New minimal supergravity and spontaneous supersymmetry breaking

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It is shown that spontaneous breaking of supersymmetry without a cosmological constant is impossible in the context of new minimal supergravity.

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I. INTRODUCTION

Some time ago the 20/20 nonminimal supergravity multiplet (e_m^a, ψ_m^α, T_α, S, b_a, $K_{\alpha\dot\alpha}$, λ_α) was studied [1]. The component form, computed to order (spinor)^2 (scalar), of the Lagrangian

$$\mathcal{L}^{20,\mathrm{II}}=-\tfrac{1}{8}\int d^2\theta\,\varepsilon_{20}\bar\Delta e^{-a\bar\Omega}e^{b\Omega}f(\phi,\bar\phi)+\int d^2\theta\,\varepsilon_{20}g(\phi)+\mathrm{H.c.} \tag{1}$$

describing type-II coupling of nonminimal supergravity to the chiral superfield was obtained and showed that spontaneous supersymmetry breaking in Minkowski space, i.e., with null vacuum energy, i.e., with a null cosmological constant, is possible in 20/20 nonminimal supergravity [2,3]. Contrary to what occurs in old minimal supergravity, nonminimal supergravity includes two types of coupling of supergravity to matter. The Lagrangians that described the couplings are referred to as type I and type II. A type-II coupling is described by the Lagrangian given in (1), and a type-I coupling is given by

$$\mathcal{L}^{20,\mathrm{I}}=-\tfrac{1}{2}\int d^2\theta\,\varepsilon_{20}\bar{s}-\tfrac{1}{8}\int d^2\theta\,\varepsilon_{20}\bar\Delta[e^{-a\bar\Omega}e^{b\Omega}\tilde f(\phi,\bar\phi)]+\int d^2\theta\,\varepsilon_{20}g(\phi)+\mathrm{H.c.} \tag{2}$$

Type-I theory is related to type-II theory by the redefinition [3]

$$\begin{aligned} wf &\to -4n+w\tilde f\ ,\\ xf &\to -2+x\tilde f\ , \end{aligned} \tag{3}$$

where

$$w=[(3n+1)a-(n+1)][(3n+1)b-2n]\ ,$$
$$x=(3n+1)(1-a+b)-2n\ .$$

The component form, to order (spinor)^2 (scalar), of the Lagrangian describing the type-II coupling of nonminimal supergravity to matter is given in Ref. [3]. The corresponding component form of the Lagrangian describing type-I coupling is obtained using Eq. (3), and is listed in the Appendix.

The new minimal supergravity multiplet (e_m^a, ψ_m^α, $\bar\psi_{m\dot\alpha}$, A_m, X_m) contains, moreover, the graviton field e_m^a and gravitino field ψ_m^α, two gauge potentials as auxiliary fields. One of them A_m is the gauge potential of a U(1) symmetry of the action (the action for the new minimal multiplet is invariant under the U(1) part of Weyl supertransformations), and the other is an antisymmetric tensor gauge potential.

The Lagrangian describing the coupling of the chiral superfield ϕ, whose components are given by [4]

$$A=\phi|_{\theta=\bar\theta=0}\ ,\quad \chi_\alpha=\tfrac{1}{2}\mathcal{D}_\alpha\phi|_{\theta=\bar\theta=0}\ ,\quad F=-\tfrac{1}{4}\mathcal{D}^2\phi|_{\theta=\bar\theta=0}\ , \tag{4}$$

to 12/12 new minimal supergravity is [2]

$$\mathcal{L}^0=-\frac{1}{2}\int d^2\theta\,\varepsilon_{12}\bar{S}-\frac{1}{8}\int d^2\theta\,\varepsilon_{12}\bar\Delta[e^{(b-1)\bar\Omega}e^{b\Omega}f(\phi,\bar\phi)]+\int d^2\theta\varepsilon_{12}g(\phi)+\mathrm{H.c.} \tag{5}$$

This Lagrangian can be obtained from the Lagrangian describing the type-I coupling of nonminimal supergravity to chiral matter by imposing the constraints $n=0$, and R invariance (the action for new minimal supergravity is U(1) invariant [5]) and the condition $1-a=b$. This means that we can obtain the component form of the Lagrangian describing the coupling of new minimal supergravity to chiral matter from the component-form type-I theory of nonminimal supergravity.

