Grand unified gauge-boson condensation on the cosmic string

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(Received 5 August 1991)

Expectation values of grand unified Higgs scalars can be strongly changed in the core of the cosmic string. We show that in certain cases such unusual Higgs structures imply the existence of nonzero classical gauge currents in the lowest-energy state of the system. This automatically triggers the condensation of the grand unified gauge bosons interacting linearly with this current, which could be either trivial or nontrivial under the $\tilde{U}(1)$ subgroup responsible for the string. For the former, the gauge-boson condensate accumulated in the core of the defect is strictly radial, while in the latter case it also acquires an azimuthal (and magnetic) component. Existence of such types of condensates on the boundaries of the expanding vacuum bubbles (which arise in high-temperature phase transitions) can play an important role in creating the present baryon asymmetry.

PACS number(s): 12.10.Gq, 12.50.Lr, 98.80.Cq

I. INTRODUCTION

Vortices [1] or cosmic strings [2] are well-known topological defects that can be formed during a possible phase transition in the early universe when the initial symmetry group G undergoes a series of hierarchical symmetry breaking. In each case, the string creating is due to some spontaneously broken subgroup $\tilde{U}(1)\subset G$ with a corresponding generator K and (if local) a gauge vector field A_μ. The phase of the vacuum expectation value (VEV) of the scalar χ responsible for $\tilde{U}(1)$ breaking changes by an integral multiple of 2π on any closed path around the string. For the infinitely long string this VEV in the cylindrical coordinates (ρ,θ,z) has the form [1]

$$\chi=\eta(\rho)e^{in\theta}, \tag{1}$$

where n is an integral winding number. The radial solution $\eta(\rho)$ must satisfy the boundary condition $\eta(\rho\to\infty)\to\mu$, where μ is the usual constant vacuum (minimum potential) solution. In the core of the string, there is a magnetic flux corresponding to the θ component of the vector potential A_θ which has a pure gauge asymptotic $(\rho\to\infty)$ form

$$A_\theta\to\frac{1}{e\rho}, \tag{2}$$

where e is a $\tilde{U}(1)$ gauge coupling constant. The characteristic peculiarity of solution (1) (as well as of the other topological objects such as monopoles and domain walls) is the restoration of the $\tilde{U}(1)$ symmetry in the $\rho\simeq 0$ region. The possible behavior of other scalar (and vector) expectation values in the core of the defect is, however, highly nontrivial. It turns out that the phase-space portrait of the vortex in realistic cases is much more complicated than in the usually discussed simplest approaches. In the present paper we investigate certain nontrivial configurations of grand unified Higgs expectation values in the vortex and show that these can induce a condensation of grand unified vector gauge bosons (e.g., X, Y, W) in the core of the cosmic string. The condensation of the gauge bosons can lead to interesting and important physical effects such as the catalysis of baryon decay. It should be remarked that the aforementioned cosmic-string-induced configuration can take place for a wide range of parameters so that the probability of their occurrence compares favorably with the usually proposed mechanisms. The simplest example is the bosonic-type superconducting string discussed by Witten in Ref. [3]. In this paper an electrically charged scalar H, interacting with the vortex (1), is considered. The scalar potential has the form

$$V(H,\chi)=V(\chi)+(f|\chi|^2-m^2)|H|^2+\lambda|H|^4 \tag{3}$$

where $V(\chi)$ is the self-interaction potential of χ minimized by $|\chi|=\Gamma$, m^2 is a mass, and λ,f are dimensionless parameters with the usual requirement $\lambda>0$ to insure that the potential is bounded from below. Witten has shown that if $f>0$, $m^2>0$, and $f\mu^2-m^2>0$, then the scalar H develops a nonzero VEV in the core of the defect while in the outer region H vanishes. This is because the effective mass (of H) $m_H^2=f|\chi|^2-m^2$ is a monotonic function of ρ and becomes negative in the core where $|\chi|$ is small tending to zero. On the other hand if $m^2<0$ and $f|\mu|^2-m^2<0$, the opposite situation prevails; H vanishes in the core, while it remains nonzero away from it.

Let us now imagine the scenario when two scalars H_a and H_b are interacting with the vortex χ (we stipulate that $|\chi|\gg|H_a|,|H_b|$ away from the core). Let us also assume that H_a and H_b belong to the same irreducible representation H of some spontaneously broken non-Abelian gauge group G. At the same time, we can place them in different representations of the subgroup $G'\subset G$, which appears unbroken at some intermediate stage of the hierarchical phase transition $G\to G'\to\cdots$. It is clear that the VEV's of H_a and H_b components of H which are influenced by the vortex will, in general, have different ρ dependence; the group G is broken and the effective masses M_a and M_b [of $H_a(\rho)$ and $H_b(\rho)$] are no longer degenerate. The VEV's are developed on different superpositions, $\langle H_{\rm in}\rangle = H_a(0)+H_b(0)$, and $\langle H_{\rm out}\rangle = H_a(\infty)+H_b(\infty)$ inside and outside of the core [where $H_a(\rho)$ and $H_b(\rho)$ are functions of ρ]. In particular, we can have $H_a(0)=0=H_b(\infty)$, and $H_b(0), H_a(\infty)$ to be nonzero. In the present paper we will be interested in those cases in which there exists some interval $\rho\in[\rho_1,\rho_2]$ with both $H_a(\rho)\neq 0$ and $H_b(\rho)\neq 0$. Notice that, because of the presence of the vortex, the derivatives $\partial H_{a,b}/\partial\rho\neq 0$, and are large if the influence of the string is strong in the interval $\rho\in[\rho_1,\rho_2]$. In this situation a necessary condition for gauge-boson condensation via our mechanism is the existence of a generator τ_a^b of G, which in the H representation transforms states H_a and H_b into each other, i.e., $H_a=\tau_a^b H_b$. If it is so, then in the region $\rho\in[\rho_1,\rho_2]$, one may expect the appearance of a nonzero expectation gauge current

$$\langle J_\mu\rangle = ig\langle [H_a^\dagger\partial_\mu(\tau_a^b H_b)-(\partial_\mu H_a^\dagger)\tau_a^b H_b]\rangle\neq 0\ , \quad (4)$$

where g is a gauge coupling constant of G. The very existence of a nonzero current implies that there exists a classical gauge field A_μ^{ab} (corresponding to the generator τ_a^b) interacting with this current; the appearance of $\langle J_\mu\rangle\neq 0$ in the lowest energy state inevitably leads to the condensation of the gauge boson A_{ab}^μ. Stated differently, the extremum of the Lagrangian (with current J_μ^{ab}) cannot be realized on the state $A_\mu^{a,b}=0$, because the Euler-Lagrange equation, satisfied by A_{ab}^μ,

$$\partial_\nu F_{\mu\nu}^{ab}+m^2 A_\mu^{ab}+\langle J_\mu^{ab}\rangle = 0 \quad (5)$$

does not permit the trivial solution $A_\mu^{ab}=0$. If the components H_a and H_b carry no charge of the generator K responsible for the string, then their VEV's are θ independent (we assume that there are no domain walls bounded by K strings). In this case the current (4) as well as the gauge-boson condensate is strictly radial. Such a case was briefly discussed in Ref. [4]. It is important to note that by the same mechanism the gauge-boson condensate could accumulate not only on the cosmic strings (which of all the topological defects seems to be the only possible candidate which could exist in the observable part of the present universe [2]), but on any other sufficiently massive vacuum defects, stable or unstable. In particular, such accumulators can play the role of the boundaries between the decaying false vacuum and the expanding bubbles of the new phase which are created in a first-order high-temperature phase transition in the early universe (e.g., see [5]). If H_a and H_b transform nontrivially under the generator K, then in some cases discussed below, nontrivial θ dependence of VEV's might also result giving a nonzero azimuthal (together with the radial) component of the current (4), and of the expectation value of the gauge vector field. Since $(\nabla\times A)_z\neq 0$ (in general), we may now expect finite magnetic flux associated with A_μ^{ab}. It may be mentioned that Ambjorn *et al.* have considered the possible condensation of W gauge bosons in the strong magnetic field created in the core of the superconducting cosmic string [6] in high energy collisions [7]. They showed that the condensation takes place because of the magnetic moment of the spin-1 W bosons due to which the vacuum of the electroweak theory cannot remain stable for sufficiently large magnetic fields [8]. Linde has also discussed the possible condensation of W bosons in superdense matter [9]. Both of these scenarios differ fundamentally from the mechanism of the grand unified gauge-boson condensation discussed in the present paper. We end this section by delineating the scope of this paper.

