PHYSICAL REVIEW D VOLUME 45, NUMBER 2 15 JANUARY 1992

Gravitational radiative corrections in $N=1$ supergravity

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(Received 6 June 1991)

In an ungauged $N=1$ supergravity theory defined on an arbitrary Kahlerian manifold we compute the divergent one-loop corrections to the bosonic part of the effective action. Although the theory is not renormalizable such a calculation may be of relevance in view of the fact that $N=1$ supergravities emerge as effective nonrenormalizable theories in the low-energy limit of some superstring models. In our calculations we have committed ourselves neither to a particular four-dimensional geometry nor to a particular Kahlerian manifold. We pay special attention to the one-loop scalar potential of the theory. We show that, by a proper redefinition of the metric, geometric objects such as scalar curvature can be made not to interact with the scalars and the definition of the potential of the theory becomes in this way unambiguous.

PACS number(s): 04.65.+e

INTRODUCTION

Locally supersymmetric theories have attracted the interest of particle physicists the last ten years [1]. These theories are capable of accommodating the gravitational interactions in a natural way and originally were thought to comprise the proper framework for the unification of all existing forces in nature. Nowadays supergravity theories still continue to be interesting local field theories since they are believed to be effective nonrenormalizable theories corresponding to the long-distance (low-energy) limit of a class of string theories [2,3]. This is the analogue of what happens in strong interactions, for instance, where $SU(2)\times SU(2)$ chiral-invariant σ models effectively describe pion dynamics at small energies compared to the inverse confinement radius of the QCD forces. Also in weak interactions nuclear β decay is correctly described by the nonrenormalizable Fermi theory which is valid for small momentum transfers compared to the mass of the intermediate vector boson. Although serious candidates as "theories of everything," the dynamics of string theories is not well understood as yet, unlike electroweak and strong interactions (QCD) where we deal with decent mathematically (renormalizable) point field theories in which the low-energy limit makes sense even if radiative corrections are taken into account.

A particular class of $N=1$ supergravity theories emerges as the low-energy limit of some superstring theories and describes effective particle interactions at Planckian energies [4]. These are often characterized by a high degree of vacuum degeneracy concealing issues that are of vital importance for phenomenology and thus for experimental tests of the theory. An important question raised is to what degree this degeneracy is lifted by loop corrections to the effective Lagrangian. The consideration of such radiative corrections will allow us, therefore, to understand better mechanisms which are basic for the physical understanding of our theory such as, for instance, supersymmetry breaking, masses of supersymmetric particles, etc. The lack of sufficient knowledge of how to treat properly the loop effects of the underlying string theory forces us to consider the effective supergravity theory suitably cut off a scale Λ if gravitational radiative effects are to be taken into account. Λ designates the characteristic scale which sets the onset of new physics and in the case of the string theories we are considering it is slightly larger than the Planck scale m_P.

Many authors have considered loop corrections in various field-theory models coupled to gravity [5–15]. In this work we consider an $N=1$ supergravity model with nonminimal kinetic terms for the matter fields and focus our attention mainly on the calculation of the effective potential up to next-to-leading-order terms in the cutoff scale Λ. In the context of an $N=1$ supergravity model the effective action for the scalar field up to $\ln\Lambda^2$ terms has been calculated by other authors for theories defined on particular Kahlerian manifolds. In these calculations the radiative effects of the graviton and gravitino fields were either ignored or were considered in cases where the four-dimensional background geometry had some particular form [16–21]. Our aim in this paper is to consider the complete one-loop corrections for the scalar part of an $N=1$ supergravity theory defined on an arbitrary Kahler manifold without committing ourselves to a particular four-dimensional geometry. For terms involving derivatives of the scalars we limit ourselves to the calculation of those terms which are quadratic in the cutoff scale Λ. The calculation of $\ln\Lambda^2$ terms in this case is very tedious, owing to the complex form of the fermionic propagators in a curved background, and it lies beyond the scope of this paper.

Some of the results presented in this work have been also derived by other authors, our main contribution being the space-time curvature-dependent terms of the gravitino loop contributions as well as the logarithmically divergent contributions to the same terms of the matter fermions. The graviton loop contributions for a scalar field theory coupled to gravity were first calculated in Ref. [14]. The divergent contributions to the effective scalar Lagrangian in a flat-space-time background were given by Burton, Gaillard, and Jain [18] based upon results obtained earlier by Jain [18]. The quadratically

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divergent part of the spin-$\frac{1}{2}$ fermion contribution, which depends on the space-time curvature, was calculated by Binetruy and Gaillard [18]. In this reference, as well as in Ref. [22], one can find the complete divergent scalar contribution to the curvature- and field-dependent terms.

This paper is organized as follows. In Sec. I we present the model under consideration. In Sec. II we discuss the matter fermion contribution to the effective action and in Sec. III the corresponding contributions of the gravitino field and supersymmetry ghosts are considered. Section IV deals with the matter fermion-gravitino mixing and the contribution of this coupling to the one-loop effective Lagrangian is calculated. In Sec. V we discuss the contributions of both scalars and gravitons as well as those of the general coordinate ghosts. The effective bosonic Lagrangian is discussed in Sec. VI and in Sec. VII we consider a particular $N=1$ supergravity model of the no-scale type relevant to a class of superstring theories. Finally we end up with the conclusions.

I. THE MODEL

The reference Lagrangian is the ungauged $N=1$ supergravity theory involving D chiral multiplets (z^i, x_L^i, h^i) $i=1,2,\ldots,D$ [23]. After the elimination of the auxiliary fields the Lagrangian is

$$\begin{aligned}\mathcal{L}^{N=1}=&\frac{e}{2}\mathcal{R}+eg^{\mu\nu}(-\mathcal{G}_{i\bar{j}})\partial_\mu z^i\partial_\nu z^{\bar{j}}-ee^{-\mathcal{G}}(\mathcal{G}_i\mathcal{G}_{\bar{j}}\mathcal{G}^{i\bar{j}}+3)\\&+[ie(-\mathcal{G}_{\bar{i}j})\bar{x}^i_L \not{D} x^j_L+ie(\tfrac{1}{2}\mathcal{G}_{i\bar{j}}\mathcal{G}_k+\mathcal{G}_{ik\bar{j}})\bar{x}^i_R\not{\partial}z^k x^j_R+ee^{-\mathcal{G}/2}(\mathcal{G}_{ij}-\mathcal{G}_i\mathcal{G}_j-\mathcal{G}_l\mathcal{G}^{l\bar{k}}\mathcal{G}_{\bar{k}ij})\bar{x}^i_R x^j_L+\text{H.c.}]\\&+\left[\frac{\epsilon^{\mu\nu\rho\sigma}}{2}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma+iee^{-\mathcal{G}/2}\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu-\tfrac{1}{8}\epsilon^{\mu\nu\rho\sigma}(\mathcal{G}_i\partial_\sigma z^i-\text{H.c.})\bar{\psi}_\mu\gamma_\nu\psi_\rho\right]\\&+(e\mathcal{G}_{i\bar{j}}\bar{\psi}_{\mu L}\not{\partial}z^i\gamma^\mu x^j_R+iee^{-\mathcal{G}/2}\bar{\psi}_{\mu L}\gamma^\mu\mathcal{G}_i x^i_L+\text{H.c.})\,,\end{aligned}\tag{1}$$

where natural units are used $m_P(=1/\sqrt{8\pi G_N})=1$ and we have omitted four-fermion terms which will not contribute to one-loop calculations once the background fermion fields are taken to be zero. Actually we are mainly interested in the scalar part of the one-loop effective action and these interactions are therefore of no relevance. The fermionic part can be found by supersymmetrizing the bosonic part of the Lagrangian if so wished. Throughout we use the Minkowski metric with the signature $(+,-,-,-)$ and this is the reason the Lagrangian above differs from that usually given in the literature by unimportant factors. $\mathcal{G}_{i\bar{j}}$ above stands for $\partial^2\mathcal{G}/\partial z^i\partial z^j$ where $\mathcal{G}$ is the Kahler potential. For a manifestly covariant expression for the effective action we have to expand in normal coordinates [24] around the background fields z_i^c, $g^c_{\mu\nu}$ which satisfy the classical equations of motion in the presence of source terms J^i, $J^{\mu\nu}$. At the one-loop level only the bosonic terms need be expanded since the bilinear in the fermion terms contribute through fermion loops to the effective action. The details of such an expansion can be found in Ref. [22]. Actually the bosonic part of the $N=1$ supergravity theory resembles that of a σ model coupled to gravity for which the one-loop gravitational connections have been already studied in detail [22]. In the following section when considering the graviton and scalar field contributions to the effective action these results will be used.

