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Quaternion scalar field

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We discuss the extension of a version of *quaternion* quantum mechanics to field theory and in particular to the simplest example, the free scalar field. A previous difficulty with the conservation of four-momentum for the "anomalous" bosonic particles is resolved.

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I. INTRODUCTION

Among the various suggestions for adapting relativistic quantum mechanics to the use of an underlying quaternion number system [1–4] the one proposed by one of the authors [5] claims a number of positive aspects and some negative ones. Working in first quantization, the free-particle Dirac equation can be written as

$$(\gamma^{\mu}\partial_{\mu}|i-m)\Psi=0 , \tag{1}$$

where by definition all quaternion operators must be decomposed into a left/right structure $A|b$ such that

$$(A|b)\Psi\equiv A\Psi b . \tag{2}$$

This requirement follows from the fact that A, Ψ, and possibly even b may be noncommuting quaternions. In particular the momentum operator (Hermitian for the *complex* scalar product) is defined by

$$p^{\mu}\Psi\equiv(\partial^{\mu}|i)\Psi=\partial^{\mu}\Psi i . \tag{3}$$

The quaternion γ matrices satisfy the standard Dirac condition[1]

$$\{\gamma^{\mu},\gamma^{\nu}\}=2g^{\mu\nu} \tag{4}$$

and have 2×2 irreducible quaternion representations. Our positioning of the imaginary i factors together with the Hermiticity of the Hamiltonian $H=(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m)$ guarantees the conservation of the norm and the commutativity of H with $\mathbf{p}$. We emphasize that a characteristic of this formalism is the absolute need of a complex scalar product $(\Psi,\Phi)_c$ defined in terms of the quaternion counterpart (Ψ,Φ) by

$$(\Psi,\Phi)_c\equiv\tfrac{1}{2}[(\Psi,\Phi)-i(\Psi,\Phi)i] . \tag{5}$$

All matrix elements must be given as complex scalar products, first introduced in 1984 by Horwitz and Biedenharn [4] in order to define consistently multiparticle quaternion states.

Among the positive features of this formalism we wish to list three.

(1) The appearance of all four standard Dirac free-particle solutions notwithstanding the two-component structure of the wave function (an example of the doubling of solutions with quaternions).

(2) The reproduction in quaternion calculations of the standard QED results. A nontrivial result given the explicit differences in certain spinor identities and the necessary modification of the trace theorems [6].

(3) The appearance in the nonrelativistic Schrödinger equation of two complex orthogonal solutions corresponding to spin up and spin down, without the need to pass "by hand" to the Schrödinger-Pauli equation. *A belated theoretical discovery of spin.*

This last point would be an impressive argument in favor of the use of quaternions if this doubling of solutions did not occur also in bosonic equations, where it has obviously nothing to do with spin. For example, we find two solutions (for a given four-momentum) for the Klein-Gordon equation and idem for the Maxwell equation, with the result that, in addition to the normal photon, we "discover" an *anomalous* photon. Anomalous because it appeared up to now that this photon violated the conservation of four-momentum [6].

In this paper we extend our studies to field theory for two reasons. First, because only field theory provides a consistent formal structure for particle physics and second, because it will permit us to clearly explain and correct the above-mentioned difficulty with four-momentum conservation for the anomalous photon. For simplicity we shall restrict our attention to the scalar field but the results are immediately applicable to the vector field and in particular to the massless Maxwell field.

In the next section we recall briefly the theory of the noninteracting scalar field. In the subsequent section we generalize it to the quaternion fields, and in particular to the anomalous scalar field, by means of a non-Hermitian association. In Sec. IV we describe how to define a Her-

[1]This condition actually "defines" the Dirac equation; we recall that other conditions such as that of the formally similar Duffin-Kemmer equation also exist.

mitian anomalous field. Our conclusions are drawn in Sec. V.