Since the spontaneous breaking of local supersymmetry with null vacuum energy is possible in nonminimal supergravity, one would hope that this situation would not change for the case of new minimal supergravity. However, since new minimal supergravity is R invariant (contrary to nonminimal supergravity), the question naturally arises: Could the R invariance of the Lagrangian affect the breaking of supersymmetry? The reply, in principle, appears to be yes if one considers the fact that R invariance does not permit the appearance of type-$\psi_m\sigma^{mn}\psi_n$ terms. This work shows that, contrary to the cases of old and nonminimal supergravities, the spontaneous breaking of supersymmetry with a vanishing cosmological constant is impossible in new minimal supergravity.

The organization of this paper is as follows. In Sec. II we obtain the component form of the Lagrangian that describes the coupling of new minimal supergravity to chiral matter. Section III is devoted to the study of spontaneous breaking of supersymmetry in new minimal supergravity. A brief discussion of the results obtained and an Appendix close this work.

II. COMPONENT FORM OF THE LAGRANGIAN

The component form of the Lagrangian describing the coupling of new minimal supergravity to chiral matter is obtained imposing the constraints $n=0$ and R invariance on the Eqs. (A1)–(A4) and using the definition

$$K_a-\bar{K}_a=3X_a+b_a+\sigma_a^{\alpha\dot{\alpha}}T_\alpha\bar{T}_{\dot{\alpha}}\ , \quad (6)$$

$$b_a=e_a^m A_{m+ie_a^m}(\psi_m^\alpha T_\alpha-\bar{\psi}_{m\dot{\alpha}}\bar{T}^{\dot{\alpha}})\ , \quad (7)$$

and the condition $1-a=b$. The detailed Lagrangian is lengthy but straightforward to obtain and there is no need to present it. Now we must cast the Lagrangian into its canonical form by means of the following accustomed procedure.

(a) Elimination of the auxiliary fields using their Euler-Lagrange equations:

$$\begin{aligned} X_a&=\frac{b}{2(1-bf)(1-bf+3b^2f)}e_a^m[bf\partial_m\lambda(x)+i(f_A\partial_m A-f_{\bar{A}}\partial_m\bar{A}^-)]\ ,\\ A_m&=\frac{-1}{2(1-bf)}[\partial_m\lambda(x)-i(f_A\partial_m A-f_{\bar{A}}\partial_m\bar{A})]\ ,\\ F&=-\frac{g_{\bar{A}}}{f_{A\bar{A}}}\ . \end{aligned} \quad (8)$$

(b) Diagonalization of the kinetic term using the field redefinition

$$\psi_m^\alpha=\psi_m'^\alpha-\frac{i}{\sqrt{2}}\frac{b}{1-bf}f_{\bar{A}}\bar{X}_{\dot{\alpha}}\sigma_m{}^{\alpha\dot{\alpha}}\ . \quad (9)$$

(c) In order to eliminate the Brans-Dicke-type coupling of the scalar field to the curvature scalar R, we now perform the rescaling

$$\begin{aligned} &e=\frac{\hat{e}}{\Lambda^4}\ ,\quad \psi_m^\alpha=\hat{\psi}_m^\alpha\Lambda^{-1/2}\ ,\quad e_m^a=\hat{e}_m^a\Lambda^{-1}\ ,\\ &\chi^\alpha=\hat{\chi}^\alpha\Lambda^{1/2}\ ,\quad \varepsilon^{klmn}=\hat{\varepsilon}^{klmn}\Lambda^4\ ,\quad \Lambda^2=1-bf\ . \end{aligned} \quad (10)$$

