Probing the WWZ vertex at a $\sqrt{s}=500$ GeV e^+e^- collider

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(Received 23 July 1991)

We study the precision measurement of the WWZ^0 vertex in the process $e^+e^-\to\nu\bar{\nu}\mu^+\mu^-$ at a $\sqrt{s}=500$ GeV e^+e^- collider. We find that this process can be used to make a precise determination of the trilinear gauge boson couplings. Assuming an integrated luminosity of 2 fb^{-1}, in the reaction $e^+e^-\to\nu\bar{\nu}\mu^+\mu^-$, κ_Z can be measured to 10% at the 95% C.L. and λ_Z to ±0.07 at 95% C.L. With the appropriate cuts on $M_{\mu^+\mu^-}$, $\Delta\kappa_Z=^{+0.3}_{-0.4}$ and $\Delta\lambda_Z=^{+0.4}_{-0.6}$ at 95% C.L. *independent* of deviations in κ_γ and λ_γ.

PACS number(s): 13.10.+q, 12.15.Ji, 13.40.Fn, 14.80.Er

I. INTRODUCTION

The standard model of the electroweak interactions [1] is consistent with all experimental measurements [2]. Although the fermion–gauge-boson sector of the theory is being tested to ever increasing precision [3]; the gauge boson couplings have yet to be subjected to stringent direct experimental measurements. At present, crude constraints exist on the $WW\gamma$ couplings from $W\gamma$ production in $p\bar{p}$ collisions at the Fermilab $p\bar{p}$ collider Tevatron [4] and in the near future, measurement of the γWW vertex will be possible at the DESY ep collider HERA through single W production [5] and high p_T photons [6]. Statistics will be the main limiting factor and will not allow a measurement better than 50%. Soon afterwards, the CERN e^+e^- collider LEP 200 will study both the $WW\gamma$ and WWZ^0 couplings through W pair production: $e^+e^-\to W^+W^-$ [7]. A drawback of this process is that both vertices contribute roughly equally to the W pair production cross section because the photon and the Z boson are both far off mass shell. This implies that only a linear combination of the two vertices will effectively be measured. It is therefore important that independent measurements of these couplings be made in order to complement the LEP 200 analysis. In this paper we study the sensitivity of the process $e^+e^-\to\nu\bar{\nu}Z^0$ to anomalous couplings of the WWZ^0 vertex, which can be studied at the "Next Linear Collider" (NLC): an e^+e^- collider with $\sqrt{s}=500$ GeV or more [8,9].

II. THE WWZ^0 VERTEX

Within the standard model, the $WW\gamma$ and WWZ vertices are uniquely determined by $SU(2)_L\times U(1)$ gauge invariance so that a precise measurement of the vertices comprises a stringent test of the gauge structure of the theory. The most general Lorentz and CP-invariant WWV vertex compatible with electromagnetic gauge invariance is customarily parametrized as [10]

$$\mathcal{L}_{\rm eff}=-ig_V\left\{(W^\dagger_{\mu\nu}W^\mu V^\nu-W^\dagger_\mu V_\nu W^{\mu\nu})+\kappa_V W^\dagger_\mu W_\nu F^{\mu\nu}+\frac{\lambda_V}{M_W^2}W^\dagger_{\lambda\mu}W^\mu_\nu F^{\nu\lambda}\right\},\tag{1}$$

where V represents either the photon or Z^0 and W^μ, represents the W^- fields, and where $g_\gamma=e$ and $g_{Z^0}=e\cot d_W$. As usual, $W_{\mu\nu}=\partial_\mu W_\nu-\partial_\nu W_\mu$ and $F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu$ denote the W and photon (or Z^0) field-strength tensors. We note that the presence of the W-boson mass in the λ_V term is *ad hoc* and one could argue that a scale for new physics (Λ) of several TeV's is more appropriate than this much smaller energy scale. We will conform to the customary parametrization and will not address this issue further.

In the static limit, the parameters κ_γ and λ_γ are related to the anomalous magnetic dipole moment (μ_W) and the electric quadrupole moment (Q_W) of the W by

$$\mu_W=\frac{e}{2M_W}(1+\kappa_\gamma+\lambda_\gamma),\quad Q_W=-\frac{e}{M_W^2}(\kappa_\gamma-\lambda_\gamma)\tag{2}$$

with similar relations for the WWZ^0 vertex. At tree level the standard model predicts $\kappa_V=1$ and $\lambda_V=0$. Higher order corrections to μ_W and Q_W have been calculated [11] and the results are in the 2% range in the minimal standard model and in the 3% range in supersymmetric models [12].