II. THE SCALAR FIELD

In standard field theory [7] one begins by assuming a unique vacuum $|0\rangle$ satisfying

$$P_\mu|0\rangle=0 \ , \tag{6}$$

$$J_{\mu\nu}|0\rangle=0 \ , \tag{7}$$

$$\varphi|0\rangle=0 \ , \tag{8}$$

$$D|0\rangle=0 \ , \tag{9}$$

where P_μ and $J_{\mu\nu}$ are the generators of the Poincaré group, φ represents any charge operator and D a generic discrete operator. This state is also assumed normalized:

$$\langle 0|0\rangle=1 \ . \tag{10}$$

The Hermitian free scalar field is assigned the Lagrangian density

$$\mathcal{L}=-\tfrac{1}{2}(m^2\phi^2-\partial_\mu\phi\partial^\mu\varphi) \ , \tag{11}$$

which upon variation yields the Klein-Gordon field equation

$$(\Box+m^2)\phi(x)=0 \tag{12}$$

and the "conjugate momentum"

$$\pi^\mu(x)\equiv\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}=\partial^\mu\phi(x) \ . \tag{13}$$

The fields are assumed to satisfy the canonical equal-time commutation relations

$$[\dot\phi(x),\phi(y)]_{x_0=y_0}=-i\delta^3(\mathbf{x}-\mathbf{y}) \ , \tag{14}$$

$$[\phi(x),\phi(y)]_{x_0=y_0}=0 \ , \tag{15}$$

$$[\dot\phi(x),\dot\phi(y)]_{x_0=y_0}=0 \ . \tag{16}$$

The energy-momentum density tensor is given by

$$T^{\mu\nu}=\partial^\mu\phi\partial^\nu\phi+\tfrac{1}{2}g^{\mu\nu}m^2\phi^2-\tfrac{1}{2}g^{\mu\nu}\partial_\lambda\phi\partial^\lambda\phi \ . \tag{17}$$

Consequently,

$$H\equiv p^0\equiv\int T^{00}d^3x=\tfrac{1}{2}\int(\partial_\mu\phi\partial_\mu\phi+m^2\phi^2)d^3x \ , \tag{18}$$

$$p^i=\int\dot\phi\partial^i\phi d^3x \ , \tag{19}$$

where H and p^i are the energy and momentum generators for translations in four-dimensional space-time.

Expanding $\phi(x)$ in terms of first-quantized solutions of the field equations (which for the scalar field corresponds formally to the Fourier decomposition) we obtain

$$\phi(x)=\frac{1}{V^{1/2}}\sum_{\mathbf{k}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}}}(C_{\mathbf{k}}e^{-ikx}+C_{\mathbf{k}}^\dagger e^{ikx}) \tag{20}$$

where the field has been normalized within a box of volume V and $k^0=\omega_{\mathbf{k}}=\sqrt{\mathbf{k}^2+m^2}$. Only the operators $C_{\mathbf{k}}$ and $C_{\mathbf{k}}^\dagger$ appear because ϕ is a Hermitian field. The field equal-time commutation relations then determine the canonical commutation relations for the annihilation and creation operators:

$$[C_{\mathbf{k}},C_{\mathbf{k}'}^\dagger]=\delta_{\mathbf{k}\mathbf{k}'} \tag{21}$$

and these in turn may be used to calculate the general (not necessarily equal-time) field commutation relations:

$$[\phi(x),\phi(y)]=-i\Delta(x-y) \tag{22}$$

where $\Delta(x)$ is the well-known Schwinger function.

At this stage, in order to eliminate unwanted negative-energy solutions, one imposes the condition

$$C_{\mathbf{k}}|0\rangle=0 \ . \tag{23}$$

As a consequence, the Hamiltonian is positive definite and is given by

$$H=\tfrac{1}{2}\sum_{\mathbf{k}}(C_{\mathbf{k}}C_{\mathbf{k}}^\dagger+C_{\mathbf{k}}^\dagger C_{\mathbf{k}})\omega_k \ . \tag{24}$$

The zero energy of the vacuum contained in this expression is eliminated by normal ordering of the fields in the Lagrangian and elsewhere, and henceforth this will always be implicitly assumed.