(d) In order to obtain a real gravitino mass after supersymmetry breaking, we perform a phase transformation of the fermion fields which consists of opposite chiral rotations which are allowed provided $g\neq 0$:

$$\psi_m^\alpha=\left[\frac{g}{\bar{g}}\right]^{1/4}\tilde{\psi}_m^\alpha\ ,\quad \chi^\alpha=\left[\frac{g}{\bar{g}}\right]^{-1/4}\tilde{\chi}^\alpha\ . \quad (11)$$

The Lagrangian, considering terms to (spinors)^2 (scalar) is

$$\begin{aligned} \frac{1}{e}\mathcal{L}_B^0=&-\frac{1}{2}R-\frac{1}{\Lambda^2}\left[f_{A\bar{A}}-\frac{b}{2\Lambda^2}\frac{3b-1}{1-bf+3b^2f}f_Af_{\bar{A}}\right]_A\partial_m A\partial^m\bar{A}\\ &-\frac{1}{4\Lambda^4 f}\left[3b^2+\frac{bf(3b-1)}{1-bf+3b^2f}\right](f_A^2\partial_m A\partial^m A+f_{\bar{A}}^2\partial_m\bar{A}\partial^m\bar{A})-\frac{ib}{2\Lambda^4(1-bf+3b^2f)}(f_A\partial_m A-f_{\bar{A}}\partial_m\bar{A})\partial^m\lambda\\ &-\frac{b^2f}{4\Lambda^4(1-bf+3b^2f)}\partial_m\lambda\partial^m\lambda-\frac{1}{\Lambda^4}\frac{g_A\bar{g}_{\bar{A}}}{f_{A\bar{A}}}\ , \end{aligned} \quad (12)$$

$$\begin{aligned} \frac{1}{e}\mathcal{L}_F^0=&\tfrac{1}{4}\varepsilon^{klmn}[\bar{\psi}_k\bar{\sigma}_l\psi_{mn}-\psi_k\sigma_l\bar{\psi}_{mn}]-\frac{i}{\Lambda^2}(f_{A\bar{A}}+\frac{3b^2}{1-bf}f_Af_{\bar{A}})\chi\sigma^m\partial_m\bar{\chi}\\ &-\frac{i}{\sqrt{2}\Lambda^3}\left[\left[\frac{\bar{g}}{g}\right]^{1/2}g_A\chi\sigma^m\bar{\psi}_m-\left[\frac{g}{\bar{g}}\right]^{1/2}\bar{g}_{\bar{A}}\psi_m\sigma^m\bar{\chi}\right]+\frac{1}{\Lambda^3}\left[\frac{\bar{g}}{g}\right]^{1/2}\alpha\chi^2+\frac{1}{\Lambda^3}\left[\frac{g}{\bar{g}}\right]^{1/2}\bar{\alpha}\bar{\chi}^2\ , \end{aligned} \quad (13)$$

where

$$\alpha=\frac{-2b}{\sqrt{2}(1-bf)}g_Af_A-\frac{1}{2}f_{AA\bar{A}}\bar{F}-\frac{1}{2}g_{AA}\ ,$$

and λ is a real field coming from the addition of a Lagrange multiplier term to the bosonic sector of the action. The bosonic sector has been analyzed in Ref. [2], where two cases were considered: (1) $b=g=0$ and (2) $b\neq 0$, g arbitrary. Case (1) does not interest us due to the condition $g=0$.