In composite models, however, all four parameters are essentially free. One must rely, for example, on unitarity [13] or experimental observables such as $(g-2)_\mu$ [14], the masses of the gauge bosons [15], and the partial widths of the Z boson [16] to constrain the parameters.

III. THE PROCESS $e^+e^- \to \nu\bar{\nu}\mu^+\mu^-$

The process $e^+e^- \to \nu\bar{\nu}\mu^+\mu^-$ offers the possibility of measuring the WWZ^0 vertex without interference from the $WW\gamma$ vertex by making appropriate kinematic cuts on the $\mu^+\mu^-$ invariant mass. In total, there are 28 distinct Feynman diagrams which contribute to the process $e^+e^- \to \nu\bar{\nu}\mu^+\mu^-$ of which we are interested in only one; the W^+W^- fusion to a Z^0 boson followed by the decay $Z^0 \to \mu^+\mu^-$, which is shown in Fig. 1. Nevertheless, to properly take into account the nonresonant background and maintain gauge invariance our calculation includes all diagrams for arbitrary values of κ_V and λ_V, where V denotes either γ or Z^0.

To evaluate the cross sections and distributions we used the CALKUL helicity amplitude technique [17] to obtain expressions for the matrix elements and used Monte Carlo techniques to perform the phase-space integration [18]. In our results we take $\alpha = 1/128$, $\sin^2\theta_W = 0.23$, $M_Z = 91.172$ GeV, $\Gamma_Z = 2.5$ GeV, $M_W = 81.0$ GeV, and $\Gamma_W = 2.1$ GeV.

The signal we are considering consists of an energetic $\mu^+\mu^-$ pair plus missing transverse momentum $\not{p}_T$ due to the neutrinos coming from the original beam electron and positron. We require $\not{p}_T > 10$ GeV. This eliminates the potential background of $e^+e^- \to e^+e^-\mu^+\mu^-$ via two photons where the e^+ and e^- escape down the beam pipe. In addition we require that $E_{\mu^\pm} > 10$ GeV and to take into account detector acceptance we require that the μ^+ and μ^- be at least 10° from the beam line. Our conclusions are not sensitive to the exact values of these cuts. In Fig. 2 we show the cross section for $e^+e^- \to \nu\bar{\nu}\mu^+\mu^-$ as a function of $\sqrt{s}$ for the above cuts and for several different cuts on the $\mu^+\mu^-$ invariant mass ($M_{\mu^+\mu^-}$). We note that the cross section does not go down as does the QED point cross section $\sigma(e^+e^- \to \mu^+\mu^-)$ for the energy range under consideration so one gains by going to higher $\sqrt{s}$. In the remainder of this paper we will concentrate on the NLC e^+e^- collider machine parameters of $\sqrt{s} = 500$ GeV, $\mathcal{L} = 10^{32}\text{cm}^{-2}\text{sec}^{-1}$ yielding a yearly integrated luminosity of $L = 2$ fb^{-1} (which results in of order 1000 events/yr for each unit of $R = 4\pi\alpha^2/3s$, the point cross section).

The cross section for no cuts on $M_{\mu^+\mu^-}$ (solid line) is considerable as it is dominated by low-invariant-mass events due to the photon pole which appears in many of

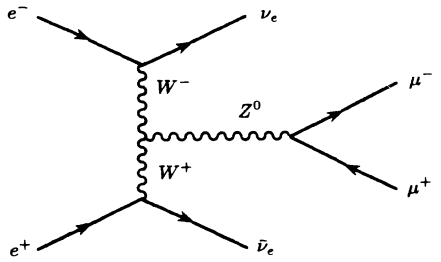

FIG. 1. Feynman diagram for the process $e^+e^- \to \nu\bar{\nu}Z^0$ followed by $Z^0 \to \mu^+\mu^-$, which includes the WWZ^0 vertex.