The one-loop effective action ΔS_{eff} is given by

$$e^{i\Delta S_{\text{eff}}}=\int\mathcal{D}\phi_i\exp\left[i\int\phi\Delta^i_{\text{op}}\phi\right]\,,\tag{2}$$

where ϕ_i stand collectively for all fields involved and Δ^i_{op} for differential operators which depend on the background values $g_{\mu\nu}{}^c$ and z^i of the graviton and scalars, respectively. Δ^i_{op} is of first order for the fermions being, in general, of the form $\Gamma^\mu\mathcal{D}_\mu+X$ with $\mathcal{D}_\mu$ being a covariant derivative while for scalars it is quadratic having a form $A^{\mu\nu}\mathcal{D}_\mu\mathcal{D}_\nu+Y$. There are standard techniques in the literature for how one computes the determinants associated with these operators which are necessary for the calculation of the effective action, as is obvious from Eq. (2). Our first task towards this goal is to isolate the fermion (spin $\frac{1}{2}$, spin $\frac{3}{2}$) and boson (spin 0, spin 2) bilinear terms in order to know the exact forms of Δ^i_{op}. As we shall see the mixing of spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ fermions will complicate the situation a little, in the specific gauge we intend to employ, and this coupling will be considered as an interaction whose contribution to the effective action is computed diagramatically. In such an approach one needs to know the expression of the fermionic propagators in an arbitrary curved background. The details of how one obtains the adiabatic expansion of these propagators around a specific space-time point is presented in the Appendix.

Since the bosonic contributions have been already calculated for a σ model coupled to gravity we begin our discussion by first considering the fermionic contributions starting from the spin-$\frac{1}{2}$ fields involved.

II. MATTER FERMION CONTRIBUTION

Ignoring its mixing with the gravitino field the bilinear in the spin-$\frac{1}{2}$ fermion part of the Lagrangian is

$$e^{-1}\mathcal{L}^{(1/2)}=i(-\mathcal{G}_{i\bar{j}})\bar{x}^j_L\gamma^\mu\mathcal{D}_\mu x^i_L+\bar{x}^i_R\mathcal{M}_{ij}x^j_L+\text{H.c.}\tag{3}$$

where

$$\mathcal{D}_\mu x_L^i \equiv D_\mu x_L^i + (\partial_\mu z^k)\Gamma^i_{jk} x_L^j + A_\mu x_L^i$$

is the full covariant derivative. $\Gamma^i_{jk} = \mathcal{G}^{i\bar{l}}\mathcal{G}_{\bar{l}jk}$ is the connection of the Kahler manifold and $\mathcal{G}^{i\bar{l}}$ stands for the inverse of the Kahler metric $\mathcal{G}_{i\bar{l}}$. The gauge field $A_\mu \equiv (i/2)\mathrm{Im}(\mathcal{G}_k \partial_\mu z^k)$ is associated with the Kahler transformations

$$x^i \to e^{(F-F^*)\gamma_5/4} x^i, \quad \psi_\mu \to e^{-(F-F^*)\gamma_5/4}\psi_\mu ,$$

$$K \to K + F + F^*, \quad W \to e^{-F} W$$

under which the Kahler potential $\mathcal{G} \equiv K + \ln|W|^2$ remains invariant.

Notice that x^i and ψ_μ carry opposite chiral charges under these chiral rotations.

The fermion mass matrix $\mathcal{M}_{ij}$ appearing in Eq. (3) is given by

$$\mathcal{M}_{ij} = e^{-\mathcal{G}/2}(\mathcal{G}_{ij} - \mathcal{G}_i\mathcal{G}_j - \mathcal{G}_l \mathcal{G}^{l\bar{k}}\mathcal{G}_{ij\bar{k}})$$

$[m_P \equiv (8\pi G_N)^{-1/2} = 1]$. Introducing flat indices $(-\mathcal{G}_{i\bar{j}}) = E_i{}^a E_{\bar{j}}{}^{\bar{b}} \eta_{a\bar{b}}$ and defining $x_L^i = E_a^i x_L^a$ the Lagrangian (3) is brought into the form

$$e^{-1}\mathcal{L}^{(1/2)} = i\eta_{a\bar{b}} \bar{x}_L^{\,b} \gamma^\mu \mathcal{D}_\mu x_L^a + \bar{x}_R^{\,\bar{a}} \mathcal{M}_{ab} x_L^b + \text{H.c.} , \qquad (3')$$

where

$$\mathcal{D}_\mu x_L^a \equiv D_\mu x_L^a + \hat{\omega}_\mu{}^q{}_b x_L^b + + A_\mu x_L^a .$$

$E^a{}_i$, $E^{\bar{a}}{}_{iibar}$ are used to convert curved $(i, iibar)$ to flat indices $(a, \bar{a})$ and vice versa in the usual fashion. The spin connection $\hat{\omega}_\mu{}^a{}_b$ is

$$\hat{\omega}_\mu{}^a{}_b = E^a{}_j(\partial_\mu E^j{}_b + \partial_\mu z^k \Gamma^j_{k\lambda} E_b^\lambda) .$$

Equation (3′) can be also written in the form

$$e^{-1}\mathcal{L}^{(1/2)} = \frac{i}{2}\epsilon_{ab}\bar{y}^a \gamma^\mu \mathcal{D}_\mu y^b + \frac{1}{2}\bar{y}^a M_{ab} y^b \qquad (3'')$$

with the Majorana fermion y^a defined as

$$y^a \equiv \sqrt{2}\begin{bmatrix} x_L^a \\ x_R^{\bar{a}} \end{bmatrix}$$

and ϵ_{ab} such that $\epsilon_{ab} \equiv n_{a\bar{b}} (\epsilon_{11} = n_{1\bar{1}}, \epsilon_{22} = n_{2\bar{2}}$ etc).

The mass matrix M_{ab} is related to $\mathcal{M}_{ab}$ by

$$M_{ab} \equiv \frac{1-\gamma_5}{2}\mathcal{M}_{ab} + \frac{1+\gamma_5}{2}\mathcal{M}_{\bar{a}\bar{b}}$$

while the covariant derivative $\mathcal{D}_\mu y^a$ is now $D_\mu y^a + \Omega_\mu{}^a{}_b y^b$. The connection $\Omega_\mu{}^a{}_b$ incorporates both the connection associated with reparametrization invariance $z^i \to f^i(z)$, $x_L^i \to (df^i/dz^j)x_L^j$ as well as the connection A_μ related to the chiral transformations we have talked about before. For the calculation of the effective action we shall need, among other things, the expression for $\ln\det(i\epsilon_{ab}\gamma^\mu\mathcal{D}_\mu + M_{ab})$ which, following standard techniques, is found by squaring first-order operators [14–16,25]. In the case at hand we find

$$(i\epsilon^{ab}\mathcal{D} + \tilde{M}^{ab})(i\epsilon_{bc}\mathcal{D} + M_{bc}) = -\delta^a_b\left[\mathcal{D}^2 - \frac{\mathcal{R}}{4}\right] + i\sigma\cdot F^a_b(\Omega) + i\epsilon^{ac}\gamma^\nu\mathcal{D}_\nu M_{cb} - \epsilon^{ar}\epsilon^{cs}\hat{M}_{rs}M_{cb}$$

where $\tilde{M}^{ab} \equiv -\epsilon^{ad}\hat{M}_{dc}\epsilon^{cb}$. $\hat{M}_{ab}$ in Eq. (4) is of the same form as M_{ab} with left-, $L = (1-\gamma_5)/2$, and right-, $R = (1+\gamma_5)/2$, handed operators interchanged $(L \rightleftarrows R)$. The field strength $F_{\mu\nu}{}^a{}_b(\Omega)$ associated with the connection Ω is

$$F_{\mu\nu}{}^a{}_b(\Omega) = D_\mu \Omega_\nu{}^a{}_b + \Omega_\mu{}^a{}_c \Omega_\nu{}^c{}_b - (\mu \rightleftarrows \nu) . \qquad (5)$$

The divergent parts of the first-order operators $(i\mathcal{D} + \tilde{M})$ and $(i\mathcal{D} + M)$ coincide so that up to $\ln\Lambda^2$ we have

$$\ln\det(i\epsilon_{ab}\gamma^\mu\mathcal{D}_\mu + M_{ab}) = \tfrac{1}{2}\ln\det(-\delta^a{}_b\mathcal{D}^2 - X^a{}_b)$$

where $-X^a{}_b$ is the right-hand side of Eq. (4) with $-\delta^a{}_b\mathcal{D}^2$ omitted. Therefore the coefficients $\alpha_0(\frac{1}{2})$, $\alpha_1(\frac{1}{2})$, $\alpha_2(\frac{1}{2})$ associated with the quartic, quadratic, and logarithmic divergences of the operator $(i\epsilon_{ab}\mathcal{D} + M_{ab})$ are found from the corresponding coefficients of the $\delta^a{}_b\mathcal{D}^2 + X^a{}_b$ operator.