Case $b\neq 0$, g arbitrary. We consider the particular case $b=\frac{1}{3}$. After the introduction of the Kahler potential $K=-3\ln\Lambda^2$ with $\Lambda^2=1-\frac{1}{3}f$, the associated function $G=-K-\ln g\bar{g}$, and after the redefinition of the scalar field A, $A=A'+\theta\lambda$, with

$$\theta=\frac{iK_{\bar{A}}e^{K/3}[f(G_{A\bar{A}}-\frac{1}{3}K_AK_{\bar{A}})-\frac{1}{6}K_AK_{\bar{A}}]}{6f(G_{A\bar{A}}+\frac{1}{3}K_AK_{\bar{A}})^2+K_A^2K_{\bar{A}}^2/6f},$$

we obtain the Lagrangian

$$\frac{1}{e}\mathcal{L}_B^0=-\frac{1}{2}R+\left[G_{A\bar{A}}+\frac{1}{3}K_AK_{\bar{A}}\right]\partial_mA\partial^m\bar{A}+\frac{K_A^2}{12f}\partial_mA\partial^mA+\frac{K_{\bar{A}}^2}{12f}\partial_m\bar{A}\partial^m\bar{A}+\tau\partial_m\lambda\partial^m\lambda$$
$$+\frac{g_A}{g}\frac{\bar{g}_{\bar{A}}}{\bar{g}}\frac{e^G}{G_{A\bar{A}}+\frac{1}{3}K_AK_{\bar{A}}}, \tag{14}$$

$$\frac{1}{e}\mathcal{L}_F^0=\frac{1}{4}\varepsilon^{klmn}(\bar{\psi}_k\bar{\sigma}_l\psi_{mn}-\psi_k\sigma_l\bar{\psi}_{mn})+iG_{A\bar{A}}\chi\sigma^m\mathcal{D}_m\bar{\chi}-\frac{i}{\sqrt{2}}e^{-G/2}\left[\left[\frac{g_A}{g}\right]\chi\sigma^m\bar{\psi}_m-\left[\frac{\bar{g}_{\bar{A}}}{\bar{g}}\right]\psi_m\sigma^m\bar{\chi}\right]$$
$$+e^{-G/2}\left[\frac{\alpha}{g}\chi^2+\frac{\bar{\alpha}}{\bar{g}}\bar{\chi}^2\right], \tag{15}$$

where

$$\tau=[12(G_{A\bar{A}}+\tfrac{1}{3}K_AK_{\bar{A}})f+K_AK_{\bar{A}}]\frac{K_AK_{\bar{A}}e^{2K/3}[f(G_{A\bar{A}}-\frac{1}{3}K_AK_{\bar{A}})-\frac{1}{6}K_AK_{\bar{A}}]^2}{6f[6f(G_{A\bar{A}}+\frac{1}{3}K_AK_{\bar{A}})^2+K_A^2K_{\bar{A}}^2/6f]^2}$$
$$+\frac{1}{3}e^{2K/3}\left[\frac{f}{12}+\frac{K_AK_{\bar{A}}[f(G_{A\bar{A}}-\frac{1}{3}K_AK_{\bar{A}})-\frac{1}{6}K_AK_{\bar{A}}]}{6f(G_{A\bar{A}}+\frac{1}{3}K_AK_{\bar{A}})^2+K_A^2K_{\bar{A}}^2/6f}\right].$$

Finally we can say that the kinetic term of the bosonic sector can be put approximately into diagonal form by considering, as in Ref. [3], $f_A\cong k$ (where k is the gravitational constant) and $f_{A\bar{A}}=1$. This means that K_A terms are suppressed by a factor $(Mp)^2$ compared to the $K_{A\bar{A}}$ term.