In Sec. II we develop the above idea through the simple but realistic example of minimal SU(5) grand unified theory (GUT). We analyze the behavior of grand unification Higgs VEV's in the vicinity of a vortex field formed due to breaking of an additional $\tilde{\rm U}(1)$ group under which SU(5) scalars are assumed to be trivial. We show that for a certain range of parameters, the grand unified phase-transition patterns can change and lead to an accumulation (condensation) of (strictly radial) grand unification gauge bosons like X, Y, and W.

In Sec. III, we study the behavior of the vacuum expectation value of scalars carrying nontrivial K-charge of the external vortex field.

In Sec. IV we consider the gauge-boson condensation which might take place when the scalar multiplet responsible for the string formation is nonzero in the core. As a realistic example, we consider an SU(5)-string which might exist during an intermediate stage of the high temperature phase transition in the early universe. We also show that in those stages of evolution of the universe, the aforementioned gauge-boson (e.g., X_μ) condensate could readily form.

II. SU(5) HIGGS SECTOR WITH ZERO $\tilde{\rm U}(1)$ CHARGE

A. Adjoint 24-plet in the vortex field

Here we consider symmetry-breaking patterns in minimal SU(5) GUT with an additional $\tilde{\rm U}(1)$ symmetry responsible for the string creation. We begin with the simplest model in which all the ordinary SU(5) Higgs scalars in the adjoint (24-dimensional, to be denoted by Σ_i^k) as well as in the fundamental (5-dimensional, H_i, $i,k=1-5$) representation are trivial under the $\tilde{\rm U}(1)$ symmetry. In this situation, the only possible (renormalizable) coupling between SU(5) and $\tilde{\rm U}(1)$ Higgs bosons are the self-conjugate interactions of the type [SU(5) indices are suppressed]

$$[f_\Sigma({\rm Tr}\Sigma^2)+f_H(H^\dagger H)]|\chi|^2\ . \quad (6)$$

Near the string ($\rho<R$, where R is a typical radius $\sim\mu^{-1}$) where the vortex field has nonzero gradients, the effective mass terms for Σ and H will be modified causing a change

in their VEV's. Far from the string this influence is absent since $|\chi| \to$const as $\rho \to \infty$, and SU(5) Higgs fields must develop their usual VEV's leading to the hierarchical breaking pattern $SU(5) \to SU(3) \otimes SU(2) \otimes U(1) \to SU(3) \otimes U(1)$.

Let us consider the situation in detail beginning with the analysis of Σ's behavior (neglecting H) in the vortex field. The most general SU(5)-invariant renormalizable self-coupling potential for the adjoint Σ_i^k has the form [suppressing SU(5) indices]

$$V_\Sigma = \frac{M^2}{2}\mathrm{Tr}\Sigma^2 + \frac{h}{4}(\mathrm{Tr}\Sigma^2)^2 + \frac{\mu}{3}\mathrm{Tr}\Sigma^3 + \frac{\lambda}{4}\mathrm{Tr}\Sigma^4 \qquad (7)$$

where M and μ are masses, and h and λ are dimensionless parameters.

It is well known that Σ_i^k, a vector in the 24-dimensional representation space, can be brought by means of SU(5) transformations into the simple diagonal form $\Sigma = \mathrm{diag}(\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, \Sigma_5)$. This makes it a vector in the 4-dimensional ($\mathrm{Tr}\Sigma = 0$) orbit space allowing a representation $\Sigma_i^k = C^\alpha{}_i{}^k A^\alpha$ where $C^\alpha{}_i{}^k$ are real and $A^\alpha (\alpha = 1-4)$ are diagonal generators from SU(5) Cartan subalgebra, which at the same time form an orthonormal basis ($\mathrm{Tr}A^\alpha = 0, \mathrm{Tr}A^\alpha A^\beta = \delta_{\alpha\beta}$) in the orbit space. As soon as Σ is viewed as a vector in the orbit space, it can be described by its length $(\mathrm{Tr}\Sigma^2)^{1/2} = (\sum_\alpha C_\alpha^2)^{1/2}$, and directional angles $C_\alpha / (\mathrm{Tr}\Sigma^2)^{1/2}$.

The potential V_Σ contains three independent SU(5)-invariants $\mathrm{Tr}\Sigma^2, \mathrm{Tr}\Sigma^3$, and $\mathrm{Tr}\Sigma^4$. First of these $\mathrm{Tr}\Sigma^2$, is also an SU(24)-invariant and does not distinguish between directions of Σ in representation as well as in the orbit spaces. The other two depend on the length as well as the direction of Σ. For an arbitrary fixed $\mathrm{Tr}\Sigma^2 = a^2$, they take different values for different orbits and determine which direction in the orbit space is energetically more favorable.

The potential (4) has been keenly investigated [10] and it was found that its minima for any values of parameters λ, h, M, μ is realized on those directions in orbit space which break SU(5) (if it is broken at all) down to one of the subgroups, $SU(3) \otimes SU(2) \otimes U(1)$ or $SU(4) \otimes U(1)$ with respective configurations

$$\Sigma = \mathrm{diag}(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2})\sqrt{\tfrac{2}{15}}\,a\ , \quad a = \frac{(1/\sqrt{30})\mu \pm \sqrt{\mu^2/30 - 4\mu^2(h + \lambda\frac{7}{30})}}{2(h + \lambda\frac{7}{30})}\ , \qquad (8)$$

$$\Sigma = \mathrm{diag}(1,1,1,1,-4)\frac{1}{\sqrt{20}}a\ , \quad a = \frac{(3/\sqrt{20})\mu + \sqrt{\frac{9}{20}\mu - 4\mu^2(h + \lambda\frac{13}{20})}}{2(h + \lambda\frac{13}{20})}\ . \qquad (9)$$

For fixed $\mathrm{Tr}\Sigma^2 = a^2$ the orbit-direction-dependent part of the potential V_Σ

$$V' = \frac{\mu}{3}\mathrm{Tr}\Sigma^3 + \frac{\lambda}{4}\mathrm{Tr}\Sigma^4 \qquad (10)$$

takes the form (respectively for the two cases)

$$V_1 = \mu(-\tfrac{1}{6}\sqrt{\tfrac{2}{15}})a^3 + \lambda\tfrac{7}{120}a^4 \qquad (11)$$

and

$$V_1 = \mu(-\tfrac{1}{20})a^3 + \lambda\tfrac{13}{80}a^4\ . \qquad (12)$$

Note that the sign of the parameter μ plays no role in determining the SU(5) breaking pattern, since for any sign of μ, a adjusts its sign in such a way that $\mu\mathrm{Tr}\Sigma^3$ remains negative. So without any loss of generality, we can assume $\mu > 0$. Then one can easily see that if $\lambda < 0$ the minima of V_Σ corresponds to (9) [with residual symmetry $SU(4) \otimes U(1)$] for all a. This unrealistic case is not of interest for us. When $\lambda > 0$, the minimum of V_Σ is realized on the direction $(111-\frac{3}{2}-\frac{3}{2})$ [Eq. (11)] if $a > a_0$ and on $(1111-4)$ [Eq. (12)] if $a < a_0$, where $a_0 = (\mu/\lambda)\frac{4}{3}[(\frac{9}{20})^{1/2} - (\frac{1}{30})^{1/2}][\frac{13}{20} - \frac{7}{30}]^{-1}$. For $a = a_0$, V_Σ has two directly degenerate minima (11) and (12) with equal vacuum energy. If the interaction with the vortex field χ is absent, then a has the same value in the whole universe, and the breaking pattern has to be $SU(5) \to SU(3) \otimes SU(2) \otimes U(1)$.