In fact writing as

$$\ln\det(i\epsilon_{ab}\mathcal{D} + M_{ab}) = -i\int d^4x\sqrt{-g}\left[\frac{\Lambda^4}{2}\alpha_0(\tfrac{1}{2}) + \Lambda^2\alpha_1(\tfrac{1}{2}) + \ln\Lambda^2\alpha_2(\tfrac{1}{2})\right] \qquad (6)$$

where Λ^2 is the momentum cutoff, the coefficients $\alpha_i(\frac{1}{2}), i = 0,1,2$ are found to be [5–8]

$$(4\pi)^2\alpha_0(\tfrac{1}{2}) = 2D ,$$

$$(4\pi)^2\alpha_1(\tfrac{1}{2}) = -\frac{D}{6}\mathcal{R} - 2m_P^{-4}\mathcal{M}_{ij}\mathcal{M}^{ji} , \qquad (7)$$

$$(4\pi)^2\alpha_2(\tfrac{1}{2}) = \tfrac{1}{360}[D(\tfrac{7}{2}\mathcal{R}^2_{\mu\nu\rho\sigma} - 4\mathcal{R}^2_{\mu\nu} + \tfrac{5}{2}\mathcal{R}^2) + 60m_P^{-4}\mathcal{R}(\mathcal{M}_{ij}\mathcal{M}^{ji}) + 360m_P^{-8}(\mathcal{M}_{ij}\mathcal{M}^{jk}\mathcal{M}_{k\lambda}\mathcal{M}^{\lambda i}) + \cdots] .$$

The total derivatives in the α_2's encountered throughout this paper will not be explicitly shown as they do not contribute to the effective action. The ellipsis in α_2 denotes terms depending on ∂z which will not concern us. Note that in Eq. (7) we have reestablished dimensions. $\mathcal{M}_{ij}$ has a dimension of mass but $\mathcal{M}^{ij}=(-\mathcal{G}^{i\bar{\alpha}})(-\mathcal{G}^{j\bar{\beta}})\mathcal{M}_{\bar{\alpha}\bar{\beta}}$ has five. $\mathcal{G}$ is dimensionless so that $\mathcal{G}_{i\bar{j}}$ has dimensions of inverse mass squared. D in Eq. (5) is the number of chiral multiplets involved.

We now proceed to determine the corresponding coefficients for the spin-$\frac{3}{2}$ and supersymmetry ghosts.

III. GRAVITINO AND SUSY GHOST CONTRIBUTIONS

The bilinear in the gravitino field terms are

$$\mathcal{L}^{(3/2)}=\tfrac{1}{2}\bar{\psi}_\mu\gamma_5\gamma_\nu\mathcal{D}_\rho\psi_\sigma\epsilon^{\mu\nu\rho\sigma}+iem_Pe^{-\mathcal{G}/2}\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu\ , \tag{8}$$

where the derivative $\mathcal{D}_\rho$ involves the connection A^μ associated with the Kahler transformation discussed in the previous section. The supersymmetry gauge fixing $\gamma^\mu\psi_\mu=\xi$ which we intend to use can be implemented by averaging over gauges using an operator $\mathcal{M}$. This has an effect on the appearance of the gauge-fixing term $\mathcal{L}_{\text{gf}}=(i/2)\bar{\psi}^\mu\gamma_\mu\mathcal{M}\gamma_\nu\psi^\nu$ along with the simultaneous appearance of the determinant $(\det\mathcal{M})^{-1/2}$ in addition to the usual Faddeev-Popov determinant $(\det\Delta_{\text{FP}})^{-1}$ [26]. For convenience we may use $\mathcal{M}\propto\Delta_{\text{FP}}$ so that we deal with only one determinant $(\det\mathcal{M})^{-3/2}$. Actually this gauge-fixing procedure has been employed in Ref. [25] in a gauged O(2) supergravity model. With $\gamma^\mu\psi_\mu=\xi$, Δ_{FP} is given by $i\mathcal{D}\!\!\!/+2m_{3/2}$ $(m_{3/2}\equiv m_Pe^{-\mathcal{G}/2})$ where $\mathcal{D}_\rho$ again involves the connection A_μ. With $\mathcal{M}=\frac{1}{2}\Delta_{\text{FP}}$ one has

$$\mathcal{L}^{(3/2)}+\mathcal{L}_{\text{gf}}=\frac{e}{2}\bar{\psi}_\mu\Delta^{\mu\nu}(\tfrac{3}{2})\psi_\nu \tag{8'}$$

where

$$\Delta^{\mu\nu}(\tfrac{3}{2})=\frac{i}{2}\gamma^\nu\gamma^\rho\gamma^\mu\mathcal{D}_\rho+g^{\mu\nu}m_{3/2}\ .$$

In flat four-dimensional space-time with $m_{3/2}=$const and omitting the A_μ connection, the inverse of $\Delta^{\mu\nu}(\frac{3}{2})$, in momentum space, is

$$G_{\nu\lambda}(\tfrac{3}{2})=\frac{1}{2}\frac{\gamma_\lambda p\!\!\!/\gamma_\nu-2m_{3/2}\eta_{\nu\lambda}}{p^2-m_{3/2}^2}\ . \tag{9}$$

The contribution of all fermions, except those arising from the mixing of gravitino and matter fermions, to the effective action is read from

$$e^{i\Delta S_{\text{eff}}}=\det{}^{1/2}\Delta(\tfrac{1}{2})\det{}^{1/2}\Delta(\tfrac{3}{2})/\det{}^{3/2}\Delta_{\text{FP}}$$

which yields

$$\Delta S_{\text{eff}}=-\frac{i}{2}[\ln\det\Delta(\tfrac{1}{2})+\ln\det\Delta(\tfrac{3}{2})-3\ln\det\Delta_{\text{FP}}(\text{ghost})]$$

or

$$\Delta S_{\text{eff}}=\tfrac{1}{2}\int d^4x\sqrt{-g}\left[\frac{\Lambda^4}{2}\alpha_0(f)+\Lambda^2\alpha_1(f)+\ln\Lambda^2\alpha_2(f)\right] \tag{10}$$

where we have kept terms up to $\ln\Lambda^2$ and

$$\alpha_i(f)=-[\alpha_i(\tfrac{1}{2})+\alpha_i(\tfrac{3}{2})-3\alpha_i(\text{ghost})],\quad i=0,1,2\ . \tag{10'}$$