Of particular importance for us in what follows is the observation that the field equations, while very important, are not the dynamical equations of motion in second quantization. Since the fields are operators this privilege is reserved for the Heisenberg (first-order) equations

$$-i\partial_\nu\phi(x)=[p_\nu,\phi(x)] \ , \tag{25}$$

in particular,

$$-i\partial\frac{\phi(x)}{\partial t}=[H,\phi(x)] \ . \tag{26}$$

Notice what would happen if we make the "wrong" operator-frequency assignments:

$$\phi'=\frac{1}{V^{1/2}}\sum_{\mathbf{k}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}}}(C_{\mathbf{k}}^\dagger e^{-ikx}+C_{\mathbf{k}}e^{ikx}) \ ; \tag{27}$$

this corresponds to the interchange $C_{\mathbf{k}}\leftrightarrow C_{\mathbf{k}}^\dagger$ everywhere. Formally H does not change, but ϕ' would not then satisfy the Heisenberg equations. This is readily corrected; we need only change the sign of $\mathcal{L}$ and hence of H. Then, however, we would recognize that ϕ' corresponds to only negative-energy states. In other words, the sign and more generally the multiplicative coefficients in the Lagrangian densities are determined uniquely by the Heisenberg equations, while they are completely arbitrary as far as the field equations are concerned. We emphasize the highly restrictive nature of the requirement that negative energies be excluded. In the above case ϕ' is a nonacceptable solution of the field equations. We shall base some of our subsequent results on this principle.

We finally recall that while this Hermitian field ϕ is by itself chargeless, a couple of Hermitian fields ϕ_1, and ϕ_2 with the same mass, may be combined in non-Hermitian complex fields $\phi(x)\neq\phi^\dagger(x)$ with the Lagrangian density

$$\mathcal{L}=-(m^2\phi^\dagger\phi-\partial_\mu\phi^\dagger\partial^\mu\phi) \ , \tag{28}$$

energy-momentum tensor

$$T^{\mu\nu}=\partial^\mu\phi^\dagger\partial^\nu\phi+\partial^\nu\phi^\dagger\partial^\mu\phi+g^{\mu\nu}m^2\phi^\dagger\phi-g^{\mu\nu}\partial_\lambda\phi^\dagger\partial^\lambda\phi \ , \tag{29}$$

and Hermitian charge operator

$$\varphi=-ie\int(\dot{\phi}^\dagger\phi-\phi^\dagger\dot{\phi})d^3x \ . \tag{30}$$

In the presence of an interaction term $\sim\phi^\dagger\rho+\text{H.c.}$ the field equation becomes

$$(\Box+m^2)\phi(x)=\rho(x) \tag{31}$$

and if ϕ is non-Hermitian, the source term $\rho(x)$ must also be non-Hermitian.

For vector fields this results in the fact that Hermitian bosonic fields such as the Maxwell field A^μ couple to Hermitian currents J^μ, while non-Hermitian bosonic fields such as $W_\mu^\pm$ couple to non-Hermitian currents $J_\mu^\mp$. Thus electromagnetic matrix elements have the structure JJ, while weak interactions have the well-known $J^\dagger J$ form. Of course the latter would coincide with the former if J were Hermitian. The importance of this observation lies in the fact that the anomalous scalar will be identified with a neutral but *non-Hermitian* field in the next section.

III. THE QUATERNION SCALAR FIELD

There are two solutions to the Klein-Gordon equation in first quantization. The first is the standard solution

$$\phi(x)=e^{-ikx} \ , \tag{32}$$

while its quaternion companion is given by

$$\tilde{\phi}(x)=je^{-ikx} \ . \tag{33}$$

The position of the j factor is fixed by the requirement that $\tilde{\phi}$ correspond to an eigenstate of four-momentum with eigenvalues k_μ.