Under this condition the bosonic sector of the Lagrangian may be written as

$$\frac{1}{e}\mathcal{L}_B^0=-\tfrac{1}{2}R+(G_{A\bar{A}}+\tfrac{1}{3}K_AK_{\bar{A}})\partial_mA\partial^m\bar{A}+\tau\partial_m\lambda\partial^m\lambda$$
$$+\frac{g_A}{g}\frac{\bar{g}_{\bar{A}}}{\bar{g}}\frac{e^G}{G_{A\bar{A}}+\frac{1}{3}K_AK_{\bar{A}}}. \tag{16}$$

III. SPONTANEOUS SUPERSYMMETRY BREAKING

We start with the transformation laws of local supersymmetry which, after redefinition of the gravitino (9), the Weyl transformation (10), the phase transformation (11), and considering

$$\xi_\alpha=\hat{\xi}_\alpha\Lambda^{-1/2}\left[\frac{g}{\bar{g}}\right]^{1/4}$$

become

$$\delta_\xi e_m^a=i(\psi_m\sigma^a\bar{\xi}-\xi\sigma^a\bar{\psi}_m),$$
$$\delta_\xi\psi_m^\alpha=-2\mathcal{D}_m\xi^\alpha+\frac{i}{3}e^{-G/2}\frac{K_{\bar{A}}g_A}{g(G_{A\bar{A}}+\frac{1}{3}K_AK_{\bar{A}})}\bar{\xi}_{\dot{\alpha}}\bar{\sigma}_m^{\dot{\alpha}\alpha},$$
$$\delta_\xi A=-\sqrt{2}\xi\chi, \tag{17}$$
$$\delta\chi_\alpha=-\sqrt{2}e^{-G/2}\frac{\bar{g}_{\bar{A}}}{\bar{g}(G_{A\bar{A}}+\frac{1}{3}K_AK_{\bar{A}})}\xi_\alpha+\cdots.$$

To discuss spontaneous symmetry breakdown we consider the transformation law of χ_α:

$$\delta\chi_\alpha=-2F\xi_\alpha\quad\text{with } F=\frac{\bar{g}_{\bar{A}}}{\bar{g}}\frac{e^{-G/2}}{G_{A\bar{A}}+\frac{1}{3}K_AK_{\bar{A}}}.$$

We know that spontaneous breakdown occurs if auxiliary fields receive a nonzero vacuum expectation value and their corresponding fermionic partner is identified as the Goldstone fermion [6].

In view of the fact that physical parameters such as mass and the cosmological constant can be extracted from the scalar potential V and from the fermionic part that constrains only quadratic fermionic terms, we now write such terms explicitly:

$$V=\frac{g_A}{g}\frac{\bar{g}_{\bar{A}}}{\bar{g}}\frac{e^{-G}}{G_{A\bar{A}}+\frac{1}{3}K_AK_{\bar{A}}}\ ,$$

$$\frac{1}{e}\mathcal{L}^0_{(2)F}=\frac{i}{\sqrt{2}}e^{-G/2}\left[\left[\frac{g_A}{g}\right]\chi\sigma^m\bar{\psi}_m-\left[\frac{\bar{g}_{\bar{A}}}{\bar{g}}\right]\psi_m\sigma^m\bar{\chi}\right]+e^{-G/2}\left[\frac{\alpha}{g}\chi^2+\frac{\bar{\alpha}}{\bar{g}}\bar{\chi}^2\right]. \quad (18)$$

Because of the fact that $G_{A\bar{A}}+\frac{1}{3}K_AK_{\bar{A}}$ should be nonzero so that the model is well defined, we see from $\mathcal{L}_{(2)F}$ and from the transformation law for the χ_α field that g_A/g is relevant for discussing the breaking of local supersymmetry.

If g_A/g is nonzero, the χ_α field is mixed with the gravitino field and transforms like a Goldstone spinor field. Since we are interested in symmetry breaking in Minkowski space, we require the cancellation of the cosmological constant. For this reason we set the potential to zero in the minimum; i.e., we require that $V=0$ in the minimum. Thus,

$$V_0=\frac{g^0_A}{g^0}\frac{\bar{g}^0_{\bar{A}}}{\bar{g}^0}\frac{e^{-G^0}}{G^0_{\bar{A}A}+\frac{1}{3}K^0_AK^0_{\bar{A}}}=0\ .$$

This implies that $g^0_A/g_0=0$ since $e^{-G^0}/(G^0_{\bar{A}A}+\frac{1}{3}K^0_AK^0_{\bar{A}})\neq 0$. But if this condition is satisfied, then the term that mixes χ_α and ψ_m disappears from the Lagrangian. Also this gives $\delta\chi_\alpha=0+\cdots$; i.e., χ_α does not transform like a Goldstone field and consequently the breaking of supersymmetry cannot take place when $V_0=0$, i.e., with a null cosmological constant. This means that the breakdown of supersymmetry in Minkowski space is not possible in new minimal supergravity.

