Multiplicity dependence of intermittent behavior and clustering production of hadrons

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(Received 17 June 1991; revised manuscript received 16 September 1991)

The multiplicity dependence of intermittency phenomena is discussed in connection with clustering production of hadrons. It is shown that the decrease of intermittency indices with increasing multiplicity is due to statistical fluctuation of such clusters.

PACS number(s): 13.85.Hd; 05.45.+b; 12.40.Ee

In recent years particle density fluctuations in small phase space have been extensively studied [1]. To study the intermittency in multiparticle production processes, Bialas and Peschanski [2] proposed that one should measure the scaled factorial moments. For an event with n hadrons in a rapidity interval ΔY the qth order of these is defined by

$$F_q(n) = \frac{1}{M}\sum_{m=1}^{M} \frac{M^q k_m(k_m-1)\cdots(k_m-q+1)}{n(n-1)\cdots(n-q+1)} . \tag{1}$$

In Eq. (1) the rapidity interval ΔY is divided into M bins of size $\delta y = \Delta Y/M$. Here k_m is the number of particles in the mth bin. This method was originally proposed to study the fluctuations of hadrons in high multiplicity events (e.g., in high-energy heavy ion and cosmic ray experiments). In practice, however, it is necessary to average over an ensemble of events with different multiplicities. Experimentally it is observed that the scaled factorial moments F_q averaged over an ensemble of events increase with decreasing bin size. In the region of small bin size, e.g., in rapidity region $0.1 < \delta y < 1.0$ power laws

$$F_q \propto (\delta y)^{-\varphi_q} \tag{2}$$

are observed not only in heavy ion and cosmic ray experiments, but also in hadronic and leptonic collisions [3]. It is interesting that the intermittency index φ_q is the largest in e^+e^- annihilation and the smallest in nucleus–nucleus collision. Since the average multiplicity in e^+e^- annihilation is lower than that in nucleus–nucleus collision, one would like to know how the intermittent behavior depends on the multiplicity.

In fact, the UA1 Collaboration [4] has studied the intermittent behavior in different multiplicity intervals in $\bar{p}p$ collision at c.m. energy 630 GeV and found that the effects are stronger in sample of events with lower multiplicities. This observation can not be explained by the commonly used Monte Carlo codes or models which are tuned to account for the general features of multiparticle production (see Ref. [4]). Some of these models predict that the effects should be stronger in a higher multiplicity sample which is *qualitatively* in disagreement with experimental data.

Several approaches were made to understand this striking feature recently. Some authors [5] explained such dependence with a random superposition of uncorrelated samples. It was also claimed that the origin of this dependence lies in the existence of a multitude of independent multiplicity distributions over the impact parameter plane in a geometrical picture [6]. For others [7] it is an indication for the local parton–hadron duality. These discussions are, however, only qualitative, semiqualitative, or by analogy. In this paper we present a quantitative calculation in the picture of clustering production.

The concept of hadronic clusters was proposed in the early 1970s in connection with short-range correlations [8] and rapidity-gap distributions [9]. However, little about cluster properties is understood.

In recent papers [10] we proposed a set of sum rules for such clusters to establish the links between clusters and directly measurable quantities, e.g., the charged multiplicity distribution. In this paper we will show that the intermittency phenomenon is related to the clustering production of hadrons and demonstrate how information on clusters can be obtained by studying the intermittency in different multiplicity intervals.

We note that the method proposed by Bialas and Peschanski [2] is not only useful in studying fluctuations, but also very useful in investigating clustering behavior of hadron production, because fluctuations (described by factorial moments F_q) and correlations [correlation functions $C(y_1, y_2, \ldots, y_q)$] are directly related to each other [11].

It should be emphasized here that the cluster notion we discuss here should not be identified with ordinary hadronic resonances. A cluster is a group of correlated hadrons which are independent from other hadrons pro-

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duced in the same process. It could be a clan, a string, a chain [12], or an intermittency source [13], whatever produces correlated hadrons.