The (-1) in front of Eq. (10') is there because we deal with fermions and the $\alpha_i(\frac{1}{2})$'s were given in the previous section. The corresponding coefficients $\alpha_i(\frac{3}{2})$, $\alpha_i(\text{ghosts})$ for the gravitino and SUSY ghosts are found by "squaring" first-order operators as was done for the case of spin-$\frac{1}{2}$ fermions. For the $\Delta^{\mu\nu}(\frac{3}{2})$ operator one seeks $\tilde{\Delta}_{\nu\lambda}(\frac{3}{2})$ so that in the product $\Delta(\frac{3}{2})\tilde{\Delta}(\frac{3}{2})$ linear in the covariant derivative $\mathcal{D}_\mu$ terms do not appear. Choosing $\tilde{\Delta}_{\lambda\nu}(\frac{3}{2})$ as

$$\tilde{\Delta}_{\lambda\nu}(\tfrac{3}{2})=\tfrac{1}{2}(-i\gamma_\nu\gamma^\rho\gamma_\lambda\mathcal{D}_\rho+2g_{\nu\lambda}m_{3/2})$$

we have

$$\begin{aligned}\Delta^{\mu\nu}_{\text{op}}(\tfrac{3}{2})&\equiv\Delta^{\mu\lambda}(\tfrac{3}{2})\tilde{\Delta}_\lambda{}^\nu(\tfrac{3}{2})\\&=g^{\mu\nu}\mathcal{D}^2+\frac{i}{2}\sigma^{\mu\nu}\mathcal{R}+\mathcal{R}^{\mu\nu}+i\sigma^{\alpha\beta}\mathcal{R}^{\mu\nu}{}_{\alpha\beta}\\&\quad+\gamma_5\{\sigma^{\mu\nu},\sigma^{\alpha\beta}\}F_{\alpha\beta}(A)+i(\mathcal{R}^\mu{}_\lambda\sigma^{\nu\lambda}-\mathcal{R}^\nu{}_\lambda\sigma^{\mu\lambda})\\&\quad+\frac{i}{2}\gamma^\nu\gamma^\rho\gamma^\mu(\partial_\rho m_{3/2})+g^{\mu\nu}m_{3/2}^2\end{aligned} \tag{11}$$

which is of the desired form. Similarly for the ghost operator $\Delta(\text{ghosts})=\Delta_{\text{FP}}$,

$$\begin{aligned}\Delta_{\text{op}}(\text{ghosts})&\equiv\Delta(\text{ghosts})\tilde{\Delta}(\text{ghosts})\\&=(i\mathcal{D}\!\!\!/+2m_{3/2})(-i\mathcal{D}\!\!\!/+2m_{3/2})\\&=\mathcal{D}^2-i\sigma^{\mu\nu}F_{\mu\nu}(A)+4m_{3/2}^2\ .\end{aligned} \tag{12}$$

From Eqs. (11) and (12) we infer the coefficients $\alpha_i(\frac{3}{2})$, $\alpha_i(\text{ghosts})$ appearing in Eq. (10). These are given by, using pertinent formulas [5–8],

$$\begin{aligned}(4\pi)^2\alpha_0(\tfrac{3}{2})&=8\ ,\\(4\pi)^2\alpha_1(\tfrac{3}{2})&=-\tfrac{2}{3}\mathcal{R}-8m_{3/2}^2\ ,\\(4\pi)^2\alpha_2(\tfrac{3}{2})&=\tfrac{1}{360}(106\mathcal{R}^2_{\mu\nu\rho\sigma}-16\mathcal{R}^2_{\mu\nu}+10\mathcal{R}^2\\&\quad+240m_{3/2}^2\mathcal{R}+1440m_{3/2}^4+\cdots)\ ,\end{aligned} \tag{13}$$

$$\begin{aligned}(4\pi)^2\alpha_0(\text{ghosts})&=2\ ,\\(4\pi)^2\alpha_1(\text{ghosts})&=-\frac{\mathcal{R}}{6}-8m_{3/2}^2\ ,\\(4\pi)^2\alpha_2(\text{ghosts})&=\tfrac{1}{360}(-\tfrac{7}{2}\mathcal{R}^2_{\mu\nu\rho\sigma}-4\mathcal{R}^2_{\mu\nu}+\tfrac{5}{2}\mathcal{R}^2\\&\quad+240m_{3/2}^2\mathcal{R}+5760m_{3/2}^4+\cdots)\ .\end{aligned} \tag{14}$$

The ellipses in Eqs. (13) and (14) denote terms involving derivatives of the scalars z^i or total derivatives and will not concern us. In order to find the full fermionic contribution to the effective action we also need to consider the mixing terms coupling matter fermions to the gravitino field. This will be discussed in the following section.

IV. SPIN-$\frac{1}{2}$–GRAVITINO MIXING CONTRIBUTIONS

So far in our considerations we have left out the mixing term

$$\mathcal{L}^{1/2-3/2}=\sqrt{-g}\,\bar{\psi}_\mu(m_P i e^{-\mathcal{G}/2}\mathcal{G}_i+m_P\mathcal{G}_{i\bar{j}}\partial z^{\bar{j}})\gamma^\mu x_L^i+\text{H.c.} \tag{15}$$

which does contribute to the effective action, through the graphs shown in Fig. 1.

A straightforward calculation yields that their contribution gives rise to the following coefficients α_i:

$$(4\pi)^2\alpha_0(a)=0\ ,$$

$$(4\pi)^2\alpha_1(a)=8(\mathcal{G}_{i\bar{j}}\partial z^i\cdot\partial z^{\bar{j}})-4m_{3/2}^2(\mathcal{G}^{i\bar{j}}\mathcal{G}_i\mathcal{G}_{\bar{j}})\ ,$$

$$\begin{aligned}(4\pi)^2\alpha_2(a)=2m_{3/2}^2m_P^2[&-m_P^{-6}(\mathcal{G}^{\bar{i}}\mathcal{G}^{j}\mathcal{M}_i{}^{\lambda}\mathcal{M}_{\lambda j}+\text{H.c.})\\&-2m_P^{-4}m_{3/2}(\mathcal{G}^{i}\mathcal{G}^{j}\mathcal{M}_{ij}+\text{H.c.})\\&+2m_{3/2}^2m_P^{-2}(\mathcal{G}^{i\bar{j}}\mathcal{G}_i\mathcal{G}_{\bar{j}})]\\&-\tfrac{2}{3}m_{3/2}^2\mathcal{R}(\mathcal{G}^{i\bar{j}}\mathcal{G}_i\mathcal{G}_{\bar{j}})\ ,\end{aligned} \tag{16a}$$

$$\begin{aligned}&(4\pi)^2\alpha_0(b)=(4\pi)^2\alpha_1(b)=0\ ,\\&(4\pi)^2\alpha_2(b)=-2m_{3/2}^4(\mathcal{G}^{i\bar{j}}\mathcal{G}_i\mathcal{G}_{\bar{j}})^2\ .\end{aligned} \tag{16b}$$

As in our previous considerations in α_2 we have not considered terms that depend on derivatives of the scalars z^i. The last term in $\alpha_2(a)$ arising from the graph (1a) depends on the scalar curvature $\mathcal{R}$ and is nonvanishing in curved space-time. For its calculation we need the adiabatic expansion of both graviton and spin-$\frac{1}{2}$ propagators, as well as expansion of the vertices involved. This expansion is given in the Appendix.

The last step in our calculation is to consider the contributions of scalars, gravitons, and general coordinate transformation (GCT) ghosts. This is the issue of the following section.

V. SCALARS, GRAVITONS, AND GCT GHOSTS CONTRIBUTIONS

The spin-0, spin-2, and general coordinate ghost contributions for a σ model coupled to gravity have been already calculated [22]. The reference Lagrangian is

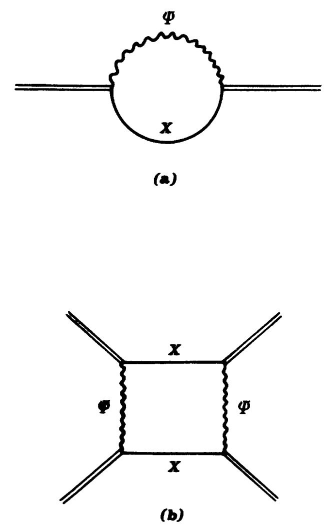

FIG. 1. Graphs contributing to the coefficients α_i involving gravitino-matter fermion mixing vertices.

$$\mathcal{L}=\frac{1}{2k^2}\sqrt{-g}\,\mathcal{R}+\frac{\sqrt{-g}}{2}g^{\mu\nu}G_{ij}\partial_\mu\phi^i\partial_\nu\phi^j-\sqrt{-g}\,V(\phi^i)\ , \tag{17}$$

where ϕ^i here are real fields. Writing z^i as $z^i=(A^i+iB^i)/\sqrt{2}$ the bosonic part of the supersymmetric Lagrangian can be cast into the above form. In the simple case of just one chiral multiplet z, for instance, we deal with two real fields ϕ^1,ϕ^2 defined as $z=(\phi^1+i\phi^2)/\sqrt{2}$ and G_{ij} is found to be related to the Kahler manifold metric $\mathcal{G}_{z\bar{z}}$ by $G_{11}=G_{22}=\mathcal{G}_{z\bar{z}}$, $G_{12}=0$. The only nonvanishing Ricci tensor components of the Kahlerian manifold are $\mathcal{R}_{z\bar{z}}=\mathcal{R}_{\bar{z}z}=\mathcal{R}_{11}=\mathcal{R}_{22}$.