IV. CONCLUDING REMARKS

We have obtained the component form of the Lagrangian for new minimal supergravity coupled to a chiral multiplet, and studied classically the spontaneous breaking of supersymmetry. We have found that the mentioned spontaneous breaking of supersymmetry without a cosmological constant is impossible in the context of new minimal supergravity. This result might help in the search for a reply to the question: Which is the appropriate supergravity multiplet compatible with the low-energy field-theory limit of superstring theory [7–11]?

In order to have a theory free of gravitational and Yang-Mills anomalies, Green and Schwarz modified the system of 10-dimensional $N=1$ supersymmetric Yang-Mills theory coupled to supergravity (viewed as the low-energy effective field theory of superstring theory) by choosing as a gauge group either SO(32) or $E_8\times E_8$ and by adding suitable local interactions. These interactions are higher-derivative terms parametrized in terms of Chern-Simons forms both for the Yang-Mills gauge potentials and for the Lorentz connection; i.e., they are higher-curvature terms. When these higher-curvature terms required by the Green-Schwarz modification are studied in the context of an $N=1$ supergravity theory, one encounters that the auxiliary field equations are nonlinear and therefore there is in general more than one solution.

Then we have the problem of giving a physical interpretation to the new solutions. If the auxiliary fields turn out to be propagating degrees of freedom, these new solutions should be regarded as new physical vacua corresponding to nontrivial values for the new fields, which, being the auxiliary components of the initial multiplet, imply broken supersymmetry. Thus higher-derivative supergravity potentially contains a new nonperturbative mechanism for breaking supersymmetry.

Thus we have that in new minimal supergravity (with a vanishing cosmological constant) supersymmetry is classically not broken, but when higher-order curvature terms are considered it could be broken by some quantum effects such as the condensation of fermions.

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APPENDIX

In Ref. [3] the component form is given to order (spinor)^2 (scalar) of the Lagrangian describing type-II coupling of 20/20 supergravity to chiral matter. Here we list the corresponding Lagrangian of the type-I theory:

$$\begin{aligned}\frac{1}{e}\mathcal{L}^{20,\mathrm{I}}_{K/B}=&\frac{1}{4}(-2+xf)R-\frac{1}{2}R\left[\frac{T^2}{S}\left[-4n+wf-2n(-2+xf)+(x+2n)f_A\frac{F}{S}+2(3n+1)\frac{g}{S}\right]+\mathrm{H.c.}\right]\\&-f_{A\bar{A}}\partial_mA\partial^m\bar{A}-\frac{1}{6}(-2+xf)b_ab^a-\frac{i}{3}b^ae_a^m(f_A\partial_mA-f_{\bar{A}}\partial_m\bar{A})+(x+2n)\left[f_A\frac{K_a^2F}{S}+f_{\bar{A}}\frac{\bar{K}_a^2\bar{F}}{\bar{S}}\right]\\&-\frac{1}{6}(3n+1)(3n-1)(-2+xf)(K_a^2+\bar{K}_a^2)+2(3n+1)\left[\bar{g}\frac{K_a^2}{S}+g\frac{\bar{K}_a^2}{\bar{S}}\right]\\&+\frac{1}{3}[(3n-1)^2(2-xf)+6(4n-wf)]K_a\bar{K}^a+\frac{i}{2}\left[x-\frac{2}{3}\right]K^ae_a^m[(3n-1)f_A\partial_mA+(3n+1)f_{\bar{A}}\partial_m\bar{A}]\\&+\frac{i}{2}\left[x-\frac{2}{3}\right]\bar{K}^ae_a^m[(3n+1)f_A\partial_mA+(3n-1)f_{\bar{A}}\partial_m\bar{A}]+4[2n(-2+xf)-(4n+wf)]\frac{K_a^2}{S}\frac{\bar{K}_a^2}{\bar{S}}\ ,\end{aligned} \quad (A1)$$