Now let us discuss what happens when the interaction with χ is switched on; the effective mass (of Σ) $M(\rho) = M^2 + f|\chi|^2$, now ρ dependent, forces radial variation on $\mathrm{Tr}\Sigma^2 = a^2$ so that the system can go through different symmetry phases at different ρ. If f_Σ is chosen to be negative, $\mathrm{Tr}\Sigma^2 = a^2$ decreases with decreasing $|\chi|$. Here with different choices of potential parameters one has to consider two possibilities:

(a) $a > a_0$ everywhere. The group SU(5) is now broken down to $SU(3) \otimes SU(2) \otimes U(1)$ in all space.

(b) If the influence of the vortex is sufficiently strong, then at some point $\rho = \rho_0$, $a(\rho)$ could become smaller than its critical value a_0, i.e., we could have $a > a_0$ for $\rho > \rho_0$, and $a < a_0$ for $\rho < \rho_0$. If this tendency of the effective potential were strong enough, the system would, by a first-order phase transition (under ρ), go to the $SU(4) \otimes U(1)$-invariant phase.

To discuss the latter in more detail, it is possible to choose a convenient (Cartan algebra) basis consisting of the following $SU(3) \otimes SU(2) \otimes U(1)$ generators:

$$U(1) \to A^1 = \mathrm{diag}[1,1,1,-\tfrac{3}{2},-\tfrac{3}{2}]\sqrt{\tfrac{2}{15}}\ ,$$

$$SU(2) \to A^2 = \mathrm{diag}[0,0,0,1-1]\frac{1}{\sqrt{2}}\ ,$$

$$SU(3) \to \begin{cases} A^3 = \mathrm{diag}[-2,1,1,0,0]\frac{1}{\sqrt{6}} \\ A^4 = \mathrm{diag}[0,1,-1,0,0]\frac{1}{\sqrt{2}}\ . \end{cases} \qquad (13)$$

In terms of (13), we can express $\Sigma = C_1 A^1 \equiv a A^1$ for $\rho > \rho_0$ $(a > a_0)$, and $\Sigma = C_1 A^1 + C_2 A^2 \equiv a[(\frac{3}{8})^{1/2}A^1 + (\frac{25}{40})^{1/2}A^2]$ for $\rho < \rho_0 (a < a_0)$. At the point $\rho \to 0$, the direction of Σ at the orbit level cannot be determined by the potential alone because V_Σ has two minima: (11) and (12). The former, being deeper, is the true minimum for $\rho > \rho_0$. As $\rho \to 0$ $(a \to a_{\min})$, the energy of the minimum

(11) increases while that of (12) decreases. At $\rho=\rho_0$, the potentials become degenerate and as ρ decreases further, (12) becomes the deeper of the two.

At the point $\rho=\rho_0$, one has a phase transition with corresponding shifts $\Delta C_1=[1-(\frac{3}{8})^{1/2}]a_0$, and $\Delta C_2=(\frac{25}{40})^{1/2}a_0$; the vacuum solutions $C_1(\rho)$ and $C_2(\rho)$ are some smooth functions of ρ with solitonlike behavior in the vicinity of the critical point ρ_0. Despite the fact that $dC_{1,2}/d\rho\neq 0$, $C_{1,2}(\rho)\neq 0$ near ρ_0 gauge-boson condensation does not take place since there is no SU(5) generator which connects C_1 and C_2.

B. Higgs 5-plet included

The most general form of the Higgs potential, containing self-coupling of H, and its interaction with Σ and χ, is

$$V_{\Sigma H\chi}=m_1^2H^\dagger H+f_HH^\dagger H|\chi|^2+\mu_1H^\dagger\Sigma H+\lambda_1H^\dagger\Sigma^2H+h_1(H^\dagger H)^2+\lambda_2(H^\dagger H)\mathrm{Tr}\Sigma^2 \ . \quad (14)$$

As soon as the Σ's nonzero VEV breaks SU(5) symmetry down to SU(3)⊗SU(2)⊗U(1), the mass degeneracy between the colored triplet ($T_i=H_1, i=1,2,3$) and the weak doublet ($D_\alpha=H_i, \alpha=i-3, i=4,5$) components of H is removed. The effective masses of T and D are obtained by substituting the VEV (11) in Eq. (8),

$$\begin{aligned} m_T^2&=m_1^2+f_H|\chi|^2+[\tfrac{2}{15}\lambda_1+\lambda_2]a^2+(\tfrac{2}{15})^{1/2}\mu_1a \ , \\ m_D^2&=m_1^2+f_H|\chi|^2+[\tfrac{3}{10}\lambda_1+\lambda_2]a^2-(\tfrac{3}{10})^{1/2}\mu_1a \end{aligned} \quad (15)$$

where the parameters have to be fine tuned (by some mechanism) in such a way that $m_W^2\sim m_D^2<0$ (m_W is the weak scale ~ 100 GeV) in order to obtain the correct scales for electroweak symmetry breaking. At the same time, one must demand $m_T^2\sim a^2>0$ ($a\sim 10^{15-16}$ GeV as the GUT mass scale) which will prevent an unacceptably fast proton decay.

With the aforementioned mechanism, m_T^2 and m_D^2 become functions of ρ through $|\chi|$ and a, and can vary strongly near the string core. In fact, for a wide range of parameters, it is possible to arrange $m_T^2<0$, $m_D^2>0$ on the string, and thus condensation of the colored triplet can take place on the vortex line. As an illustration, one may propose the choice $m_1^2=0$, $f_H=0$, $A\equiv\frac{2}{15}\lambda_1+\lambda_2>0$, and $B\equiv\frac{3}{10}\lambda_1+\lambda_2<0$ which is consistent for $\lambda_1<0$, $\lambda_2>0$, and $-\frac{2}{15}\lambda_1<\lambda_2<-\frac{3}{10}\lambda_1$, and leads to

$$\begin{aligned} m_T^2&=Aa^2-\mu'a \ , \\ m_D^2&=-Ba^2+\tfrac{3}{2}\mu'a \end{aligned} \quad (16)$$

where $\mu'=-(\frac{2}{15})^{1/2}\mu_1>0$. For correct magnitudes of the doublet-triplet splitting arising out of the string interactions, we must demand $-Ba^2+(\frac{3}{2})\mu'a<0$, $-B[a-\frac{3}{2}\mu'/B]\sim m_W^2/a$, and $m_T^2>0$ and sufficiently large.

Equations (16) admit a variety of possibilities. Let us begin with analyzing the case when $a>a_0$ everywhere. It should be possible, then, to arrange $m_T^2<0$, $m_D^2>0$ if a is sufficiently smaller than μ' with the resulting condensation of T on the string. The general behavior of m_D^2 and m_T^2 can be readily discussed. As $\rho\to 0$ with a approaching $a_{\min}$, $m_D^2(m_T^2)$ has a tendency to increase (decrease) in regions close to the string. Two distinct situations may emerge. If

$$\mu'/2A<\sigma_{\min}<\mu'/A<\tfrac{3}{2}\mu'/B<a_{\max} \ ,$$

m_D^2 can become positive before m_T^2 goes negative, making D vanish before T develops a VEV. In this case, both the VEV's are absent in a cylindrical annulus around the string.