We choose the following gauge fixing for the general coordinate transformations [14,15]:

$$\mathcal{L}_{\text{GCT}}^{\text{gf}}=\frac{\sqrt{-g}}{4}(\nabla_\mu\bar{h}^{\mu\nu}-2k\partial^\nu\phi^i\xi^jG_{ij})^2$$

where $\bar{h}^{\mu\nu}=h^{\mu\nu}-g^{\mu\nu}(\text{Tr}h)/2$. $h^{\mu\nu}$ denotes the fluctuations of the graviton field around the classical background $g^{\mu\nu}{}_c$ and ξ_i are the scalar field quantum fluctuations.

With these in mind the scalar and graviton contributions can be directly read from the conclusions reached in Ref. [22]. Actually the bosonic contributions yield, for the coefficients α_i,

$$(4\pi)^2\alpha_0(0+2)=2(D+1)\ ,$$

$$(4\pi)^2\alpha_1(0+2)=\frac{D-23}{3}\mathcal{R}+[20m_P^{-4}V+2\partial z^i\partial z^{\bar{j}}(\mathcal{R}_{i\bar{j}}+2\mathcal{G}_{i\bar{j}})-2m_P^{-2}D^iD_iV]\ ,$$

$$\begin{aligned}(4\pi)^2\alpha_2(0+2)=\tfrac{1}{180}\{&2(106+D)\mathcal{R}_{\mu\nu\rho\sigma}^2-2(D-359)\mathcal{R}_{\mu\nu}^2+5(D-29)\mathcal{R}^2+20m_P^{-4}V^2+m_P^{-4}[(D_iD_jV)^2+(D^iD_iV)^2]\\&-(m_P^{-2}/3)\mathcal{R}(D_iD^iV)-\tfrac{26}{3}m_P^{-2}\mathcal{R}V-8m_P^{-4}D_iVD^iV+\cdots\}\ .\end{aligned} \tag{18}$$

As a partial check of the correctness of these results notice the coefficients of $\mathcal{R}^2{}_{\mu\nu\rho\sigma}$, $\mathcal{R}^2{}_{\mu\nu}$, and $\mathcal{R}$ of the graviton contributions to α_2 agree with those of Cristensen and Duff [8]. Also the terms $20m_P{}^{-4}V^2$, $-\frac{26}{3}m_P{}^{-2}\mathcal{R}V$ agree with those given by Fradkin and Tseytlin [25] (see the coefficients β_4, β_5 in Table I of that reference).

VI. THE EFFECTIVE LAGRANGIAN

Summing up the various contributions we get $\alpha_0(\text{total})=0$ as expected because the number of bosonic degrees of freedom matches that of fermionic ones. The coefficient $\alpha_1(\text{total})$ is nonvanishing given by

$$(4\pi)^2\alpha_1(\text{total})=\frac{D-5}{2}\mathcal{R}+2\,\mathrm{Tr}(M_F^\dagger M_F)-\mathrm{Tr}M_S^2-4m_{3/2}^2+4k^2V+2(\mathcal{R}_{i\bar{j}}+6\mathcal{G}_{i\bar{j}})\partial z^i\partial z^{\bar{j}}+5(4k^2V-\mathcal{R}) \tag{19}$$

where $k=m_P^{-1}$ and $\mathrm{Tr}M_F^\dagger M_F\equiv m_P^{-4}\mathcal{M}_{ij}\mathcal{M}^{ji}$, $\mathrm{Tr}M_S^2\equiv 2m_P^{-2}D_iD^iV$. Up to terms that vanish when $z^i=$const and $R=4k^2V$ this expression for α_i is identical to that obtained by other authors [16,17]. In those references the effective action was calculated expanding around a constant background field $\bar{z}^i$ satisfying the zeroth-order Einstein equation $\mathcal{R}(g)=4k^2V(\bar{z})$ with the omission of tadpole terms. Therefore (19) generalizes previous results for arbitrary scalar and metric background fields, in an $N=1$ supergravity model with nonminimal kinetic terms.

Concerning the coefficient $\alpha_2(\text{total})$ we find, using the relation $k^2\partial_\alpha V=2m_{3/2}^2\mathcal{G}_\alpha+k^2m_{3/2}\mathcal{M}_{\alpha\beta}\mathcal{G}^\beta$,

$$\begin{aligned}(4\pi)^2\alpha_2(\text{total})=&\frac{D+41}{48}\mathcal{R}^2_{\mu\nu\rho\sigma}+\frac{D+47}{48}\mathcal{R}^2_{\mu\nu}+4k^4V^2-\frac{23}{3}k^2V\mathcal{R}-\frac{2}{3}m_{3/2}^2\mathcal{R}\\&+(14m_{3/2}^4-\mathrm{Tr}M_F^\dagger M_F M_F^\dagger M_F+\tfrac{1}{2}\mathrm{Tr}M_S^4)-12k^4(D_iV)^2\\&-\frac{\mathcal{R}}{6}(\mathrm{Tr}M_F^\dagger M_F+\mathrm{Tr}M_S^2)+4k^2m_{3/2}^2(\mathcal{G}^iD_iV+\text{H.c.})\\&+[4E_{\mu\nu}^2+\tfrac{5}{3}\mathcal{R}(\mathcal{R}-4k^2V)+4m_{3/2}^2(\mathcal{R}-4k^2V)+(\mathcal{R}-4k^2V)^2]+\cdots\end{aligned} \tag{20}$$

where we have defined

$$\mathrm{Tr}M_F^\dagger M_F M_F^\dagger M_F\equiv k^8\mathcal{M}_{ij}\mathcal{M}^{ik}\mathcal{M}_{k\lambda}\mathcal{M}^{\lambda i},\quad \mathrm{Tr}M_S^4\equiv k^4[(D_iD_jV)^2+(D_iD^iV)^2]$$

and

$$E_{\mu\nu}=\mathcal{R}_{\mu\nu}-\frac{g_{\mu\nu}}{4}\mathcal{R}\ .$$

The bottom line in Eq. (20) vanishes in Einstein spaces and when the background fields satisfy $\mathcal{R}=4k^2V$, as considered in Refs. [16] and [17]. Also there are terms depending on (ϑ_iV) which also vanish if z^i are constant fields satisfying the classical equations of motion. The rest turn out to be identical to those found in Ref. [17] except a slight difference occurring in the coefficient of the $\mathcal{R}^2$ term. Actually we find $(D+47)\mathcal{R}^2/48$ instead of $(D+43)\mathcal{R}^2/48$ given in that reference.

The one-loop effective Lagrangian for the spin-0 and spin-2 fields is then

$$\mathcal{L}^{1\,\text{loop}}=\mathcal{L}^{N=1}+\frac{\sqrt{-g}}{2}[\Lambda^2\alpha_1(\text{total})+(\ln\Lambda^2)\alpha_2(\text{total})]\ , \tag{21}$$

where terms up to $\ln\Lambda^2$ have been retained. In our calculations however we have not considered $(\ln\Lambda^2)$ contributions of terms involving derivatives of the scalar fields z^i. The Lagrangian (21) with α_1, α_2 as given by Eqs. (19) and (20), respectively, generalizes previous results. In our considerations the Kahler manifold was considered nonflat ($\mathcal{G}_{i\bar{j}}\neq\delta_{i\bar{j}}$); that is, the kinetic terms were allowed to have a nonminimal form. Also the scalar background field z^i was taken arbitrary unlike Refs. [16,17] where it was considered constant. A nonconstant value for z^i is necessary if one wants to know the renormalization of the Kahler metric as we shall see. Also in our considerations we have not committed ourselves to any particular form for four-dimensional geometry unlike Refs. [16,17] where only Einstein four-dimensional spaces were considered. Our expression for the Lagrangian (21) coincides on shell with that obtained in Ref. [17] in the minimal case with constant z^i. This offers a check of the correctness of our calculations and also shows that on-shell results are gauge independent since we have adopted a gauge fixing different from that employed in Refs. [16,17].