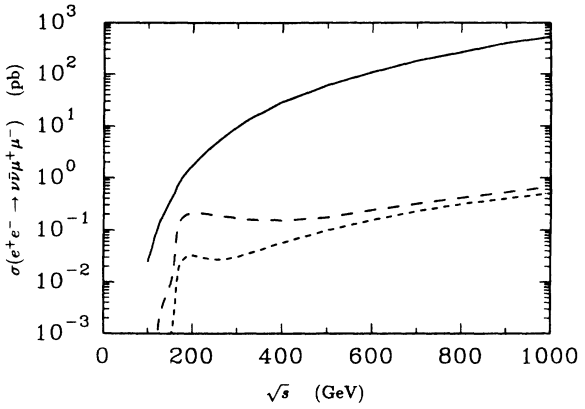

FIG. 2. $\sigma(e^+e^- \to \nu\bar{\nu}\mu^+\mu^-)$ as a function of $\sqrt{s}$ for $\theta_{\mu^\pm} > 10°$. The solid line is for no cut on $M_{\mu^+\mu^-}$, the long-dashed line for $M_{\mu^+\mu^-} > 25$ GeV, and the short-dashed line for 86 GeV $< M_{\mu^+\mu^-} < 96$ GeV.

the Feynman diagrams, including photon bremstrahlung and the diagram of Fig. 1 with the Z^0 replaced by a photon. Imposing a cut $M_{\mu^+\mu^-} > 25$ GeV eliminates this pole and reduces the cross section substantially (long-dashed line). Finally, if we impose the cut that $M_{\mu^+\mu^-}$ lies within 5 GeV of the Z pole (short-dashed line) we can separate the effects of the WWZ^0 from those of the $WW\gamma$ vertex on the cross section. We verified that with this last cut on the invariant mass of the lepton pair, the cross section is insensitive (σ varies by less than 1%) to variations in κ_γ and λ_γ of ± 0.2 around their standard model (SM) values while varying κ_z from -2 to 1.5 and λ_z from -1 to 1. Furthermore, the different distributions that we used remained unchanged by these variations in the photon parameters. For larger variations in the photon sector, the cross section varies by less than 2% and 6% for variations of ± 0.5 in κ_γ and λ_γ, respectively. As a point of reference, the statistical error for 2

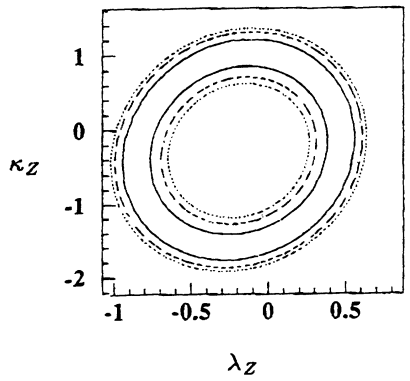

FIG. 3. Confidence level contours for consistency of the cross section $\sigma(e^+e^- \to \mu^+\mu^-\nu\bar{\nu})$ with the standard-model prediction as a function of κ_Z and λ_Z using the cuts described in the text. The solid line is the 68% C.L., the dashed line is the 90% C.L., and the dotted line is the 95% C.L.

fb^{-1} integrated luminosity corresponds to an uncertainty in σ of roughly 7% so that at the 1σ level the process at hand is insensitive to variations in κ_γ and λ_γ of ± 0.5.

Our interest in this process is to see how precisely κ_Z and λ_Z can be measured as a test of the gauge structure of the standard model. We therefore impose the cut 86 GeV $< M_{\mu^+\mu^-} <$ 96 GeV. To quantify the sensitivity to κ_Z and λ_Z we begin by considering the effect of varying κ_Z and λ_Z on the cross section $\sigma(e^+e^- \to \mu^+\mu^-\nu\bar{\nu})$ with the cuts described above. We test for consistency with the standard model based on the statistical errors expected from integrated luminosities of 2 and 5 fb^{-1}. In Fig. 3 we show 68%, 90%, and 95% confidence level contours for consistency with the standard model in the κ_Z-λ_Z plane based on 2 fb^{-1} integrated luminosity. The region between the rings is consistent with the standard model. Note that there are regions in the parameter space which deviate substantially from the standard model but have cross sections which are consistent with that of the standard model. Varying one parameter at a time we find $\Delta\kappa_Z = {}^{+0.3}_{-0.4}$ ($\lambda_Z = 0$) and $\Delta\lambda_Z = {}^{+0.4}_{-0.6}$ ($\kappa_Z = 1.0$), which gives a much different picture of the allowed parameter space. This underlines the importance of performing a global analysis of all free parameters to obtain a realistic test of the theory and the pitfalls of varying one parameter at a time.