More interesting is the case $\frac{3}{2}(\mu'/B)<\mu'/A$ for which there exists an interval $\rho\epsilon[\rho_1,\rho_2]$ in which both m_T^2 and m_D^2 are negative. One may note that such a choice of parameters does not correspond to the correct doublet-triplet splitting away from the core. However, for our simplest illustrative example, it does not matter. One can readily show that in more complicated cases, e.g., when $a_0<a_{\min}$ (see below) in SU(5) GUT, or in larger GUT's [SO(10), $E_6\cdots$] in which a variety of Higgs scalar multiplets with VEV's ($\sim 10^{15}$ GeV) inducing the D-T splitting occur, this incompatibility can be avoided for a wide range of parameters.

For further analysis relevant to this region, it is convenient to write effective potentials for T and D components in the form

$$V=m_T^2[|T|^2+|D|^2]+h_1[|T|^2+|D|^2]^2+(m_D^2-m_T^2)|D|^2 \ . \quad (17)$$

By means of the residual SU(3)⊗SU(2)⊗U(1) transformation, one can always convert the triplet and the doublet components in such a manner that each one of these has only one nonzero component, e.g., $T_1=T(H_1), D_2=D(H_5)$. The subsequent behavior, thus, can be discussed in terms of the two nonzero components. Evidently, the first two terms are invariant under all SU(5) (in this context only $T\rightleftarrows D$) rotations. Consequently, the minimum of the first two terms of Eq. (11), $|T|^2+|D|^2=-m_T^2/2h_1$ is continuously degenerate. The last term, however, breaks the degeneracy imparting VEV on either the D [for $m_D^2-m_T^2<0$] or on the $T(m_D^2-m_T^2>0)$ component.

For $m_D^2=m_T^2$, the symmetry is restored and one obtains degenerate minima in which the direction of VEV's on the T-D plane cannot be fixed on the potential level. In this situation the phase portrait of the string is the following: In the region $\rho>\rho_c$ the lowest energy state of the system is usually SU(3)⊗U(1)-symmetric with the corresponding configuration of SU(5) Higgs VEV's

$$\begin{aligned} H&=(0,0,0,0,H_5) \ , \\ \Sigma&=\mathrm{diag}[1+\epsilon,1+\epsilon,1+\epsilon,-\tfrac{3}{2}+\epsilon,-\tfrac{3}{2}+\epsilon]a \end{aligned} \quad (18)$$

where $\epsilon\sim H_5/a$ is a small correction arising due to the back reaction of the 5-plet on Σ's VEV. As ρ increases the energetic balance between various minima of the potential $V[\Sigma,H]$ changes. Starting from the point $\rho<\rho_c$, the system begins to prefer the SU(2)⊗SU(2)⊗U(1)-invariant vacuum

$$\begin{aligned} H&=(H_1,0,0,0,0) \ , \\ \Sigma&=\mathrm{diag}(1-4\epsilon',1+\epsilon',1+\epsilon',-\tfrac{3}{2}+c',-\tfrac{3}{2}+\epsilon')a \end{aligned} \quad (19)$$

where $\epsilon' \sim (H_1/a)$. At the point ρ_c the first-order phase transition $SU(3)\otimes U(1) \to SU(1)\otimes SU(2)\times U(1)$ relative to the parameter ρ takes place. So that in the neighborhood of the point $\rho=\rho_c$ the Higgs expectation values correspond to the intermediate $SU(2)\otimes U(1)$-symmetric state

$$\begin{aligned} &H=[H_1,0,0,0,H_5]\ ,\\ &\Sigma=\mathrm{diag}(1+\epsilon-4\epsilon',1+\epsilon+\epsilon',1+\epsilon+\epsilon'\ ,\\ &\qquad -\tfrac{3}{2}+\epsilon+\epsilon',-\tfrac{3}{2}+\epsilon'-4\epsilon)a\ . \end{aligned} \tag{20}$$

Notice that due to the gauge freedom, we can always choose H_1 and H_5 to be real and positive. Qualitative behavior of H_1 and H_5 is shown in Fig. 1. Since $\partial H_1/\partial\rho<0$ and $\partial H_5/\partial\rho>0$, the expectation value of the current [g is SU(5) gauge constant]

$$J_\mu = ig(H_1\partial_\mu H_5 - H_5\partial_\mu H_1) \tag{21}$$

is nonvanishing in this region. This current corresponds to the nondiagonal SU(5)-generator τ_1^5 which in the fundamental representation transforms the components H_1 and H_5 into each other. Existence of a nonzero current in the lowest energy state precipitates a condensate of the corresponding gauge field in the boundary between $SU(3)\otimes U(1)$ and $SU(2)\otimes SU(2)\otimes U(1)$ symmetric phases. To examine the situation in some detail, let us substitute the $SU(2)\otimes U(1)$-invariant VEV's (20) of Higgs scalars in the gauge-field-dependent part of the Lagrangian,

$$\mathcal{L}=|\partial_\mu H_i - igA^k_{\mu,i}H_k|^2+|\partial_\mu\Sigma_i^k - ig[A\Sigma]_i^k|^2-\tfrac{1}{4}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} \tag{22}$$

where $\mu,\nu=0,1,2,3$ are the Lorentz, and $i,k=1,\ldots,5$ are the SU(5) indices, $A^i_{\mu,k}$ is a 24-plet, SU(5) gauge-boson potential matrix, and $F^k_{\mu\nu,i}=\partial_\mu A^k_{\nu i}-\partial_\nu A^k_{\mu i} - ie[A_\mu A_\nu]_i^k$ is the field tensor. The relevant part of the energy density including that of the gauge fields has the following form:

$$E=\tfrac{1}{4}\mathrm{tr}F_{\mu\nu}F^{\mu\nu}+E_1+E_2+g^2A_k^i A_i^k|\Sigma_i^i-\Sigma_k^k|^2 \tag{23}$$

with

$$E_1=|\partial_\mu H_5+gI_\mu H_1|^2+|\partial_\mu H_1 - gI_\mu H_5|^2$$

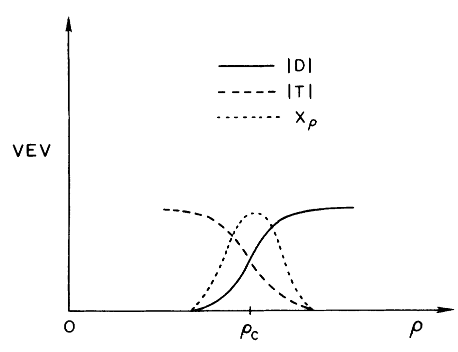

FIG. 1. Highly qualitative behavior of $|T|,|D|$, and the X_ρ condensate (VEV's) near $\rho\simeq\rho_c$, the critical point.