To begin with, let us consider an event with N clusters in the rapidity interval ΔY and assume that every cluster may decay into $0, 1, 2, \ldots, c$ charged hadrons distributed in ΔY. Let $N_w(l|N)$ be the number of clusters, each of which contributes l charged hadrons to a rapidity bin of size δy (or a rapidity window) in the N-cluster event. Then, the number of charged hadrons n_w in the bin δy (the subscript w stands for window or bin) is given by

$$n_w = \sum_{l=0}^{c} l N_w(l|N). \tag{3}$$

By definition, the following equation should also be satisfied:

$$N = \sum_{l=0}^{c} N_w(l|N). \tag{4}$$

It should be noticed here that the number of charged hadrons n_w changes with the bin size, while the number of cluster N is independent of the bin.

Let $\alpha_w(l)$ be the average probability for the production of one of the $N_w(l|N)$ clusters which contributes l charged hadrons in the bin δy, and $P(N)$ be the probability for producing N clusters in ΔY; then the probability for observing n_w charged hadrons in the bin is given by the sum of weighted multinomial distributions [10]:

$$P_w(n_w) = \sum_N \sum{}' \; P(N) \left(N! \Big/ \prod_{l=0}^{c} N_w(l|N)! \right) \times \prod_{l=0}^{c} \alpha_w(l)^{N_w(l|N)} , \tag{5}$$

where the sum is first taken over $N_w(l|N)$. The prime on the summation sign indicates that the conditions given in Eqs. (3) and (4) should be satisfied.

The average probability $\alpha_w(l)$ for having a cluster which contributes l charged hadrons in the given bin is a useful quantity. On the one hand, it is closely related to the intrinsic properties of the clusters and, on the other hand, it reflects the relative importance of different kinds of clusters contributing to the given bin. Equation (5) connects the hadronic clusters and the experimentally directly measurable multiplicities. Because the cluster multiplicity distribution $P(N)$ and the cluster parameters $\alpha_w(l)$ are unknown, similar to the discussion in [10] we solve Eq. (5) with successive approximations. Before we do this, let us define the moments for both clusters in the rapidity interval and hadrons in the bin.

In the rapidity interval ΔY the average cluster number $\langle N \rangle$ is defined by

$$\langle N \rangle = \sum NP(N) \tag{6}$$

and the scaled factorial moments of clusters $\mathcal{F}_q$ by

$$\mathcal{F}_q \equiv \frac{\langle N(N-1)\cdots(N-q+1)\rangle}{\langle N \rangle^q} = \frac{\sum N(N-1)\cdots(N-q+1)P(N)}{(\sum NP(N))^q} . \tag{7}$$

In the bin of size δy the average hadron multiplicity $\langle n_w \rangle$ is defined by

$$\langle n_w \rangle = \frac{\sum n_w P_w(n_w)}{\sum P_w(n_w)} \tag{8}$$

and the scaled factorial moments of hadrons F_q by

$$F_q \equiv \frac{\langle n_w(n_w-1)\cdots(n_w-q+1)\rangle}{\langle n_w \rangle^q} = \frac{\sum n_w(n_w-1)\cdots(n_w-q+1)P_w(n_w)}{[\sum n_w P_w(n_w)]^q} . \tag{9}$$

From Eq. (5) we see that the average multiplicity of cluster $\langle N \rangle$ and that of hadrons $\langle n_w \rangle$ is related in the following way:

$$\langle n_w \rangle = \langle N \rangle \sum_{l=1}^{c} l\alpha_w(l) . \tag{10}$$

The following relations between the scaled factorial moments F_q and $\mathcal{F}_q$ can be also readily shown from Eq. (5):

$$F_2 = \mathcal{F}_2 + \frac{A_2}{\langle N \rangle} , \tag{11}$$

$$F_3 = \mathcal{F}_3 + \frac{3A_2}{\langle N \rangle}\mathcal{F}_2 + \frac{A_3}{\langle N \rangle^2} , \tag{12}$$

$$F_4 = \mathcal{F}_4 + \frac{6A_2}{\langle N \rangle}\mathcal{F}_3 + \frac{4A_3 + 3A_2^2}{\langle N \rangle^2}\mathcal{F}_2 + \frac{A_4}{\langle N \rangle^3} , \tag{13}$$