From the form of α_1, α_2 it is evident that $L^{1\,\text{loop}}$ involves terms that mix geometry-dependent terms, such as scalar curvature, for instance, with terms involving scalar fields z^i. Because of this it is not clear how one defines the scalar potential of the theory when gravitational corrections are taken into account. We shall show that by a redefinition of the metric

$$g_{\mu\nu}=\hat{g}_{\mu\nu}+\Delta_{\mu\nu}\ , \tag{22}$$

with properly chosen $\Delta_{\mu\nu}$, one is led to an effective Lagrangian $L(\hat{g}_{\mu\nu},z^i)$ which does not involve $\mathcal{R}^2{}_{\mu\nu}$ and $\mathcal{R}^2$ terms and in which the scalar fields do not mix with the scalar curvature $\mathcal{R}$. In the $L(\hat{g}_{\mu\nu},z^i)$ theory one easily identifies unambiguously the potential terms as that part not involving derivatives of z^i. $\Delta_{\mu\nu}$ appearing in Eq. (22) is assumed to have the form

$$\Delta_{\mu\nu}=\alpha\mathcal{R}_{\mu\nu}+\beta g_{\mu\nu}\mathcal{R}+g_{\mu\nu}P(z^i)+\partial_\mu z^i\partial_\nu z^{\bar{j}}f_{i\bar{j}}+g_{\mu\nu}\partial z^i\partial z^{\bar{j}}K_{i\bar{j}} \quad (23)$$

where the constants α,β,f_{ij}, K_{ij}, and P are determined in a way that will become clear in the sequel.

Dropping a term $[(D+41)/48]\,(\mathcal{R}^2{}_{\mu\nu\rho\sigma}-4\mathcal{R}^2{}_{\mu\nu}+\mathcal{R}^2)$ in the Lagrangian (21), which is a total derivative, one is left with only $\mathcal{R}^2{}_{\mu\nu}$ and $\mathcal{R}^2$ which by the redefinition (22) of the metric can be made to vanish and the new theory involves no $\mathcal{R}^2{}_{\mu\nu},\mathcal{R}^2$ terms. This can be achieved if the constants α and β are taken as

$$d=\frac{k^2}{16\pi^2}\left[\frac{D+89}{12}\right]\ln\Lambda^2\,,$$
$$\beta=\frac{k^2}{16\pi^2}\left[-\frac{D+49}{24}\right]\ln\Lambda^2\,. \quad (24)$$

Then the coefficients f_{ij}, K_{ij} can be determined if cancellation of $R^{\mu\nu}\vartheta_\mu z^i\vartheta_\nu z^j$ and $R\,\vartheta z^i\vartheta z^j$ is demanded. However they are found to depend on $\ln\Lambda^2$ and since we have ignored altogether $\ln\Lambda^2$ terms involving derivatives ϑz^i we can ignore them from the rest of our discussion. The last term to be determined is the $P(z^i)$ which is uniquely fixed if one demands the absence of terms mixing scalars and scalar curvature. With these in mind the final form of the one-loop Lagrangian is found to be

$$e^{-1}\mathcal{L}^{1\,\mathrm{loop}}(\hat{g}_{\mu\nu},z^i)=\frac{\hat{\mathcal{R}}}{2k_R^2}+m_P^2(-\mathcal{G}^R_{i\bar{j}})\partial z^i\partial z^{\bar{j}}-V+\frac{\Lambda^2}{32\pi^2}(24k^2V+2\,\mathrm{Tr}M_F^\dagger M_F-\mathrm{Tr}M_S^2-4m_{3/2}^2)$$
$$+\frac{\ln\Lambda^2}{32\pi^2}\Bigg[14m_{3/2}^4-\mathrm{Tr}(M_F^\dagger M_F)^2+\tfrac{1}{2}\mathrm{Tr}M_S^4+k^2V(\tfrac{40}{3}m_{3/2}^2-\tfrac{2}{3}\mathrm{Tr}M_F^\dagger M_F-\tfrac{2}{3}\mathrm{Tr}M_S^2)$$
$$+k^4\left[\frac{D}{3}-11\right]V^2-12k^4D_iVD^iV+4(\mathcal{G}^iD_iV+\mathrm{H.c.})k^2m_{3/2}^2\Bigg] \quad (25)$$

where all quantities refer to the new metric $\hat{g}_{\mu\nu}$ and the dressed Newton's constant k_R and Kahler metric $\mathcal{G}^R_{i\bar{j}}$ are given by

$$k_R^{-2}=k^{-2}+\left[\frac{D-15}{32\pi^2}\right]\Lambda^2\,,$$
$$\mathcal{G}^R_{i\bar{j}}=\mathcal{G}_{i\bar{j}}-\frac{k_R^2\Lambda^2}{16\pi^2}(\mathcal{R}_{i\bar{j}}+6\mathcal{G}_{i\bar{j}})\,. \quad (26)$$

The Lagrangian (25) is our final expression for the effective theory up to logarithmic terms in the cutoff scale Λ^2. We should remind the reader that we have not bothered to calculate $\ln\Lambda^2$ terms which depend on derivatives of the scalars z^i. The scalar potential V is unambiguously determined by

$$V^{1\,\mathrm{loop}}(z^i,z^{\bar{j}})=V-\frac{\Lambda^2}{32\pi^2}[A]-\frac{\ln\Lambda^2}{32\pi^2}[B] \quad (27)$$

where the expressions $[A]$, $[B]$ in Eq. (27) are the same as those appearing in Eq. (25) multiplying $\Lambda^2/32\pi^2$ and $\ln\Lambda^2/32\pi^2$, respectively.

VII. APPLICATION TO THE NO-SCALE MODELS

As an application of our results we consider the case of a no-scale model [27] whose Kahler potential $\mathcal{G}$ is given by [4]

$$\mathcal{G}=\ln(S+S^\dagger)+3\ln(T+T^\dagger-z^iz^{\bar{i}})-\ln|W(z^i)+W(S)|^2\,. \quad (28)$$

The S and T fields are singlets under the gauge group but z^i are in general nonsinglets (observable fields). The vacuum expectation value (VEV) of the S field is related to the grand unified coupling constant at energies near the Planck scale by $\langle S\rangle=g_{\mathrm{GUT}}^{-2}$. Models whose Kahler potential is given by Eq. (28) emerge from a class of string theories and are believed to describe the dynamics of particle interactions as one approaches Planck energies from above. At the tree level the scalar potential of this theory is positive definite having minima at $\langle z_i\rangle=0$, $\langle S\rangle\neq0$ with a zero cosmological constant leaving, however, undetermined the VEV of the field T which is related to the compactification radius. The low-energy scalars z_i ought to have a vanishing VEV at Planck scales otherwise we may have breaking of the underlying electroweak gauge symmetry of the order of m_P owing to the fact that z_i are nonsinglets.

In the model under consideration the expression for $\mathrm{Tr}M_S^2-2\,\mathrm{Tr}M_F^\dagger M_F$ is found to be

$$\mathrm{Tr}M_S^2-2\,\mathrm{Tr}M_F^\dagger M_F=2(k^2V-3m_{3/2}^2)-\frac{2e^{-G}}{(s+s^\dagger)}\left|W(s)-(s+s^\dagger)\frac{\partial W}{\partial s}\right|^2 \quad (29)$$

where $G=3\ln(T+T^\dagger-z^iz^{\bar{i}})$. In view of this the one-loop corrected potential of Eq. (27) takes the form

$$V^{1\,\mathrm{loop}}=V-\frac{\Lambda^2}{32\pi^2}\left[22k^2V+2m_{3/2}^2+\frac{2e^{-G}}{s+s^\dagger}\left|W-(s+s^\dagger)\frac{\partial W}{\partial s}\right|^2\right] \quad (30)$$

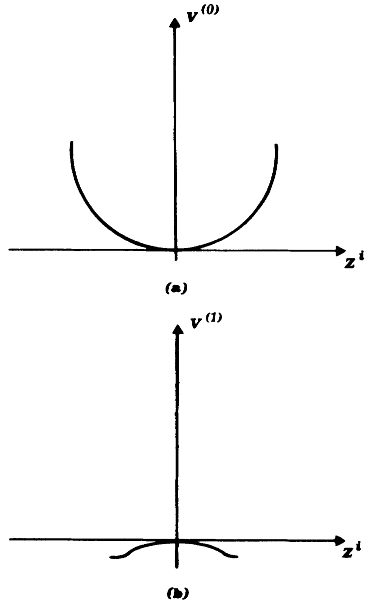

FIG. 2. Behavior of the (a) tree-level and (b) one-loop corrected potential given in Eq. (31). The contributions of the $\ln\Lambda^2$ terms has been omitted.

where we have not included the $\ln\Lambda^2$ terms. In order to investigate the behavior of the potential given by Eq. (30) we shall stay on the tree-level minimum $\langle S\rangle\neq 0$ and consider fluctuations in the direction of z^i around the tree-level VEV $\langle z_i\rangle=0$, that is, $z_i=\langle z_i\rangle+\Delta z$ with Δz denoting deviations from the tree-level value. Since V is zero at the tree-level VEV $\langle z_i\rangle=0$, $\langle S\rangle\neq 0$, and with vanishing scalar masses for z_i's, a particular characteristic of these models, we have, up to one-loop order,

$$V^{1\,\mathrm{loop}}(\langle s\rangle,\langle z\rangle+\Delta z)=-\frac{\Lambda^2}{32\pi^2}(2m_{3/2}^2)+O(\ln\Lambda^2) \quad (31)$$

for sufficiently small Δz_i, which is negative for any value of the scalar field T.