and

$$\begin{aligned} E_2=\ &g^2|A^1_{\mu1}H_1+R_\mu H_5|^2+g^2|R_\mu H_1+A^5_{\mu5}H_5|^2\\ &+g^2|A^1_{\mu2}H_1+A^5_{\mu2}H_5|^2+g^2|A^1_{\mu3}H_1+A^5_{\mu3}H_5|^2\\ &+g^2|A^1_{\mu4}H_1+A^5_{\mu4}H_5|^2\ , \end{aligned}$$

where R_μ and I_μ are respectively the real and imaginary parts of the gauge boson $A^5_{\mu1}=A^{1*}_{\mu5}=(R_\mu+iI_\mu)(1/\sqrt{2})$ (well-known proton-decay-mediating X^+_μ boson). Let us now seek the form of classical vector fields which will minimize energy in the region under consideration. For the present purpose, our interest pertains only to those gauge field components which do not commute with H's expectation value on (20) (such components have at least one SU(5) index equal to 1 or 5). All others are zero if they correspond to the broken generators, or can be eliminated by means of a gauge transformation. Evidently, the term E_2 is minimized by any configuration of $R, A^i_5, A^i_1\,(i=2,3,4)$ satisfying the conditions

$$\begin{aligned} &A^1_{\mu1}=-\frac{H_5}{H_1}R_\mu\ ,\quad A^5_{\mu5}=-\frac{H_1}{H_5}R_\mu\ ,\quad A^1_{\mu2}=-\frac{H_5}{H_1}A^5_{\mu2}\ ,\\ &A^1_{\mu3}=-\frac{H_5}{H_1}A^5_{\mu3}\ ,\quad A^1_{\mu4}=-\frac{H_5}{H_1}A^5_{\mu4}\ . \end{aligned} \tag{24}$$

However, for a simultaneous minimization of the last term in (22), only the trivial solution

$$A^i_{\mu5}=A^i_{\mu1}=R=0\ (i=2,3,4) \tag{25}$$

is acceptable. Otherwise, there will be nonvanishing contributions to the energy from this term since $\Sigma_1^1-\Sigma_i^i\neq0$ if $i\neq1$ and $\Sigma_5^5-\Sigma_i^i\neq0$ if $i\neq5$ in the $SU(2)\otimes U(1)$ symmetric state (20). Thus we are left with only one possible nonzero field I_μ. It can be seen easily that in contrast to other gauge fields, I_μ cannot vanish in the lowest energy state. This is due to the nonzero current (21) which generates an effective linear term for I in (22). Forgetting for a moment the term $\mathrm{tr}F^2_{\mu\nu}$ in (22), one finds the following

$$\langle I_\rho\rangle=\frac{iJ_\rho}{2m^2(\rho)},m^2(\rho)=g^2[|H_1|^2+|H_5|^2+(|\Sigma_1^1-\Sigma_5^5|^2)^2] \tag{26}$$

minimum-energy solution. Since all scalar VEV's depend exclusively on the radial variable ρ, the only surviving part of the current (20) is the radial component $J_\rho=ig(H_1\partial H_5/\partial\rho - H_5\partial H_1/\partial\rho)$ implying that only the radial component I_ρ of the classical solution (25) has ρ dependence. Therefore $F_{\mu\nu}=0$ for such a configuration and in fact (18) is the true vacuum solution for given VEV's of H_5, H_1, and Σ. Existence of nonzero expectation value of the gauge boson can be seen directly from the Lagrange equation for X^+_μ,

$$\partial_\nu f^+_{\mu\nu}+m^2X_\mu - g^2[X_\mu(X^+_\nu X^+_\nu)-X^+_\mu(X^+_\nu X_\nu)]-J_\mu=0 \tag{27}$$

which forbids the trivial solution $\langle X \rangle = 0$ in the region where $\langle J_\mu \rangle \neq 0$. Qualitative behavior of X_ρ condensate is shown in Fig. 1.

Existence of the $\langle X_\rho \rangle$ condensate is not a gauge artifact and it cannot be eliminated by means of local SU(5) transformations. We may attempt to eliminate the radial classical current J_ρ by means of the following local (ρ-dependent) orthogonal (since H_1 and H_5 are chosen real) rotations in the 1-5 subplane,

$$H \to \exp[i\tau_1^5 \alpha(\rho)]H = (0,0,0,0,\sqrt{H_1^2+H_5^2}) \quad (28)$$

where $\alpha(\rho) = \arctan[H_1(\rho)/H_5(\rho)]$ is a gauge rotation parameter and τ_1^5 is the appropriate antisymmetric SU(5) generator which can be represented by a 2×2 submatrix

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

in the 1-5 plane. However, this transformation immediately generates a nonzero ρ-dependent element

$$\Sigma_1^5(\rho) = \tfrac{1}{2}[\Sigma_5^5(\rho) - \Sigma_1^1(\rho)]\sin 2\alpha(\rho) \quad (29)$$

of the 24-plet which, in turn, creates another radial current [instead of (20)]

$$J'_\rho = ig[(\partial_\rho \Sigma_i^1)\Sigma_5^i - (\partial_\rho \Sigma_5^i)\Sigma_i^1] \quad (30)$$

interacting with the same gauge boson $X_\mu^i(A_i^5)$; the boson condensate emerges unharmed. For similar reasons, gauge fields of the usual vortex (1) away from the core cannot be eliminated everywhere by means of local U(1) transformations because of a nontrivial winding number. Of course, in our case the SU(5) sector by itself does not carry any topological number; the nontrivial phase structure of the system is generated from "outside" by the vortex which forces the SU(5) Higgs scalars to form nontopological soliton-type configurations in its own core.

The SU(5)-phase portrait of the string may be considerably more complicated. For example if $a_{\min} < a_0$, then at some point $\rho_0[a(\rho_0) = a_0]$ the phase transition, with either one of the possible jumps of Σ's VEV:

$$\Sigma = a_0\sqrt{2/15}(1,1,1,-\tfrac{3}{2}-\tfrac{3}{2}) \to \begin{cases} a_0\sqrt{\tfrac{1}{20}}(1,1,1,1,-4)\,, \\ a_0\sqrt{\tfrac{1}{20}}(1,1,1,-4,1)\,, \\ a_0\sqrt{\tfrac{1}{20}}(4,1,1,1,1)\,, \end{cases} \quad (31)$$

can take place. (Back reaction from the 5-plet VEV is neglected.) This phase transition, depending on the choice of parameters, can induce variety of respective phase transitions in the fundamental representation sector, e.g.,

$$\begin{aligned} &(H_1,0,0,0,0) \to (0,0,0,0,H_5)\,, \\ &(0,0,0,0,H_5) \to (0,0,0,H_4,0)\,, \\ &(0,0,0,H_4,0) \to (H_1,0,0,0,0)\,, \end{aligned} \quad (32)$$

leading to the condensation of X, W, Y gauge bosons.

III. BEHAVIOR OF SCALAR VEV WITH NONTRIVIAL CHARGE

In many realistic cases, the H_a and H_b components which compose the current (1) (source of nonzero gauge boson expectation value), have nontrivial charges under the $\tilde{\mathrm{U}}(1)$ subgroup responsible for the string. Before investigating such cases, one needs to understand the possible behavior of some scalar H transforming nontrivially under $\tilde{\mathrm{U}}(1)$ in the vortex field. Let y be the $\tilde{\mathrm{U}}(1)$ charge associated with H (the charge is measured in units of χ's charge). The field H is influenced by χ in two ways: (1) directly by the $\tilde{\mathrm{U}}(1)$-gauge vector field, and (2) through a $\tilde{\mathrm{U}}(1)$-invariants coupling in the potential. For simplicity, we assume $H \ll \chi$, so that the back influence of H on χ can be neglected. This assumption will be quite valid for realistic cases when $\tilde{\mathrm{U}}(1)$ is embedded, for example, in some GUT symmetry group. Let us at first consider the case when the Higgs charge is an integral multiple of χ's charge, i.e., when y is an integer. We assume the topological charge $n=1$. While going around the string, χ's phase changes according to the $\tilde{\mathrm{U}}(1)$ transformation [θ measures the angle] $\chi(\theta) = \chi(0)\exp[i\theta]$, while the corresponding $\tilde{\mathrm{U}}(1)$ transformation for H is

