these models' connection with QCD dynamics is still very tentative so that none of them have general acceptance. Surprisingly, this model dependence will in the end disappear because of the tight constraints imposed by the new data. For definiteness we proceed first with the QCD-inspired approach [5]. After including quarks as well as gluons, contributions to the total cross section of a constant term, Regge type terms, and a $\ln s$ term also appear, which render the asymptotic QCD behavior of Eq. (1) more complicated. They are, however, essential when one attempts to apply these ideas to the transitional energy regime we previously discussed. We have refitted the model, including the new Tevatron data for σ_{tot}, ρ, and B. We used the formalism of Ref. [9]. Fine-tuning of parameters represented

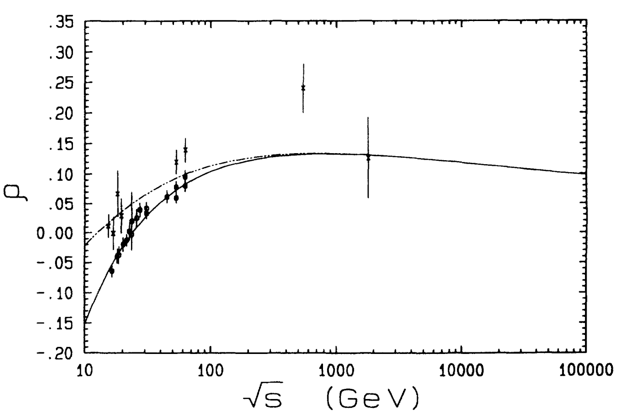

FIG. 2. ρ, the calculated and measured ratios $\text{Re}f(0)/\text{Im}f(0)$, for $\bar{p}p$ (dashed curve and crosses) and pp (full curve and circles), vs $\sqrt{s}$, the energy, in GeV.

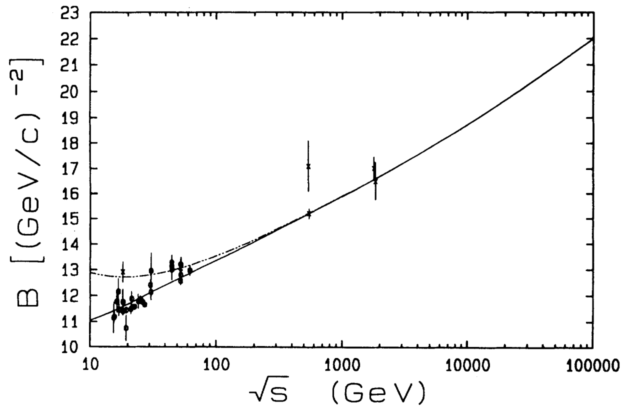

FIG. 3. B, the calculated and measured nuclear slope parameters in $(\mathrm{GeV}/c)^{-2}$, for $\bar{p}p$ (dashed curve and crosses) and pp (full curve and circles) vs $\sqrt{s}$, the energy, in GeV.

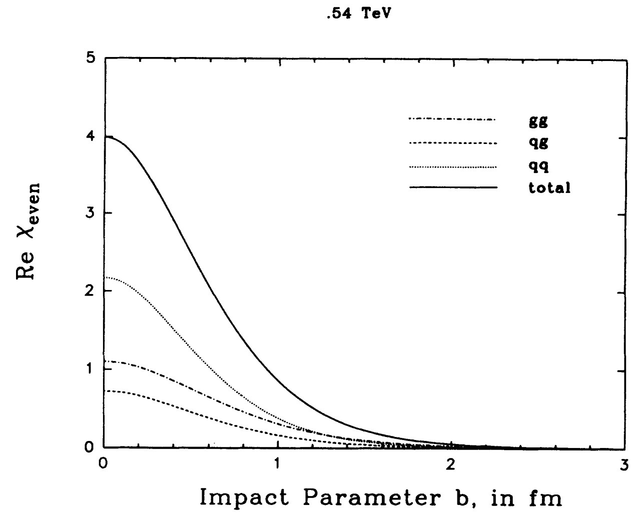

FIG. 5. The real portion of the even eikonal, $\mathrm{Re}\chi_{\mathrm{even}}$ vs b, the impact parameter distance, in fm, at 540 GeV. The dotted curve is the quark-quark (qq) contribution, the dashed curve is the quark-gluon (qg) contribution, the dashed-dotted curve is the gluon-gluon contribution, and the solid curve is the summed (total) contribution.

the only modification necessary to accommodate the new data. No parameters have qualitatively changed. The data were simultaneously fit for σ_{tot}, ρ, and B, for both pp and $\bar{p}p$ collisions, in the energy region 15–1800 GeV, and the results are shown in Figs. 1–3, respectively. The technique of fitting forward scattering parameters allows one to predict the elastic scattering cross section $d\sigma/dt$ vs $|t|$, the four-momentum transfer squared, as well. We show the results for three typical energies, 1.8, 16, and 40 TeV, in Fig. 4. As expected, the curvature C becomes increasingly negative with energy, being zero at 1800 GeV, and becoming pronouncedly negative at 40 TeV. Further, we note the dip structure clearly moving to lower $|t|$ with increasing s, along with a very flat secondary maximum building up at large $|t|$.