Fortunately, there is more information available beyond the cross-section measurement. We analyzed numerous distributions of kinematic variables and found that the angular distribution of the outgoing muons with respect to each other ($\theta_{\mu^+\mu^-}$) and the p_T distribution of the Z are particularly sensitive to anomalous couplings. In Figs. 4(a) and 4(b) we show the angular distributions and p_{TZ} distributions, respectively, for several representative values of κ_Z and λ_Z which in Fig. 3 appeared to be consistent with the standard model on the basis of the cross section measurement. One clearly sees that the angular distributions and p_{TZ} distributions for those values of κ_Z and λ_Z are substantially different. To quantify this we divide the angular distribution into 3 regions: $-1.0 < \theta_{\mu^+\mu^-} < -0.5$, $-0.5 < \theta_{\mu^+\mu^-} < -0.1$, and $-0.1 < \theta_{\mu^+\mu^-} < 0.75$ and perform a χ^2 analysis. The 68%, 90%, and 95% confidence level contours are shown in Figs. 5(a) and 5(b) based on 2 and 5 fb^{-1} integrated luminosity, respectively. Similarly, we divided the p_{TZ} distribution into three bins: $p_{TZ} < 80$ GeV, 80 GeV $< p_{TZ} <$ 120 GeV and 120 GeV $< p_{TZ} <$ 240 GeV. The 68%, 90%, and 95% confidence level bounds are shown in Figs. 5(c) and 5(d) for integrated luminosities of 2 and 5 fb^{-1}, respectively. This additional information from the distributions substantially restricts the allowed region in parameter space that is consistent with the standard model. Varying one parameter at a time leads roughly to $0.4 < \kappa_Z < 1.5$ and $-0.6 < \lambda_Z < 0.5$ (95% C.L. and using 2 fb^{-1}), it is clear from these figures that the realistic limits are broader: κ_Z extends from 0 to 1.5 and λ_Z from -0.6 to 0.5.

With real data one would likely do better in the sense that rather than utilizing general binnings of the distributions to quantify deviations from the standard model, one could choose physics cuts that would optimize the sensitivity to anomalous observations.

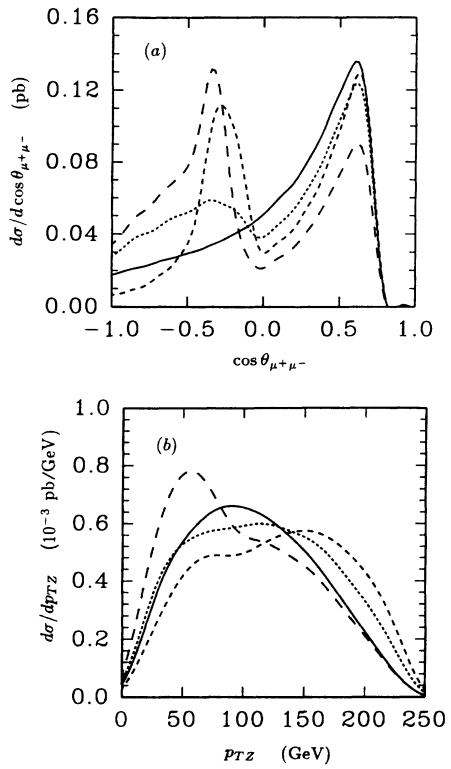

FIG. 4. (a) The angular distribution of the outgoing muons with respect to each other. (b) The transverse-momentum distribution of the reconstructed Z^0. In both figures the solid line is the standard model prediction, the long-dashed line is for $\kappa = -0.4$, $\lambda = -0.7$, the short-dashed line is for $\kappa = -0.4$, $\lambda = +0.5$, and the dotted line is for $\kappa = -1.7$, $\lambda = 0.0$.

If deviations from the standard model were found, the additional information in the distributions would be crucial to determine the values of κ_Z and λ_Z. For example, if κ_Z and λ_Z turned out to be -0.5 and -0.8, respectively, values of $\kappa_Z = -0.5$ and $\lambda_Z = 0.5$ or $\kappa_Z = -1.5, \lambda_Z = -0.25$ would be excluded at better than 99.99% C.L.

IV. SUMMARY

We have examined how well the WWZ^0 trilinear gauge-boson coupling could be determined at a $\sqrt{s} = 500$ GeV e^+e^- collider and have demonstrated how the WWZ^0 vertex can be measured *nearly independently* of the $WW\gamma$ vertex. We have shown that varying only one parameter at a time can lead to misleading conclusions. The measurement of the cross section by itself would lead to $0.6 < \kappa_Z < 1.3$ and $-0.6 < \lambda_Z < 0.4$ when varying one parameter at a time (with 2 fb^{-1}) while it leads to a broad ellipse in parameter space that extends from $-2.0 < \kappa_Z < 1.5$ to $-1 < \lambda_Z < 0.8$. By considering the angular distribution of the two leptons with respect to each other and the transverse momentum of the Z boson,