$$H(\theta) = \exp(iy\theta)H(0)\,.$$

Since the model is locally $\tilde{\mathrm{U}}(1)$-symmetric, it is obvious to look for the solution of H in the form $H = \xi(\rho)e^{iy\theta + ic}$ with $c=$const, and $\xi(\rho) \to$ const as $\rho \to \infty$. Now the kinetic term of H in cylindrical coordinates away from the vortex (where radial derivatives are negligible) is seen to be

$$\frac{|H|^2}{\rho^2}\left[\frac{\partial\omega}{d\theta} - y\right]^2 \quad (33)$$

where ω denotes the phase of H, and where we have used the gauge field asymptotic condition [Eq. (2)]. Notice that periodicity in θ demands that

$$\omega(2\pi) - \omega(0) = 2\pi\kappa \quad (34)$$

(where κ is an integer) be satisfied for a single-valued VEV of the scalar field H. Since y is an integer, then clearly the minimizing solution

$$\omega(\theta) = y\theta + c \quad (35)$$

(with arbitrary constant c) satisfies the constraint (34) and is acceptable. This solution also minimizes possible non-self-conjugate phase-dependent couplings between H and χ which for $y=2$ (for example) can be expressed as (χ has unit charge)

$$mH\chi^{+2} + m^+H^+\chi^2 = 2|m||H||\chi|^2\cos(\omega - 2\theta + \beta) \quad (36)$$

(where β is the phase of the parameter $m = |m|e^{i\beta}$) and has a minimum for (35) with $y=2$ and $c = \pi(2k+1) - \beta$.

Thus for integral y, both the kinetic and the potential energies are minimized by a $\omega(\theta)$ which satisfies the periodicity constraint.

The situation changes drastically for nonintegral y because the solution (35) which mathematically minimizes the kinetic energy (3) (as well as a possible phase-dependent coupling in the potential) no longer respects the physical constraint (34).

We assume that there are no non-self-conjugate terms in the potential $V(\chi,H)$ which, in our case, is given by Eq. (3). Note that if $y\neq\pm\frac{1}{2}$, $\pm\frac{1}{3}$ then Eq. (3) represents the most general, locally $\tilde{\mathrm{U}}(1)$-invariant renormalizable potential. For $y=\frac{1}{2}(\frac{1}{3})$, one could add additional non-self-conjugate terms, $\chi H^{\dagger 2}+\mathrm{H.c.}$ or $(\chi H^{\dagger 3}+\mathrm{H.c.})$. It can be shown that the existence of such terms lead to the appearance of domain walls as soon as the respective residual discrete symmetry $Z_2\equiv\{H\to e^{i\pi m}H, m=0,1\}$, or $Z_3\equiv\{H\to e^{i(2\pi/3)m}H, m=0,1,2\}$ is broken by the VEV of H. These walls are bounded by χ strings. However, the non-self-conjugate terms can be eliminated from the potential since the relevant counter terms are not induced radiatively by any interaction. This is guaranteed by the additional global symmetry $\mathrm{U}(1)_H\times\mathrm{U}(1)_\chi$ which prevails if the aforementioned terms are absent. The kinetic term for H in polar coordinates is

$$\left[\frac{\partial|H|}{\partial\rho}\right]^2+\frac{1}{\rho^2}\left[\frac{\partial|H|}{\partial\theta}\right]^2 +|H|^2\left[\frac{1}{\rho}\frac{\partial\omega}{\partial\theta}-eA_\theta\right]^2+|H|^2\left[\frac{\partial\omega}{\partial\rho}\right]^2 . \quad (37)$$

Since the solution $|H|$ minimizing the potential goes to a constant, and A_θ has the asymptotic form given by Eq. (2), it is obvious that we look for a solution (away from the core) of the form $H=\xi e^{i\omega(\theta)}$, where $\xi=|H|_\infty$ and $\omega(\theta)$ is the appropriate function which minimizes

$$G=\int_0^{2\pi}\left[\frac{\partial\omega}{\partial\theta}-y\right]^2 d\theta \quad (38)$$

subject to (34). Variation of (38) leads to the extremum condition

$$\frac{\partial^2\omega}{\partial\theta^2}=0 \quad (39)$$

with the solution

$$\omega=k\theta+C , \quad (40)$$

where k is an integer minimizing $|k-y|$. This result can be appreciated in the following manner. The already mentioned full global symmetry of the model, $\mathrm{U}(1)_\chi\times\mathrm{U}(1)\ \mathrm{U}(1)_H$ [together with local $\tilde{\mathrm{U}}(1)$] is broken down to $\mathrm{U}(1)_H$ by χ's VEV responsible for the local string formation. Being broken by H, the residual global $\mathrm{U}(1)_H$ gives rise to a global string with a different topological charge k, and to a Goldstone boson expressed in the arbitrariness of C. The energy of the solution $H=\xi e^{i\theta k+C}$ diverges logarithmically with radius as $\rho\to\infty$ [R is a measure of the string radius]

$$E(\rho)\approx 2\pi|H|_\infty^2(k-y)^2\ln\frac{\rho}{R} , \quad (41)$$

as in the case of the standard global string [11]. In typical realistic cases, the scalar H, in addition, transforms nontrivially under some larger symmetry group G (unbroken by χ). Thus, the total symmetry breaking induced by H is $G\times\mathrm{U}(1)_H\to G'$. It might be that this breaking, by itself, admits no string solution. This is the case if the manifold $G\times\mathrm{U}(1)_H/G'$ is simply connected, and then the solution $H=\xi e^{i(k\theta+c)}$ is a nontopological vortex. Such a structure, by itself, is topologically unstable and can be smoothly deformed to the trivial solution ($k=0$) if coupling with A_θ is switched off ($y\to 0$).

The next important issue is the behavior of the function $\xi(\rho)$ in the core of the defect. Of particular interest is the case when the minimum of $|k-y|$ corresponds to $k=0$ [if $k\neq 0$ then $\xi(0)=0$]. The integral

$$\int_0^{2\pi}\left[\frac{d\omega}{d\theta}-\rho A_\theta(\rho)ey\right]^2 d\theta \quad (42)$$

for any fixed ρ is minimized by the functions of type (41). Such solutions with different k cannot be smoothly transformed into one another if $\xi(\rho)=0$. The solution with $k=0$, must, for $\rho\to\infty$, behave as $H=\xi(\rho)e^{ic}$ with an arbitrary constant phase c. The energy density of this field configuration is

$$E=\left[\frac{\partial\xi}{\partial\rho}\right]^2+\xi^2W(\rho)+\lambda\xi^4 , \quad (43)$$

with

$$W(\rho)=(eyA_\theta)^2+(f|\chi|-m^2) . \quad (44)$$

Behavior of $\xi(\rho)$ is evidently determined by the tendency of the function $W(\rho)$. For $m^2>0$, the coefficient $W(\rho)$ is negative at the origin since $A(0)=0$ and $\chi(0)=0$. However, this is not sufficient to have $\xi(0)\neq 0$, since $W(\rho)$ is not, in general, negatively determined. Corresponding to the realistic example which we will discuss in Sec. IV is the case when $f\mu^2-m^2>0$. In this situation, the existence of a solution with $\xi(0)\neq 0$ can be investigated by checking the stability of the trivial solution. Following (3), let us consider small fluctuations of the form $\xi(t,x,y,z)=e^{i\Delta t}\xi(x,y)$ in the string background, the equation for ξ is a two-dimensional Schrödinger equation (linearized)

$$\left[-\frac{d^2}{dx^2}-\frac{d^2}{dy^2}\right]\xi+W(\rho)\xi=\Delta^2\xi . \quad (45)$$