For the second-quantized field we should expand in both, but for simplicity let us consider only the anomalous field. Our sole constraint will be that the Lagrangian density be unique and valid both for ϕ and $\tilde{\phi}$. Hence we must employ that given in the previous section:

$$\mathcal{L}=-\tfrac{1}{2}(m^2\tilde{\phi}^2-\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}) \ . \tag{34}$$

However, this assumed a Hermitian field and we immediately see that the Hermitian of the negative frequency

$$(je^{-ikx})^\dagger=-je^{-ikx} \tag{35}$$

does *not* yield the positive-frequency term.

After various attempts it becomes obvious that we must employ the more general form

$$\mathcal{L}=-\tfrac{1}{2}(m^2\tilde{\phi}^\dagger\tilde{\phi}-\partial_\mu\tilde{\phi}^\dagger\partial^\mu\tilde{\phi}) \tag{36}$$

which reduces to the previous form if $\tilde{\phi}$ is Hermitian. The field $\tilde{\phi}$ is now *defined* (here $C_{\mathbf{k}}$ refers to the anomalous particle) as

$$\tilde{\phi}=\frac{1}{V^{1/2}}\sum_{\mathbf{k}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}}}(C_{\mathbf{k}}je^{-ikx}+C_{\mathbf{k}}^\dagger je^{ikx}) \ . \tag{37}$$

We assume that $C_{\mathbf{k}}$, $C_{\mathbf{k}}^\dagger$ commute with the quaternion number field (commutation with the complex numbers has always been implicitly assumed). In fact, in order to maintain the canonical commutation relations for these creation-annihilation operators, we must assume either commutation or anticommutation relations with j:

$$\begin{aligned}[][C_{\mathbf{k}},C_{\mathbf{k}'}^\dagger]&=\delta_{\mathbf{k}\mathbf{k}'}\\ &\Longrightarrow -j[C_{\mathbf{k}},C_{\mathbf{k}'}^\dagger]j=-j\delta_{\mathbf{k}\mathbf{k}'}j\\ &\Longrightarrow [-jC_{\mathbf{k}}j,-jC_{\mathbf{k}'}^\dagger j]=\delta_{\mathbf{k}\mathbf{k}'}\\ &\Longrightarrow -jC_{\mathbf{k}}j=\pm C_{\mathbf{k}} \ ;\end{aligned} \tag{38}$$

i.e., the possibility

$$-jC_{\mathbf{k}}j=\pm C_{\mathbf{k}}^\dagger \tag{39}$$

is excluded.

It is to be noted that $\tilde{\phi}$ is not Hermitian; in fact,

$$\tilde{\phi}^+=\frac{1}{V^{1/2}}\sum_{\mathbf{k}}\frac{-1}{\sqrt{2\omega_{\mathbf{k}}}}(C_{\mathbf{k}}^\dagger je^{-ikx}+C_{\mathbf{k}}je^{ikx}) \ . \tag{40}$$

This field has the "wrong" frequency associations. We recall the example of ϕ' in the previous section. With quaternions the Heisenberg equations of motion become

$$\partial_\nu A=[p_\nu,A]|i \ ; \tag{41}$$

this follows from the fact that the formal solution of the equation

$$\partial\frac{\Psi}{\partial t}i=H\Psi \tag{42}$$

is given by

$$\Psi(t)=e^{-Ht|i}\Psi(0) \tag{43}$$

so that in practice we must replace iH by $H|i$ in all relevant equations, and hence ip_ν by $p_\nu|i$.