The quadratically divergent terms of Eq. (31) are exactly the same as those found by Breit, Ovrut, and Segre and independently by Binetruy and Gaillard (in Ref. [19]). The fact that the Λ^2 contributions are negative may lead to the conclusion that the potential is unstable. Omitting the $\ln\Lambda^2$ terms the shape of the one-loop corrected potential is as shown in Fig. 2, indicating an unstable behavior which may lead to an unpleasant electroweak gauge symmetry breaking at the Planck scale. However it has been shown by Binetruy and Gaillard (last reference in Ref. [19]) that the sign of the quadratically divergent contributions is undetermined depending on the regularization scheme adopted and therefore any conclusion based solely on the sign of only the terms appearing in Eq. (31) is misleading. In addition to this the contributions of subleading terms should be also considered for a complete study of the behavior of the potential. Actually it has been shown that the instability generated by the quadratically divergent corrections is raised if one takes into account the $\ln\Lambda^2$ terms and the gauge-nonsinglet scalar fields are forced to acquire a vanishing vacuum expectation value preventing an unpleasant breaking of the electroweak symmetry at the Planck scale (Breit *et al.*, Binetruy and Gaillard in Ref. [19]).

CONCLUSIONS

In this work we have calculated the divergent one-loop corrections in an $N=1$ supergravity theory defined on an arbitrary Kahler manifold. In particular the scalar potential has been calculated up to logarithmic terms in the cutoff parameter Λ while for the scalar kinetic terms only the quadratic terms have been retained. Our findings are useful especially for models inspired by a class of string theories where radiative corrections near the Planck scale may shed light on unresolved problems such as how supersymmetry breaking is achieved, what the scalar and gaugino masses are, and so on. These issues are closely related to phenomenology and are therefore of primary importance for the experimental check of the theory. In this case Λ plays the role of the characteristic scale beyond which the underlying string dynamics takes over and in order of magnitude is somewhat larger than the Planck scale m_P. The effective $N=1$ supergravity theory is seen as an effective tree-level Lagrangian describing the particle dynamics for energies $E\lesssim m_P$ and therefore in all radiative corrections the loop integrals encountered should be properly truncated at scales $\Lambda\sim m_P$.

In the course of our one-loop analysis we have allowed for the most general four-dimensional geometry ($g_{\mu\nu}$) and Kahler manifold geometry which is defined by the metric $\mathcal{G}_{i\bar{j}}=\partial^2\mathcal{G}/\partial z^i\partial z^{\bar{j}}$, $\mathcal{G}$ being the Kahler potential. Since we deal with a theory relevant to Planck energies the gravitational radiative effects also should be taken into account and therefore the graviton field should be considered quantized. In our considerations no commitment has been made on the form of the four-dimensional background $g_{\mu\nu}$ and hence our results are very general. Our results for the scalar potential agree on shell with those of other authors who in a different gauge examined an $N=1$ supergravity theory defined on a flat Kahler manifold in Einstein four-dimensional spaces. This consists of a check for the correctness of our calculations and it shows also the on-shell gauge independence of the physical results.

Moreover we have shown that by a redefinition of the metric $g_{\mu\nu}\rightarrow\hat{g}_{\mu\nu}$ one is led to an equivalent theory described by the Lagrangian $\mathcal{L}(\hat{g}_{\mu\nu},z^i)$ in which the one-loop potential $V(z^i,z^{\bar{i}})$ is directly read. Actually in $\mathcal{L}(\hat{g}_{\mu\nu},z^i)$, four-dimensional geometric quantities such as scalar curvature, Ricci tensors, etc., couple only to derivatives of the scalar fields z^i and not to z^i themselves. Therefore by putting derivative-dependent terms equal to zero we get the scalar potential of our theory. As an application we have considered an $N=1$ supergravity model of the no-scale type inspired by a particular class of superstring theories.

Note added in proof. After the completion of this work we became aware of the following papers in which the issue of the on-shell gauge independence in particular field-theory models is also discussed: I. L. Buchbinder and S. D. Odintsou, Phys. Lett. B **228**, 104 (1989); S. D. Odintsou, Fortschr. Phys. **38**, 5 (1990).

APPENDIX

1. Spin-$\frac{1}{2}$ propagator

For the Majorana fermions y^a defined in Sec. II the propagator satisfies the equation

$$(i\epsilon_{ab}\mathcal{D}+M_{ab})S^{bc}(x,x')=(-g)^{-1/2}\delta_a{}^c\delta^{(4)}(x-x')\ . \quad \text{(A1)}$$

One expands in normal coordinates with the origin at x'. Defining $\bar{S}^{ab}$ by $S^{ab}\equiv(-g)^{-1/4}\bar{S}^{ab}(-g')^{-1/4}$ one has the equation

$$(1-\tfrac{1}{12}y^\alpha y^\beta \mathcal{R}_{\alpha\beta}+\cdots)\Bigg[\epsilon_{ac}\left[i\gamma_m\delta^{m\mu}\partial_\mu\bar{S}^{cb}-\frac{i}{6}\gamma_m\mathcal{R}^m{}_\alpha{}^\mu{}_\beta y^\alpha y^\beta\partial_\mu\bar{S}^{cb}-\tfrac{1}{2}\gamma_m\delta^{m\mu}\sigma^{\alpha\beta}\mathcal{R}_{\mu\alpha\beta\lambda}y^\lambda\bar{S}^{cb}\right]$$
$$+(M_{ac}+y^\lambda\partial_\lambda M_{ac}+\cdots)\bar{S}^{cb}$$
$$+i\gamma_m\delta^{m\mu}(\Omega_{\mu ac}+y^\lambda\partial_\lambda\Omega_{\mu ac})\bar{S}^{cb}-\frac{i}{6}\epsilon_{ac}\gamma^\rho\mathcal{R}_{\rho\alpha}y^\alpha\bar{S}^{cb}\Bigg]+\cdots=\delta_a{}^b\delta^{(4)}(y)\ . \quad \text{(A2)}$$

The ellipses in (A2) stand for terms not relevant to our calculation and y^a denotes the x^a space-time point in the normal coordinate system ($y=0$ for x'). The details of such an adiabatic expansion for the propagator can be found in the literature [5–7]. By taking the Fourier transform of $\bar{S}_{ab}(y)$,

$$\bar{S}_{ab}(y)=\frac{1}{(2\pi)^4}\int d^4k\, e^{-iky}\tilde{S}_{ab}(k)$$

we have, from (A2),

$$(\epsilon_{ac}\not{k}+M_{ac}+i\gamma^\mu\Omega_{\mu ac})\tilde{S}^{cb}+\epsilon_{ac}\Bigg[\frac{1}{12}\mathcal{R}^{\alpha\beta}\gamma^\mu\frac{\partial^2}{\partial k^\alpha\partial k^\beta}(k_\mu\tilde{S}^{cb})+\frac{1}{6}\gamma_m\mathcal{R}^m{}_\alpha{}^\mu{}_\beta\frac{\partial^2}{\partial k^\alpha\partial k^\beta}(k_\mu\tilde{S}^{cb})$$
$$+\frac{i}{2}\gamma^\mu\sigma^{\alpha\beta}\mathcal{R}_{\mu\alpha\beta\lambda}\left[\frac{\partial\tilde{S}^{cb}}{\partial k^\lambda}\right]-\frac{1}{6}\gamma^\mu\mathcal{R}_{\mu\lambda}\left[\frac{\partial\tilde{S}^{cb}}{\partial k^\lambda}\right]\Bigg]$$
$$-i\left[\partial_\lambda M_{ac}+i\gamma^\mu\partial_\lambda\Omega_{\mu ac}\right]\frac{\partial\tilde{S}^{cb}}{\partial k_\lambda}+\cdots=\delta_a{}^b \quad \text{(A3)}$$