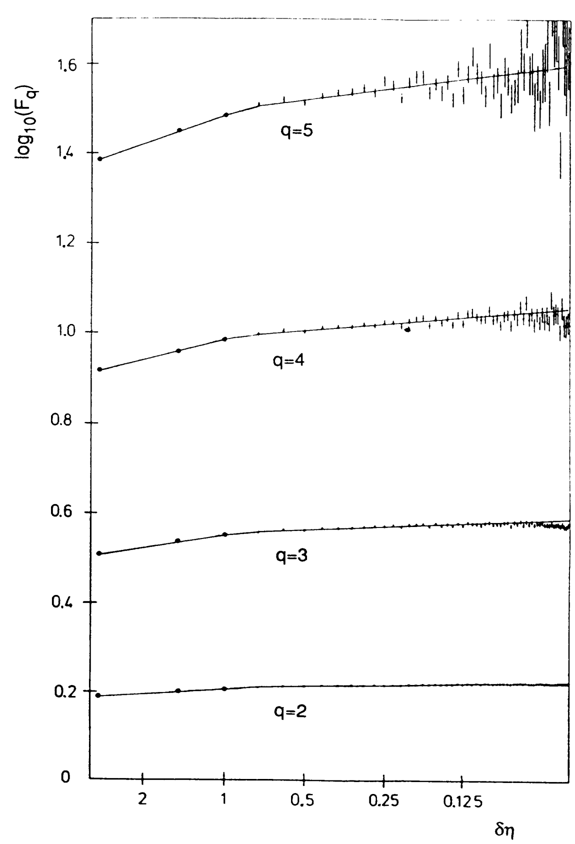

FIG. 1. $\log_{10} F_q$ vs $\delta\eta$. The data are taken from Ref. [4] and the solid curves are the fitting results.

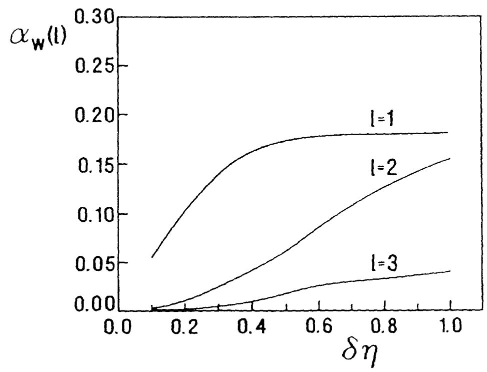

FIG. 2. The fitted cluster parameters $\alpha_w(l)$ as functions of bin size $\delta\eta$.

$$F_5 = \mathcal{F}_5 + \frac{10A_2}{\langle N\rangle}\mathcal{F}_4 + \frac{10A_3 + 15A_2^2}{\langle N\rangle^2}\mathcal{F}_3 + \frac{4A_4 + 10A_2A_3}{\langle N\rangle^3}\mathcal{F}_2 + \frac{A_5}{\langle N\rangle^4} \quad (14)$$

These, and similar equations (called sum rules) in which higher order $\mathcal{F}_q$ and F_q are involved are the basis for the successive approximation. Here the $\langle N\rangle$, $\mathcal{F}_q$ and F_q are given in Eqs. (6), (7), and (9), respectively. The A_i are defined by

$$A_i = \frac{\sum_{l=1}^{c} l(l-1)\cdots(l-i+1)\alpha_w(l)}{(\sum_{l=1}^{c} l\alpha_w(l))^i}, \qquad i \geq 2 \quad (15)$$

We note that the A_i are completely determined by the cluster parameter $\alpha_w(l)$. They depend on the bin size δy, while the scaled factorial moments for cluster $\mathcal{F}_q$, and the