Assuming that the i on the right-hand side (RHS) of Eq. (41) commutes with the vacuum (and hence with all physical states) we note that while $\tilde{\phi}$ satisfies the Heisenberg equations, $\tilde{\phi}^\dagger$ does not. Again $\tilde{\phi}^\dagger$ resembles the ϕ' of the previous section. Furthermore, the structure $\tilde{\phi}^\dagger\tilde{\phi}$ brings the j factors in direct contact, so that the Lagrangian density is formally equivalent to the standard structure (see Sec. II). Since $\tilde{\phi}$ is not Hermitian it couples to non-Hermitian sources. In analogy, the interaction matrix elements of the anomalous photon will resemble those of weak interactions $\sim\tilde{J}^\dagger\tilde{J}$ and again the troublesome j factors will cancel directly or complex conjugate *all* the exponential factors. In either case, standard four-momentum conservation is obtained. We recall that it was the assumption of the photoniclike $\tilde{J}\tilde{J}$ structure for the anomalous photon which led to a $jJjJ$ anomalous interaction and consequent difficulties with four-momentum conservation.

Needless to say, the Hamiltonian is the same as for the standard Hermitian scalar field ϕ. Less obvious is the fact that the charge is still zero with a non-Hermitian $\tilde{\phi}$. This can most easily be seen by noting that, unlike the case of the complex scalar field, we have only one type of particle operator in $\tilde{\phi}$ while the charge operator in standard field theory is essentially due to cross terms $i(C_1 C_2^\dagger - C_2 C_1^\dagger)$ which vanish when $C_1 = C_2$.

We also observe that the presence of the j factors modifies the field commutation relations. This raises a major difficulty with quaternion field theory. The presence of noncommuting wave functions means that we cannot maintain simultaneously the canonical field commutation relations and the canonical creation-annihilation operator commutation relations. One of these must be abandoned. In order to reproduce the spin-statistics relation and in particular the Pauli exclusion principle we choose to retain the latter (considered as more fundamental) and to relinquish the canonical field commutation relations. This complicates numerous proofs, but does not invalidate demonstrations such as causality, etc. In the next section we shall bypass this difficulty for the scalar field but we warn that it will represent a "trick" that is not extendable to fermion fields.

IV. THE HERMITIAN ANOMALOUS FIELD

There is one aspect of quaternion field theory that we have ignored in the previous section. It was perhaps already evident in the modified form of the Heisenberg equation. The field operators must be defined in general as "bared" operators, so that the Heisenberg equations are meaningful operator identities. Now from its explicit form H is evidently a left-acting operator. So $\partial_\nu \phi$ must bring down the right-acting i. Thus our fields should have been written as

$$\phi = \frac{1}{V^{1/2}} \sum_{\mathbf{k}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} (C_{\mathbf{k}} | e^{-ikx} + C_{\mathbf{k}}^\dagger | e^{ikx}) , \tag{44}$$

and for the anomalous field (here we employ a tilde to distinguish the creation-annihilation operators) as

$$\tilde{\phi} = \frac{1}{V^{1/2}} \sum_{\mathbf{k}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} (\tilde{C}_{\mathbf{k}} j | e^{-ikx} + \tilde{C}_{\mathbf{k}}^\dagger j | e^{ikx}) \tag{45}$$

or alternatively as

$$\tilde{\phi}' = \frac{1}{V^{1/2}} \sum_{\mathbf{k}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} (\tilde{C}_{\mathbf{k}} | j e^{-ikx} + \tilde{C}_{\mathbf{k}}^\dagger | j e^{ikx}) . \tag{46}$$

For this anomalous scalar field the choice may be made so as to reobtain the canonical field commutation relations; i.e., we can choose the $\tilde{\phi}$ structure [Eq. (45)] so as to keep separate the j factor from the exponential plane-wave functions. Then, using the fact that

$$(A|a)(B|b) = AB|ab \tag{47}$$

we find that, when a and b are commuting functions (complex in this case),

$$[A|a, B|b] = [A,B]|ab . \tag{48}$$

Since j has been assumed to commute with the $\tilde{C}_{\mathbf{k}}(\tilde{C}_{\mathbf{k}}^\dagger)$ the commutation reduces to that of the $\tilde{C}_{\mathbf{k}}$, and the standard result is obtained up to an overall sign due to the j^2 factor. Even this sign difference may be avoided by redefining $\tilde{\phi}$ as

$$\tilde{\phi} = \frac{1}{V^{1/2}} \sum_{\mathbf{k}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} (\tilde{C}_{\mathbf{k}} j | i e^{-ikx} + \tilde{C}_{\mathbf{k}}^\dagger j | i e^{ikx}) . \tag{49}$$