If there is a normalizable bound state solution with $\Delta^2<0$, then we can say that state with $\xi(0)\neq 0$ is unstable. It was shown in Ref. [3] that such a solution exists for $y=0$ (see also Ref. [12]). In the case $y\neq 0$, the possibility of a solution with $\xi(0)\neq 0$ depends on the balance between the strength and range of the attractive potential in the region $W(\rho)<0$ (around the origin), and the repulsive tendency of the energetical barrier in the region $W(\rho)>0$. Usually in the realistic cases, H transforms nontrivially under some other Abelian gauge group $\mathrm{U}(1)'$ with a corresponding generator τ, and gauge field B_μ. In Ref. [3], it is shown that in the kinetic term of H, the influence from the vortex gauge field, A_μ,

$$|H|^2(\partial_\mu\omega-eyA_\mu-e'y'B'_\mu)^2 \quad (46)$$

[where e' and y' are the U(1)$'$-gauge coupling constant and hypercharge, respectively], can be compensated by B_μ. This will happen if the potential of $H \sim \lambda^{-1}$ overcomes the magnetic energy $\sim (1/e')^2$ associated with B_μ. In such a case $\xi(0) \neq 0$, and the magnetic flux in the core of the string corresponds to the superposition of $\tilde{\mathrm{U}}(1)$ and U(1)$'$ generators:

$$k' = e'y'k - ey\tau \ , \tag{47}$$

which remains unbroken in the core.

IV. TOPOLOGICALLY NONTRIVIAL CLASSICAL CURRENTS, GAUGE-BOSON CONDENSATION

Our further analysis is based on the fact that some scalar H with nontrivial y-charge can (under conditions discussed above) have nonvanishing VEV in the core of the vortex χ. The case of our interest is the one in which both the scalars χ (responsible for the string) as well as H (having nonzero expectation value in the string core) belong to the same irreducible representation of a certain gauge group G. To develop the argument, it will be necessary to stipulate the existence of a (nondiagonal) generator τ (of G) which transforms states χ and H into each other, $\chi = \tau H$. If it is so, then in the false vacuum of the vortex, there definitely exists a classical current

$$J_\mu = ig[\chi^\dagger \partial_\mu H - (\partial_\mu \chi^\dagger) H] \tag{48}$$

with nonvanishing θ and ρ components given by

$$J_\rho = ige^{-i\theta}\left[\eta \frac{\partial \xi}{\partial \rho} - \xi \frac{\partial \eta}{\partial \rho}\right] \ , \tag{49}$$

$$J_\theta = -g\frac{e^{-i\theta}}{\rho}\eta\xi \ , \tag{50}$$

with ξ assumed real. We also consider only the winding number $n=1$ strings. Accordingly, the expectation value of the gauge boson (corresponding to τ), which inevitably will be created by this current, will have both θ and ρ components. To demonstrate the working of this mechanism, we consider the realistic example of the string which might be formed in minimal SU(5)-GUT without any additional $\tilde{\mathrm{U}}(1)$ symmetry. It is obvious that standard pattern of symmetry breaking

$$\mathrm{SU}(5) \to \mathrm{SU}(3)\otimes \mathrm{SU}(2) \otimes \mathrm{U}(1) \to \mathrm{SU}(3) \otimes \mathrm{U}(1) \tag{51}$$

admits no string solution in this model. However, it is well known (e.g., see Refs. [13–15]) that the phase transition chain at the finite temperature T need not follow the symmetry breaking patterns characteristic of zero temperature. The configuration of Higgs VEV's, which minimizes the scalar effective potential at finite temperature, differs from the VEV configurations at $T=0$. This is because the effective masses of Higgs fields are temperature dependent. High temperature approximation to the potential of any Higgs scalar H can be obtained by adding to the potential the term [16]

$$V(T,H) = V(H) + fT^2|H|^2 \tag{52}$$

where the parameter f is a function of the gauge coupling constants and of the quartic coupling of Higgs field, and can be taken to be approximately constant over a wide range of temperatures. From (52), it is clear that the changing of the scalar effective mass (and VEV) due to the finite temperature is very similar to the changes which occur in the extra vortex field χ. In the high temperature limit, therefore, the temperature dependent effective potential for SU(5) Higgs fields can be obtained by the simple replacement $|\chi| \to T$ in the equations of Sec. II. And, of course, f_Σ and S_H now must be considered as appropriate functions of gauge and quartic coupling constants. We shall now consider the following high temperature phase transition pattern [14,15]

$$SU(5) \xrightarrow{T_0} \mathrm{SU}(4) \otimes \mathrm{U}(1) \xrightarrow{T_1} \mathrm{SU}(4) \xrightarrow{T_2} \mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1) \xrightarrow{T_3} \mathrm{SU}(3) \otimes \mathrm{U}(1) \ . \tag{53}$$

In this situation T-dependent phase structure of the Higgs fields is the following: Above the critical temperature $T_c \sim M/f_\Sigma$ the effective mass of the 24-plet $M^2(T) > 0$, and thus the universe is in a SU(5)-symmetric phase. As the universe expands and cools below T_c, the 24-plet develops the VEV [Eq. (9)], and the universe passes on to the SU(4)$\otimes$U(1) symmetric phase. Further cooling decreases $M^2(T)$ (increasing $a^2 = \mathrm{tr}\Sigma^2$) and at some temperature $T_2[a(T_2) = a_0]$ a first-order phase transition takes place, and the SU(4)$\otimes$U(1) phase becomes metastable, and finally the original 24-plet passes to the SU(3)$\otimes$SU(2)$\otimes$U(1) symmetric phase. This phase structure has been studied by many authors, and it was found that at some intermediate temperatures $T_1 (T_0 > T_1 > T_2)$, the fundamental 5-plet H can develop a VEV,

$$H = (H_1, 0, 0, 0, 0) \ , \tag{54}$$

which makes the existence of an intermediate SU(4) phase possible.

At the temperature T_2, as soon as the 24-plet goes to a SU(3)$\otimes$SU(2)$\otimes$U(1)-phase, the mass square of H becomes positive and the VEV of the 5-plet vanishes.

Cosmic strings of interest to the current work are created by the H_1 component at the second stage of phase transition (53). These strings are founded by the monopole-antimonopole pairs formed at the previous stage of the phase transition, and they disappear at a temperature T_2 together with the SU(4)-symmetric vacuum; the present universe is free of them. It is worth mentioning that the (quasi)stable topological defects existing only in the limited range of temperature $T_1 > T > T_2$, and disappearing as the temperature falls ($T < T_2$) are well known in literature (e.g., see Ref. [13]). We now show that in the early universe such topological structures might serve as templates for the SU(5) gauge-boson ($X, Y, V, \ldots$) condensation. We are primarily interested in understanding the quasistationary picture at some fixed temperature $T(T_1 > T > T_2)$. So we neglect all effects related with the temperature gradients. Substituting the SU(4)$\otimes$U(1)-invariant VEV (9) of the 24-plet in the part of potential including H(14) [replacing

$M^2 \to M^2 + f_\Sigma T^2 = M^2(T)$, and insuring that all parameters pertain to the temperature T], one finds the following effective masses of the singlet H_5, and the 4-plet $H_i (i=1,2,\ldots,4)$ fragments of H in the SU(4)⊗U(1)-invariant universe (due to the unbroken local freedom we can consider only one component of the 4-plet, e.g., H_1):

$$m_{H_1}^2 = -m_1^2 + f_H T^2 + (\lambda_2 + \tfrac{1}{20}\lambda_1) a(T)^2 + \frac{1}{\sqrt{20}} \mu_1 a(T) , \tag{55}$$

and

$$m_{H_5}^2 = -m_1^2 + f_H T^2 + (\lambda_2 + \tfrac{4}{5}\lambda_1) a(T)^2 + (\tfrac{4}{5})^{1/2} \mu_1 a(T) .$$