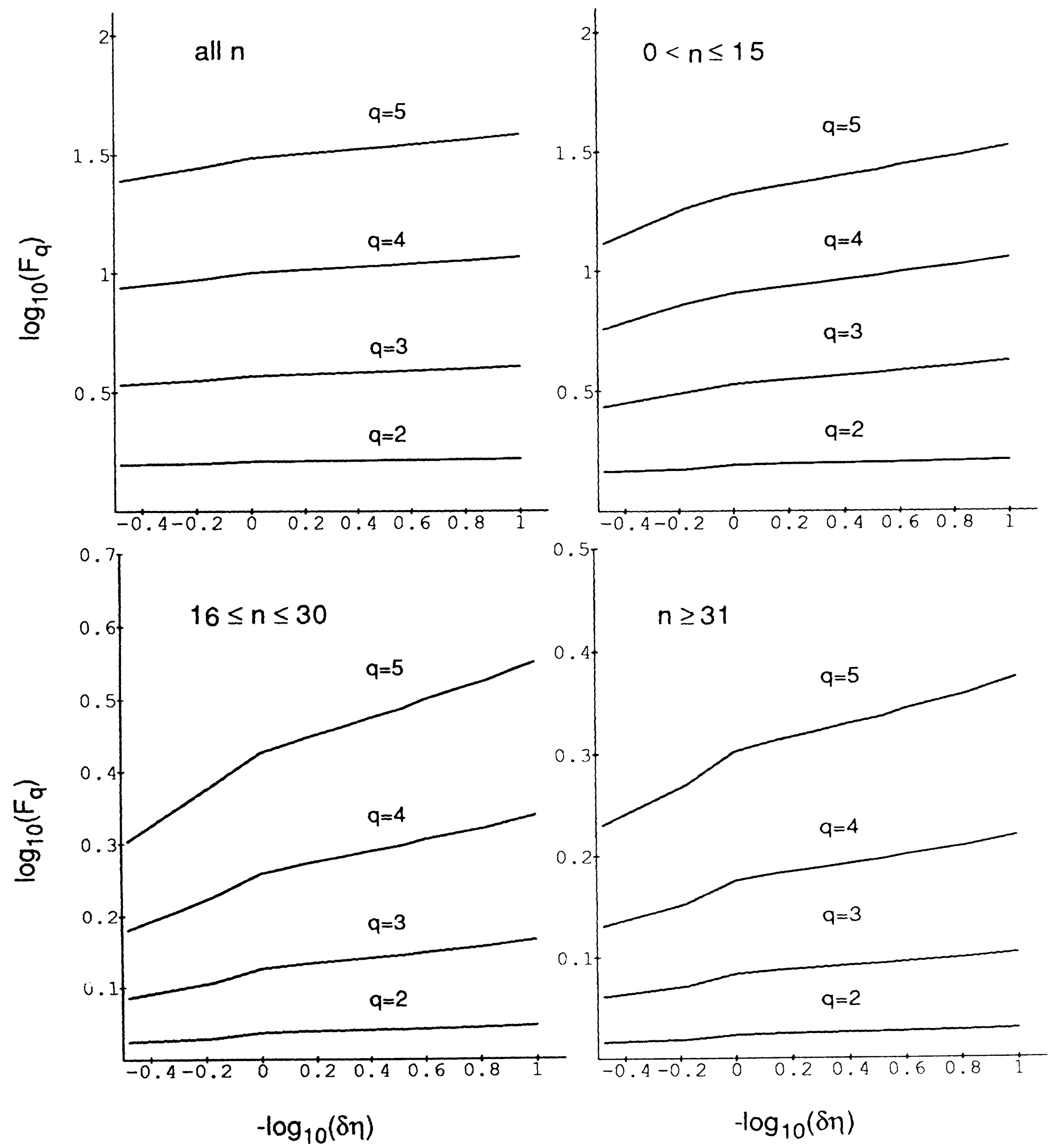

FIG. 3. The $\log_{10}F_q$ – $\log_{10}\delta\eta$ plots for different multiplicity intervals.