We note that with this definition $\tilde{\phi}$ has become complex Hermitian. In fact, by the definition of complex Hermiticity (complex a),

$$\langle \phi | (A|a)^\dagger | \psi \rangle_c \equiv \langle \psi | (A|a) | \phi \rangle_c^* \tag{50}$$

$$= \langle \psi | A | \phi \rangle_c^* a^* \tag{51}$$

$$\equiv \langle \phi | A^\dagger | \psi \rangle_c a^* \tag{52}$$

$$= \langle \phi | (A^\dagger | a^*) | \psi \rangle_c \tag{53}$$

for any $|\psi\rangle, |\phi\rangle$. Thus,

$$(A|a)^\dagger = A^\dagger | a^* . \tag{54}$$

We also note that now $\tilde{\phi}^\dagger$ has the correct frequency assignments (the j factor no longer complex conjugates the plane wave factors) and thus satisfies the corresponding Heisenberg equations.

With this definition of the anomalous scalar field $\tilde{\phi}$ (the bar in ϕ is of little interest) we no longer have the solution of the four-momentum conservation propounded in the last section, but in compensation the j factors never pass through the plain wave functions since the corresponding currents take the form

$$\tilde{J} \sim j|J . \tag{55}$$

The reason that we present this second solution as merely an alternative and not necessarily *the* solution is because this neat separation of complex and quaternion (j,k) factors breaks down for fermion fields, where it is impossible to maintain the canonical field commutation relations. The solution given in the previous section has the advantage of avoiding bars and distinguishes the anomalous field by its non-Hermiticity.

V. CONCLUSIONS

In this paper we have taken a tentative step within the realm of quaternion quantum field theory. Our considerations have been limited to the neutral (not necessarily Hermitian) free scalar field. As a result we have resolved a previous problem (four-momentum nonconservation) of the anomalous photon. Indeed we have presented two alternative definitions of the anomalous scalar field which lead to four-momentum conservation. These results are immediately extendable to the anomalous photon.

The first conclusion that we have to make is that the anomalous fields do not disappear in field theory unlike the negative-energy solutions. So we are obliged to take them seriously. We then observe that there is no evidence to date for any anomalous photon. The simplest argument for this is the normalization of blackbody radiation—the well-known factor 2 that counts the photon degrees of freedom. If two photons existed this factor should be four.[2] There is a way to avoid this conclusion,

[2]One of us (P.R.) thanks Prof. Massimo Testa for this observation.

identical to that which would permit a very small photon mass, i.e., a third longitudinal helicity state, namely the unknown thermalization time for any missing degree of freedom. This is equivalent to saying that the anomalous photon could have a very small coupling to standard matter and hence be both rare and practically invisible to observation.

There is however another possibility which merits consideration. In the Salam-Weinberg theory one begins with *two* massless "photons," one of which picks up its mass from spontaneous symmetry breaking. The question posed is if our anomalous photon can be identified with the Z^0 particle or, at least, can our anomalous vector be given sufficient mass by the same mechanism to explain its nonobservation?

With reference to the Salam-Weinberg model we wish to note that group invariance of a given Lagrangian density naturally takes the form of a tensor product with quaternions

$$A_L \otimes B_R \ , \tag{56}$$

where A_L and B_R are left and right group elements, respectively. The standard group SU(2)⊗U(1), which is of course for complex representations, reads $\mathrm{U}_q(1)\otimes\mathrm{U}_c(1)$ for quaternions, where

$$\mathrm{U}_q(1) \sim e^{-(\alpha i+\beta j+\gamma k)} \quad (\alpha,\beta,\gamma\in R) \ , \tag{57}$$

$$\sim \mathrm{SU}_c(2) \ . \tag{58}$$

This possibility is currently under investigation.

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