In general, the impact parameter amplitude $a(s,b)$ is given by

$$a(s,b) = \frac{i}{2}\left(1 - e^{-\chi(s,b)}\right), \tag{2}$$

where b is the transverse distance in impact parameter space, and $\chi(s,b)$ is the eikonal [9, 10]. The nuclear amplitude $f_N(s,t)$ is given by

$$f_N(s,t) = 2\int b\, db\, J_0\left(b\sqrt{-t}\right)\, a(s,b), \tag{3}$$

and the total cross section and the differential elastic scattering cross section are given by

$$\sigma_{\mathrm{tot}} = 4\pi\, \mathrm{Im}\, f_N(s,t),$$
$$\frac{d\sigma}{dt} = \pi\left|f_N(s,t)\right|^2, \tag{4}$$

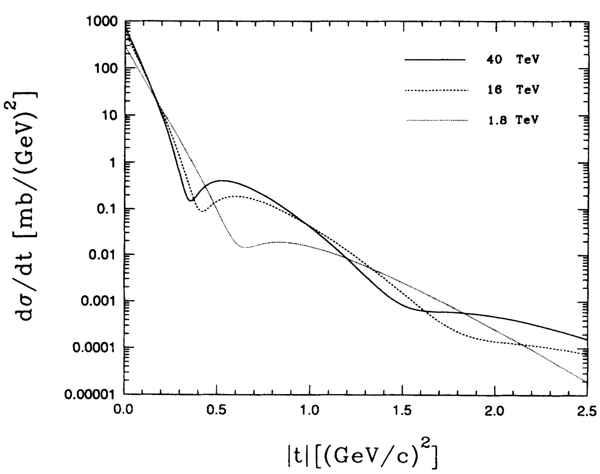

FIG. 4. $d\sigma/dt$, the elastic differential scattering cross section in $\mathrm{mb}/(\mathrm{GeV}/c)^2$, vs $|t|$, the squared four-momentum transfer, for energies of 1.8 TeV (dotted curve), 16 TeV (dashed curve), and 40 TeV (solid curve).

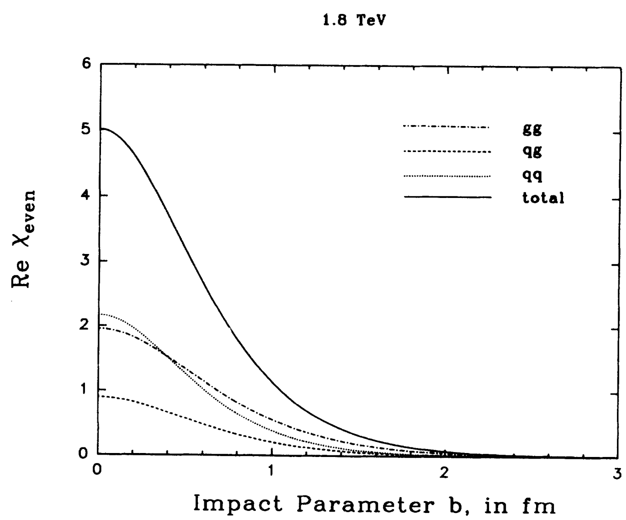

FIG. 6. The real portion of the even eikonal, $\mathrm{Re}\chi_{\mathrm{even}}$ vs b, the impact parameter distance, in fm, at 1.8 TeV. The legends are the same as in Fig. 5.