TABLE I. The average multiplicity $\langle N\rangle$ and scaled factorial moments $\mathcal{F}_q$ of clusters in dependence of different multiplicity intervals.

	$\langle N\rangle$	$\mathcal{F}_2$	$\mathcal{F}_3$	$\mathcal{F}_4$	$\mathcal{F}_5$
$n \leq 15$	3.21	1.09	1.16	1.07	0.79
$16 \leq n \leq 30$	10.1	0.94	0.84	0.71	0.57
$n \geq 31$	17.7	0.97	0.94	0.90	0.88

average cluster number $\langle N\rangle$ do not, because they are defined in the rapidity interval ΔY. However, according to Eqs. (6) and (7) $\mathcal{F}_q$ and the $\langle N\rangle$ depend on the multiplicity intervals from which the events are selected.

To carry out the successive approximation, let us, first of all, consider a simple case. We assume that every cluster decays into only *one* charged hadron; i.e., the charged hadrons are produced independently. From Eq. (15) we see that all A_i disappear. From Eqs. (11)–(14) we get $F_q = \mathcal{F}_q$. Since the $\mathcal{F}_q$ are independent of bin size, all the intermittency indices φ_q equal zero. That is, there is no intermittency in this case, as it should be, because the pure statistical fluctuations of hadrons are eliminated by the scaled factorial moments [2].

We assume now that in the rapidity interval ΔY each cluster decays into *two* charged hadrons. This ansatz is based on the fact that there are indications that charged hadrons are produced via neutral clusters which decay into a pair of oppositely charged hadrons [14]. In this approximation, the $\langle N\rangle$ and scaled factorial moments for clusters $\mathcal{F}_i$ can be readily calculated from the multiplicity distribution in the rapidity interval ΔY. The results for different multiplicity samples are listed in Table I. By adjusting the cluster parameters $\alpha_w(i)$ which depend on the bin size δy, power laws of F_q given in Eq. (2) can be obtained. In Fig. 1 we show the fitted F_q as a function of pseudorapidity bin size $\delta\eta$ together with UA1 data [4].

In Fig. 2 we show the fitted parameter $\alpha_w(l)$ as a function of bin size $\delta\eta$ in the region of $0.1 \leq \delta\eta \leq 1.0$ where the linear fits will be made. In this approximation $\alpha_w(2)$ will increase to 1 at $\delta\eta = \Delta Y = 3.0$, while other $\alpha_w(l)$'s will become zero. For simplicity, all the $\alpha_w(i)$ with $i \geq 4$ were taken to be zero in all rapidity bin sizes. The reasons for this are twofold. First, only large clusters contribute to the A_i with large i [see Eq. (15)] whose contribution to F_q is very much suppressed by the large average cluster number $\langle N\rangle$ [see Eqs. (11)–(14)]. Second, numerical calculation (see Fig. 2) shows the higher the l, the smaller the $\alpha_w(l)$. One expects, therefore, the probability for the occurrence of large clusters is small. This guarantees also the stability of the results. The decrease of $\alpha_w(l)$ with decreasing $\delta\eta$ can be most easily understood from Eq. (10) which is always satisfied in fitting F_q. Because the $\langle n_w\rangle$ decreases with $\delta\eta$ and the $\langle N\rangle$ is independent of $\delta\eta$ the sum of $\alpha_w(l)$ must decrease with $\delta\eta$.

It should be mentioned that once $\alpha_w(l)$ is known in fitting data of a specific multiplicity sample, such as one with no cut on multiplicity shown above, F_q for other multiplicity samples are then uniquely determined by $\langle N\rangle$ and $\mathcal{F}_q$ which depend on the sample (see Table 1). In other words, if $\alpha_w(l)$ is known one can predict the multiplicity dependence of F_q.

To compare with data we made the same cuts on n as UA1 did. By using the values for $\langle N\rangle$ and $\mathcal{F}_q$ listed in Table I we calculated the corresponding F_q as functions of $\delta\eta$. The results for different multiplicity intervals are compared in Fig. 3. Similar to the case where no cuts on n are applied, intermittency is observed for different n cuts in small bin size $0.1 < \delta\eta < 1.0$.