$$\frac{1}{e}\mathcal{L}^{20,\mathrm{I}}_{SP/B}=-\tfrac{1}{4}(-4n+wf)S\bar{S}+f_{A\bar{A}}F\bar{F}+Fg_A+\bar{F}\bar{g}_{\bar{A}}-\tfrac{1}{4}(x-2n)(f_A\bar{S}+f_{\bar{A}}S\bar{F})-\tfrac{1}{2}(3n-1)(gS+\bar{g}\bar{S})\ , \qquad \text{(A2)}$$

$$\begin{aligned}\frac{1}{e}\mathcal{L}^{20,\mathrm{I}}_{K/F}=&-\tfrac{1}{8}(-2+xf)\varepsilon^{klmn}(\bar{\psi}_k\bar{\sigma}_l\psi_{mn}-\psi_k\sigma_l\bar{\psi}_{mn})-if_{A\bar{A}}\chi\sigma^m\mathcal{D}_m\bar{\chi}-2i(-4n+wf)T\sigma^m\mathcal{D}_m\bar{T}\\&+\frac{x+2n}{\sqrt{2}(3n+1)}(f_A\chi\sigma^{mn}\psi_{mn}+f_{\bar{A}}\bar{\chi}\bar{\sigma}^{mn}\bar{\psi}_{mn})+\frac{2(x+2n)}{3n+1}\left[f_A\frac{F}{S}T\sigma^{mn}\psi_{mn}+f_{\bar{A}}\frac{\bar{F}}{\bar{S}}\bar{T}\bar{\sigma}^{mn}\bar{\psi}_{mn}\right]\\&-\frac{i(3n-1)(x+2n)}{\sqrt{2}(3n+1)}(f_A\chi\sigma^m\mathcal{D}_m\bar{T}+f_{\bar{A}}T\sigma^m\mathcal{D}_m\bar{\chi})-\frac{2i(3n-1)(x+2n)}{3n+1}\left[f_A\frac{F}{S}+f_{\bar{A}}\frac{\bar{F}}{\bar{S}}\right]T\sigma^m\mathcal{D}_m\bar{T}\\&+4\left[\frac{\bar{g}}{S}T\sigma^{mn}\psi_{mn}+\frac{g}{\bar{S}}\bar{T}\bar{\sigma}^{mn}\bar{\psi}_{mn}\right]-4i(3n-1)\left[\frac{\bar{g}}{S}+\frac{g}{\bar{S}}\right]T\sigma^m\mathcal{D}_m\bar{T}\ ,\end{aligned} \qquad \text{(A3)}$$