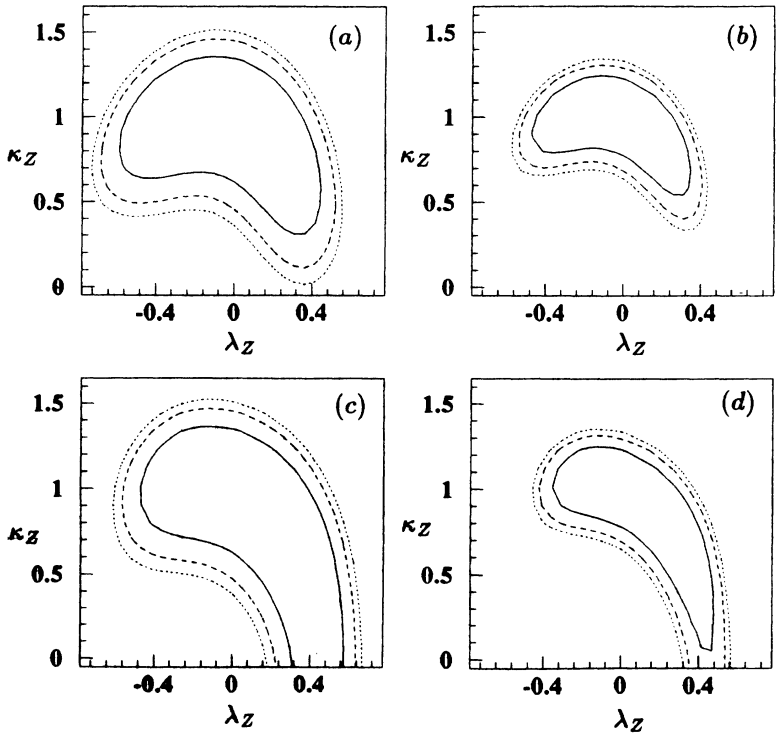

FIG. 5. Confidence-level contours for consistency with the standard model as a function of κ_Z and λ_Z (a) based on the angular distribution of the μ^+ and μ^- with respect to each other for integrated luminosity of 2 fb^{-1}, (b) for 5 fb^{-1}, (c) based on the transverse momentum distribution of the reconstructed Z^0 for 2 fb^{-1}, (d) 5 fb^{-1}. In all cases the solid line is the 68% C.L., the dashed line is the 90% C.L., and the dotted line is the 95% C.L.

this ellipse is substantially reduced to $0.0<\kappa_Z<1.5$ and $-0.6<\lambda_Z<0.5$. As such, the NLC would be a valuable tool in disentangling the WWZ^0 vertex from the $WW\gamma$ vertex.

The process at hand is richer than the results we just mentioned. Indeed, the most dramatic effects of anomalous gauge-boson couplings do not appear at the poles but as interference between the photon and Z^0 away from either resonance. We found that in the regions 25 GeV $<M_{\mu^+\mu^-}<86$ GeV and 96 GeV $<M_{\mu^+\mu^-}<500$ GeV the interference between the photon and the Z boson leads to sizable effects when varying κ_γ or λ_γ in addition to κ_Z and λ_Z [8]. Thus, at the Z-boson pole, one can measure κ_Z and λ_Z and off the Z^0 pole, one can also observe deviations in κ_γ and λ_γ. For example, using the $M_{\mu^+\mu^-}$ bins, 25 GeV $<M_{\mu^+\mu^-}<86$ GeV, 86 GeV $<M_{\mu^+\mu^-}<96$ GeV, 96 GeV $<M_{\mu^+\mu^-}<110$ GeV, and 110 GeV $<M_{\mu^+\mu^-}<400$ GeV, we found that at 95% C.L. $\Delta\kappa_Z=^{+0.13}_{-0.7}$, $\Delta\lambda_Z=\pm0.07$, $\Delta\kappa_\gamma=^{+0.21}_{-0.07}$, and $\Delta\lambda_\gamma=^{+0.08}_{-0.10}$ varying one parameter at a time. However, as we saw above, one should be cautious how one interprets the results when varying only one parameter at a time. Nevertheless, the NLC offers the possibility of measuring all four C- and P-conserving parameters of a general VWW vertex. A complete analysis is in progress and will be presented elsewhere [19].

ACKNOWLEDGMENTS

This research was supported in part by the Natural Science and Engineering Research Council of Canada. G.C. thanks the Superconducting Supercollider Center, TRIUMF, and the I.S.E.M. at S.M.U. for the use of their computing facilities. S.G. and R.L. thank the University of Toronto ZEUS and CDF Collaborations and the University of Guelph for the use of their computing facilities.

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