In Fig. 4 the intermittency indices fitted in the region $0.1 < \delta\eta < 1.0$ for different multiplicity intervals are shown together with data [4]. The calculated results are in good agreement with the data. From the experimental data on the average multiplicity and the factorial moments [4] in ΔY the multiplicity distribution can be parametrized with a certain distribution, with the help of which higher factorial moments can be calculated. We did this with a negative binomial which is widely used to describe the multiplicity distributions in restricted rapidity regions [15]. As an example we predicted φ_5 by using $\mathcal{F}_5$ obtained from the negative binomial distribution. This is also shown in Fig. 4.

As mentioned above, the $\alpha_w(l)$ does not depend on n cuts; the multiplicity dependence of F_q is described *completely* by $\mathcal{F}_q$ and $\langle N\rangle$ of clusters given in Eqs. (6)–(15). When one compares the results for different n intervals shown in Fig. 3, one finds that F_q decreases systematically with increasing n interval, which results in the decrease of intermittency indices. From Eqs. (11)–(14) and Table I we see that there are two reasons for this.

One reason is that due to the truncation of multiplicity, the fluctuation of N becomes smaller. So F_q is lowered systematically. As listed in Table I, some $\mathcal{F}_q$ are even smaller than 1. In the interval $16 \leq n \leq 30$, two sides of the distribution are truncated; the fluctuations are expected to be the smallest. Indeed, one sees there is a dip in F_q for this interval in Table 1.

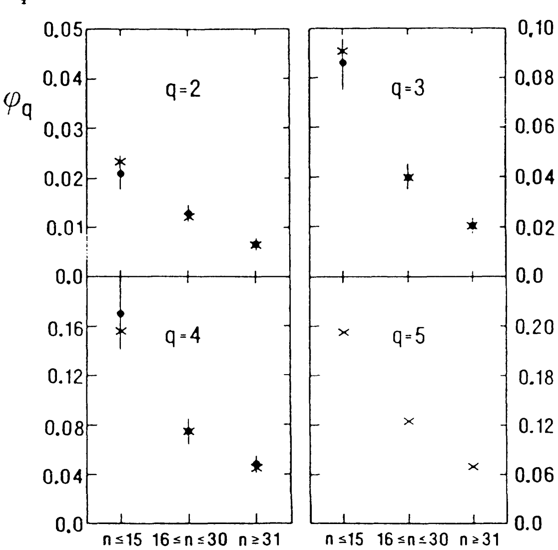

FIG. 4. The multiplicity dependence of intermittency indices φ_q. The solid circles are the UA1 data [4] and the crosses are the calculated results in the region $0.1 < \delta\eta < 1.0$. The φ_5 is the prediction.

The other reason is the fast increase of cluster number with hadron numbers. The large number of clusters N not only lowers the F_q overall, but also suppress the slopes strongly.

Based on these results the following conclusions and remarks can be made.

(A) Our quantitative study suggests that the multiplicity dependence of intermittency phenomena in hadron-hadron collisions is due to the statistical fluctuations of hadronic clusters. Because, in general, the production mechanism(s) and the properties of clusters are different in different kinds of collisions at different energies, it does not mean that the energy dependence of φ_q and its dependence on kinds of collisions can be described by the multiplicity dependence of φ_q as expected in [5].

(B) The intermittency phenomena can emerge in high-energy multiparticle production processes, even if the fluctuation of clusters is *purely* statistical. This differs from the statistical fluctuation of hadrons where, as aforementioned, there is no intermittency.

(C) From present experiment we could not get information on large clusters. To study large clusters and their contribution to intermittency phenomena, the investigation of scaled factorial moments as functions of multiplicity in smaller rapidity intervals should be very useful.

The author is indebted to A. Bialas, I. Dremin, T.C. Meng, I. Sarcevic, and Z.N. Zhou for helpful discussions.

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