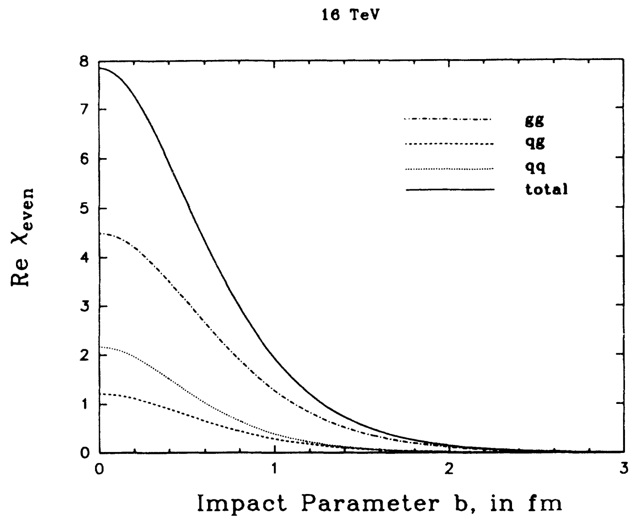

FIG. 7. The real portion of the even eikonal, Re$\chi_{\rm even}$ vs b, the impact parameter distance, in fm, at 16 TeV. The legends are the same as in Fig. 5.

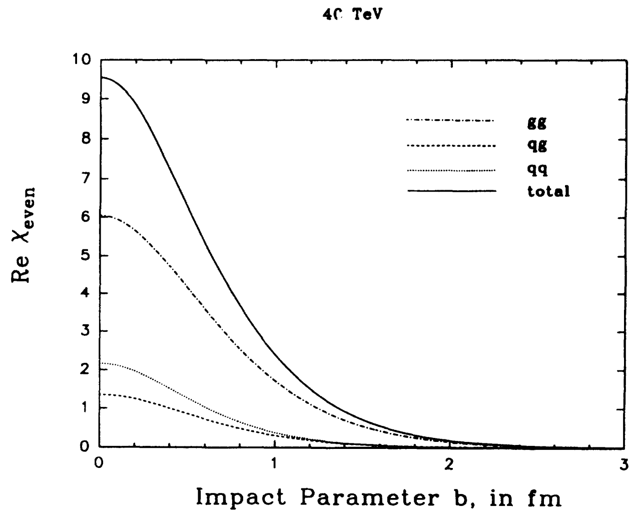

FIG. 8. The real portion of the even eikonal, Re$\chi_{\rm even}$ vs b, the impact parameter distance, in fm, at 40 TeV. The legends are the same as in Fig. 5.

respectively. In the process of fitting, we must obtain even and odd eikonals $\chi_{\rm even}(s,b)$ and $\chi_{\rm odd}(s,b)$, where $\chi = \chi_{\rm even} + \chi_{\rm odd}$. At large s, which is the domain considered here, $\chi_{\rm odd}(s,b)$ vanishes. We have plotted Re $\chi_{\rm even}(s,b)$, the real portion of the even eikonal [mainly responsible for the amplitude $a(s,b)$ in Eq (2), since χ is almost real, and $\chi_{\rm odd}$ vanishes at high energies] versus b for the energies 540 GeV, 1800 GeV, 16 TeV and 40 TeV in Fig. 5–8, respectively. We note that for 540 GeV, the contribution to the eikonal labeled qq (for quark-quark interactions) predominates over the contribution labeled gg (for glue-glue interactions). The qq contribution is basically constant with energy at reasonably high energies, whereas the gg contribution asymptotically grows as a power of s, i.e., s^{J-1}, and is responsible for the ultimate $\ln^2 s$ behavior of the cross section. We see that these two contributions are about *equal* at 1800 GeV, which is further strong evidence that this energy provides "the first look at the asymptotic nucleon," a thought first expounded by us in Ref. [5]. As we go up in energy, we see the gg contribution dominating, as expected asymptotically. We note the disklike behavior exhibited in the elastic scattering $d\sigma/dt$, at the highest energies shown in Fig. 4. The term qg, the quark-gluon interaction, which has a dominant $\ln s/s_0$ dependence (where s_0 is a scale factor), is found to be small, even at the highest energies.

The total cross sections at the LHC and SSC are of prime interest, for both the machine operation and physics results to be expected at these super colliders. Our fits give $\sigma_{\rm tot}({\rm LHC}) = 107 \pm 4\,{\rm mb}$ at $\sqrt{s} = 16$ TeV and $\sigma_{\rm tot}({\rm SSC}) = 121 \pm 5\,{\rm mb}$ at $\sqrt{s} = 40$ TeV, where the quoted error is the statistical error due to the errors in the fitted parameters. We comment that the central SSC value of 121 mb is nicely bracketed from above by 135 mb, the Regge pole upper limit, and from below by 118 mb, the extrapolation of an asymptotic $\ln s$ fit. The central value is identical to the prediction of Landshoff and Donnachie [7]. There is an equally nice result at the LHC energy, with the Regge pole upper limit of 116 mb and the $\ln s$ lower limit of 105 mb tightly bracketing our QCD prediction of 107 mb. These tight limits give us more confidence in our ability to make accurate total cross-section extrapolations to future colliders at substantially higher energies.

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