In general, neither $m_{H_1}^2$ nor $m_{H_5}^2$ is forbidden, by any constraints, to become negative [14]. We can thus look for a range of parameters for which both $m_{H_1}^2, m_{H_5}^2 < 0$. Then, the necessary condition for H_1 to develop a VEV is

$$m_{H_1}^2 < m_{H_5}^2 . \tag{56}$$

If this is so, then the universe goes to the SU(4)-symmetric phase, and the VEV of H_1 is

$$|H_1|^2 = -\frac{m_{H_1}^2}{2h_1} . \tag{57}$$

The back influence from H_1 on Σ is expressed through a modification of a. Since H_1 creates the string, it has the standard form (we consider a string with top number $n=1$)

$$H_1 = \eta(\rho) e^{i\theta} \tag{58}$$

with $\eta(0)=0$ and, $\eta(\infty)=H_1$. The magnetic flux in the string corresponds to the generator

$$K = \mathrm{diag}(1, -\tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{1}{4}) .$$

What is the behavior of the H_5 component in the core of this vortex? Conditional minima of the potential which [in SU(4)⊗U(1) phase] has the form:

$$V(H, H_5) = m_{H_1}^2 |H_1|^2 + m_{H_5}^2 |H_5|^2 + h_1 (|H_1|^2 + |H_5|^2)^2 ,$$

subject to $H_1=0$ is at $|H_5|^2 = -(m_{H_5}^2)/2h_1 (m_{H_5}^2 < 0)$. We neglect by radiative splitting in quartic term. The potential energy, thus, tends to make $H_5 \neq 0$ in the core of the defect. But since H_5 has hypercharge $y = -\frac{1}{4}$ under the generator K (in units of H_1's charge), it is difficult for H_5 to develop a VEV in the core (see Sec. III), because of the influence of the vortex gauge field A_μ in the kinetic term of H_5. This influence, however, can be compensated by some other U(1)⊂SU(4) gauge field B_μ interacting with H_5 [with corresponding generators $\tau = \mathrm{diag}(0, \tau_2, \tau_3, \tau_4, \tau_5)$] if the magnetic energy of B is less than the Higgs energy of H_5. This is just the case if [3] $h^{-1} < e^{-2}$ where e is the SU(5) gauge-coupling constant pertaining to the temperature T. The magnetic energy, proportional to $\mathrm{tr}\tau^2$, has to be minimized subject to the conditions $\tau_5 = -K_5$, $\tau_1 = 0$. The generators B turn out to be $B = \mathrm{diag}(0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1)$. Finally $H_5 = \xi(\rho)$ is monotonic function of ρ with $\xi(0) \neq 0$ and $\partial\xi(0)/\partial\rho = 0$ and the magnetic flux in the core is aligned in the directions $\tau + K = (1, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 0)$; a superposition of color, weak hypercharge, and electricity. To see that the above picture leads to the touted X_μ-boson condensation we have to check the stability of the system at $X_\mu = 0$. The X_μ-dependent part of the Lagrangian has the form

$$\begin{aligned} L(X) = &-\tfrac{1}{2} |D_\mu X_\nu - D_\nu X_\mu|^2 - m_x^2 X_\mu^+ X_\mu - i g f_{\mu\nu} X_\mu^+ X_\nu \\ &+ \tfrac{1}{2} g^2 [X_\mu^{+2} X_\nu^2 - (X_\mu^+ X_\mu)^2] \\ &+ X_\mu^+ i g (H_1^+ \partial_\mu H_5 - H_5 \partial_\mu H_1^+ \\ &\qquad - i e A'_\mu H_5 H_1^+) + \text{h.c.} . \end{aligned} \tag{59}$$

Here $m_x^2 = g^2(|H_1|^2 + |H_5|^2 + |\Sigma_1^1 - \Sigma_5^5|^2)$ is the mass of the X_μ boson, $D_\mu = \partial_\mu - i g A'_\mu$, and $f_{\mu\nu} = \partial_\mu A'_\nu - \partial_\partial A'_\mu$ is the magnetic field of the gauge boson A'_μ corresponding to the generator $(1, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 0)$ which generates magnetic flux in the core of the string. The last term is linear in X_μ and $X_\mu = 0$ is definitely unstable in the core; the nonvanishing source

$$J_\rho = i e (H_1^+ \partial_\mu H_5 - H_5 \partial_\mu H_1^+ - i e B_\mu H_5 H_1^+) \neq 0 \tag{60}$$

tends to preserve the linear term in X_μ, resisting any attempts by the system to settle to $X_\mu = 0$. The source (60) has nonzero ρ and θ components

$$J_\rho = i g e^{-i\theta} \left[\eta \frac{\partial \xi}{\partial \rho} - \xi \frac{\partial \eta}{\partial \rho} \right] , \tag{61}$$

$$J_\theta = g e^{-i\theta} \xi \eta \left[-\frac{1}{\rho} + g B_\theta \right] , \tag{62}$$

since $B_\theta \neq (g\rho)^{-1}$ in the core. Note that the term $f_{\mu\nu} X_\mu^+ X_\nu$ is an "anomalous" magnetic moment term associated with the vortex gauge field B_μ. This term is unbounded from below and, in principle, can also lead to the condensation of X_μ in the vortex magnetic field in the same way as the interaction with the normal magnetic field associated with the superconducting string induces the condensation of the W bosons [6]. In this paper, however, we do not investigate this interesting possibility.

V. SUMMARY

In the present paper we have shown that under certain well-defined conditions, the cosmic string (vortex) exciting (in its core) VEV's of grand-unification Higgs scalars can also create nonzero classical currents in the false vacuum of the defect. This current, in turn, leads to the accumulation of a nonzero expectation value of the corresponding grand-unification gauge bosons in the vortex field. The scalars forming the classical current could be either trivial or nontrivial under the $\tilde{\mathrm{U}}(1)$ group responsible for the string. In the former case (discussed in Sec. III), this current has a nontopological nature implying that similar condensation could take place on other vacuum defects (topologically stable or unstable) like the domain walls and monopoles provided the grand-unification Higgs sector has sufficiently strong interactions with the scalar χ representing the defect.

It is well known that in the high temperature era, the universe could pass through various exotic phases before reaching its usual SU(3)⊗SU(2)⊗U(1) symmetric phase which, below the electroweak scale (100 GeV), undergoes the transition to SU(3)⊗U(1) symmetric vacuum. Determined by the choice of parameters as well as by the initial grand unified symmetry group [SU(5),SU(12),E(6), . . .], the phase transition chain can be quite complicated. In principle, during the high temperature symmetry breaking, the phase transition of the type SU(3)⊗U(1) →SU(2)⊗SU(2)⊗U(1) discussed in Sec. II, with rapid transfer of VEV from H_1 to H_5 of the fundamental representation, can also occur. In minimal GUT, for example, one could imagine the following scenario of the phase state of the universe.

At some temperature T_c, the phase SU(3)⊗U(1) corresponding to the form (18) of the Higgs VEV becomes unstable and will decay via the creating of the expanding bubbles of the SU(2)⊗SU(2)⊗U(1) symmetric vacuum [with scalar VEV's of the form (19)]. On the boundary between two phases, the VEV's have the form (20), but the parameter ρ, now, must be understood as the coordinate transverse to the boundary surface. If the bubbles are expanding, then the scalar VEV's acquire a time dependence and, in turn, add a time component to the current (created on the bubble surface), as well as to the induced classical X_μ-boson field to their already existing space components.

Appearance of the X-boson expectation value during the high temperature phase transitions can play an important role in creating the observed baryon-antibaryon asymmetry of the universe.

ACKNOWLEDGMENTS

We thank Dr. Z. Berezhiani, Dr. T. Bibilashvili, Dr. J. Chkareuli, Dr. M. Shaposhnikov, and Dr. S. Mufson for discussions.

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