$$\begin{aligned}\frac{1}{e}\mathcal{L}^{20,\mathrm{I}}_{(2)F}=&\frac{-3in(x-2/3)}{2\sqrt{2}(3n+1)}(f_A\bar{S}\chi\sigma^m\bar{\psi}_m-f_{\bar{A}}S\psi_m\sigma^m\bar{\chi})-\tfrac{1}{2}(f_{AA\bar{A}}\bar{F}\chi^2+f_{A\bar{A}\bar{A}}F\bar{\chi}^2)-\tfrac{1}{8}(x-2n)(f_{AA}\bar{S}\chi^2+f_{\bar{A}\bar{A}}S\bar{\chi}^2)\\&-\frac{3n(x-2/3)}{2\sqrt{2}(3n+1)}(f_A\chi\lambda+f_{\bar{A}}\bar{\chi}\bar{\lambda})-\tfrac{1}{2}(-4n+wf)(T\lambda+\bar{T}\bar{\lambda})-\frac{i}{2}(-4n+wf)(S\psi_m\sigma^m\bar{T}-\bar{S}T\sigma^m\bar{\psi}_m)\\&+\frac{i(3n-1)(x+2n)}{2(3n+1)}\left[f_{\bar{A}}\frac{S\bar{F}}{\bar{S}}\psi_m\sigma^m\bar{T}-f_A\frac{\bar{S}F}{S}T\sigma^m\bar{\psi}_m\right]+\frac{1}{2\sqrt{2}}(2x-5n-1)f_{A\bar{A}}(\bar{F}T\chi+F\bar{T}\bar{\chi})\\&-\frac{(3n-1)(x+2n)}{2(3n+1)}\left[f_A\frac{F}{S}T\lambda+f_{\bar{A}}\frac{\bar{F}}{\bar{S}}\bar{T}\bar{\lambda}\right]-\frac{1}{2}(3n+1)(x-2n)(f_{\bar{A}}FT^2+f_AF\bar{T}^2)\\&+(x+2n)f_{A\bar{A}}\left[\frac{F\bar{F}}{S}T^2+\frac{F\bar{F}}{\bar{S}}\bar{T}^2\right]-\frac{1}{2}(5n-1)(-4n+wf)(\bar{S}T^2+S\bar{T}^2)\\&+\frac{1}{\sqrt{2}}\left[-\frac{1}{4}(13n-3)x+(2nx+w)-\frac{x+2n}{8(3n+1)}[6(n-1)^2-(3n+1)(3n-1)]\right](f_A\bar{S}T\chi+f_{\bar{A}}S\bar{T}\bar{\chi})\\&+\left[(2nx-w)-\frac{x+2n}{2(3n+1)}[3(n-1)^2+(3n+1)(3n-1)]\right]\left[f_A\frac{S\bar{F}}{S}T^2+f_{\bar{A}}\frac{S\bar{F}}{\bar{S}}\bar{T}^2\right]\\&-(\bar{g}\psi_m\sigma^{mn}\psi_n+g\bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n)-\frac{1}{2}(g_{AA}\chi^2+\bar{g}_{\bar{A}\bar{A}}\bar{\chi}^2)-i(3n-1)\left[g\frac{S}{\bar{S}}\psi_m\sigma^m\bar{T}-\bar{g}\frac{\bar{S}}{S}T\sigma^m\bar{\psi}_m\right]\\&+i(3n-1)(\bar{g}\psi_m\sigma^m\bar{T}-gT\sigma^m\bar{\psi}_m)+\frac{i}{2}(\bar{g}_{\bar{A}}\psi_m\sigma^m\bar{\chi}-g_A\chi\sigma^m\bar{\psi}_m)-\frac{1}{2\sqrt{2}}(13n-3)(g_AT\chi+\bar{g}_{\bar{A}}\bar{T}\bar{\chi})\\&-(3n-1)\left[\frac{\bar{g}}{S}T\lambda+\frac{g}{\bar{S}}\bar{T}\bar{\lambda}\right]+2(3n+1)\left[\bar{g}_{\bar{A}}\frac{\bar{F}}{S}T^2+g_A\frac{F}{S}\bar{T}^2\right]-4n(3n-1)(gT^2+\bar{g}\bar{T}^2)\\&-(3n-1)(5n-1)\left[\bar{g}\frac{\bar{S}}{S}T^2+g\frac{S}{\bar{S}}\bar{T}^2\right]\ ,\end{aligned} \qquad \text{(A4)}$$

where

$$w=[(3n+1)a-(n+1)][(3n+1)b-2n]\ ,\quad x=(3n+1)(1-a+b)-2n\ .$$

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