where in (A3) the γ^μ's encountered are $\gamma^\mu=\gamma^m\delta^\mu{}_m$; that is, they are actually flat. We are interested in that part of $\tilde{S}_{ab}(k)$ which contributes to quadratic (Λ^2) and logarithmic ($\ln\Lambda^2$) divergencies through the graphs shown in Fig. 1. Inverting thus (A3) order by order by writing

$$\tilde{S}_{ab}(k)=\tilde{S}^{(0)}_{ab}(k)+\tilde{S}^{(2)}_{ab}(k)+\cdots \quad \text{(A4)}$$

we find

$$S^{(0)}_{ab}(k)=\epsilon_{ab}\frac{\not{k}}{k^2}-\frac{\not{k}}{k^2}(i\gamma^\mu\Omega_{\mu ab}+M_{ab})\frac{\not{k}}{k^2}+\cdots\ , \quad \text{(A5a)}$$

$$S^{(2)}_{ab}(k)=-\frac{\not{k}}{k^2}\epsilon_{ac}\left[\frac{1}{12}\gamma^\mu\mathcal{R}^{\alpha\beta}+\frac{1}{6}\gamma_m\mathcal{R}^{m\alpha\mu\beta}\right]\frac{\partial^2}{\partial k^\alpha\partial k^\beta}(k_\mu\tilde{S}^{c}_{(0)b})$$
$$-\frac{\not{k}}{k^2}\left[\frac{i}{2}\epsilon_{ac}\gamma^\mu\sigma^{\alpha\beta}\mathcal{R}_{\mu\alpha\beta\lambda}-\frac{1}{6}\epsilon_{ac}\gamma^\mu\mathcal{R}_{\mu\lambda}-i\partial_\lambda M_{ac}+\gamma^\mu\partial_\lambda\Omega_{\mu ac}\right]\frac{\partial}{\partial k_\lambda}\tilde{S}^{c}_{(0)b}\ . \quad \text{(A5b)}$$

Terms other than those shown in (A5) carry higher powers of k_μ in the denominator and lead to finite contributions for which we are not interested.

2. Spin-$\frac{3}{2}$ propagator

In the gauge employed the gravitino propagator satisfies the equation

$$\Delta^{\mu\lambda}(\tfrac{3}{2})G_{\lambda\nu}(x,x')\equiv\left[\frac{i}{2}\gamma^\lambda\gamma^\rho\gamma^\mu\mathcal{D}_\rho+g^{\mu\lambda}m_{3/2}\right]G_{\lambda\nu}(x,x')=(-g)^{-1/2}\delta^{(4)}(x-x')\delta^\mu{}_\nu \quad \text{(A6)}$$

[see Eq. (8), main text] which by defining $\bar{G}$ as $G_{\mu\nu}(x,x')=(-g)^{-1/4}\bar{G}_{\mu\nu}(x,x')(-g')^{-1/4}$ and by using normal coordinates originating at x', as in the spin-$\frac{1}{2}$, case yields, in momentum space,

$$\left[\frac{1}{2}\gamma^\lambda k\!\!\!/\gamma^\mu+n^{\mu\lambda}m_{3/2}-\frac{i}{2}\gamma^\lambda\gamma_5 A\!\!\!/\gamma^\mu\right]\tilde G_{\lambda\nu}$$
$$+\left[\frac{i}{4}\gamma^\lambda\gamma^\rho\gamma^\mu\sigma^{\alpha\beta}\mathcal{R}_{\rho\alpha\beta\rho'}+\frac{1}{3}\gamma^\mu\mathcal{R}^\lambda_{\rho'}-in^{\mu\lambda}\partial_{\rho'}m_{3/2}-\frac{1}{2}\gamma^\lambda\gamma^\rho\gamma^\mu(\partial_{\rho'}A_\rho)\gamma_5-\frac{1}{12}\mathcal{R}^{\rho\rho'}\gamma^\lambda\gamma_\rho\gamma^\mu\right]\frac{\partial\tilde G_{\lambda\nu}}{\partial k_{\rho'}}$$
$$+m_{3/2}(\tfrac{1}{3}\mathcal{R}^{\mu\alpha\lambda\beta}+\tfrac{1}{12}\eta^{\mu\lambda}\mathcal{R}^{\alpha\beta})\frac{\partial^2\tilde G_{\lambda\nu}}{\partial k^\alpha\partial k^\beta}$$
$$+\tfrac{1}{12}[\tfrac{1}{2}\mathcal{R}^{\alpha\beta}\gamma^\lambda\gamma^\rho\gamma^\mu+(\mathcal{R}^{\sigma\alpha\lambda\beta}\gamma_\sigma\gamma^\rho\gamma^\mu+\mathcal{R}^{\sigma\alpha\rho\beta}\gamma^\lambda\gamma_\sigma\gamma^\mu+\mathcal{R}^{\sigma\alpha\mu\beta}\gamma^\lambda\gamma^\rho\gamma_\sigma)]\frac{\partial^2}{\partial k^\alpha\partial k^\beta}(k_\rho\tilde G_{\lambda\nu})=\delta^\mu{}_\nu\,. \qquad \text{(A7)}$$

Putting

$$\tilde G_{\mu\nu}(k)=\tilde G^{(0)}_{\mu\nu}(k)+\tilde G^{(2)}_{\mu\nu}(k)+\cdots$$

we can solve iteratively (A7) and we get

$$\tilde G^{(0)}_{\lambda\nu}(k)=\tilde D_{\lambda\nu}(k)+\tilde D_{\lambda\alpha}(k)\left[\frac{i}{2}\gamma^\beta\gamma_5 A\!\!\!/\gamma^\alpha\right]\tilde D_{\beta\nu}(k)-\frac{m_{3/2}}{k^2}n_{\lambda\nu}+\cdots,\quad \tilde D_{\lambda\nu}(k)\equiv\tfrac{1}{2}\gamma_\nu\frac{k\!\!\!/}{k^2}\gamma_\lambda\,, \qquad \text{(A8a)}$$

$$\tilde G^{(2)}_{\mu\nu}(k)=-G^{(0)}_{\mu k}(k)\Bigg\{[\tfrac{1}{16}\gamma^\lambda\gamma^\rho\gamma^k\gamma^{k'}\gamma^\sigma\mathcal{R}_{\rho\rho'k'\sigma}+\tfrac{1}{3}\gamma^k\mathcal{R}^\lambda{}_{\rho'}+\tfrac{1}{2}\gamma^\lambda\gamma^\rho\gamma^\mu(\partial_{\rho'}A_\rho)\gamma^5-in^{k\lambda}(\partial_{\rho'}m_{3/2})-\tfrac{1}{12}\gamma^\lambda\gamma^\rho\gamma^k\mathcal{R}_{\rho\rho'}]\frac{\partial}{\partial k_{\rho'}}\tilde G^{(0)}_{\lambda\nu}$$
$$+\tfrac{1}{12}[\tfrac{1}{2}\gamma^\lambda\gamma^\rho\gamma^k\mathcal{R}^{\alpha\beta}+(\gamma_\sigma\gamma^\rho\gamma^k\mathcal{R}^{\sigma\alpha\lambda\beta}+\gamma^\lambda\gamma_\sigma\gamma^k\mathcal{R}^{\sigma\alpha\rho\beta}+\gamma^\lambda\gamma^\rho\gamma_\sigma\mathcal{R}^{\sigma\alpha k\beta})]\frac{\partial^2}{\partial k^\alpha\partial k^\beta}(k_\rho\tilde G^{(0)}_{\lambda\nu})$$
$$+m_{3/2}(\tfrac{1}{3}\mathcal{R}^{k\alpha\lambda\beta}+\tfrac{1}{12}n^{k\lambda}\mathcal{R}^{\alpha\beta})\frac{\partial^2 G^{(0)}_{\lambda\nu}}{\partial k^\alpha\partial k^\beta}\Bigg\}\,. \qquad \text{(A8b)}$$

The "curved" pieces $\tilde S^{(2)}(k)$, $\tilde G^{(2)}{}_{\mu\nu}(k)$ of the spin-$\frac{1}{2}$–spin-$\frac{3}{2}$ propagators [Eqs. (A5b), (A8b)] contribute to $\ln\Lambda^2$ divergences through the graph shown in Fig. 1(a).

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